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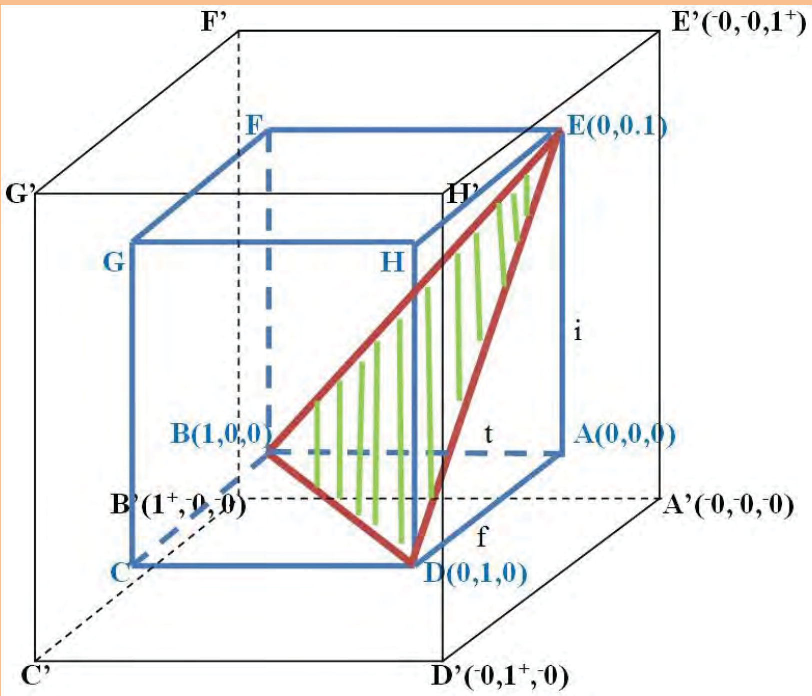

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*"*Neutrosophic Sets and Systems*"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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On Neutrosophic Soft Topological Space

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Abstract: In this paper, the concept of connectedness and compactness on neutrosophic soft topological space have been introduced along with the investigation of their several characteristics. Some related theorems have been established also. Then, the notion of neutrosophic soft continuous mapping on a neutrosophic soft topological space and it's properties are developed here.

Keywords : Connectedness and compactness on neutrosophic soft topological space, Neutrosophic soft continuous mapping.

1 Introduction

Zadeh's [1] classical concept of fuzzy sets is a strong mathematical tool to deal with the complexity generally arising from uncertainty in the form of ambiguity in real life scenario. Researchers in economics, sociology, medical science and many other several fields deal daily with the vague, imprecise and occasionally insufficient information of modeling uncertain data. For different specialized purposes, there are suggestions for nonclassical and higher order fuzzy sets since from the initiation of fuzzy set theory. Among several higher order fuzzy sets, intuitionistic fuzzy sets introduced by Atanassov [2] have been found to be very useful and applicable. But each of these theories has it's different difficulties as pointed out by Molodtsov [3]. The basic reason for these difficulties is inadequacy of parametrization tool of the theories.

Molodtsov [3] presented soft set theory as a completely generic mathematical tool which is free from the parametrization inadequacy syndrome of different theory dealing with uncertainty. This makes the theory very convenient, efficient and easily applicable in practice. Molodtsov [3] successfully applied several directions for the applications of soft set theory, such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration and probability etc. Now, soft set theory and it's applications are progressing rapidly in different fields. Shabir and Naz [4] presented soft topological spaces and defined some concepts of soft sets on this spaces and separation axioms. Moreover, topological structure on fuzzy, fuzzy soft, intuitionistic fuzzy and intuitionistic fuzzy soft set was defined by Coker [5], Li and Cui [6], Chang [7], Tanay and Kandemir [8], Osmanoglu and Tokat [9], Neog et al. [10], Varol and Aygun [11], Bayramov and Gunduz [12,13]. Turanh and Es [14] defined compactness in intuitionistic fuzzy soft topological spaces.

The concept of **Neutrosophic Set (NS)** was first introduced by Smarandache [15,16] which is a generalisation of classical sets, fuzzy set, intuitionistic fuzzy set etc. Later, Maji [17] has introduced a combined concept **Neutrosophic soft set (NSS)**. Using this concept, several mathematicians have produced their research works in different mathematical structures for instance Arockiarani et al.[18,19], Bera and Mahapatra [20], Deli [21,22], Deli and Broumi [23], Maji [24], Broumi and Smarandache [25], Salama and Alblowi [26], Saroja and Kalaichelvi [27], Broumi [28], Sahin et al.[29]. Later, this concept has been modified by Deli and Broumi [30]. Accordingly, Bera and Mahapatra [31-36] have developed some algebraic structures over the neutrosophic soft set.

The present study introduces the notion of connectedness, compactness and neutrosophic soft continuous mapping on a neutrosophic soft topological space. Section 2 gives some preliminary necessary definitions which will be used in rest of this paper. The notion of connectedness and compactness on neutrosophic soft topological spaces along with investigation of related properties have been introduced in Section 3 and Section 4, respectively. The concept of neutrosophic soft continuous mapping has been developed in Section 5. Finally, the conclusion of the present work has been stated in Section 6.

2 Preliminaries

In this section, we recall some necessary definitions and theorems related to fuzzy set, soft set, neutrosophic set, neutrosophic soft set, neutrosophic soft topological space for the sake of completeness.

Unless otherwise stated, E is treated as the parametric set through out this paper and $e \in E$, an arbitrary parameter.

2.1 Definition [31]

1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t - norm if $*$ satisfies the following conditions :

(i) $*$ is commutative and associative.

(ii) $*$ is continuous.

(iii) $a * 1 = 1 * a = a,\ \forall a \in [0,1]$.
(iv) $a * b \leq c * d$ if $a \leq c,\ b \leq d$ with $a,b,c,d \in [0,1]$.

A few examples of continuous t-norm are $a * b = ab, a * b = \min\{a,b\}, a * b = \max\{a+b-1, 0\}$.

2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t - conorm (s - norm) if $\diamond$ satisfies the following conditions :

(i) $\diamond$ is commutative and associative.
(ii) $\diamond$ is continuous.
(iii) $a \diamond 0 = 0 \diamond a = a,\ \forall a \in [0,1]$.
(iv) $a \diamond b \leq c \diamond d$ if $a \leq c,\ b \leq d$ with $a,b,c,d \in [0,1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab, a \diamond b = \max\{a,b\}, a \diamond b = \min\{a+b, 1\}$.

2.2 Definition [15]

Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is $T_A, I_A, F_A : X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$ and so, $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2.3 Definition [3]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \to P(U)$ is a mapping.

2.4 Definition [17]

Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F : A \to NS(U)$ is a mapping.

This concept has been modified by Deli and Broumi [30] as given below.

2.5 Definition [30]

Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \to NS(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(U)$ and therefore it can be written as a set of ordered pairs,

$$N = \{(e, \{< x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) >: x \in U\}) : e \in E\}$$

where $T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) \in [0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_N(e)}(x) + I_{f_N(e)}(x) + F_{f_N(e)}(x) \leq 3$ is obvious.

2.5.1 *Example*

Let $U = \{h_1, h_2, h_3\}$ be a set of houses and $E = \{e_1(\text{beautiful}), e_2(\text{wooden}), e_3(\text{costly})\}$ be a set of parameters with respect to which the nature of houses are described. Let,

$$f_N(e_1) = \{< h_1, (0.5, 0.6, 0.3) >, < h_2, (0.4, 0.7, 0.6) >, < h_3, (0.6, 0.2, 0.3) >\};$$
$$f_N(e_2) = \{< h_1, (0.6, 0.3, 0.5) >, < h_2, (0.7, 0.4, 0.3) >, < h_3, (0.8, 0.1, 0.2) >\};$$
$$f_N(e_3) = \{< h_1, (0.7, 0.4, 0.3) >, < h_2, (0.6, 0.7, 0.2) >, < h_3, (0.7, 0.2, 0.5) >\};$$

Then $N = \{[e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)]\}$ is an NSS over (U, E). The tabular representation of the NSS N is as :

Table 1 : Tabular form of NSS N.

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$
h_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
h_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
h_3	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)

2.6 Definition [30]

1. The complement of a neutrosophic soft set N is denoted by N^c and is defined by

$$N^c = \{(e, \{< x, F_{f_N(e)}(x), 1 - I_{f_N(e)}(x), T_{f_N(e)}(x) >: x \in U\}) : e \in E\}$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if $\forall e \in E$ and $\forall x \in U$,

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x).$$

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

2.7 Definition [30]

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as :

$$N_3 = \{(e, \{< x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x) >: x \in U\}) : e \in E\}$$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \diamond T_{f_{N_2}(e)}(x), I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x), F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x)$.

2. Their intersection is denoted by $N_1 \cap N_2 = N_4$ and is defined as :

$$N_4 = \{(e, \{< x, T_{f_{N_4}(e)}(x), I_{f_{N_4}(e)}(x), F_{f_{N_4}(e)}(x) >: x \in U\}) : e \in E\}$$

where $T_{f_{N_4}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x), I_{f_{N_4}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x), F_{f_{N_4}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x)$.

2.8 Definition [33]

1. Let M, N be two NSSs over (U, E). Then $M - N$ may be defined as, $\forall x \in U$, $e \in E$,

$$M - N = \{< x, T_{f_M(e)(x)} * F_{f_N(e)(x)}, I_{f_M(e)(x)} \diamond (1 - I_{f_N(e)}(x)), F_{f_M(e)}(x) \diamond T_{f_N(e)}(x) >\}$$

2. A neutrosophic soft set N over (U, E) is said to be null neutrosophic soft set if $T_{f_N(e)}(x) = 0, I_{f_N(e)}(x) = 1, F_{f_N(e)}(x) = 1, \forall e \in E, \forall x \in U$. It is denoted by ϕ_u.

A neutrosophic soft set N over (U, E) is said to be absolute neutrosophic soft set if $T_{f_N(e)}(x) = 1, I_{f_N(e)}(x) = 0, F_{f_N(e)}(x) = 0, \forall e \in E, \forall x \in U$. It is denoted by 1_u.

Clearly, $\phi_u^c = 1_u$ and $1_u^c = \phi_u$.

2.9 Definition [33]

Let *NSS*(U, E) be the family of all neutrosophic soft sets over U via parameters in E and $\tau_u \subset NSS(U, E)$. Then τ_u is called neutrosophic soft topology on (U, E) if the following conditions are satisfied.

(i) $\phi_u, 1_u \in \tau_u$
(ii) the intersection of any finite number of members of τ_u also belongs to τ_u.
(iii) the union of any collection of members of τ_u belongs to τ_u.

Then the triplet (U, E, τ_u) is called a neutrosophic soft topological space. Every member of τ_u is called τ_u-open neutrosophic soft set. An NSS is called τ_u-closed iff it's complement is τ_u-open. There may be a number of topologies on (U, E). If τ_{u^1} and τ_{u^2} are two topologies on (U, E) such that $\tau_{u^1} \subset \tau_{u^2}$, then τ_{u^1} is called neutrosophic soft strictly weaker (coarser) than τ_{u^2} and in that case τ_{u^2} is neutrosophic soft strict finer than τ_{u^1}. Moreover *NSS*(U, E) is a neutrosophic soft topology on (U, E).

2.9.1 *Example*

1. Let $U = \{h_1, h_2\}, E = \{e_1, e_2\}$ and $\tau_u = \{\phi_u, 1_u, N_1, N_2, N_3, N_4\}$ where N_1, N_2, N_3, N_4 being NSSs are defined as following :

$$\begin{aligned} f_{N_1}(e_1) &= \{< h_1, (1, 0, 1) >, < h_2, (0, 0, 1) >\}, \\ f_{N_1}(e_2) &= \{< h_1, (0, 1, 0) >, < h_2, (1, 0, 0) >\}; \end{aligned}$$

$$\begin{aligned} f_{N_2}(e_1) &= \{< h_1, (0, 1, 0) >, < h_2, (1, 1, 0) >\}, \\ f_{N_2}(e_2) &= \{< h_1, (1, 0, 1) >, < h_2, (0, 1, 1) >\}; \end{aligned}$$

$$\begin{aligned} f_{N_3}(e_1) &= \{< h_1, (1, 1, 1) >, < h_2, (0, 1, 1) >\}, \\ f_{N_3}(e_2) &= \{< h_1, (0, 1, 0) >, < h_2, (0, 1, 1) >\}; \end{aligned}$$

$$\begin{aligned} f_{N_4}(e_1) &= \{< h_1, (1, 1, 0) >, < h_2, (1, 1, 0) >\}, \\ f_{N_4}(e_2) &= \{< h_1, (1, 0, 0) >, < h_2, (0, 1, 1) >\}; \end{aligned}$$

Here $N_1 \cap N_1 = N_1, N_1 \cap N_2 = \phi_u, N_1 \cap N_3 = N_3, N_1 \cap N_4 = N_3, N_2 \cap N_2 = N_2, N_2 \cap N_3 = \phi_u, N_2 \cap N_4 = N_2, N_3 \cap N_3 = N_3, N_3 \cap N_4 = N_3, N_4 \cap N_4 = N_4$ and $N_1 \cup N_1 = N_1, N_1 \cup N_2 = 1_u, N_1 \cup N_3 = N_1, N_1 \cup N_4 = 1_u, N_2 \cup N_2 = N_2, N_2 \cup N_3 = N_4, N_2 \cup N_4 = N_4, N_3 \cup N_3 = N_3, N_3 \cup N_4 = N_4, N_4 \cup N_4 = N_4$;

Corresponding t-norm and s-norm are defined as $a * b = \max\{a + b - 1, 0\}$ and $a \diamond b = \min\{a + b, 1\}$. Then τ_u is a neutrosophic soft topology on (U, E) and so (U, E, τ_u) is a neutrosophic soft topological space over (U, E).

2. Let $U = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau_u = \{\phi_u, 1_u, N_1, N_2, N_3\}$ where N_1, N_2, N_3 being NSSs over (U, E) are defined as follow :

$$f_{N_1}(e_1) = \{< x_1, (1.0, 0.5, 0.4) >, < x_2, (0.6, 0.6, 0.6) >, < x_3, (0.5, 0.6, 0.4) >\},$$
$$f_{N_1}(e_2) = \{< x_1, (0.8, 0.4, 0.5) >, < x_2, (0.7, 0.7, 0.3) >, < x_3, (0.7, 0.5, 0.6) >\};$$
$$f_{N_2}(e_1) = \{< x_1, (0.8, 0.5, 0.6) >, < x_2, (0.5, 0.7, 0.6) >, < x_3, (0.4, 0.7, 0.5) >\},$$
$$f_{N_2}(e_2) = \{< x_1, (0.7, 0.6, 0.5) >, < x_2, (0.6, 0.8, 0.4) >, < x_3, (0.5, 0.8, 0.6) >\};$$
$$f_{N_3}(e_1) = \{< x_1, (0.6, 0.6, 0.7) >, < x_2, (0.4, 0.8, 0.8) >, < x_3, (0.3, 0.8, 0.6) >\},$$
$$f_{N_3}(e_2) = \{< x_1, (0.5, 0.8, 0.6) >, < x_2, (0.5, 0.9, 0.5) >, < x_3, (0.2, 0.9, 0.7) >\};$$

The t-norm and s-norm are defined as $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$. Here $N_1 \cap N_1 = N_1, N_1 \cap N_2 = N_2, N_1 \cap N_3 = N_3, N_2 \cap N_2 = N_2, N_2 \cap N_3 = N_3, N_3 \cap N_3 = N_3$ and $N_1 \cup N_1 = N_1, N_1 \cup N_2 = N_1, N_1 \cup N_3 = N_1, N_2 \cup N_2 = N_2, N_2 \cup N_3 = N_2, N_3 \cup N_3 = N_3$. Then τ_u is a neutrosophic soft topology on (U, E) and so (U, E, τ_u) is a neutrosophic soft topological space over (U, E).

3. Let *NSS*(U, E) be the family of all neutrosophic soft sets over (U, E). Then $\{\phi_u, 1_u\}$ and *NSS*(U, E) are two examples of the neutrosophic soft topology over (U, E). They are called, respectively, indiscrete (trivial) and discrete neutrosophic soft topology. Clearly, they are the smallest and largest neutrosophic soft topology on (U, E), respectively.

2.10 Definition [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$ be arbitrary. Then the interior of M is denoted by M^o and is defined as :

$$M^o = \cup\{N_1 : N_1 \text{ is neutrosophic soft open and } N_1 \subset M\}$$

i.e., it is the union of all open neutrosophic soft subsets of M.

2.10.1 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M, P \in NSS(U, E)$. Then,
(i) $M^o \subset M$ and M^o is the largest open set.
(ii)$M \subset P \Rightarrow M^o \subset P^o$.
(iii) M^o is an open neutrosophic soft set i.e., $M^o \in \tau_u$.
(iv) M is neutrosophic soft open set iff $M^o = M$.
(v) $(M^o)^o = M^o$.
(vi)$(\phi_u)^o = \phi_u$ and $1_u^o = 1_u$.
(vii) $(M \cap P)^o = M^o \cap P^o$.
(viii) $M^o \cup P^o \subset (M \cup P)^o$.

2.11 Definition [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$ be arbitrary. Then the closure of M is denoted by $\overline{M}$ and is defined as :

$$\overline{M} = \cap\{N_1 : N_1 \text{ is neutrosophic soft closed and } N_1 \supset M\}$$

i.e., it is the intersection of all closed neutrosophic soft supersets of M.

2.11.1 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M, P \in NSS(U, E)$. Then,
(i) $M \subset \overline{M}$ and $\overline{M}$ is the smallest closed set.
(ii) $M \subset P \Rightarrow \overline{M} \subset \overline{P}$.
(iii) $\overline{M}$ is closed neutrosophic soft set i.e., $\overline{M} \in \tau_u^c$.
(iv) M is neutrosophic soft closed set iff $\overline{M} = M$.
(v) $\overline{\overline{M}} = \overline{M}$.
(vi) $\overline{\phi_u} = \phi_u$ and $\overline{1_u} = 1_u$.
(vii) $\overline{M \cup P} = \overline{M} \cup \overline{P}$.
(viii) $\overline{M \cap P} \subset \overline{M} \cap \overline{P}$.

2.11.2 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M \in NSS(U, E)$. Then, (i) $(\overline{M})^c = (M^c)^o$ (ii)$(M^o)^c = \overline{(M^c)}$

2.12 Definition [33]

1. A neutrosophic soft point in an NSS N is defined as an element $(e, f_N(e))$ of N, for $e \in E$ and is denoted by e_N, if $f_N(e) \notin \phi_u$ and $f_N(e') \in \phi_u, \forall e' \in E - \{e\}$.
2. The complement of a neutrosophic soft point e_N is another neutrosophic soft point e_N^c such that $f_N^c(e) = (f_N(e))^c$.
3. A neutrosophic soft point $e_N \in M$, M being an NSS if for the element $e \in E$, $f_N(e) \leq f_M(e)$.

2.12.1 *Example*

Let $U = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Then,
$e_{1N} = \{< x_1, (0.6, 0.4, 0.8) >, < x_2, (0.8, 0.3, 0.5) >, < x_3, (0.3, 0.7, 0.6) >\}$
is a neutrosophic soft point whose complement is
$e_{1N}^c = \{< x_1, (0.8, 0.6, 0.6) >, < x_2, (0.5, 0.7, 0.8) >, < x_3, (0.6, 0.3, 0.3) >\}$.
For another NSS M defined on same (U, E), let,
$f_M(e_1) = \{< x_1, (0.7, 0.4, 0.7) >, < x_2, (0.8, 0.2, 0.4) >, < x_3, (0.5, 0.6, 0.5) >\}$.
Then, $f_N(e_1) \leq f_M(e_1)$ i.e., $e_{1N} \in M$.

2.13 Definition [33]

Hausdorff space : Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E). For two distinct neutrosophic soft points e_K, e_S, if there exists disjoint neutrosophic soft open sets M, P such that $e_K \in M$ and $e_S \in P$ then (U, E, τ_u) is called T_2 space or Hausdorff space.

2.13.1 *Example*

Let $U = \{h_1, h_2\}$, $E = \{e\}$ and $\tau_u = \{\phi_u, 1_u, M, P\}$ where M, P being neutrosophic soft subsets of N are defined as following :

$$f_M(e) = \{< h_1, (1, 0, 1) >, < h_2, (0, 0, 1) >\};$$
$$f_P(e) = \{< h_1, (0, 1, 0) >, < h_2, (1, 1, 0) >\};$$

Then τ_u is a neutrosophic soft topology on (U, E) with respect to the t-norm and s-norm defined as $a * b = \max\{a + b - 1, 0\}$ and $a \diamond b = \min\{a + b, 1\}$. Here $e_M \in M$ and $e_P \in P$ with $e_M \neq e_P$ and $M \cap P = \phi_u$.

2.14 Definition [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) where τ_u is a topology on (U, E) and $M \in NSS(U, E)$ an arbitrary NSS. Suppose $\tau_M = \{M \cap N_i : N_i \in \tau_u\}$. Then τ_M forms also a topology on (U, E). Thus (U, E, τ_M) is a neutrosophic soft topological subspace of (U, E, τ_u).

2.14.1 *Example*

Let us consider the example (2) in [2.9.1]. We define $M \in NSS(U, E)$ as following :

$$f_M(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.7, 0.3, 0.2) >, < x_3, (0.5, 0.5, 0.7) >\};$$
$$f_M(e_2) = \{< x_1, (0.6, 0.3, 0.5) >, < x_2, (0.4, 0.7, 0.6) >, < x_3, (0.8, 0.3, 0.5) >\};$$

We denote $M \cap \phi_u = \phi_M, M \cap 1_u = 1_M, M \cap N_1 = M_1, M \cap N_2 = M_2, M \cap N_3 = M_3$; Then M_1, M_2, M_3 are given as following :

$f_{M_1}(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.6, 0.6, 0.6) >, < x_3, (0.5, 0.6, 0.7) >\};$
$f_{M_1}(e_2) = \{< x_1, (0.6, 0.4, 0.5) >, < x_2, (0.4, 0.7, 0.6) >, < x_3, (0.7, 0.5, 0.6) >\};$
$f_{M_2}(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.5, 0.7, 0.6) >, < x_3, (0.4, 0.7, 0.7) >\};$
$f_{M_2}(e_2) = \{< x_1, (0.6, 0.6, 0.5) >, < x_2, (0.4, 0.8, 0.6) >, < x_3, (0.5, 0.8, 0.6) >\};$
$f_{M_3}(e_1) = \{< x_1, (0.4, 0.6, 0.8) >, < x_2, (0.4, 0.8, 0.8) >, < x_3, (0.3, 0.8, 0.7) >\};$
$f_{M_3}(e_2) = \{< x_1, (0.5, 0.8, 0.6) >, < x_2, (0.4, 0.9, 0.6) >, < x_3, (0.2, 0.9, 0.7) >\};$

Here $M_1 \cap M_2 = M_2, M_1 \cap M_3 = M_3, M_2 \cap M_3 = M_3$ and $M_1 \cup M_2 = M_2, M_1 \cup M_3 = M_3, M_2 \cup M_3 = M_3$. Then $\tau_M = \{\phi_M, 1_M, M_1, M_2, M_3\}$ is neutrosophic soft subspace topology on (U, E).

2.15 Theorem [33]

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E) and $M, N \in NSS(U, E)$. Then,
(i) If $ß_u$ is a base of τ_u then $ß_M = \{B \cap M : B \in ß_u\}$ is a base for the topology τ_M.
(ii) If Q is closed NSS in M and M is closed NSS in N, then Q is closed in N.
(iii) Let $Q \subset M$. If $\overline{Q}$ is the closure of Q then $\overline{Q} \cap M$ is the closure of Q in M.
(iv) An NSS $M \in NSS(U, E)$ is an open NSS iff M is a neighbourhood of each NSS N contained in M.

2.16 Proposition (De-Morgan's law)[33]

Let N_1, N_2 be two neutrosophic soft sets over (U, E). Then,
(i) $(N_1 \cup N_2)^c = {N_1}^c \cap {N_2}^c$ (ii) $(N_1 \cap N_2)^c = {N_1}^c \cup {N_2}^c$.

3 Connectedness

In this section, the concept of connectedness on neutrosophic soft topological space has been introduced with suitable example. Some related theorems have been developed in continuation.

3.1 Definition

Two neutrosophic soft sets N_1, N_2 of a neutrosophic soft topological space (U, E, τ_u) over (U, E) are said to be separated if
(i) $N_1 \cap N_2 = \phi_u$ and (ii) $\overline{N_1} \cap N_2 = \phi_u$ or $N_1 \cap \overline{N_2} = \phi_u$.

3.2 Definition

Let (U, E, τ_u) be a neutrosophic soft topological space over (U, E). Then a pair of nonempty neutrosophic soft open sets N_1, N_2 is called a neutrosophic soft separation of (U, E, τ_u) if $1_u = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi_u$.

In the Example (1) of [2.9.1], the pair N_1, N_2 is a neutrosophic soft separation of (U, E, τ_u) as $1_u = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi_u$.

3.3 Definition

A neutrosophic soft topological space (U, E, τ_u) is said to be neutrosophic soft connected if there does not exist a neutrosophic soft separation of (U, E, τ_u). Otherwise, (U, E, τ_u) is called neutrosophic soft disconnected.

The topological space in the Example (2) of [2.9.1] is connected but (1) of [2.9.1] is disconnected.

3.4 Theorem

A neutrosophic soft topological space (U, E, τ_u) is said to be neutrosophic soft disconnected iff there exists a nonempty proper neutrosophic soft subset of 1_u which is both neutrosophic soft open and neutrosophic soft closed.

Proof. Let $M \subset 1_u, M \neq \phi_u$ and M is both neutrosophic soft open and closed. Then $M^c \subset 1_u, M^c \neq \phi_u$ and M^c is both neutrosophic soft open and closed, also. Let $P = M^c$. Then $\overline{M} = M$ and $\overline{P} = P$. Thus 1_u can be expressed as the union of two separated neutrosophic soft sets M, P and so, is neutrosophic soft disconnected.

Conversely, let 1_u be neutrosophic soft disconnected. Then there exists nonempty neutrosophic soft open sets N_1, N_2 such that $1_u = N_1 \cup N_2$ and $N_1 \cap N_2 = \phi_u$. Then $N_1 = N_2^c$ i.e., N_1 is closed, also. Similarly, $N_2 = N_1^c$ and so, N_2 is closed.

3.5 Theorem

A neutrosophic soft topological space (U, E, τ_u) is said to be neutrosophic soft connected iff there exists neutrosophic soft sets in $NSS(U, E)$ which are both neutrosophic soft open and neutrosophic soft closed, are ϕ_u and 1_u.

Proof. Let (U, E, τ_u) be a connected neutrosophic soft topological space. For contrary, we suppose that M is both neutrosophic soft open and closed different from $\phi_u, 1_u$. Then M^c is also both neutrosophic soft open and closed different from $\phi_u, 1_u$. Also $M \cap M^c = \phi_u$ and $M \cup M^c = 1_u$. Therefore M, M^c is a neutrosophic soft separation of 1_u. This is a contradiction. So, the only neutrosophic soft closed and open sets in $NSS(U, E)$ are ϕ_u and 1_u.

Conversely, let M, P be a neutrosophic soft separation of (U, E, τ_u). Then $M \neq N$ i.e., $M = P^c$, otherwise $M = 1_u$ implies $P = \phi_u$, a contradiction. This shows that M is both neutrosophic soft open and neutrosophic soft closed different from $\phi_u, 1_u$. This is a contradiction. Hence, (U, E, τ_u) is connected.

3.6 Theorem

If the neutrosophic soft sets N_1, N_2 form a neutrosophic soft separation of (U, E, τ_u) and if (U, E, τ_M) is a neutrosophic soft connected subspace of (U, E, τ_u), then $M \subset N_1$ or $M \subset N_2$.

Proof. Here $N_1, N_2 \in \tau_u$ such that $N_1 \cap N_2 = \phi_u$ and $N_1 \cup N_2 = 1_u$. Then $N_1 \cap M, N_2 \cap M \in \tau_M$ as (U, E, τ_M) is a neutrosophic soft topological subspace of (U, E, τ_u). Now $(N_1 \cap M) \cap (N_2 \cap M) = (N_1 \cap N_2) \cap M = \phi_u \cap M = \phi_u$ and $(N_1 \cap M) \cup (N_2 \cap M) = (N_1 \cup N_2) \cap M = 1_u \cap M = M$. Thus the pair $N_1 \cap M, N_2 \cap M$ would constitute a neutrosophic soft separation of (U, E, τ_M), a contradiction.

Hence, one of $N_1 \cap M$ and $N_2 \cap M$ is empty and so M is entirely contained in one of them.

3.7 Theorem

Let (U, E, τ_M) be a neutrosophic soft topological subspace of (U, E, τ_u). A separation of (U, E, τ_M) is a pair of disjoint nonempty neutrosophic soft sets M_1, M_2 whose union is M such that $M_1 \cap \overline{M_2} = \phi_u$ and $M_2 \cap \overline{M_1} = \phi_u$.

Proof. Suppose M_1, M_2 forms a separation of (U, E, τ_M). Then M_1 is both neutrosophic soft open and closed subset of M by Theorem [3.4]. The neutrosophic soft closure of M_1 in M is $\overline{M_1} \cap M$ by Theorem [2.19]. Since M_1 is neutrosophic soft closed in M then $M_1 = \overline{M_1} \cap M$. It implies $\overline{M_1} \cap M_2 = (\overline{M_1} \cap M) \cap M_2 = M_1 \cap M_2 = \phi_u$. Similarly, $\overline{M_2} \cap M_1 = \phi_u$.

Conversely, let $M = M_1 \cup M_2$ with $M_1 \cap M_2 = \phi_u$ such that $\overline{M_1} \cap M_2 = \phi_u$ and $\overline{M_2} \cap M_1 = \phi_u$. Then $M \cap \overline{M_1} = \phi_u$ and $M \cap \overline{M_2} = \phi_u \Rightarrow M_1, M_2$ are neutrosophic soft closed in M. Also $M_1 = M_2^c$ implies both are neutrosophic soft open in M.

3.8 Theorem

Let (U, E, τ_M) be a connected neutrosophic soft subspace of (U, E, τ_u). If (U, E, τ_P) be any neutrosophic soft subspace of (U, E, τ_u) such that $M \subset P \subset \overline{M}$, then (U, E, τ_P) is also neutrosophic soft connected.

Proof. Let the neutrosophic soft set P satisfy the hypothesis. If possible, let P_1, P_2 form a neutrosophic soft separation of (U, E, τ_P). Then $M \subset P_1$ or $M \subset P_2$. Let $M \cap P_1 = \phi_u$. So $M \subset P_1^c$ and P_1^c is closed NSS. It implies $M \subset P \subset \overline{M} \subset P_1^c \Rightarrow P \subset P_1^c \Rightarrow P \cap P_1 = \phi_u$. This is a contradiction to the fact that $P_1 \cup P_2 = P$. Hence, (U, E, τ_P) is neutrosophic soft connected.

3.9 Theorem

Arbitrary union of connected neutrosophic soft subspaces of (U, E, τ_u) having nonempty intersection is also neutrosophic soft connected.

Proof. Let $\{(U, E, \tau_{N_i}) : i \in \Gamma\}$ be a class of connected neutrosophic soft subspaces of (U, E, τ_u) with nonempty intersection. Let $\tau_M = \cup_i(\tau_{N_i})$. If possible, we take a neutrosophic soft separation P, Q of (U, E, τ_M). For each i, $P \cap N_i$ and $Q \cap N_i$ are disjoint neutrosophic soft open sets in the subspace such that their union is N_i. Since each (U, E, τ_{N_i}) is connected, any of $P \cap N_i$ and $Q \cap N_i$ must be empty. Let $P \cap N_i = \phi_u \Rightarrow Q \cap N_i = N_i \Rightarrow N_i \subset Q, \forall i \in \Gamma \Rightarrow \cup_i N_i \subset Q \Rightarrow M \subset Q \Rightarrow P \cup Q \subset Q \Rightarrow P$ is empty, a contradiction. So, (U, E, τ_M) is neutrosophic soft connected.

3.10 Theorem

Arbitrary union of a family of connected neutrosophic soft subspaces of (U, E, τ_u) such that one of the members of the family has nonempty intersection with every member of the family, is neutrosophic soft connected.

Proof. Let $\{(U, E, \tau_{N_i}) : i \in \Gamma\}$ be a class of connected neutrosophic soft subspaces of (U, E, τ_u) and N_k be a fixed member such that $N_k \cap N_i \neq \phi_u$ for each $i \in \Gamma$. Let $M_i = N_k \cup N_i$. Then by Theorem [3.9], (U, E, τ_{M_i}) is a neutrosophic soft connected for each $i \in \Gamma$. Now, $\cup_i M_i = \cup_i(N_k \cup N_i) = (N_k \cup N_1) \cup (N_k \cup N_2) \cup \cdots = N_k \cup (N_1 \cup N_2 \cup \cdots) = \cup_i N_i$ and $\cap_i M_i = \cap_i(N_k \cup N_i) = (N_k \cup N_1) \cap (N_k \cup N_2) \cap \cdots = N_k \cup (N_1 \cap N_2 \cap \cdots) \neq \phi_u$.

This completes the theorem.

4 Compactness

Here, the notion of compactness on neutrosophic soft topological space is developed with some basic theorems.

4.1 Definition

Let (U, E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. A family $\Omega = \{Q_i : i \in \Gamma\}$ of neutrosophic soft sets is said to be a cover of M if $M \subset \cup Q_i$.

If every member of that family which covers M is neutrosophic soft open then it is called open cover of M. A subfamily of Ω which also covers M is called a subcover of M.

4.1.1 Definition

Let (U, E, τ_u) be a neutrosophic soft topological space and $M \in \tau_u$. Suppose Ω be an open cover of M. If Ω has a finite subcover which also covers M then M is called neutrosophic soft compact.

4.1.2 *Example*

In the Example (1) of [2.9.1], $1_u = \cup_{i=1}^{4} N_i$. So $\{N_1, N_2, N_3, N_4\}$ is an open cover of (U, E, τ_u). Also, $1_u = N_1 \cup N_2$ or $1_u = N_1 \cup N_4$. So (U, E, τ_u) is neutrosophic soft compact topological space.

4.2 Theorem

Let (U, E, τ_u) be a neutrosophic soft compact topological space and M be a neutrosophic soft closed set of that space. Then M is also compact.

Proof. Let $\Omega = \{Q_i : i \in \Gamma\}$ be an open cover of M.

Then $\{Q_i\} \cup M^c$ is an open cover of (U, E, τ_u), obviously. Since (U, E, τ_u) is compact so there exists a finite subcover of $\{Q_i\} \cup M^c$ such that

$$\begin{aligned} & 1_u = Q_1 \cup Q_2 \cup \cdots \cup Q_n \cup M^c \\ \Rightarrow\ & M \subset 1_u = Q_1 \cup Q_2 \cup \cdots \cup Q_n \cup M^c \\ \Rightarrow\ & M \subset Q_1 \cup Q_2 \cup \cdots \cup Q_n \text{ as } M \cap M^c = \phi_u. \end{aligned}$$

Hence, M has a finite subcover and so is compact.

4.3 Theorem

Let (U, E, τ_u) be a neutrosophic soft Hausdorff topological space and M be a neutrosophic soft compact set belonging to that space. Then M is a closed NSS.

Proof. Let $e_K \in M^c$ be a neutrosophic soft point. Then for each $e_S \in M$, we have $e_K \neq e_S$. So by definition of Hausdorff space, there are disjoint neutrosophic soft open sets N_K, N_S so that $e_K \in N_K$ and $e_S \in N_S$. Let $\{N_S : e_S \in M\}$ be a neutrosophic soft open cover of M. Since M is neutrosophic soft compact so it has a finite subcover, say, $\{N_{S_1}, N_{S_2}, \cdots N_{S_n}\}$ i.e., $M \subset N_{S_1} \cup N_{S_2} \cup \cdots \cup N_{S_n} = P$, say. Then P is neutrosophic soft open.

Let $Q = N_{K_1} \cap N_{K_2} \cap \cdots \cap N_{K_n}$ where each N_{K_i} is open NSS corresponding to $e_{K_i} \in M^c$. Now, $N_{S_i} \cap N_{K_i} = \phi_u \Rightarrow N_{S_i} \cap Q = \phi_u$ for each i. Then $P \cap Q = (N_{S_1} \cup N_{S_2} \cup \cdots \cup N_{S_n}) \cap Q = (N_{S_1} \cap Q) \cup (N_{S_2} \cap Q) \cup \cdots \cup (N_{S_n} \cap Q) = \phi_u$. Since $M \subset P$ and $P \cap Q = \phi_u$, so $M \cap Q = \phi_u \Rightarrow Q \subset M^c$ and Q is open NSS. This implies M^c is open NSS i.e., M is closed.

4.4 Theorem

A neutrosophic soft topological space is compact iff each family of neutrosophic soft closed sets with the finite intersection property has a nonempty intersection.

Proof. Let (U, E, τ_u) be a compact neutrosophic soft topological space. Consider $\Omega = \{Q_i : i \in \Gamma\}$ be a family of closed NSSs such that $\cap_i Q_i = \phi_u$. We show Ω can not have finite intersection property. Let $\Delta = \{Q_i^c : Q_i \in \Omega, i \in \Gamma\}$. Then Δ is an open cover of (U, E, τ_u) such that there exists a finite subcover $\{Q_1^c, Q_2^c, \cdots, Q_n^c\}$. Now $\cap_{i=1}^n Q_i = 1_u - (Q_1^c \cup Q_2^c \cup \cdots \cup Q_n^c) = 1_u - 1_u = \phi_u$ by Definition [2.8]. Hence, the 'if part' holds.

Next assume that (U, E, τ_u) is not compact. Then, a neutrosophic soft open cover $\{Q_i : i \in \Gamma\}$, say, of (U, E, τ_u) has no finite subcover i.e., $Q_1 \cup Q_2 \cup \cdots \cup Q_n \neq 1_u$. This implies $Q_1^c \cap Q_2^c \cap \cdots \cap Q_n^c \neq \phi_u$ by Definition [2.8] and Proposition [2.16]. Thus $\{Q_i^c : i \in \Gamma\}$ has finite intersection property. Then by hypothesis, $\cap_i Q_i^c \neq \phi_u$ and $\cup_i Q_i \neq 1_u$ which is a contradiction. Hence, (U, E, τ_u) is compact.

5 Neutrosophic soft continuous mappings

In this section, first we define neutrosophic soft mapping, then define image and pre-image of an NSS under a neutrosophic soft mapping. In continuation, we introduce the notion of neutrosophic soft continuous mapping in a neutrosophic soft topological space along with some of it's properties.

In rest of the paper, if M be an NSS over U via parameter set E, we write (M, E), an NSS over U i.e., $(M, E) = \{< e, f_M(e) >: e \in E\}$.

5.1 Definition

Let, $\varphi : U \to V$ and $\psi : E \to E$ be two functions where E is the parameter set for each of the crisp sets U and V. Then the pair (φ, ψ) is called an NSS function from (U, E) to (V, E). We write, $(\varphi, \psi) : (U, E) \to (V, E)$.

5.1.1 Definition

Let (M, E) and (N, E) be two NSSs defined over U and V, respectively and (φ, ψ) be an NSS function from (U, E) to (V, E). Then,

(1) The image of (M, E) under (φ, ψ), denoted by $(\varphi, \psi)(M, E)$, is an NSS over V and is defined as :
$(\varphi, \psi)(M, E) = (\varphi(M), \psi(E)) = \{< \psi(a), f_{\varphi(M)}(\psi(a)) >: a \in E\}$ where $\forall b \in \psi(E), \forall y \in V$.

$$T_{f_{\varphi(M)}(b)}(y) = \begin{cases} \max_{\varphi(x)=y} \max_{\psi(a)=b} [T_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 0 \ , & \text{otherwise.} \end{cases}$$

$$I_{f_{\varphi(M)}(b)}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(a)=b} [I_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \ , & \text{otherwise.} \end{cases}$$

$$F_{f_{\varphi(M)}(b)}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(a)=b} [F_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \ , & \text{otherwise.} \end{cases}$$

(2) The pre-image of (N, E) under (φ, ψ), denoted by $(\varphi, \psi)^{-1}(N, E)$, is an NSS over U and is defined by :
$(\varphi, \psi)^{-1}(N, E) = (\varphi^{-1}(N), \psi^{-1}(E))$ where $\forall a \in \psi^{-1}(E), \forall x \in U$.

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x) &= T_{f_N(\psi(a))}(\varphi(x)) \\ I_{f_{\varphi^{-1}(N)}(a)}(x) &= I_{f_N(\psi(a))}(\varphi(x)) \\ F_{f_{\varphi^{-1}(N)}(a)}(x) &= F_{f_N(\psi(a))}(\varphi(x)) \end{aligned}$$

If ψ and φ are injective (surjective), then (φ, ψ) is injective (surjective).

5.1.2 Proposition

Let, $(\varphi, \psi) : (U, E) \to (V, E)$ be a neutrosophic soft mapping and (M_1, E) and (M_2, E) be two NSSs defined over U. Then the followings hold.

(1) $(M_1,E) \subseteq (\varphi,\psi)^{-1}[(\varphi,\psi)(M_1,E)]$
(2) $[(\varphi,\psi)(M_1,E)]^c \subseteq (\varphi,\psi)(M_1,E)^c$, if φ is surjective.
(3) $(\varphi,\psi)[(M_1,E) \cup (M_2,E)] = (\varphi,\psi)(M_1,E) \cup (\varphi,\psi)(M_2,E)$
(4) $(\varphi,\psi)[(M_1,E) \cap (M_2,E)] = (\varphi,\psi)(M_1,E) \cap (\varphi,\psi)(M_2,E)$

Proof.
(1) $(\varphi,\psi)^{-1}[(\varphi,\psi)(M_1,E)] = (\varphi,\psi)^{-1}[\varphi(M_1),\psi(E)] = [\varphi^{-1}(\varphi(M_1)),\psi^{-1}(\psi(E))]$. Then for $a \in \psi^{-1}(\psi(E))$ and $x \in U$, we have, $T_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x) = T_{f_{\varphi(M_1)}(\psi(a))}(\varphi(x)) = \max_{\varphi(x)} \max_{\psi(a)}[T_{f_M(a)}(x)]$. Now, $T_{f_M(a)}(x) \leq \max_{\varphi(x)} \max_{\psi(a)}[T_{f_M(a)}(x)] = T_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x)$.
Similarly, $I_{f_M(a)}(x) \geq I_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x)$ and $F_{f_M(a)}(x) \geq F_{f_{\varphi^{-1}(\varphi(M_1))}(a)}(x)$.
Hence, $(M_1,E) \subseteq (\varphi,\psi)^{-1}[(\varphi,\psi)(M_1,E)]$.

(2) Suppose, φ is surjective mapping. Here, $[(\varphi,\psi)(M_1,E)]^c = [(\varphi(M_1))^c,\psi(E)]$ and $(\varphi,\psi)(M_1,E)^c = [\varphi(M_1^c),\psi(E)]$. For $b \in \psi(E)$ and $y \in V$, we have, $T_{f_{(\varphi(M_1))^c}(b)}(y) = F_{f_{(\varphi(M_1))}(b)}(y) = \min_{\varphi(x)=y} \min_{\psi(a)=b}[F_{f_{M_1}(a)}(x)]$. But, $T_{f_{\varphi(M_1^c)}(b)}(y) = \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1^c}(a)}(x)] = \max_{\varphi(x)=y} \max_{\psi(a)=b}[F_{f_{M_1}(a)}(x)]$. Thus, $T_{f_{(\varphi(M_1))^c}(b)}(y) \leq T_{f_{\varphi(M_1^c)}(b)}(y) \cdots\cdots\cdots$ (i)
Similarly, $F_{f_{(\varphi(M_1))^c}(b)}(y) \geq F_{f_{\varphi(M_1^c)}(b)}(y) \cdots\cdots\cdots$ (ii)
Finally, $I_{f_{(\varphi(M_1))^c}(b)}(y) = 1 - I_{f_{(\varphi(M_1))}(b)}(y) = 1 - \min_{\varphi(x)=y} \min_{\psi(a)=b}[I_{f_{M_1}(a)}(x)]$ and $I_{f_{\varphi(M_1^c)}(b)}(y) = \min_{\varphi(x)=y} \min_{\psi(a)=b}[I_{f_{M_1^c}(a)}(x)] = \min_{\varphi(x)=y} \min_{\psi(a)=b}[1 - I_{f_{M_1}(a)}(x)]$.
This shows, $I_{f_{(\varphi(M_1))^c}(b)}(y) \geq I_{f_{\varphi(M_1^c)}(b)}(y) \cdots\cdots\cdots$ (iii)

This completes the 2nd part.

(3) Let, $(M_1,E) \cup (M_2,E) = (M,E)$.
Then, $(\varphi,\psi)[(M_1,E) \cup (M_2,E)] = (\varphi,\psi)(M,E) = [\varphi(M),\psi(E)]$. So, for $b \in \psi(E)$ and $y \in V$, we have,

$$\begin{aligned} T_{f_{\varphi(M)}(b)}(y) &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_M(a)}(x)] \\ &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) \diamond T_{f_{M_2}(a)}(x)] \end{aligned}$$

Next, $(\varphi,\psi)(M_1,E) \cup (\varphi,\psi)(M_2,E) = [\varphi(M_1) \cup \varphi(M_2),\psi(E)] = [P,\psi(E)]$, say. Then,

$$\begin{aligned} & T_{f_P(b)}(y) \\ = \;& T_{f_{\varphi(M_1)}(b)}(y) \diamond T_{f_{\varphi(M_2)}(b)}(y) \\ = \;& \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x)] \diamond \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_2}(a)}(x)] \\ = \;& \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) \diamond T_{f_{M_2}(a)}(x)] \end{aligned}$$

Thus, $T_{f_{\varphi(M)}(b)}(y) = T_{f_P(b)}(y)$. Similar results also hold for I, F.

This completes the proof of part (3).

(4) Let, $(M_1,E) \cap (M_2,E) = (M,E)$.
Then, $(\varphi,\psi)[(M_1,E) \cap (M_2,E)] = (\varphi,\psi)(M,E) = [\varphi(M),\psi(E)]$. So, for $b \in \psi(E)$ and $y \in V$, we have,

$$\begin{aligned} T_{f_{\varphi(M)}(b)}(y) &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_M(a)}(x)] \\ &= \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) * T_{f_{M_2}(a)}(x)] \end{aligned}$$

Next, $(\varphi,\psi)(M_1,E) \cap (\varphi,\psi)(M_2,E) = [\varphi(M_1) \cap \varphi(M_2),\psi(E)] = [Q,\psi(E)]$, say. Then,

$$\begin{aligned} & T_{f_Q(b)}(y) \\ = \;& T_{f_{\varphi(M_1)}(b)}(y) * T_{f_{\varphi(M_2)}(b)}(y) \\ = \;& \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x)] * \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_2}(a)}(x)] \\ = \;& \max_{\varphi(x)=y} \max_{\psi(a)=b}[T_{f_{M_1}(a)}(x) * T_{f_{M_2}(a)}(x)] \end{aligned}$$

Thus, $T_{f_{\varphi(M)}(b)}(y) = T_{f_Q(b)}(y)$. Similar results also hold for I, F.

This ends the last part.

5.1.3 Proposition

Let, $(\varphi,\psi) : (U,E) \to (V,E)$ be a neutrosophic soft mapping and (N_1,E) and (N_2,E) be two NSSs defined over V. Then the followings hold.
(1) $(\varphi,\psi)[(\varphi,\psi)^{-1}(N_1,E)] = (N_1,E)$, if (φ,ψ) is surjective.
(2) $[(\varphi,\psi)^{-1}(N_1,E)]^c = (\varphi,\psi)^{-1}(N_1,E)^c$
(3) $(\varphi,\psi)^{-1}[(N_1,E) \cup (N_2,E)] = (\varphi,\psi)^{-1}(N_1,E) \cup (\varphi,\psi)^{-1}(N_2,E)$
(4) $(\varphi,\psi)^{-1}[(N_1,E) \cap (N_2,E)] = (\varphi,\psi)^{-1}(N_1,E) \cap (\varphi,\psi)^{-1}(N_2,E)$

Proof. We shall prove (2) and (3), only. The others can be proved similarly.
(2) Here, $[(\varphi,\psi)^{-1}(N_1,E)]^c = [(\varphi^{-1}(N))^c,\psi^{-1}(E)]$. Then, for $a \in \psi^{-1}(E), x \in U$,

$$\begin{aligned} T_{f_{(\varphi^{-1}(N))^c}(a)}(x) = F_{f_{\varphi^{-1}(N)}(a)}(x) = F_{f_N(\psi(a))}(\varphi(x)), \\ I_{f_{(\varphi^{-1}(N))^c}(a)}(x) = 1 - I_{f_{\varphi^{-1}(N)}(a)}(x) = 1 - I_{f_N(\psi(a))}(\varphi(x)), \\ F_{f_{(\varphi^{-1}(N))^c}(a)}(x) = T_{f_{\varphi^{-1}(N)}(a)}(x) = T_{f_N(\psi(a))}(\varphi(x)). \end{aligned}$$

Next, $(\varphi,\psi)^{-1}(N_1,E)^c = [\varphi^{-1}(N_1^c),\psi)^{-1}(E)]$. Then,

$$\begin{aligned} T_{f_{\varphi^{-1}(N^c)}(a)}(x) = T_{f_{N^c}(a)}(x) = F_{f_N(\psi(a))}(\varphi(x)), \\ I_{f_{\varphi^{-1}(N^c)}(a)}(x) = I_{f_{N^c}(a)}(x) = 1 - I_{f_N(\psi(a))}(\varphi(x)), \\ F_{f_{\varphi^{-1}(N^c)}(a)}(x) = F_{f_{N^c}(a)}(x) = T_{f_N(\psi(a))}(\varphi(x)). \end{aligned}$$

Hence, the result is proved.

(3) Let, $(N_1,E) \cup (N_2,E) = (N,E)$.
Then, $(\varphi,\psi)^{-1}[(N_1,E) \cup (N_2,E)] = (\varphi,\psi)^{-1}(N,E) = [\varphi^{-1}(N),\psi^{-1}(E)]$. So, for $a \in \psi^{-1}(E)$ and $x \in U$, we have,

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x) &= T_{f_N(\psi(a))}(\varphi(x)) \\ &= T_{f_{N_1}(\psi(a))}(\varphi(x)) \diamond T_{f_{N_2}(\psi(a))}(\varphi(x)) \end{aligned}$$

Next, $(\varphi,\psi)^{-1}(N_1,E) \cup (\varphi,\psi)^{-1}(N_2,E) = [\varphi^{-1}(N_1) \cup \varphi^{-1}(N_2), \psi^{-1}(E)] = [R, \psi^{-1}(E)]$, say. Then,

$$\begin{aligned} T_{f_{R(a)}}(x) &= T_{f_{\varphi^{-1}(N_1)}(a)}(x) \diamond T_{f_{\varphi^{-1}(N_2)}(a)}(x) \\ &= T_{f_{N_1}(\psi(a))}(\varphi(x)) \diamond T_{f_{N_2}(\psi(a))}(\varphi(x)) \end{aligned}$$

Thus, $T_{f_{\varphi^{-1}(N)}(a)}(x) = T_{f_{R(a)}}(x)$. Similar results also hold for I, F.

This completes the proof of part (3).

5.2 Definition

Let $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a mapping where (U,E,τ_u) and (V,E,τ_v) be two neutrosophic soft topological spaces.

(1) For each neutrosophic soft open set $(M,E) \in (U,E,\tau_u)$, if the image $(\varphi,\psi)(M,E)$ is open in (V,E,τ_v) then (φ,ψ) is said to be neutrosophic soft open mapping.

(2) For each neutrosophic soft closed set $(Q,E) \in (U,E,\tau_u)$, if the image $(\varphi,\psi)(Q,E)$ is closed in (V,E,τ_v) then (φ,ψ) is said to be neutrosophic soft closed mapping.

5.3 Theorem

Let, (U,E,τ_u) and (V,E,τ_v) be two neutrosophic soft topological spaces and $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a mapping. Then,

(1) (φ,ψ) is a neutrosophic soft open mapping iff for each neutrosophic soft set $(M,E) \in (U,E,\tau_u)$, there be hold $(\varphi,\psi)(M,E)^o \subset [(\varphi,\psi)(M,E)]^o$.

(2) (φ,ψ) is a neutrosophic soft closed mapping iff for each neutrosophic soft set $(Q,E) \in (U,E,\tau_u)$, there be hold $\overline{[(\varphi,\psi)(Q,E)]} \subset (\varphi,\psi)\overline{(Q,E)}$.

Proof. **(1)** Let (φ,ψ) is a neutrosophic soft open mapping and $(M,E) \in (U,E,\tau_u)$. Then $(M,E)^o$ is a neutrosophic soft open set and $(M,E)^o \subset (M,E)$. Since (φ,ψ) is a neutrosophic soft open mapping, $(\varphi,\psi)(M,E)^o$ is neutrosophic soft open in (V,E,τ_v). Then $(\varphi,\psi)(M,E)^o \subset (\varphi,\psi)(M,E)$. But $[(\varphi,\psi)(M,E)]^o$ is the largest open NSS contained in $(\varphi,\psi)(M,E)$. Hence, $(\varphi,\psi)(M,E)^o \subset [(\varphi,\psi)(M,E)]^o$ is obtained.

Conversely, suppose (M,E) be an open NSS in (U,E,τ_u) such that the given condition holds. Then $(M,E) = (M,E)^o$ and so $(\varphi,\psi)(M,E) = (\varphi,\psi)(M,E)^o \subset [(\varphi,\psi)(M,E)]^o \subset (\varphi,\psi)(M,E)$. Hence, $[(\varphi,\psi)(M,E)]^o = (\varphi,\psi)(M,E)$. This ends the proof.

(2) Let (φ,ψ) is a neutrosophic soft closed mapping and $(Q,E) \in (U,E,\tau_u)$. Then $\overline{(Q,E)}$ is a neutrosophic soft closed set and $(Q,E) \subset \overline{(Q,E)}$. Since (φ,ψ) is a neutrosophic soft closed mapping, $(\varphi,\psi)\overline{(Q,E)}$ is neutrosophic soft closed in (V,E,τ_v). Then $(\varphi,\psi)(Q,E) \subset (\varphi,\psi)\overline{(Q,E)}$. But $\overline{[(\varphi,\psi)(Q,E)]}$ is the smallest closed NSS containing $(\varphi,\psi)(Q,E)$. Hence, $\overline{[(\varphi,\psi)(Q,E)]} \subset (\varphi,\psi)\overline{(Q,E)}$ is obtained.

Conversely, suppose (Q,E) be a closed NSS in (U,E,τ_u) such that the given condition holds. Then $\overline{(Q,E)} = (Q,E)$ and so $(\varphi,\psi)(Q,E) \subset \overline{[(\varphi,\psi)(Q,E)]} \subset (\varphi,\psi)\overline{(Q,E)} = (\varphi,\psi)(Q,E)$. Hence, $\overline{[(\varphi,\psi)(Q,E)]} = (\varphi,\psi)(Q,E)$. This completes the proof.

5.4 Definition

Let, (U,E,τ_u) and (V,E,τ_v) be two neutrosophic soft topological spaces. Then $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ is said to be a neutrosophic soft continuous mapping if for each $(N,E) \in \tau_v$, the inverse image $(\varphi,\psi)^{-1}(N,E) \in \tau_u$ i.e., the inverse image of each open NSS in (V,E,τ_v) is also open in (U,E,τ_u).

5.4.1 *Example*

For two neutrosophic soft topological spaces (U,E,τ_u) and (V,E,τ_v), let $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a mapping.

(1) If τ_v is the neutrosophic soft indiscrete topology on V, then (φ,ψ) is a neutrosophic soft continuous mapping.

(2) If τ_u is the neutrosophic soft discrete topology on U, then (φ,ψ) is a neutrosophic soft continuous mapping.

(3) Let, $U = \{u_1,u_2,u_3\}, V = \{v_1,v_2,v_3\}, E = \{e_1,e_2\}, \tau_v = \{\phi_v, 1_v, (N_1,E), (N_2,E)\}, \tau_u = \{\phi_u, 1_u, (M_1,E), (M_2,E), (M_3,E)\}$, where $(N_1,E),(N_2,E)$ are as follows :

$$\begin{aligned} f_{N_1}(e_1) &= \{< v_1,(0.8,0.5,0.6) >, < v_2,(0.5,0.7,0.6) >, < v_3,(0.4,0.7,0.5) >\}; \\ f_{N_1}(e_2) &= \{< v_1,(0.7,0.6,0.5) >, < v_2,(0.6,0.8,0.4) >, < v_3,(0.5,0.8,0.6) >\}; \\ f_{N_2}(e_1) &= \{< v_1,(0.6,0.6,0.7) >, < v_2,(0.4,0.8,0.8) >, < v_3,(0.3,0.8,0.6) >\}; \\ f_{N_2}(e_2) &= \{< v_1,(0.5,0.8,0.6) >, < v_2,(0.5,0.9,0.5) >, < v_3,(0.2,0.9,0.7) >\}; \end{aligned}$$

and $(M_1,E),(M_2,E),(M_3,E)$ are given as followings :

$$\begin{aligned} f_{M_1}(e_1) &= \{< u_1,(0.8,0.4,0.5) >, < u_2,(0.7,0.5,0.6) >, < u_3,(0.7,0.7,0.3) >\}; \\ f_{M_1}(e_2) &= \{< u_1,(1.0,0.5,0.4) >, < u_2,(0.5,0.6,0.4) >, < u_3,(0.6,0.6,0.6) >\}; \\ f_{M_2}(e_1) &= \{< u_1,(0.5,0.8,0.6) >, < u_2,(0.2,0.9,0.7) >, < u_3,(0.5,0.9,0.5) >\}; \\ f_{M_2}(e_2) &= \{< u_1,(0.6,0.6,0.7) >, < u_2,(0.3,0.8,0.6) >, < u_3,(0.4,0.8,0.8) >\}; \\ f_{M_3}(e_1) &= \{< u_1,(0.7,0.6,0.5) >, < u_2,(0.5,0.8,0.6) >, < u_3,(0.6,0.8,0.4) >\}; \\ f_{M_3}(e_2) &= \{< u_1,(0.8,0.5,0.6) >, < u_2,(0.4,0.7,0.5) >, < u_3,(0.5,0.7,0.6) >\}; \end{aligned}$$

The t-norm and s-norm in both τ_u, τ_v are defined as $a * b = \min\{a,b\}$ and $a \diamond b = \max\{a,b\}$. Consider the mapping (φ,ψ)

as : $\varphi(u_1) = v_1, \varphi(u_2) = v_3, \varphi(u_3) = v_2$ and $\psi(e_1) = e_2, \psi(e_2) = e_1$. Then $(\varphi,\psi)^{-1}(N_1,E), (\varphi,\psi)^{-1}(N_2,E) \in \tau_u$.

For convenience, the calculation of $(\varphi,\psi)^{-1}(N_1,E)$ is provided for one parameter. The others are in similar way.

$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = T_{f_{N_1}(e_2)}(v_1) = 0.7$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = I_{f_{N_1}(e_2)}(v_1) = 0.6$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = F_{f_{N_1}(e_2)}(v_1) = 0.5$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = T_{f_{N_1}(e_2)}(v_3) = 0.5$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = I_{f_{N_1}(e_2)}(v_3) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = F_{f_{N_1}(e_2)}(v_3) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = T_{f_{N_1}(e_2)}(v_2) = 0.6$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = I_{f_{N_1}(e_2)}(v_2) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = F_{f_{N_1}(e_2)}(v_2) = 0.4$$

5.4.2 Proposition

Let $(\varphi,\psi) : (U,E,\tau_u) \to (V,E,\tau_v)$ be a neutrosophic soft continuous mapping. Then for each $e \in E$, $(\varphi,\psi) : (U,\tau_u^e) \to (V,\tau_v^e)$ is a neutrosophic continuous mapping.

Proof. Let, $(N,E) \in \tau_v$. Since (φ,ψ) be a neutrosophic soft continuous mapping, so $(\varphi,\psi)^{-1}(N,E) \in \tau_u$. It implies $(\varphi,\psi)^{-1}(\{< e, f_N(e) >: e \in E\}) \in \tau_u$ i.e., $(\varphi,\psi)^{-1}(< e, f_N(e) >) \in \tau_u^e$ for $< e, f_N(e) > \in \tau_v^e$. This follows the theorem.

But the converse does not hold. The following example shows the fact.

Let, $U = \{u_1,u_2,u_3\}, V = \{v_1,v_2,v_3\}, E = \{e_1,e_2\}, \tau_v = \{\phi_v, 1_v, (N_1,E),(N_2,E)\}, \tau_u = \{\phi_u, 1_u, (M_1,E),(M_2,E),(M_3,E)\}$, where $(N_1,E),(N_2,E)$ are as follows :

$$f_{N_1}(e_1) = \{< v_1,(0.8,0.5,0.6) >, < v_2,(0.5,0.7,0.6) >, < v_3,(0.4,0.7,0.5) >\};$$
$$f_{N_1}(e_2) = \{< v_1,(0.7,0.6,0.5) >, < v_2,(0.6,0.8,0.4) >, < v_3,(0.5,0.8,0.6) >\};$$
$$f_{N_2}(e_1) = \{< v_1,(1.0,0.5,0.4) >, < v_2,(0.6,0.6,0.6) >, < v_3,(0.5,0.6,0.4) >\};$$
$$f_{N_2}(e_2) = \{< v_1,(0.8,0.4,0.5) >, < v_2,(0.7,0.7,0.3) >, < v_3,(0.7,0.5,0.6) >\};$$

and $(M_1,E),(M_2,E),(M_3,E)$ are given as follows :

$$f_{M_1}(e_1) = \{< u_1,(0.6,0.6,0.6) >, < u_2,(0.5,0.6,0.4) >, < u_3,(1.0,0.5,0.4) >\};$$
$$f_{M_1}(e_2) = \{< u_1,(0.7,0.7,0.3) >, < u_2,(0.7,0.5,0.6) >, < u_3,(0.8,0.4,0.5) >\};$$
$$f_{M_2}(e_1) = \{< u_1,(0.5,0.7,0.6) >, < u_2,(0.4,0.7,0.5) >, < u_3,(0.8,0.5,0.6) >\};$$
$$f_{M_2}(e_2) = \{< u_1,(0.5,0.9,0.5) >, < u_2,(0.2,0.9,0.7) >, < u_3,(0.5,0.8,0.6) >\};$$
$$f_{M_3}(e_1) = \{< u_1,(0.5,0.6,0.6) >, < u_2,(0.4,0.7,0.4) >, < u_3,(0.9,0.5,0.5) >\};$$
$$f_{M_3}(e_2) = \{< u_1,(0.6,0.8,0.4) >, < u_2,(0.5,0.8,0.6) >, < u_3,(0.7,0.6,0.5) >\};$$

The t-norm and s-norm in both τ_u, τ_v are defined as $a * b = \min\{a,b\}$ and $a \diamond b = \max\{a,b\}$. Define a neutrosophic soft mapping (φ,ψ) as : $\varphi(u_1) = v_2, \varphi(u_2) = v_3, \varphi(u_3) = v_1$ and $\psi(e_1) = e_1, \psi(e_2) = e_2$. We now calculate $(\varphi,\psi)^{-1}(N_1,E)$.

$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = T_{f_{N_1}(e_1)}(v_2) = 0.5$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = I_{f_{N_1}(e_1)}(v_2) = 0.7$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_1) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_1)) = F_{f_{N_1}(e_1)}(v_2) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = T_{f_{N_1}(e_1)}(v_3) = 0.4$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = I_{f_{N_1}(e_1)}(v_3) = 0.7$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_2) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_2)) = F_{f_{N_1}(e_1)}(v_3) = 0.5$$
$$T_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = T_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = T_{f_{N_1}(e_1)}(v_1) = 0.8$$
$$I_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = I_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = I_{f_{N_1}(e_1)}(v_1) = 0.5$$
$$F_{f_{\varphi^{-1}(N_1)}(e_1)}(u_3) = F_{f_{N_1}(\psi(e_1))}(\varphi(u_3)) = F_{f_{N_1}(e_1)}(v_1) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_2)}(u_1) = T_{f_{N_1}(\psi(e_2))}(\varphi(u_1)) = T_{f_{N_1}(e_2)}(v_2) = 0.6$$
$$I_{f_{\varphi^{-1}(N_1)}(e_2)}(u_1) = I_{f_{N_1}(\psi(e_2))}(\varphi(u_1)) = I_{f_{N_1}(e_2)}(v_2) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_2)}(u_1) = F_{f_{N_1}(\psi(e_2))}(\varphi(u_1)) = F_{f_{N_1}(e_2)}(v_2) = 0.4$$
$$T_{f_{\varphi^{-1}(N_1)}(e_2)}(u_2) = T_{f_{N_1}(\psi(e_2))}(\varphi(u_2)) = T_{f_{N_1}(e_2)}(v_3) = 0.5$$
$$I_{f_{\varphi^{-1}(N_1)}(e_2)}(u_2) = I_{f_{N_1}(\psi(e_2))}(\varphi(u_2)) = I_{f_{N_1}(e_2)}(v_3) = 0.8$$
$$F_{f_{\varphi^{-1}(N_1)}(e_2)}(u_2) = F_{f_{N_1}(\psi(e_2))}(\varphi(u_2)) = F_{f_{N_1}(e_2)}(v_3) = 0.6$$
$$T_{f_{\varphi^{-1}(N_1)}(e_2)}(u_3) = T_{f_{N_1}(\psi(e_2))}(\varphi(u_3)) = T_{f_{N_1}(e_2)}(v_1) = 0.7$$
$$I_{f_{\varphi^{-1}(N_1)}(e_2)}(u_3) = I_{f_{N_1}(\psi(e_2))}(\varphi(u_3)) = I_{f_{N_1}(e_2)}(v_1) = 0.6$$
$$F_{f_{\varphi^{-1}(N_1)}(e_2)}(u_3) = F_{f_{N_1}(\psi(e_2))}(\varphi(u_3)) = F_{f_{N_1}(e_2)}(v_1) = 0.5$$

Thus $(\varphi,\psi)^{-1}(N_1,E) \notin \tau_u$ though $(\varphi,\psi)^{-1}(N_2,E) = (M_1,E)$. So $(\varphi,\psi)^{-1}$ is not neutrosophic soft continuous. Now,

$$\begin{aligned}\tau_u^{e_1} &= \{(0,1,1),(1,0,0), f_{M_1}(e_1), f_{M_2}(e_1), f_{M_3}(e_1)\},\\ \tau_u^{e_2} &= \{(0,1,1),(1,0,0), f_{M_1}(e_2), f_{M_2}(e_2), f_{M_3}(e_2)\};\\ \tau_v^{e_1} &= \{(0,1,1),(1,0,0), f_{N_1}(e_1), f_{N_2}(e_1)\},\\ \tau_v^{e_2} &= \{(0,1,1),(1,0,0), f_{N_1}(e_2), f_{N_2}(e_2)\};\end{aligned}$$

Then, $(\varphi,\psi) : (U,\tau_u^{e_1}) \to (V,\tau_v^{e_1})$ is neutrosophic continuous mapping because $(\varphi,\psi)^{-1}[f_{N_1}(e_1)] = f_{M_2}(e_1)$ and $(\varphi,\psi)^{-1}[f_{N_2}(e_1)] = f_{M_1}(e_1)$.
Similarly, $(\varphi,\psi) : (U,\tau_u^{e_2}) \to (V,\tau_v^{e_2})$ is neutrosophic continuous mapping as : $(\varphi,\psi)^{-1}[f_{N_1}(e_2)] = f_{M_3}(e_2)$ and $(\varphi,\psi)^{-1}[f_{N_2}(e_2)] = f_{M_1}(e_2)$.

5.5 Theorem

For two neutrosophic soft topological spaces (U, E, τ_u) and (V, E, τ_v), let $(\varphi, \psi) : (U, E, \tau_u) \to (V, E, \tau_v)$ be a neutrosophic soft mapping. Then the following conditions are equivalent.

(1) (φ, ψ) is neutrosophic soft continuous mapping.
(2) The inverse image of a closed NSS in (V, E, τ_v) is closed in (U, E, τ_u).
(3) For each $(M, E) \in NSS(U, E)$, $(\varphi, \psi)\overline{(M, E)} \subset \overline{(\varphi, \psi)(M, E)}$.
(4) For each $(N, E) \in NSS(V, E)$, $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}\overline{(N, E)}$.
(5) For each $(N, E) \in NSS(V, E)$, $(\varphi, \psi)^{-1}(N, E)^o \subset [(\varphi, \psi)^{-1}(N, E)]^o$.

Proof. $(1) \Rightarrow (2)$
Let, (Q, E) be a closed NSS in (V, E, τ_v). Then $(Q, E)^c \in \tau_v$ and so by (1), $(\varphi, \psi)^{-1}(Q, E)^c \in \tau_u$. But $(\varphi, \psi)^{-1}(Q, E)^c = ((\varphi, \psi)^{-1}(Q, E))^c$. So $(\varphi, \psi)^{-1}(Q, E)$ is a closed NSS in (U, E, τ_u).

$(2) \Rightarrow (3)$
Let, $(M, E) \in NSS(U, E)$. Since $(M, E) \subset (\varphi, \psi)^{-1}((\varphi, \psi)(M, E))$ and $(\varphi, \psi)(M, E) \subset \overline{(\varphi, \psi)(M, E)}$, we have $(M, E) \subset (\varphi, \psi)^{-1}((\varphi, \psi)(M, E)) \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$. Obviously, $\overline{(\varphi, \psi)(M, E)}$ is closed in (V, E, τ_v). Then by (2), $(\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$ is closed in (U, E, τ_u). But, since $(M, E) \subset \overline{(M, E)}$ and $\overline{(M, E)}$ is the smallest closed NSS, so $(M, E) \subset \overline{(M, E)} \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$. This implies $(\varphi, \psi)\overline{(M, E)} \subset (\varphi, \psi)[(\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})]$ i.e., $(\varphi, \psi)\overline{(M, E)} \subset \overline{(\varphi, \psi)(M, E)}$ is obtained.

$(3) \Rightarrow (4)$
Let, $(N, E) \in NSS(V, E)$ and $(\varphi, \psi)^{-1}(N, E) = (M, E)$. Then $\overline{(\varphi, \psi)^{-1}(N, E)} = \overline{(M, E)}$. But by (3), we have $\overline{(M, E)} \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$ i.e., $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}(\overline{(\varphi, \psi)(M, E)})$. This shows $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}[\overline{(\varphi, \psi)((\varphi, \psi)^{-1}(N, E))}]$ i.e., $\overline{(\varphi, \psi)^{-1}(N, E)} \subset (\varphi, \psi)^{-1}\overline{(N, E)}$.

$(4) \Rightarrow (5)$
Let, $(N, E) \in NSS(V, E)$. Replacing (N, E) by $(N, E)^c$ and applying (4), we have $\overline{(\varphi, \psi)^{-1}(N, E)^c} \subset (\varphi, \psi)^{-1}(\overline{(N, E)^c})$ i.e., $[(\varphi, \psi)^{-1}(\overline{(N, E)^c})]^c \subset [\overline{(\varphi, \psi)^{-1}(N, E)^c}]^c$. By Theorem (ii) of [2.15.2], since $(N, E)^o = [\overline{(N, E)^c}]^c$, so $(\varphi, \psi)^{-1}(N, E)^o = (\varphi, \psi)^{-1}(\overline{(N, E)^c})^c = [(\varphi, \psi)^{-1}(\overline{(N, E)^c})]^c \subset [\overline{(\varphi, \psi)^{-1}(N, E)^c}]^c = [(\varphi, \psi)^{-1}(N, E)]^o$.

$(5) \Rightarrow (1)$
Let, (N, E) be an open NSS in (V, E, τ_v). Then $(N, E)^o = (N, E)$. Since $[(\varphi, \psi)^{-1}(N, E)]^o \subset (\varphi, \psi)^{-1}(N, E) = (\varphi, \psi)^{-1}(N, E)^o \subset [(\varphi, \psi)^{-1}(N, E)]^o$, so $[(\varphi, \psi)^{-1}(N, E)]^o = (\varphi, \psi)^{-1}(N, E)$ is obtained. Thus, $(\varphi, \psi)^{-1}(N, E)$ is an open NSS in (U, E, τ_u) and so (φ, ψ) is neutrosophic soft continuous mapping.

5.6 Theorem

Let, (U, E, τ_u) and (V, E, τ_v) be two neutrosophic soft topological spaces. Also let, $(\varphi, \psi) : (U, E, \tau_u) \to (V, E, \tau_v)$ be a continuous neutrosophic soft mapping. If (M, E) is neutrosophic soft compact in (U, E, τ_u), then $(\varphi, \psi)(M, E)$ is so in (V, E, τ_v).

Proof. Let $\{(N_i, E) : i \in \Gamma\}$ be a neutrosophic soft open covering of $(\varphi, \psi)(M, E)$ i.e., $(\varphi, \psi)(M, E) \subset \cup_i (N_i, E)$. Since, (φ, ψ) is neutrosophic soft continuous, $\{(\varphi, \psi)^{-1}(N_i, E) : i \in \Gamma\}$ is a neutrosophic soft open cover of (M, E). But, (M, E) is neutrosophic soft compact. So, there exists a finite subcover $\{(\varphi, \psi)^{-1}(N_i, E) : 1 \leq i \leq k\}$ such that $(M, E) \subset \cup_{i=1}^{k} (\varphi, \psi)^{-1}(N_i, E)$ hold. Hence, $(\varphi, \psi)(M, E) \subset (\varphi, \psi)[\cup_{i=1}^{k} (\varphi, \psi)^{-1}(N_i, E)] = \cup_{i=1}^{k} (\varphi, \psi)[(\varphi, \psi)^{-1}(N_i, E)] = \cup_{i=1}^{k} (N_i, E)$.

This shows that $(\varphi, \psi)(M, E)$ is covered by a finite number of member of $\{(N_i, E) : i \in \Gamma\}$. Hence, $(\varphi, \psi)(M, E)$ is neutrosophic soft compact also.

5.7 Theorem

Let, (U, E, τ_u) be a neutrosophic soft topological space and (V, E, τ_v) be a neutrosophic soft Hausdorff space. Then, a neutrosophic soft function $(\varphi, \psi) : (U, E, \tau_u) \to (V, E, \tau_v)$ is closed if it is continuous.

Proof. Let (Q, E) be any neutrosophic soft closed set in (U, E, τ_u). Then by Theorem [4.2], (Q, E) is compact NSS. Since (φ, ψ) is continuous neutrosophic soft function then $(\varphi, \psi)(Q, E)$ is compact NSS in (V, E, τ_v). As (V, E, τ_v) is neutrosophic soft Hausdorff space, so $(\varphi, \psi)(Q, E)$ is closed by Theorem [4.3].

6 Conclusion

Topology is a major sector in mathematics and it can give many relationships between other scientific area and mathematical models. The motivation of the present paper is to extend the concept of topological structure on neutrosophic soft set introduced in the paper [33]. Here, we have defined connectedness and compactness on neutrosophic soft topological space, neutrosophic soft continuous mappings. These are illustrated by suitable examples. Their several related properties and structural characteristics have been investigated. We expect, this paper will promote the future study on neutrosophic soft topological groups and many other general frameworks.

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The creation of three logical connectors to reapprove how comprehensive and effective the Neutrosophic logic is compared to the fuzzy logic and the classical logic

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Abstract. The main objective of this research is a simple attempt to suggest three new logical connectors and establish an equation a chart of truth for each of them. Secondly, and using the logical operations of these three connectors, we seek to show how comprehensive and widespread and effective is the Neutrosophic logic (NL) compared to any other logic, taking into account the Fuzzy Logic (FL) as well as the classical logic (CL) as a comparative model.

Keywords: Logical Connectives , Logical Operations , Truth Table , Classical Logic , Fuzzy Logic , Neutrosophic Logic.

1 Introduction:

To begin, it is known that the eight known logical connectors are nothing but conjunctive characters and tools in the natural language which are used to link between two sentences or more in order to form a meaningful speech. Also, it is obvious that by searching through the logic's history and as the specialists strived to build an artificial language that would be alternative for expressing reality more precisely, the thing that pushed them to make these characters and tools take the form of mathematical symbols used to link between two cases or more to build a compound case that can be judged to be truthful or false. But, since the day the American Philosopher C. S. Peirce (1839,1914) established the double negation logic that was named after him: Peirce's connector, we have not encountered any attempt to establish any other connector, and it has become common in the logic and mathematic media the use of these eight logic connectors only, which means that the natural language has only eight conjunctive characters and tools, but the truth is that it has more than that; there are also other conjunctive tools and characters which need to be mathematically written and symbolized. From this logic and the following neutrosophic mottos: "All is possible, the impossible too!; Nothing is perfect, not even the perfect!"[**1**], we have questioned why don't we try to write some of the other conjunctive characters and tools in the natural language mathematically in addition to the other eight known characters and tools. From that, we have attempted to create three logical connectors that we named as follows: probability connector, duplex probability connector, and the falsification connector. We have then chosen the dual-value classical logic and the fuzzy logic as comparative models. Our second aim is to attempt a research for other conjunctive characters and tools in the natural language and establishing it as symbolic logical connectors.

2 The three new logical connectors :

2.1 Probability connector (P) :

We can define the probability connector in one word: probability or maybe and that can be deduced from our saying: the professor came $\boldsymbol{x}$ and the professor's probability $\boldsymbol{y}$, or maybe the teacher $\boldsymbol{y}$, which means that the probability of the professor coming $\boldsymbol{y}$ ends as soon as the professor comes $\boldsymbol{x}$ so if the professor comes $\boldsymbol{x}$ and the teacher came $\boldsymbol{y}$ is truthful, and if the professor came $\boldsymbol{x}$ and the professor did not come $\boldsymbol{y}$ is also truthful. What matters is that the professor $\boldsymbol{x}$ came and it can be false only if the professor $\boldsymbol{x}$ does not come. Whether the professor $\boldsymbol{y}$ came or did not come, because $\boldsymbol{x}$ is what is important in this case. $\boldsymbol{x}$, however, is secondary and we can see the truth chart of this logical connector in the classical logic, the fuzzy logic and the neutrosophic logic as follows:

2.1.1 Classical Logic :

The result of the probability connector between the two classical propositions (A) and (B) :

$$CL(APB) = CL(A) = \Big(A - \big((\{1\} - B) - (\{1\} - B)\big)\Big)$$

The result of the probability connector between the two classical propositions (A) and (B) in the following truth table :

A	B	APB
1	1	1
1	0	1
0	1	0
0	0	0

2.1.2 Fuzzy Logic :

The result of the probability connector between the two fuzzy propositions (A) and (B) :

$$FL(APB) = FL(A) = \begin{pmatrix} \left(T_A - \left((\{1\} - T_B) - (\{1\} - T_B)\right)\right), \\ \left(F_A - \left((\{1\} - F_B) - (\{1\} - F_B)\right)\right) \end{pmatrix}$$

The result of the probability connector between the two fuzzy propositions (A) and (B) in the following truth table :

A	B	APB
(1,0)	(1,0)	(1,0)
(1,0)	(0,1)	(1,0)
(0,1)	(1,0)	(0,1)
(0,1)	(0,1)	(0,1)

2.1.3 Neutrosophic Logic :

The result of the probability connector between the two neutrosophic propositions (A) and (B) :

$$NL(APB) = NL(A) = \begin{pmatrix} \left(T_A \ominus \left((\{1\} \ominus T_B) \ominus (\{1\} \ominus T_B)\right)\right), \\ \left(I_A \ominus \left((\{1\} \ominus I_B) \ominus (\{1\} \ominus I_B)\right)\right), \\ \left(F_A \ominus \left((\{1\} \ominus F_B) \ominus (\{1\} \ominus F_B)\right)\right) \end{pmatrix}$$

The result of the probability connector between the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	APB
(1,0,0)	(1,0,0)	(1,0,0)
(1,0,0)	(0,0,1)	(1,0,0)
(0,0,1)	(0,1,0)	(0,0,1)
(0,0,1)	(1,0,0)	(0,0,1)
(0,1,0)	(0,0,1)	(0,1,0)
(0,1,0)	(0,1,0)	(0,1,0)

2.2 Duplex probability connector $(\boldsymbol{PP})$:

We can also refer to the duplex probability connector simply in word: probability or maybe, but this time at the beginning of the sentence, like saying: the probability that the professor $\boldsymbol{x}$ and the professor $\boldsymbol{y}$ come, or maybe the professor $\boldsymbol{x}$ and professor $\boldsymbol{y}$ come. Which means that both professor $\boldsymbol{x}$ and professor $\boldsymbol{y}$ coming is probable. So if they both come together, it is truthful and if they both don't come, it is truthful as well. But if one comes and the other does not, it is still truthful. What matters is that all expected cases of them coming together or not coming at all, or even having only one of them come are expected cases and are always truthful. We can see the truth chart of this logical connector in the classical logic, the fuzzy logic and the neutrosophic logic as follows:

2.2.1 Classical Logic :

The result of the duplex probability connector between the two classical propositions (A) and (B) :

$$CL(APPB) = \left((A + (\{1\} - A)) \times (B + (\{1\} - B))\right)$$

The result of the duplex probability connector between the two classical propositions (A) and (B) in the following truth table :

A	B	$APPB$
1	1	1
1	0	1
0	1	1
0	0	1

2.2.2 Fuzzy Logic :

The result of the duplex probability connector between the two fuzzy propositions (A) and (B) :

$$FL(APPB) = \begin{pmatrix} \left((T_A + (\{1\} - T_A)) \times (T_B + (\{1\} - T_B))\right), \\ \left((F_A + (\{1\} - F_A)) \times (F_B + (\{1\} - F_B))\right) \end{pmatrix}$$

The result of the duplex probability connector between the two fuzzy propositions (A) and (B) in the following truth table :

A	B	$APPB$
(1,0)	(1,0)	(1,1)
(1,0)	(0,1)	(1,1)
(0,1)	(1,0)	(1,1)
(0,1)	(0,1)	(1,1)

2.2.3 Neutrosophic Logic :

The result of the duplex probability connector between the two neutrosophic propositions (A) and (B) :

$$NL(APPB) = \begin{pmatrix} \left((T_A \oplus (\{1\} \ominus T_A)) \odot (T_B \oplus (\{1\} \ominus T_B))\right), \\ \left((I_A \oplus (\{1\} \ominus I_A)) \odot (I_B \oplus (\{1\} \ominus I_B))\right), \\ \left((F_A \oplus (\{1\} \ominus F_A)) \odot (F_B \oplus (\{1\} \ominus F_B))\right) \end{pmatrix}$$

The result of the duplex probability connector between the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$APPB$
(1,0,0)	(1,0,0)	(1,1,1)
(1,0,0)	(0,0,1)	(1,1,1)
(0,0,1)	(0,1,0)	(1,1,1)
(0,0,1)	(1,0,0)	(1,1,1)
(0,1,0)	(0,0,1)	(1,1,1)
(0,1,0)	(0,1,0)	(1,1,1)

2.3 Falsification connector ($\mathbf{0}$) :

In fact, the falsification connector is simply like us saying: I do not believe in Quantum physics or relative physics, or saying: I totally disapprove of science's results or the philosophical ones, and more precisely, this connector is what is approved of like the right to veto in the United States, i.e. the right to disapprove or falsify any case no matter how truthful or false it is and we can see that in the truth chart of this in the classical logic, the fuzzy logic and the neutrosophic logic as follows:

2.3.1 Classical Logic :

The result of the falsification connector between the two classical propositions (A) and (B) :

$$CL(A0B) = (|A - (\{1\} - A)| - |B - (\{1\} - B)|)$$

The result of the falsification connector between the two classical propositions (A) and (B) in the following truth table :

A	B	$A0B$
1	1	0
1	0	0
0	1	0
0	0	0

2.3.2 Fuzzy Logic :

The result of the falsification connector between the two fuzzy propositions (A) and (B) :

$$FL(A0B) = \begin{pmatrix} |T_A - (\{1\} - T_A)| - |T_B - (\{1\} - T_B)|, \\ |F_A - (\{1\} - F_A)| - |F_B - (\{1\} - F_B)| \end{pmatrix}$$

The result of the falsification connector between the two fuzzy propositions (A) and (B) in the following truth table :

A	B	$A0B$
(1,0)	(1,0)	(0,0)
(1,0)	(0,1)	(0,0)
(0,1)	(1,0)	(0,0)
(0,1)	(0,1)	(0,0)

2.3.3 Neutrosophic Logic :

The result of the falsification connector between the two neutrosophic propositions (A) and (B) :

$$NL(A0B) = \begin{pmatrix} |T_A \ominus (\{1\} \ominus T_A)| \ominus |T_B \ominus (\{1\} \ominus T_B)|, \\ |I_A \ominus (\{1\} \ominus I_A)| \ominus |I_B \ominus (\{1\} \ominus I_B)|, \\ |F_A \ominus (\{1\} \ominus F_A)| \ominus |F_B \ominus (\{1\} \ominus F_B)| \end{pmatrix}$$

The result of the falsification connector between the two neutrosophic propositions (A) and (B) in the following truth table :

A	B	$A0B$
(1,0,0)	(1,0,0)	(0,0,0)
(1,0,0)	(0,0,1)	(0,0,0)
(0,0,1)	(0,1,0)	(0,0,0)
(0,0,1)	(1,0,0)	(0,0,0)
(0,1,0)	(0,0,1)	(0,0,0)
(0,1,0)	(0,1,0)	(0,0,0)

3 Conclusion :

From what has been discussed previously, we can ultimately reach two points:

3.1 We see that the logical operations of the neutrosophic logic (NL) are different from the logical operations of the fuzzy logic (FL) in terms of width, comprehensiveness and effectiveness. The reason behind that is the addition of professor Florentine Samarkendah of a new field to the real values; the truth and falsity interval in (FL) and that is what he called "the indeterminacy interval" which is expressed in the function I_A or I_B in the logical operations of: (NL) as we have seen, and that is what makes (NL) gives the closest and most precise image of the hidden logical structure of the universe like it was mentioned previously.

3.2 We see from our attempt to create three new logical connectors starting from the idea that the natural language has more than eight connecting characters and tools that need to be written in the form of symbols, that the difference in natural languages means a difference and an availability of connecting characters and tools. Consequently, we should not quote connecting characters or tools from a single language like French or English, but we should take all the languages into consideration. For example: the Chinese language has 47035 characters and that number keeps increasing. So, the best decision is to collect different connecting characters and tools from the different international natural languages and give these connectors a form of symbols. Only then will the artificial language evolve progressively compared to how it is today.

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Bézier Surface Modeling for Neutrosophic Data Problems

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Abstract. The main goal of this paper is to construct Bézier surface modeling for neutrosophic data problems. We show how to build the surface model over a data sample from agriculture science after the theoretical structure of the modeling is introduced. As a sampler application for agriculture systems, we give a visualization of Bézier surface model of an estimation of a given yield of bean seeds grown in a field over a period.

Keywords: Neutrosophic logic, neutrosophic data, neutrosophic geometry, Bézier surface, geometric design

1 Introduction

The contribution of mathematical researches is fundamental and leading the science as today's technologies are rapidly developing. The geometrical improvements both model the mathematics of the objects and become geometrically most abstract concepts. In the future of science will be around the artificial intelligence. For the development of this technology, many branches of science work together and especially the topics such as logic, data mining, quantum physics, machine learning come to the forefront. Of course, the common place where these areas can cooperate is the computer environment. Data can be transferred in several ways. One of them is to transfer the data as a geometric model. The first method that comes to mind in terms of a geometric model is the Bézier technique. This method is generally used for curve and surface designs. In addition to this, it is used in many disciplines ranging from the solution of differential equations to robot motion planning.

The concretization state of obtaining meaning and mathematical results from uncertainty states (fuzzy) was introduced by Zadeh [1]. Fuzzy sets proposed by Zadeh provided a new dimension to the concept of classical sets. Atanassov introduced intuitionistic fuzzy sets dealing with membership and non-membership degrees [2]. Smarandache proposed neutrosophy as a mathematical application of the concept neutrality [3]. Neutrosophic set concept is defined with membership, non-membership and indeterminacy degrees. Neutrosophic set concept is separated from intuitionistic fuzzy set by the difference as follow: intuitionistic fuzzy sets are defined by degree of membership and non-membership degree and, uncertainty degrees by the 1- (membership degree plus non-membership degree), while degree of uncertainty is considered independently of the degree of membership and non-membership in neutrosophic sets. Here, membership, non-membership, and uncertainty (indeterminacy) degrees can be evaluated according to the interpretation in the spaces to be used, such as truth and falsity degrees. It depends entirely on subject or topic space (discourse universe). In this sense, the concept of neutrosophic set is the solution and representation of the problems with various fields.

The paths of logic and geometry sometimes intersect and sometimes separate but both deal with information. Logic is related to information about the truth of statements, and geometry deals with information about location and visualization. Classical truth considers false and true, 0 and 1. It's geometrical interpretation with boolean connectives was represented as a boolean lattice by Miller [4-5]. Futhermore, a more geometrical representation was given by the 16 elements of the affine 4-space A over the two-element Galois field GF(2) [6] as can be seen in Figure 1. The affine space is created by 0,1 and 16 operators.

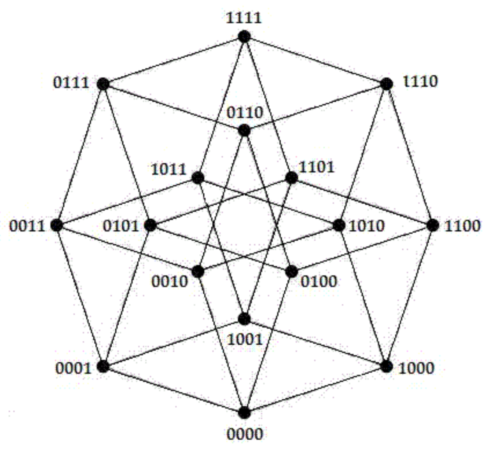

Figure 1. 16 elements of the affine 4-space A over the two-element Galois field GF(2).

Neutrosophic data has become an important instance of the expression "*think outside the box*" that goes beyond classical knowledge, accuracy and truth. Geometric approach to neutrosophic data which involve truth, falsity and indeterminacy values between the interval [0,1] provide rich mathematical structures. This paper presents an initial geometrical interpretation of neutrosophy theory.

Recently, geometric interpretations of data that have uncertain truth have presented by Wahab and friends [7-10]. They studied geometric models of fuzzy and intuitionistic fuzzy data and gave fuzzy interpolation and Bézier curve modeling. The authors of this paper presented Bézier curve modeling of neutrosophic data [11]. In this paper, we consider Bézier surface modeling of neutrosophic data problems and applications in real life.

2. Preliminaries

In this section, we first give some fundamental definitions dealing with Bézier curve and neutrosophic sets (elements). We then introduce new definitions needed to form a neutrosophic Bézier surface.

Definition 1. Let $\mathrm{P}_i, i = 0 \dots n$ are the set of points in 3-dimensional Euclidean space. Then the Bézier curve with degree n is defined by

$$\mathrm{B}(t) = \sum_{i=0}^{n} \binom{n}{i} (1-t)^{n-i} t^i \, \mathrm{P}_i, \qquad t \in [0,1].$$

where $\binom{n}{i} = \frac{n!}{(n-i)!i!}$ and the points P_i are the control points of this Bézier curve.

Definition 2. Let $\mathrm{P}_{ij}, i = 0 \dots n, j = 0 \dots m,$ are the set of points in 3-dimensional Euclidean space. Then the Bézier surface with degree nxm is defined by

$$\mathrm{B}(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} \binom{n}{i} (1-u)^{n-i} u^i \binom{m}{j} (1-v)^{m-j} v^i \mathrm{P}_{ij}, \qquad t \in [0,1].$$

where the points P_{ij} are the control points of this Bézier surface. The First-degree interpolation of these points forms a mesh and called the control polyhedron. These types of surfaces are called tensor product surfaces too. Therefore, one can show the matrix representation of a Bézier surface as

$$\mathrm{B}(u,v) = [(1-u)^n \;\; mu(1-u)^{n-1} \; \dots \; u^n][\mathrm{P}_{ij}] \begin{bmatrix} (1-v)^m \\ nv(1-u)^{m-1} \\ \vdots \\ v^m \end{bmatrix}.$$

Definition 3. Let E be a universe and $A \subseteq E$. $N = \{(x, T(x), I(x), F(x)) : x \in A\}$ is a neutrosophic element where $T_p : N \to [0,1]$ (membership function), $I_p : N \to [0,1]$ (indeterminacy function) and $F_p : N \to [0,1]$ (non-membership function).

Definition 4. Let $A^* = \{(x, T(x), I(x), F(x)): x \in A\}$ and $B^* = \{(y, T(y), I(y), F(y)): y \in B\}$

be neutrosophic elements.

$$NR = \{((x, y), T(x, y), I(x, \imath), F(x, y)): x \in A, y \in B\}$$

is a neutrosophic relation on A^* and B^*.

2.1. Neutrosophic Bézier Model

Definition 5. Neutrosophic set of P^* in space N is *NCP* (neutrosophic control point) and $P^* = \{P_i^*\}$ where $i = 0, \ldots, n$ is a set of NCPs where there exists $T_p: N \to [0, 1]$ as membership function, $I_p: N \to [0,1]$ as indeterminacy function and $F_p: N \to [0, 1]$ as non-membership function with

$$T_p(P^*) = \begin{cases} 0 & if\ P_i \notin N \\ a \in (0, 1) & if\ P_i \tilde{\in} N, \\ 1 & if\ P_i \in N \end{cases}$$

$$I_p(P^*) = \begin{cases} 0 & if\ P_i \notin N \\ b \in (0, 1) & if\ P_i \tilde{\in} N \\ 1 & if\ P_i \in N \end{cases}$$

$$F_p(P^*) = \begin{cases} 0 & if\ P_i \notin N \\ c \in (0, 1) & if\ P_i \tilde{\in} N \\ 1 & if\ P_i \in N \end{cases}$$

Definition 6. A neutrosophic Bezier curve with degree n was defined by Taş and Topal [11].

$$NB(t) = \sum_{i=0}^{n} \binom{n}{i} (1 - t)^{n-i} t^i \, NR_{p_i}, t \in [0, 1]$$

One can see there are three Bezier curves (Fig 1). The *ith* (i=0…n) control points of these curves are on the same straight line. Line geometry shows us that if we interpolate these straight lines then we get a developable (cylindrical) ruled surface. Therefore, these curves belong to a developable ruled surface that is a surface that can be transformed to a plane without tearing or stretching (Figure 2). As a result, we can say that a neutrosophic Bezier curve corresponds to a cylindrical ruled surface.

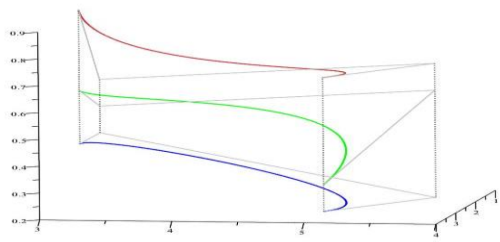

Figure 2. Neutrosophic Bézier curve: membership (green curve), non- membership (orange curve), and indeterminacy (blue curve).

Definition 7. Neutrosophic Bézier surfaces are generated by the control points from one of

$TC = \{(x, y, T(x, y)): x \in A, y \in B\}$,

$IC = \{(x, y, I(x, y)): x \in A, y \in B\}$,

$FC = \{(x, y, F(x, y)): x \in A, y \in B\}$ sets. Thus, there will be three different Bézier surface models for a neutrosophic relation and variables x and y. A neutrosophic control point relation can be defined as a set of $(n$+1$)(m$+1) points that shows a position and coordinate of a location and is used to describe three surface which are denoted by

$$NR_{P_{ij}} = \{NR_{P_{00}}, NR_{P_{01}}, \ldots, NR_{P_{nm}}\}$$

and can be written as quadruples

$$\{((x_i, y_j), T(x_i, y_j), I(x_i, y_j), F(x_i, y_j)): i = 0, \ldots, n, j = 0, \ldots, m\}$$

in order to control the shape of a curve from a neutrosophic data.

Definition 7. A neutrosophic Bézier surface with degree $n\ x\ m$ is defined by

$$\mathbf{NB}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}\binom{n}{i}(1-u)^{n-i}u^{i}\binom{m}{j}(1-v)^{m-j}v^{j}\,\mathbf{NR}_{p_{ij}}.$$

Every set of $\boldsymbol{TC}$, $\boldsymbol{IC}$ and $\boldsymbol{FC}$ determines a Bézier surface. Thus, we obtain three Bézier surfaces. A neutrosophic Bézier surface is defined by these three surfaces. So it is a set of surfaces as in its definition.

As an illustrative example, we can consider a neutrosophic data in Table 1. One can see there are three Bézier surfaces.

Example 1. Suppose that a field is a subset of two-dimensional space. By choosing a starting point (origin point) we seed certain point bean seeds. Depending on the reasons such as irrigation, rocky soil and so on, this is an estimate of the length of time that these seeds will arrive after a certain period of time. For example, we estimate each of the bean poles to reach 100 cm in length (Table 1). So we are trying to predict which parts of the land are more productive without planting yet. A yield map of the field with the data presented is obtained from the surface map of the plant.

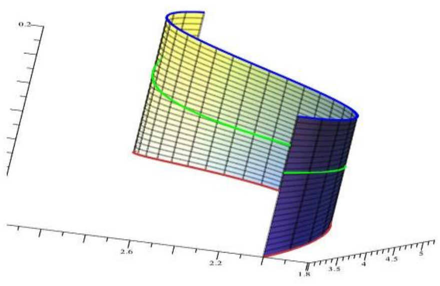

Figure 3. Neutrosophic Bézier curve and cylindrical ruled surface.

Table 1. Neutrosophic data

Bean seeds in coordinate system	Truth	Indeterminacy	Falsity
$\mathbf{P}_{00}$=(1,1)	0.53	0.45	0.56
$\mathbf{P}_{01}$=(1,2)	0.53	0.5	0.6
$\mathbf{P}_{02}$=(1,3)	0.45	0.65	0.72
$\mathbf{P}_{03}$=(1,4)	0.3	0.24	0.9
$\mathbf{P}_{10}$=(1,5)	0.72	0.5	0.6
$\mathbf{P}_{11}$=(1,6)	0.5	0.4	0.5
$\mathbf{P}_{12}$=(1,7)	0.25	0.6	0.19
$\mathbf{P}_{13}$=(1,8)	0.42	0.6	0.7
$\mathbf{P}_{20}$=(2,1)	0.91	0.33	0.4
$\mathbf{P}_{21}$=(2,2)	0.7	0.59	0.6
$\mathbf{P}_{22}$=(2,3)	0.53	0.45	0.5
$\mathbf{P}_{23}$=(2,4)	0.28	0.55	0.67
$\mathbf{P}_{30}$=(2,5)	0.43	0.65	0.7
$\mathbf{P}_{31}$=(2,6)	0.32	0.25	0.9
$\mathbf{P}_{32}$=(2,7)	0.7	0.54	0.6
$\mathbf{P}_{33}$=(2,8)	0.35	0.66	0.12

Neutrosophic Bézier surface of data in Table 1 can be illustrated in Figure 4. The surface can be turned to neutrosophic data because these surfaces are connected to the control points.

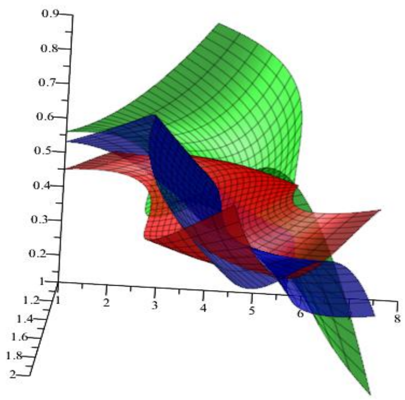

Figure 4. Neutrosophic Bézier surface according to data in Table 1.

3. Conclusions

Visualization or geometric modeling of data plays a significant role in data mining, databases, stock market, economy, stochastic processes and engineering. In this article, we have used a strong tool, the Bézier surface technique for visualizing neutrosophic data which belongs to agriculture systems. This surface model also is appropriate for statisticians, data scientists, economists and engineers. Furthermore, the differential geometric properties of this model can be investigated for classification of neutrosophic data. On the other hand, transforming the images of objects into neutrosophic data is an important problem [12]. In our model, the surface and the data can be transformed into each other by the blossoming method, which can be used in neutrosophic image processing. This and similar applications should be studied in the future.

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Neutrosophic EOQ Model with Price Break

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Abstract—Inventory control of an ideal resource is the most important one which fulfils various activities (functions) of an organisation. The supplier gives the discount for an item in the cost of units inorder to motivate the buyers (or) customers to purchase the large quantity of that item. These discounts take the form of price breaks where purchase cost is assumed to be constant. In this paper an EOQ model with price break in inventory model is developed to obtain its optimum solution by assuming neutrosophic demand and neutrosophic purchasing cost as triangular neutrosophic numbers. A numerical example is provided to illustrate the proposed model.

Keywords: Price break, neutrosophic demand, neutrosophic purchase cost, neutrosophic sets, triangular neutrosophic number.

1 Introduction

Bai and Li[1] have discussed triangular and trapezoidal fuzzy numbers in inventory model for determining the optimal order quantity and the optimal cost. The quantity discount problem has been analyzed from a buyers perspective. Hadley and Whintin[2], Peterson and Silver[3], and Starr and Miller[6] considered various discount polices and demand assumptions.

Yang and Wee[7] developed an economic ordering policy in the view of both the supplier and the buyer. Prabjot Kaur and Mahuya Deb[5] developed an intuitionistic approach for price breaks in EOQ from buyer's perspective. Smarandache[5] introduced neutrosophic set and neutrosophic logic by considering the non-standard analysis. Also, neutrosophic inventory model without shortages is introduced by M. Mullai and S. Broumi[3].

In this paper, we introduce the neutrosophic inventory models with neutrosophic price break to find the optimal solution of the model for the optimal order quantity. Also the neutrosophic inventory model under neutrosophic demand and neutrosophic purchasing cost at which the quantity discount are offered to be triangular neutrosophic number. Also the optimal order quantity for the neutrosophic total cost is determined by defining the accuracy function of triangular neutrosophic numbers.

2 Notations:

Q^N = Number of pieces per order

C_0^N = Neutrosophic Ordering cost for each order

C$_h^N$ = Neutrosophic Holding cost per unit per year

D^N = Neutrosophic Annual demand in units

3 Neutrosophic EOQ Model With Price Break:

The Neutrosophic inventory model with neutrosophic price break is introduced to find the optimal solutions for the optimal neutrosophic order quantity. Here we assume that there is no stock outs, no backlogs, replenishment is instantaneous, the neutrosophic ordering cost involved to receive an order are known and constant and purchasing values at which discounts are offered as triangular neutrosophic numbers.
Consider the following variables:
D^N: Neutrosophic yearly demand,

P^N: Neutrosophic purchasing cost

Let $D^N = (D_1^N, D_2^N, D_3^N)\ (D_1'^N, D_2^N, D_3'^N)\ (D_1''^N, D_2^N, D_3''^N)$

$P^N = (P_1^N, P_2^N, P_3^N)\ (P_1'^N, P_2^N, P_3'^N)\ (P_1''^N, P_2^N, P_3''^N)$

$P_1^N = (P_{11}^N, P_{12}^N, P_{13}^N)\ (P_{11}'^N, P_{12}^N, P_{13}'^N)\ (P_{11}''^N, P_{12}^N, P_{13}''^N)$

$P_2^N = (P_{21}^N, P_{22}^N, P_{23}^N)\ (P_{21}'^N, P_{22}^N, P_{23}'^N)\ (P_{21}''^N, P_{22}^N, P_{23}''^N)$

are non negative triangular neutrosophic numbers.

Now, we introduce the neutrosophic inventory model under neutrosophic demand and neutrosophic purchasing cost at which the quantity discounts are offered. Total neutrosophic inventory cost is given by

$$(TC)^N = D^N \otimes P^N \oplus \frac{D^N C_0^N}{Q^N} \oplus \frac{Q^N P^N \otimes I^N}{2}$$

Then the total neutrosophic inventory cost is

$$(TC)^N = \left(D_1^N P_1^N + \frac{D_1^N C_0^N}{Q^N} + \frac{Q^N P_1^N I^N}{2}, D_2^N P_2^N + \frac{D_2^N C_0^N}{Q^N} + \frac{Q^N P_2^N I^N}{2}, D_3^N P_3^N + \frac{D_3^N C_0^N}{Q^N} + \frac{Q^N P_3^N I^N}{2}\right)\left(D_1'^N P_1'^N + \frac{D_1'^N C_0^N}{Q^N} + \frac{Q^N P_1'^N I^N}{2}, D_2^N P_2^N + \frac{D_2^N C_0^N}{Q^N} + \frac{Q^N P_2^N I^N}{2}, D_3'^N P_3'^N + \frac{D_3'^N C_0^N}{Q^N} + \frac{Q^N P_3'^N I^N}{2}\right)\left(D_1''^N P_1''^N + \frac{D_1''^N C_0^N}{Q^N} + \frac{Q^N P_1''^N I^N}{2}, D_2^N P_2^N + \frac{D_2^N C_0^N}{Q^N} + \frac{Q^N P_2^N I^N}{2}, D_3''^N P_3''^N + \frac{D_3''^N C_0^N}{Q^N} + \frac{Q^N P_3''^N I^N}{2}\right)$$

The defuzzified total neutrosophic cost using accuracy function is given by

$$D(TC)^N = \frac{1}{8}\left[\left(D_1^N P_1^N + \frac{D_1^N C_0^N}{Q^N} + \frac{Q^N P_1^N I^N}{2}\right) + 2\left(D_2^N P_2^N + \frac{D_2^N C_0^N}{Q^N} + \frac{Q^N P_2^N I^N}{2}\right) + \left(D_3^N P_3^N + \frac{D_3^N C_0^N}{Q^N} + \frac{Q^N P_3^N I^N}{2}\right) + \left(D_1''^N P_1''^N + \frac{D_1''^N C_0^N}{Q^N} + \frac{Q^N P_1''^N I^N}{2}\right) + 2\left(D_2^N P_2^N + \frac{D_2^N C_0^N}{Q^N} + \frac{Q^N P_2^N I^N}{2}\right) + \left(D_3''^N P_3''^N + \frac{D_3''^N C_0^N}{Q^N} + \frac{Q^N P_3''^N I^N}{2}\right)\right]$$

To find the minimum of D(TC)^N by taking the derivative D(TC)^N and equating it to zero,

(i.e) $\frac{1}{8Q^{2N}}[(D_1^N C_0^N + 2D_2^N C_0^N + D_3^N C_0^N) + (D_1''^N C_0^N + 2D_2^N C_0^N + D_3''^N C_0^N)] + \frac{1}{16}[(P_1^N I^N + 2P_2^N I^N + P_3^N I^N) + (P_1''^N I^N + 2P_2^N I^N + P_3''^N I^N)] = 0$, we get

$$Q^N = \sqrt{\frac{2[(D_1^N C_0^N + 2D_2^N C_0^N + D_3^N C_0^N) + (D_1''^N C_0^N + 2D_2^N C_0^N + D_3''^N C_0^N)]}{[(P_1^N I^N + 2P_2^N I^N + P_3^N I^N) + (P_1''^N I^N + 2P_2^N I^N + P_3''^N I^N)]}}$$

Neutrosophic Price Break:

S.No.	Quantity	Price Per Unit (Rs)
1	0$\leq Q_1^N \leq b$	P_1^N
2	b $\leq Q_2^N$	$P_2^N (< P_1^N)$

4 Algorithm For Finding Neutrosophic Optimal Quantity and Neutrosophic Optimal Cost:

Step I:
Consider the lowest price P_2^N and determine Q_2^N by using the economic order quantity (EOQ) formula:

$$Q^N = \sqrt{\frac{2[(D_1^N C_0^N + 2D_2^N C_0^N + D_3^N C_0^N) + (D_1''^N C_0^N + 2D_2^N C_0^N + D_3''^N C_0^N)]}{[(P_1^N I^N + 2P_2^N I^N + P_3^N I^N) + (P_1''^N I^N + 2P_2^N I^N + P_3''^N I^N)]}}$$

If Q_2^N lies in the range specified, $b \geq Q_2^N$ then Q_2^N is the EOQ .The defuzzified optimal total cost $(TC)^N$ associated with Q^N is calculated as follows:

(TC)$^N = D^N * P_2^N + \frac{D^N C_0^N}{b} + \frac{b P_2^N * I^N}{2}$, by using the accuracy function $A^N = \frac{(a_1 + 2a_2 + a_3) + (a_1'' + 2a_2 + a_3'')}{8}$

Step 2:

(i) If $Q_2^N < b$, we cannot place an order at the lowest price P_2^N.
(ii) We calculate Q_1^N with price P_1^N and the corresponding total cost TC at Q^N.
(iii) If $(TC)^N b > (TC)^N Q_1^N$, then EOQ is $Q^{*N} = Q_1^N$, Otherwise $Q^{*N} = b$ is the required EOQ.

The EOQ in crisp, fuzzy and intuitionistic fuzzy sets are discussed detail in [5]. They are

(i) Crisp:

$$Q_2^* = \sqrt{\frac{2DC_0}{P_2 I}}$$

(ii) Fuzzy:

$$\widetilde{Q}_2^* = \sqrt{\frac{2(D_1C_0+2D_2C_0+D_3C_0)}{P_1I+2P_2I+P_3I}}$$

(iii) Intuitionistic fuzzy:

$$\overline{\overline{Q}}_2^* = \sqrt{\frac{2(D_1C_0+4D_2C_0+D_3C_0+D_1'C_0+D_3'C_0)}{P_1I+4P_2I+P_3I+P_1'I+P_3'I}}$$

Using these formula, the numerical example for neutrosophic set is illustrated as follows.

5 NUMERICAL EXAMPLE:

A manufacturing company issues the supply of a special component which has the following price schedule:

0 to 99 items: Rs.800 per unit

100 items and above: Rs.600 per unit

The inventory holding costs are estimated to be Rs.30/- of the value of the inventory. The procurement ordering costs are estimated to be Rs.1500 per order. If the annual requirement of the special component is 350, then compute the economic order quantity for the procurement of these items.

Solution:

(i) **Crisp Case:**

Given D = 350, P_1 = Rs.800, P_2 = Rs.600, C_0 = Rs.1500, I = 0.3

Q_2^* = 76

TC$(P_1 = 800)$ = Rs.296039

TC(b=100) = Rs.224250, which is lower than the total cost corresponding to Q_2

(ii) **Fuzzy Case:**

Given $\widetilde{D}$ = (300, 350, 400), $\widetilde{P}_1$ = (750, 800, 850)

$\widetilde{P}_2$ = (550, 600, 650), C_0 = Rs.1500, I = 0.3

$\widetilde{Q}_2^*$ = 88.192

$\widetilde{TC}(P_1 = 800)$ = Rs.297785.95

$\widetilde{TC}$(b=100) = Rs.225500, which is lower than the total cost corresponding to Q_2.

(iii) **Intuitionistic Fuzzy Case:**

Given $\overline{\overline{D}}$ = (300, 350, 400) (250, 350, 450)

$\overline{\overline{P}}_1$ = (750, 800, 850) (700, 800, 900)

$\overline{\overline{P}}_2$ = (550, 600, 650) (500, 600, 700),
C_0 = Rs.1500, I = 0.3

$\overline{\overline{Q}}_2^*$ = 88.19

$\overline{\overline{TC}}(P_1 = 800)$ = Rs.299660.85

$\overline{\overline{TC}}(b = 100)$ = Rs.227375, which is lower than the total cost corresponding to Q_2.

(iv) **Neutrosophic Case:**

Given D^N = (300, 350, 400) (250, 350, 450) (150, 350, 550)

P_1^N = (750, 800, 850) (700, 800, 900)(600, 800, 1000)

P_2^N = (550, 600, 650) (500, 600, 700)(400, 600, 800)

C_0^N = Rs.1500, I^N = 0.3

We calculate ${Q_2^*}^N$ corresponding to the lowest price 600,

$${Q_2^*}^N = \sqrt{\frac{2[(D_1^N C_0^N + 2D_2^N C_0^N + D_3^N C_0^N) + (D_1''^N C_0^N + 2D_2^N C_0^N + D_3''^N C_0^N)]}{[(P_1^N I^N + 2P_2^N I^N + P_3^N I^N) + (P_1''^N I^N + 2P_2^N I^N + P_3''^N I^N)]}}$$

= 76.376, which is less than the price break point.

Therefore, we have to determine the optimal total cost for the first price and the total cost at the price- break corresponding to the second price and compare the two.

The defuzzified optimal total cost $(TC)^N$ associated with P_1^N is calculated as follows:

$$(TC)^N(P_1^N = 800) = D^N * P_1^N + \frac{D^N C_0^N}{Q_2^N} + \frac{Q_2^N P_1^N * I^N}{2}$$
$$= Rs.306664.13$$

$$(TC)^N(b = 100) = D^N * P_2^N + \frac{D^N C_0^N}{b} + \frac{bP_2^N * I^N}{2}$$
$$= Rs.173812.5$$

which is lower than the total cost corresponding to Q_2^N.

6 Sensitivity Analysis

In this section, the analysis between intuitionistic set and neutrosophic set is tabulated and the results are compared graphically.

S.No.	Intuitionistic Demand	Neutrosophic Demand
1	(270,320,370) (220,320,420)	(270,320,370) (220,320,420) (120,320,520)
2	(280,330,380) (230,330,430)	(280,330,380) (230,330,430) (130,330,530)
3	(300,350,400) (250,350,450)	(300,350,400) (250,350,450) (150,350,550)
4	(320,370,420) (270,370,470)	(320,370,420) (270,370,470) (170,370,570)
5	(330,380,430) (280,380,480)	(330,380,430) (280,380,480) (180,380,580)

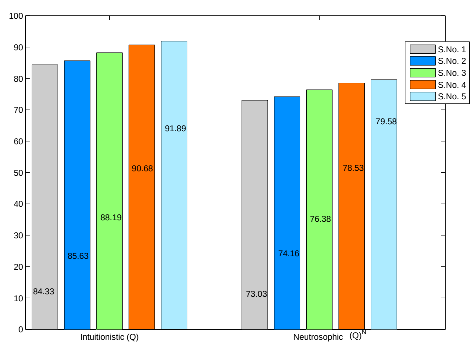

Figure 1. Analysis of economic order quantity (EOQ) between intuitionistic fuzzy set and neutrosophic set

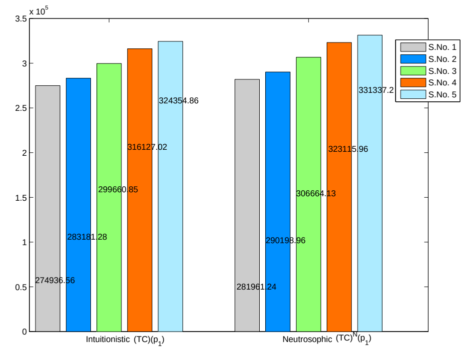

Figure 2. Analysis of first price between intuitionistic fuzzy set and neutrosophic set

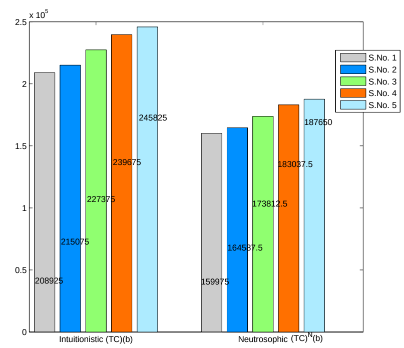

Figure 3. Analysis of price break corresponding to second price between intuitionistic fuzzy set and neutrosophic set

Conclusion

In this paper, EOQ model with price break in neutrosophic environment is introduced. An inventory model is developed for price breaks and its optimum solution is obtained by using triangular neutrosophic number. An algorithm for solving neutrosophic optimal quantity and neutrosophic optimal cost is also developed. This will be an advantage for the buyer who can easily decrease the bad cases and increase the better ones. Hence, the neutrosophic set gives the better solutions to the real world problems than fuzzy and intuitionistic fuzzy sets. In future, the various neutrosophic inventory models will be developed with various limitations such as lead time, backlogging, back order and deteriorating items, etc.

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TOPSIS Strategy for Multi-Attribute Decision Making with Trapezoidal Neutrosophic Numbers

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Abstract. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a popular strategy for Multi-Attribute Decision Making (MADM). In this paper, we extend the TOPSIS strategy of MADM problems in trapezoidal neutrosophic number environment. The attribute values are expressed in terms of single-valued trapezoidal neutrosophic numbers. The weight information of attribute is incompletely known or completely unknown. Using the maximum deviation strategy, we develop an optimization model to obtain the weight of the attributes. Then we develop an extended TOPSIS strategy to deal with MADM with single-valued trapezoidal neutrosophic numbers. To illustrate and validate the proposed TOPSIS strategy, we provide a numerical example of MADM problem.

Keywords: Single-valued trapezoidal neutrosophic number, multi-attribute decision making, TOPSIS.

1 Introduction

Multi-attribute decision making (MADM) plays an important role in decision making sciences. MADM is a process of finding the best alternative that has the highest degree of satisfaction over the predefined conflicting attributes. The preference values of alternatives are generally assessed quantitatively and qualitatively according to the nature of attributes. When the preference values are imprecise, indeterminate or incomplete, the decision maker feels comfort to evaluate the alternatives in MADM in terms of fuzzy sets [1], intuitionistic fuzzy sets [2], hesitant fuzzy sets [3], neutrosophic sets [4], etc., rather than crisp sets. A large number of strategies has been developed for MADM problems such as technique for order preference by similarity to ideal solution (TOPSIS) [5], PROMETHEE [6], VIKOR [7], ELECTRE [7, 8], AHP [9], etc. MADM problem has been studied extensively in fuzzy environment [10-14], intuitionistic fuzzy environment [15-22].

TOPSIS [5] is one of the sophisticated strategy for solving MADM. The main idea of TOPSIS is that the best alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS), simultaneously. Since its proposition, researchers have extended the TOPSIS strategy to deal with different environment. Chen [23] extended the TOPSIS strategy for solving multi-criteria decision making (MCDM) problems in fuzzy environment. Boran et al. [24] extended the TOPSIS strategy for MCDM problem in intuitionistic fuzzy environment. Zhao [25] also studied TOPSIS strategy for MADM under interval intuitionistic fuzzy environment and utilized the strategy in teaching quality evaluation. Xu [19] proposed TOPSIS strategy for hesitant fuzzy multi-attribute decision making with incomplete weight information.

However fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets have some limitations to express indeterminate and incomplete information in decision making process. Recently, single valued neutrosophic set (SVNS) [26] has been successfully applied in MADM or multi-attribute group decision [27-37]. SVNS [26] and interval neutrosophic set (INS) [38], and other hybrid neutrosophic sets have caught attention of the researchers for developing TOPSIS strategy. Biswas et al. [39] developed TOPSIS strategy for multi-attribute group decision making (MAGDM) for single valued neutrosophic environment. Sahin et al. [40] proposed another TOPSIS strategy for supplier selection in neutrosophic environment. Chi and Liu developed TOPSIS strategy to deal with interval neutrosophic sets in MADM problems. Zhang and Wu [41] proposed TOPSIS strategies for MCDM in single valued neutrosophic environment and interval neutrosophic set environment where the information about criterion weights are incompletely known or completely unknown. Ye [42] put forward TOPSIS strategy for MAGDM with single-valued neutrosophic linguistic numbers. Peng et al. [43] presented multi-attributive border approximation area comparison (MBAC), TOPSIS, and similarity measure approaches for neutrosophic MADM. Pramanik et al. [44] extended TOPSIS strategy for MADM in neutrosophic soft expert set environment. Different TOPSIS strategies [45-49] have been studied in different hybrid neutrosophic set environment.

Single valued trapezoidal neutrosophic number (SVTrNN) [50, 51] is another extension of single-valued neutrosophic sets. SVTrNN presents the situation, in which each element is characterized by trapezoidal number that has truth membership degree, indeterminate membership degree, and falsity membership degree. Recently, Deli and Şubaş [52] proposed a ranking strategy of single valued neutrosophic number and utilized this strategy in MADM problems. Biswas et al. [53] also proposed value and ambiguity based ranking strategy of single valued trapezoidal neutrosophic number and applied it to MADM.

However, TOPSIS strategy of MADM has not been studied earlier with trapezoidal neutrosophic numbers, although these numbers effectively deal with uncertain information in MADM model. In this study, our objective is to develop an MADM model, where the attribute values assume the form of SVTrNNs and the weight information of attribute is incompletely known or completely unknown. The existing TOPSIS strategy of MADM cannot handle with such situations. Therefore, we need to extend the TOPSIS strategy in SVTrNN environment.

To develop the model, we consider the following sections: Section 2 presents a preliminaries of fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, and single-valued trapezoidal neutrosophic number IFS, SVNS. Section 3 contains the extended TOPSIS strategy for MADM with SVTrNNs. Section 4 presents an illustrative example. Finally, Section 5 presents conclusion and future direction research.

2 Preliminaries

In this section, we review some basic definitions of fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, and single-valued trapezoidal neutrosophic number.

Definition 1. [1] Let X be a universe of discourse, then a fuzzy set A is defined by

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\} \quad (1)$$

which is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ is the degree of membership of the element x to the set A.

Definition 2. [54,55] A generalized trapezoidal fuzzy number A denoted by $A = (a, b, c, d; w)$ is described as a fuzzy subset of the real number $\mathbb{R}$ with membership function μ_A which is defined by

$$\mu_A(x) = \begin{cases} \dfrac{x-a)w}{b-a} & a \le x < b, \\ w & b \le x \le c, \\ \dfrac{(d-x)w}{d-c} & c < x \le d, \\ 0 & otherwise \end{cases}$$

where a, b, c, d are real number satisfying $a \le b \le c \le d$ and w is the membership degree.

Definition 3.[4] Let X be a universe of discourse. An neutrosophic sets A over X is defined by

$$A = \{x, < T_A(x), I_A(x), F_A(x) > \mid x \in X\} \quad (2)$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0,1^+[$ that is $T_A(\mathrm{x}) : \mathrm{X} \to]^-0,1^+[$, $I_A(\mathrm{x}) : \mathrm{X} \to]^-0,1^+[$ and $F_A(\mathrm{x}) : \mathrm{X} \to]^-0,1^+[$. The membership functions satisfy the following properties:

$^-0 \le T_A(\mathrm{x}) + I_A(\mathrm{x}) + F_A(\mathrm{x}) \le 3^+$.

Definition 4. [26] Let X be a universe of discourse. A single-valued neutrosophic set $\tilde{A}$ in X is given by

$$A = \{x, < T_A(x), I_A(x), F_A(x) > \mid x \in X\} \quad (3)$$

where $T_A(x) : X \to [0,1]$, $I_A(x) : X \to [0,1]$ and $F_A(x) : X \to [0,1]$ with the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent, respectively, the truth membership function, the indeterminacy membership function and the falsity membership function of the element x to the set A.

Definition 5. [50, 51] Let $\tilde{a}$ is a single valued trapezoidal neutrosophic trapezoidal number (SVNTrN). Then its truth membership function is

$$T_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a)t_{\tilde{a}}}{b-a} & a \le x < b, \\ t_{\tilde{a}} & b \le x \le c, \\ \dfrac{(d-x)t_{\tilde{a}}}{d-c} & c < x \le d, \\ 0 & otherwise \end{cases}$$

Its indeterminacy membership function is

$$_{\tilde{a}}(x)=\begin{cases}\dfrac{b-x+(x-a\ i_{\tilde{a}}}{b-a} & a\le x<b,\\ i_{\tilde{a}} & b\le x\le c,\\ \dfrac{x-c+(d-x)i_{\tilde{a}}}{d-c} & c<x\le d,\\ 0 & otherwise\end{cases}$$

and its falsity membership function is

$$_{\tilde{a}}(x)=\begin{cases}\dfrac{b-x+(x-a)f_{\tilde{a}}}{b-a} & a\le x<b,\\ f_{\tilde{a}} & b\le x\le c,\\ \dfrac{x-c+(d-x)f_{\tilde{a}}}{d-c} & c<x\le d,\\ 0 & otherwise\end{cases}$$

where $0 \le T_{\tilde{a}}(x) \le 1$, $0 \le I_{\tilde{a}}(x) \le 1$, $0 \le F_{\tilde{a}}(x) \le 1$ and $0 \le T_{\tilde{a}}(x) + I_{\tilde{a}}(x) + F_{\tilde{a}}(x) \le 3$; $a, b, c, d \in R$. Then $\tilde{a} = ([a, b, c, d]; t_{\tilde{a}}, i_{\tilde{a}}, f_{\tilde{a}})$ is called a neutrosophic trapezoidal number.

Definition 5. [50,51] Let $\tilde{a}_1 = ([a_1, b_1, c_1, d_1]; t_{\tilde{a}_1}, i_{\tilde{a}_1}, f_{\tilde{a}_1})$ and $\tilde{a}_2 = ([a_2, b_2, c_2, d_2]; t_{\tilde{a}_2}, i_{\tilde{a}_2}, f_{\tilde{a}_2})$ be two neutrosophic trapezoidal fuzzy numbers and $\lambda \ge 0$, then

1. $\tilde{a}_1 \oplus \tilde{a}_2 = ([a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; t_{\tilde{a}_1} + t_{\tilde{a}_2} - t_{\tilde{a}_1}t_{\tilde{a}_2}, i_{\tilde{a}_1} i_{\tilde{a}_2}, f_{\tilde{a}_1} f_{\tilde{a}_2})$;
2. $\tilde{a}_1 \otimes \tilde{a}_2 = ([a_1a_2, b_1b_2, c_1c_2, d_1d_2]; t_{\tilde{a}_1}t_{\tilde{a}_2}, i_{\tilde{a}_1} + i_{\tilde{a}_2} - i_{\tilde{a}_1} i_{\tilde{a}_2}, f_{\tilde{a}_1} + f_{\tilde{a}_2} - f_{\tilde{a}_1} f_{\tilde{a}_2})$;
3. $\lambda\, \tilde{a}_1 = ([\lambda\, a_1, \lambda\, b_1, \lambda\, c_1, \lambda\, d_1]; 1 - (1 - t_{\tilde{a}_1})^{\lambda}, (i_{\tilde{a}_1})^{\lambda}, (f_{\tilde{a}_1})^{\lambda}$;
4. $(\tilde{a})^{\lambda} = ([a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}]; (t_{\tilde{a}_1})^{\lambda}, 1 - (1 - i_{\tilde{a}_1})^{\lambda}, 1 - (1 - f_{\tilde{a}_1})^{\lambda})$

Definition 6. Let $\tilde{a}_1 = ([a_1, b_1, c_1, d_1]; t_{\tilde{a}_1}, i_{\tilde{a}_1}, f_{\tilde{a}_1})$ and $\tilde{a}_2 = ([a_2, b_2, c_2, d_2]; t_{\tilde{a}_2}, i_{\tilde{a}_2}, f_{\tilde{a}_2})$ be two neutrosophic trapezoidal fuzzy numbers, then the normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows:

$$d(\tilde{a}_1,\tilde{a}_2)=\frac{1}{12}\left(\begin{array}{l}|a_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|b_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|c_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|d_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\end{array}\right) \quad (4)$$

Property 1 The normalized Hamming distance measure $d(.)$ of $\tilde{a}_1$ and $\tilde{a}_2$ satisfies the following properties:

i. $d(\tilde{a}_1, \tilde{a}_2) \ge 0$,
ii. $d(\tilde{a}_1, \tilde{a}_2) = d(\tilde{a}_2, \tilde{a}_1)$,
iii. $d(\tilde{a}_1, \tilde{a}_3) \le d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3)$, where $\tilde{a}_3 = ([a_3, b_3, c_3, d_3]; t_{\tilde{a}_3}, i_{\tilde{a}_3}, f_{\tilde{a}_3})$ is a SVTrNN.

Proof:

i. The distance measure $d(\tilde{a}_1, \tilde{a}_2)$ is obviously non-negative. If $\tilde{a}_1 \approx \tilde{a}_2$ that is for $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$, $t_{\tilde{a}_1} = t_{\tilde{a}_2}$, $i_{\tilde{a}_1} = i_{\tilde{a}_2}$, and $f_{\tilde{a}_1} = f_{\tilde{a}_2}$ we have $d(\tilde{a}_1, \tilde{a}_1) = 0$. Therefore $d(\tilde{a}_1, \tilde{a}_2) \ge 0$.
ii. The proof of straightforward.
iii. The normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_3$ is defined as follows:

$$d(\tilde{a}_1,\tilde{a}_3)=\frac{1}{12}\left(\begin{array}{l}|a_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-a_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ +|b_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-b_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ +|c_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-c_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ +|d_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-d_2(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\end{array}\right)$$

$$=\frac{1}{12}\left(\begin{array}{l}\left|\begin{array}{l}a_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})\\ +a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-a_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})\end{array}\right|\\ +\left|\begin{array}{l}b_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})\\ +b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-b_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})\end{array}\right|\\ +\left|\begin{array}{l}c_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})\\ +c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-c_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})\end{array}\right|\\ +\left|\begin{array}{l}d_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})\\ +d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-d_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})\end{array}\right|\end{array}\right)$$

$$=\frac{1}{12}\left(\begin{array}{l}|a_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-a_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ |b_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-b_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ |c_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|+c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-c_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ |d_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-d_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\end{array}\right)$$

$$\le\frac{1}{12}\left(\begin{array}{l}|a_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|b_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|c_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\\ +|d_1(2+t_{\tilde{a}_1}-i_{\tilde{a}_1}-f_{\tilde{a}_1})-d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})|\end{array}\right)$$

$$+\frac{1}{12}\left(\begin{array}{l}|a_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-a_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ +|b_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-b_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ +|c_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-c_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\\ +|d_2(2+t_{\tilde{a}_2}-i_{\tilde{a}_2}-f_{\tilde{a}_2})-d_3(2+t_{\tilde{a}_3}-i_{\tilde{a}_3}-f_{\tilde{a}_3})|\end{array}\right)$$

$\le d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3)$. □

2.1 TOPSIS Strategy for MADM

The idea behind the TOPSIS strategy [5] is to find out the optimal alternative that has the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution, simultaneously. The schematic structure of classical TOPSIS strategy is presented in the following figure (see Fig. 1)

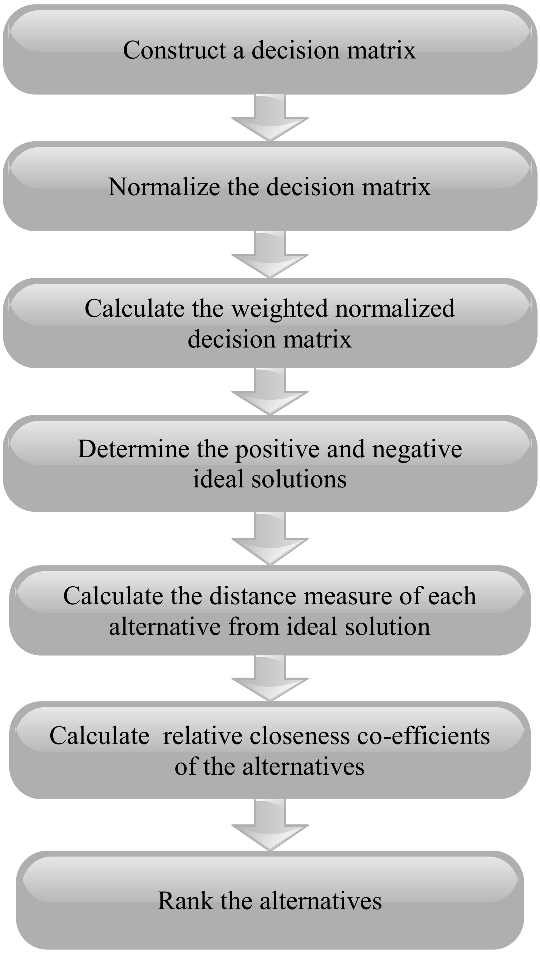

Figure 1. A schematic structure of TOPSIS strategy

3 TOPSIS strategy for multi-attribute decision making with neutrosophic trapezoidal number

In this section, we put forward a framework for determining the attribute weights and the ranking orders for all the alternatives with incomplete weight information under neutrosophic environment.

Consider a MADM problem, where $A = \{A_1, A_2, \dots, A_m\}$ is a set of m alternatives and $C = \{C_1, C_2, \dots, C_n\}$ is a set of n attributes. The attribute value of alternative $A_i(i = 1,2,\dots,m)$ over the attribute $C_j(j = 1,2,\dots,n)$ assumes the form of neutrosophic trapezoidal number $\tilde{a}_{ij} = \left([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; t_{\tilde{a}_{ij}}, i_{\tilde{a}_{ij}}, f_{\tilde{a}_{ij}}\right)$, where $0 \le t_{\tilde{a}_{ij}} \le 1$, $0 \le i_{\tilde{a}_{ij}} \le 1$, $0 \le f_{\tilde{a}_{ij}} \le 1$ and $0 \le t_{\tilde{a}_{ij}} + i_{\tilde{a}_{ij}} + f_{\tilde{a}_{ij}} \le 3$; $a, b, c, d \in R$.

Here, $t_{\tilde{a}_{ij}}$ denotes the truth membership degree, $i_{\tilde{a}_{ij}}$ denotes the indeterminate membership degree, and $f_{\tilde{a}_{ij}}$ denotes the falsity membership degree to consider the trapezoidal number $[a_{ij}, b_{ij}, c_{ij}, d_{ij}]$ as the rating values of A_i over the attribute C_j. An MADM problem can be expressed by a decision matrix in which the entries represent the evaluation information of all alternatives with respect to the attributes. Then we construct the following neutrosophic decision matrix, whose elements are SVNTrNs:

$$D = \left(\tilde{a}_{ij}\right)_{m\times n} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{pmatrix} \tag{5}$$

Due to different attribute weights, we assume that the weight vector of all attributes is given by $w = (w_1, w_2, \dots, w_n)^T$, where $0 \le w_j \le 1$, $j = 1,2,\dots,n$, and w_j is the weight of each attribute. The information about attribute weights is usually incomplete in decision making problems under uncertain environment. For convenience, we assume Δ be a set of the known weight information [56-59], where Δ can be constructed by the following forms, for $i \ne j$:

Form 1. A weak ranking: $\{w_i \ge w_j\}$;

Form 2. A strict ranking: $\{w_i - w_j \ge \alpha_j\}(\alpha_j > 0)$;

Form 3. A ranking of difference: $\{w_i - w_j \ge w_k - w_l\}$, for $j \ne k \ne l$;

Form 4. A ranking with multiples: $\{w_i \ge \alpha_j w_j\}$ $(0 \le \alpha_j \le 1)$;

Form 5. An interval form: $\{\alpha_i \le w_i \le \alpha_i + \epsilon_i\}(0 \le \alpha_j \le \alpha_i + \epsilon_i \le 1)$.

Now we develop a strategy for solving the MADM problems, in which the information about attribute weights is completely unknown or partially known and the attribute values are expressed by SVTrNNs.

The following steps are considered to develop the model.

3.1 Standardize the decision matrix

Let $D = \left(\tilde{a}_{ij}\right)_{m\times n}$ be a neutrosophic decision matrix, where the SVTrNNs $\tilde{a}_{ij} = \left(\left[a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4\right]; t_{\tilde{a}_{ij}}, i_{\tilde{a}_{ij}}, f_{\tilde{a}_{ij}}\right)$ is the rating values of alternative A_i with respect to attribute C_j. Now to eliminate the effect from different physical dimensions into decision making process, we should standardize the decision matrix $\left(\tilde{a}_{ij}\right)_{m\times n}$ based on two common types of attributes such as benefit type attribute and cost type attribute. We consider the following technique to obtain the standardized decision matrix $R=\left(\tilde{r}_{ij}\right)_{m\times n}$, in which the component r_{ij}^{k} of the entry $\tilde{r}_{ij}=\left(\left[r_{ij}^{1},r_{ij}^{2},r_{ij}^{3},r_{ij}^{4}\right];t_{\tilde{r}_{ij}},i_{\tilde{r}_{ij}},f_{\tilde{r}_{ij}}\right)$ in the matrix R are considered as:

1. For benefit type attribute:

$$\tilde{r}_{ij}=\left\{\left(\left[\frac{a_{ij}^{1}}{u_{j}^{+}},\frac{a_{ij}^{2}}{u_{j}^{+}},\frac{a_{ij}^{3}}{u_{j}^{+}},\frac{a_{ij}^{4}}{u_{j}^{+}}\right];t_{\tilde{r}_{ij}},i_{\tilde{r}_{ij}},f_{\tilde{r}_{ij}}\right)\right. \tag{6}$$

2. For cost type attribute:

$$\tilde{r}_{ij}=\left\{\left(\left[\frac{u_{j}^{-}}{a_{ij}^{4}},\frac{u_{j}^{-}}{a_{ij}^{3}},\frac{u_{j}^{-}}{a_{ij}^{2}},\frac{u_{j}^{-}}{a_{ij}^{1}}\right];t_{\tilde{r}_{ij}},i_{\tilde{r}_{ij}},f_{\tilde{r}_{ij}}\right)\right., \tag{7}$$

where $u_{j}^{+}=\max\{a_{ij}^{4}\mid i=1,2,...m\}$ and

$u_{j}^{-}=\min\{a_{ij}^{1}\mid i=1,2,...m\}$ for $j=1,2,...n.$

Then we obtain the following standardized decision matrix:

$$R=\left(\tilde{r}_{ij}\right)_{m\times n}=\begin{pmatrix}\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n}\\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn}\end{pmatrix} \tag{8}$$

3.2 Determine the attribute weight

To determine the attribute weights, we use maximum deviation strategy, which was proposed by Wang [60]. According to Wang [60],

i. The attribute that has the larger deviation value among alternatives should be assigned larger weight.
ii. The attribute having deviation value among alternatives should be assigned smaller weight.
iii. The attribute having no deviation among alternatives should be assigned zero weight.

Following the idea of maximum deviation method, we construct an optimization model to determine the optimal weights of attributes with SVTrNNs. The deviation of the alternative A_i to all the other alternatives for the attribute C_j can be defined as follows:

$d_{ij}(w)=\sum_{k=1}^{m}d(\tilde{a}_{ij},\tilde{a}_{kj})w_j$, $i=1,2,\dots,m;$ $j=1,2,\dots,n$

where

$$d(\tilde{a}_{ij},\tilde{a}_{kj})=\frac{1}{12}\begin{pmatrix}\left|a_{ij1}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj1}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\\ +\left|a_{ij2}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj2}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\\ +\left|a_{ij3}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj3}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\\ +\left|a_{ij4}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj4}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\end{pmatrix} \tag{9}$$

$$=\frac{1}{12}\sum_{p=1}^{4}\left|\begin{matrix}a_{ijp}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)\\ -a_{kjp}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\end{matrix}\right|$$

denotes the neutrosophic Hamming distance between two SVTrNNs $\tilde{a}_{ij}$ and $\tilde{a}_{kj}$.

The deviation value of all the alternatives to other alternatives for the attribute C_j can be obtained as follows:

$$D_j(w)=\sum_{i=1}^{m}d_{ij}(w)=\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{a}_{ij},\tilde{a}_{kj}\right)w_j$$
$$=\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{12}\sum_{p=1}^{4}\left|a_{ij}^{p}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)-a_{kj}^{p}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\right|\right)w_j. \tag{10}$$

Similarly, the deviation value of all the alternatives to other alternatives for all the criteria can be taken as:

$$D(w)=\sum_{j=1}^{n}D_j(w)=\sum_{j=1}^{n}\sum_{i=1}^{m}d_{ij}(w)$$
$$=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{i=1}^{m}\sum_{k=1}^{m}d\left(\tilde{a}_{ij},\tilde{a}_{kj}\right)w_j$$
$$=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\left(\frac{1}{12}\sum_{p=1}^{4}\left|\begin{matrix}a_{ij}^{p}\left(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}}\right)\\ -a_{kj}^{p}\left(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}}\right)\end{matrix}\right|\right)w_j$$

If the information about the attribute weights is partially known or completely unknown, then we propose two models to obtain the attribute weights.

3.2.1 *Information about the weights of attributes is partially known.*

In order to obtain the weight vector, we construct a non-linear programming model that maximizes all deviation values of attributes. The model can be presented as follows:

$$(M-2)\begin{cases}\max\ D(w)=\dfrac{1}{12}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{matrix}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\ -a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{matrix}\right|\right)w_j\\ \text{subject to } w\in\Delta,\quad \displaystyle\sum_{j=1}^{n}w_j=1,\ w_j\geq 0,\quad \text{for } j=1,2,..,n.\end{cases} \tag{11}$$

By solving the model (M-1), we obtain the optimal solution to be used as the weight vector.

3.2.2 *Information about the weights of attributes is unknown.*

If the information about attribute weight is completely unknown, then we can establish the following programming model:

$$(M-2)\begin{cases}\max\ D(w)=\dfrac{1}{12}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{matrix}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\ -a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{matrix}\right|\right)w_j\\ \text{subject to } w\in\Delta,\quad \displaystyle\sum_{j=1}^{n}w_j^{\,2}=1,\ w_j\geq 0,\ \text{for } j=1,2,..,n.\end{cases} \tag{12}$$

To solve the model (M-2), we develop the Lagrange function:

$$L(w,\xi)=\frac{1}{12}\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)w_j+\frac{\xi}{24}\left(\sum_{j=1}^{n}w_j^{2}-1\right) \tag{13}$$

where ξ is a real number and denoting the Lagrange multiplier variable. Then the partial derivative of L with respect to $w_j\,(j=1,2,...,n)$ and ξ are obtained as:

$$\frac{\partial L}{\partial w_j}=\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)w_j+\xi w_j=0 \tag{14}$$

$$\frac{\partial L}{\partial \xi}=\sum_{j=1}^{n}w_j^{2}-1=0 \tag{15}$$

It follows from Eq. (14) that

$$w_j=\frac{-\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)}{\xi}\quad \text{for}\quad j=1,2,...,n. \tag{16}$$

Putting the values of w_j in Eq.(15), we obtain

$$\xi^{2}=\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)\right)^{2} \tag{17}$$

$$\Rightarrow \xi=-\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)\right)^{2}}\quad \text{for}\quad \xi<0. \tag{18}$$

Then combining Eq.(16) and Eq.(18), we obtain the following formula for determining the weight of attribute $C_j\,(j=1,2,...,n)$:

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)\right)^{2}}}\,. \tag{19}$$

We make their sum into a unit by normalizing $w_j\,(j=1,2,...,n)$ and get the optimal weight of attribute $C_j\,(j=1,2,...,n)$:

$$\bar{w}_j=\frac{w_j}{\sum_{j=1}^{n}w_j}=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m}\sum_{p=1}^{4}\left(\left|\begin{array}{l}a_{ij}^{p}(2+t_{\tilde{a}_{ij}}-i_{\tilde{a}_{ij}}-f_{\tilde{a}_{ij}})\\-a_{kj}^{p}(2+t_{\tilde{a}_{kj}}-i_{\tilde{a}_{kj}}-f_{\tilde{a}_{kj}})\end{array}\right|\right)} \tag{20}$$

Then we get the normalized weight vector of attributes:

$\bar{w}=\{\bar{w}_1,\bar{w}_2,...,\bar{w}_n\}$.

3.3 Determine the ideal solutions

In the normalized decision matrix $R=\left(\tilde{r}_{ij}\right)_{m\times n}$, the neutrosophic trapezoidal local positive ideal solution (NTrPIS) and the neutrosophic trapezoidal local negative ideal solution (NTrNIS) are defined as follows

$$\tilde{r}^{+}=\left(\tilde{r}_1^{+},\tilde{r}_2^{+},...,\tilde{r}_n^{+}\right)\text{ and }\tilde{r}^{-}=\left(\tilde{r}_1^{-},\tilde{r}_2^{-},...,\tilde{r}_n^{-}\right) \tag{21}$$

where,

$$r_j^{+}=\left(\left[r_j^{1+},r_j^{2+},r_j^{3+},r_j^{4+}\right],t_j^{+},i_j^{+},f_j^{+}\right)=\left(\begin{array}{c}\left[\max_i\left(r_{ij}^{1}\right),\max_i\left(r_{ij}^{2}\right),\max_i\left(r_{ij}^{3}\right),\max_i\left(r_{ij}^{4}\right)\right];\\ \max_i\left(t_{ij}\right),\min_i\left(i_{ij}\right),\min_i\left(f_{ij}\right)\end{array}\right) \tag{22}$$

$$r_j^{-}=\left(\left[r_j^{1-},r_j^{2-},r_j^{3-},r_j^{4-}\right],t_j^{-},i_j^{-},f_j^{-}\right)=\left(\begin{array}{c}\left[\min_i\left(r_{ij}^{1}\right),\min_i\left(r_{ij}^{2}\right),\min_i\left(r_{ij}^{3}\right),\min_i\left(r_{ij}^{4}\right)\right];\\ \min_i\left(t_{ij}\right),\max_i\left(i_{ij}\right),\max_i\left(f_{ij}\right)\end{array}\right) \tag{23}$$

Moreover, the trapezoidal neutrosophic global positive ideal solution and the trapezoidal neutrosophic global trapezoidal global negative ideal solution can be directly considered as

$$r_j^{+}=\left(\left[1,1,1,1\right],1,0,0\right)\text{ and }r_j^{-}=\left(\left[0,0,0,0\right],0,1,1\right) \tag{24}$$

3.4 Determine the separation measures from ideal solutions to each alternative

The separation measures d_i^{+} and d_i^{-} of each alternative from the ideal solutions can be determined by Eq.(9), Eq.(20) and Eq.(21), respectively, as follows:

$$d_i^{+}=\sum_{j=1}^{n}w_j d\left(\tilde{r}_{ij},\tilde{r}_j^{+}\right)=\frac{1}{12}\sum_{j=1}^{n}w_j\sum_{p=1}^{4}\left(\left|\begin{array}{l}r_{ij}^{p}(2+t_{\tilde{r}_{ij}}-i_{\tilde{r}_{ij}}-f_{\tilde{r}_{ij}})\\-r_j^{p+}(2+t_{\tilde{r}_j^{+}}-i_{\tilde{r}_j^{+}}-f_{\tilde{r}_j^{+}})\end{array}\right|\right)\text{ for } i=1,2,...,m \tag{25}$$

$$d_i^{-}=\sum_{j=1}^{n}w_j d\left(\tilde{r}_{ij},\tilde{r}_j^{-}\right)=\frac{1}{12}\sum_{j=1}^{n}w_j\sum_{p=1}^{4}\left(\left|\begin{array}{l}r_{ij}^{p}(2+t_{\tilde{r}_{ij}}-i_{\tilde{r}_{ij}}-f_{\tilde{r}_{ij}})\\-r_j^{p-}(2+t_{\tilde{r}_j^{-}}-i_{\tilde{r}_j^{-}}-f_{\tilde{r}_j^{-}})\end{array}\right|\right)\text{ for } i=1,2,...,m \tag{26}$$

3.5 Determine the relative closeness co-efficient

The relative closeness co-efficient of an alternative A_i with respect to ideal alternative A^{+} is defined as the following formula:

$$RC(A_i) = \frac{d_i^-}{d_i^+ + d_i^-} \tag{27}$$

where $0 \le RC(A_i) \le 1$ for $i = 1,2,...m$. According to the closeness co-efficient $RC(A_i)$, the ranking orders of all alternatives and the best alternative can be selected. The schematic diagram of the proposed TOPSIS is presented in Figure-2.

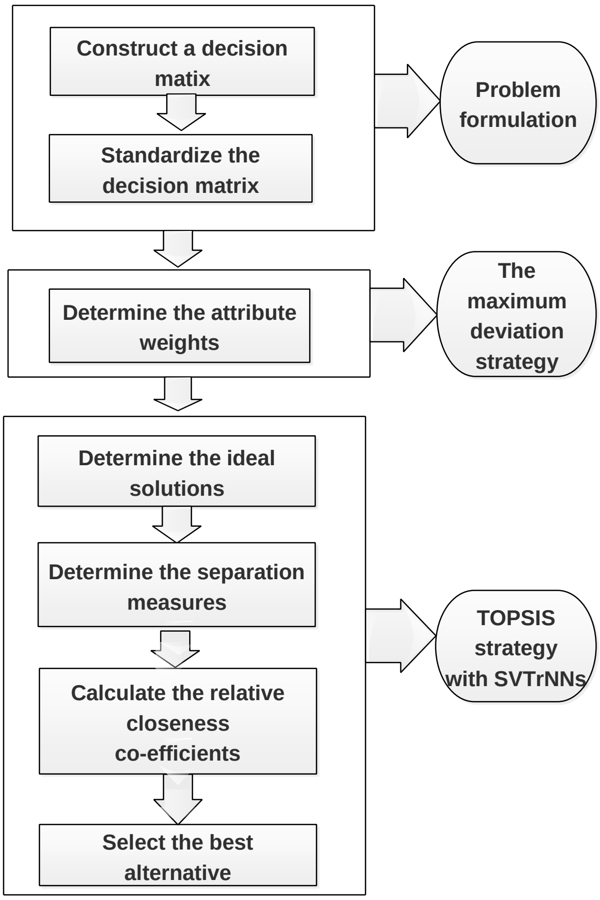

Figure 2. The schematic diagram of the proposed startegy

4 An illustrative example

In this section, we consider an illustrative example of medical representative selection problem to demonstrate and applicability of the proposed.

Consider a MADM problem, where a pharmacy company wants to recruit a medical representative. After initial scrutiny four candidates $A_i (i = 1,2,3,4)$ have been considered for further evaluation with respect to the four attributes C_j (j = 1, 2, 3, 4) namely,

1. Oral communication skill (C_1);
2. Past experience (C_2),
3. General aptitude (C_3) and
4. Self- confidence (C_4).

The decision maker evaluates the ratings of alternatives $A_i (i = 1,2,...,m)$ with respect to the attributes $C_i (i = 1,2,...,n)$ with the decision matrix $D = (\tilde{a}_{ij})_{4\times4}$ (see Table 1).

Table 1. Rating values of alternatives

	C_1	C_2
A1	([7,8,9,10]; 0.90,0.10,0.05)	([5,6,7,8]; 0.65,0.35,0.30)
A2	([5,6,7,8]; 0.65,0.35,0.30)	([6,7,8,9]; 0.80,0.20,0.15)
A3	([4,5,6,7]; 0.50,0.50,0.45)	([5,6,7,8]; 0.65,0.35,0.30)
A4	([4,5,6,7]; 0.50,0.50,0.45)	([5,6,7,8]; 0.65,0.35,0.30)
	C_3	C_4
A1	([6,7,8,9]; 0.80,0.20,0.15)	([7,8,9,10]; 0.90,0.10,0.05)
A2	([7,8,9,10]; 0.90,0.10,0.05)	([6,7,8,9]; 0.80,0.20,0.15)
A3	([4,5,6,7]; 0.50,0.50,0.45)	([4,5,6,7]; 0.50,0.50,0.45)
A4	([4,5,6,7]; 0.50,0.50,0.45)	([6,7,8,9]; 0.80,0.20,0.15)

The information of the attributes is incompletely known and the weight information is given as follows:

$$\Delta = \begin{Bmatrix} 0.20 \le w_1 \le 0.30, 0.05 \le w_2 \le 0.20, \\ 0.20 \le w_3 \le 0.35, 0.15 \le w_4 \le 0.35; \sum_{j=1}^{4} w_j = 1 \end{Bmatrix} \tag{28}$$

To determine the best alternative, we use the proposed strategy involving the following steps:

Step 1. Standardize the decision matrix

Since the selective attributes are benefit type attributes, then using Eq. (6), we have the following standardized decision matrix: $R = (\tilde{r}_{ij})_{4\times4}$ (see Table 2.)

Table 2. Standardized rating values of alternatives

	C_1	C_2
A1	([0.7,0.8,0.9,1.0]; 0.90,0.10,0.05)	([0.5,0.6,0.7,0.8]; 0.65,0.35,0.30)
A2	([0.5,0.6,0.7,0.8]; 0.65,0.35,0.30)	([0.6,0.7,0.8,0.9]; 0.80,0.20,0.15)

A3	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.5,0.6,0.7,0.8];\\0.65,0.35,0.30\end{pmatrix}$
A4	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.5,0.6,0.7,0.8];\\0.65,0.35,0.30\end{pmatrix}$
	C_3	C_4
A1	$\begin{pmatrix}[0.6,0.7,0.8,0.9];\\0.80,0.20,0.15\end{pmatrix}$	$\begin{pmatrix}[0.7,0.8,0.9,1.0];\\0.90,0.10,0.05\end{pmatrix}$
A2	$\begin{pmatrix}[0.7,0.8,0.9,1.0];\\0.90,0.10,0.05\end{pmatrix}$	$\begin{pmatrix}[0.6,0.7,0.8,0.9];\\0.80,0.20,0.15\end{pmatrix}$
A3	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\0.50,0.50,0.45\end{pmatrix}$
A4	$\begin{pmatrix}[0.4,0.5,0.6,0.7];\\0.50,0.50,0.45\end{pmatrix}$	$\begin{pmatrix}[0.6,0.7,0.8,0.9];\\0.80,0.20,0.15\end{pmatrix}$

Step 2. Determine the attribute weight

Case 1. Weight information is incompletely known.

Using the model (M-1), we construct the following single-objective programming problem:

$$\begin{cases}\max\ D(w) = 3.2133w_1 + 1.1401w_2 + 3.4250w_3 + 2.9700w_4\\ \text{subject to } w \in \Delta, \quad \sum_{j=1}^{4} w_j = 1,\ w_j \geq 0, \quad \text{for } j = 1,2,..,4.\end{cases} \tag{29}$$

Solving this model with optimization software LINGO 13, we get the optimal weight vector as $w = (0.30, 0.05, 0.35, 0.30)$.

Case 2. Weight information is completely unknown.

Following Eq.(20), we obtain the following optimal weight vector:

$\bar{w} = (0.2990, 0.1061, 0.3186, 0.2763)$.

Step 3. Determine the ideal solutions

Since the chosen attributes are benefit type attribute, then following Eq.(22) we determine the neutrosophic trapezoidal positive ideal solution as

$$A^+ = \begin{pmatrix}([0.7,0.8,0.9,1.0]; 0.90,0.10,0.05),\\ ([0.6,0.7,0.8,0.9]; 0.80,0.20,0.15),\\ ([0.7,0.8,0.9,1.0]; 0.90,0.10,0.05),\\ ([0.7,0.8,0.9,1.0]; 0.90,0.10,0.05)\end{pmatrix} \tag{30}$$

Similarly, using Eq.(23), we determine the neutrosophic trapezoidal negative ideal solution

$$A^- = \begin{pmatrix}([0.4,0.5,0.6,0.7]; 0.50,0.50,0.45),\\ ([0.5,0.6,0.7,0.8]; 0.65,0.35,0.30),\\ ([0.4,0.5,0.6,0.7]; 0.50,0.50,0.45),\\ ([0.4,0.5,0.6,0.7]; 0.50,0.50,0.45)\end{pmatrix} \tag{31}$$

Step 4. Determine the separation measures from ideal solutions to each alternative.

Case 1. Employing Eq.(25), we obtain the separation measures d_i^+ of each alternative $A_i (i = 1,2,3,4)$ from A^+:

$d(A_1, A^+) = 0.0673$, $d(A_2, A^+) = 0.1538$, $d(A_3, A^+) = 0.4792$, $d(A_4, A^+) = 0.3807$.

Similarly, using Eq.(26), we obtain the separation measures d_i^- of each alternative $A_i (i = 1,2,3,4)$ from A^-:

$d(A_1, A^-) = 0.4119$, $d(A_2, A^-) = 0.3254$, $d(A_3, A^-) = 0$, $d(A_4, A^-) = 0.0985$.

Case 2. The separation measures d_i^+ of each alternative $A_i (i = 1,2,3,4)$ from A^+:

$d(A_1, A^+) = 0.0721$, $d(A_2, A^+) = 0.1494$, $d(A_3, A^+) = 0.4615$, $d(A_4, A^+) = 0.3708$.

Similarly, the separation measures d_i^- of each alternative $A_i (i = 1,2,3,4)$ from A^-:

$d(A_1, A^-) = 0.3894$, $d(A_2, A^-) = 0.3120$, $d(A_3, A^-) = 0$, $d(A_4, A^-) = 0.0907$.

Step 5. Calculate the relative closeness coefficient.

Using Eq.(27), we calculate the relative closeness coefficient $RC(A_i)$ of alternative $A_i (i = 1,2,3,4)$ for Case 1 and Case 2, respectively. We put the result in Table 3.

Table 3. Rating values of alternatives

$RC(A_i)$	Case 1	Case 2
$RC(A_1)$	0.8596	0.8438
$RC(A_2)$	0.6790	0.6824
$RC(A_3)$	0	0
$RC(A_4)$	0.2056	0.1965

Following Table 3, we rank the alternatives $A_i (i = 1,2,3,4)$ according to the values of relative closeness coefficient $RC(A_i)$ for both cases: $A_1 \succ A_2 \succ A_4 \succ A_3$. Therefore A_1 is the best alternative.

5 Conclusions

TOPSIS strategy is a useful strategy for solving MADM problem under different environment. In this paper, we have investigated MADM problems with SVTrNNs. The weight information of attributes have been considered to be incompletely known or completely unknown. First, we have used Hamming distance measure to determine the distance measure of SVTrNNs. Second, we developed an optimization model to determine the attribute weights based on the idea of maximum deviation strategy. Third, we have extended the TOPSIS strategy for solving the MADM model with SVTrNNs. Finally, we have provided an illustrative examples to verify the feasibility and effectiveness of the proposed model. The proposed TOPSIS strategy can be extended to multi-attribute group decision making with SVTrNNs and multi-attribute decision making problem with interval trapezoidal neutrosophic numbers. The proposed TOPSIS strategy can be used in solving logistics center location selection [61, 62], weaver selection [63, 64], data mining [65], school choice [66], teacher selection [67], brick field selection [68-69), etc. under SVTrNN environment.

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Distance Measure Based MADM Strategy with Interval Trapezoidal Neutrosophic Numbers

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Abstract. In this paper, we introduce interval trapezoidal neutrosophic number and define some arithmetic operations of the proposed interval trapezoidal neutrosophic numbers. Then we consider a multiple attribute decision making (MADM) problem with interval trapezoidal neutrosophic numbers. The weight information of each attribute in the multi attribute decision making problem is expressed in terms of interval trapezoidal neutrosophic numbers. To develop distance measure based MADM strategy with interval trapezoidal neutrosophic numbers, we define normalised Hamming distance measure of the proposed numbers and develop an algorithm to determine the weight of the attributes. Using these weights, we aggregate the distance measures of preference values of each alternative with respect to ideal alternative. Then we determine the ranking order of all alternatives according to the aggregated weighted distance measures of all available alternatives. Finally, we provide an illustrative example to show the feasibility, applicability of the proposed MADM strategy with interval trapezoidal neutrosophic numbers.

Keywords: Interval trapezoidal neutrosophic number, Hamming distance measure, entropy, multi-attribute decision making.

1 Introduction

Neutrosophic set theory, pioneered by Smarandache [1], is an important tool for dealing with imprecise, incomplete, indeterminate, and inconsistent information occurred in decision making process. Neutrosophic set has three independent components: truth membership degree, indeterminate membership degree, and falsity membership degree lying in a non-standard unit interval$]^{-}0,1^{+}[$. Wang et al. [2] introduced single valued neutrosophic set which has three membership degrees and the value of each membership degree lies in [0,1]. Wang et al. [3] proposed interval neutrosophic set (INS) in which the values of its truth membership degree, indeterminacy membership degree, and falsity membership degree are intervals rather than crisp numbers. Therefore, INSs allow us flexibility in presenting neutrosophic information existing in modern decision making problem. Recently, many researchers have shown interest on possible application of INSs in the field of multi-attribute decision making (MADM) and multi-attribute group decision making (MAGDM).
Chi and Liu [4] extended technique for order preference by similarity to ideal solution (TOPSIS) strategy for MADM with INSs. Pramanik and Mondal [5] combined grey relational analysis (GRA) with MADM strategy for interval neutrosophic information and presented a novel MADM strategy in interval neutrosophic environment. Dey et al. [6] defined weighted projection measure and developed an MADM strategy for interval neutrosophic information. In the same study, Dey et al. [6] also developed an alternative strategy to solve MADM problems based on the combination of angle cosine and projection measure. Ye [7] defined some similarity measures of INSs and employed these measures in multi-criteria decision making (MCDM) problem. Pramanik et al. [8] proposed hybrid vector similarity measures of single valued neutrosophic sets as well as interval neutrosophic sets and developed two MADM strategies to solve MADM problems. Peng et al. [9] developed some aggregation operators of simplified neutrosophic sets to solve multi-criteria group decision making problem. Dey et al. [10] extended grey relational analysis strategy for solving weaver selection problem in interval neutrosophic environment. Zhang et al. [11] proposed an outranking strategy for MCDM problem with neutrosophic sets. Dalapati et al. [12] proposed cross entropy measure of INSs and employed the measure in solving MADGM problem.

However, the domain of single valued and interval neutrosophic set considered is a discrete set. Ye [13], and Şubaş [14] introduced single valued trapezoidal neutrosophic number (SVTrNN), where each element is expressed by trapezoidal numbers that has a truth membership degree, an indeterminate membership degree, and a falsity membership degree. Biswas et al. [15] also introduced SVTrNN in which each membership degree is charactrized by normalized trapezoidal fuzzy number. In the same study, Biswas et al. [15] proposed value and ambiguity based ranking strategyand applied this strategy to MADM problem. Deli and Şubaş [16] also proposed a ranking strategy of single valued neutrosophic number and utilized this strategy in MADM problems. Deli and Şubaş [17] developed some weighted geometric operators of triangular neutrosophic numbers to solve MADM problem. Ye [13] proposed two weighted aggregation operators of trapezoidal neutrosophic numbers and applied them to MADM problem. Liu and Zhang [18] presented some Maclaurin symmetric mean operators for single-valued trapezoidal neutrosophic numbers and discussed their applications to group decision making. Liang et al. [19] utilized preference relationon to solve MCDM strategy with SVTrNN. Basset et al. [20] intregated the analytical heirarchy process into Delphi framework based group decision making model with trapezoidal neutrosophic numbers.

However due to complexity of decision making problem, decision makers may face difficulties to express their opinion with the single valued truth membership degree, indeterminacy membership degree, and the falsity membership in neutrosophic environment. Then, it is easy to express their opinion in terms of three membership degrees with an interval number rather than exact real number. Therefore, we have an opportunity to investigate a trapezoidal neutrosophic number that has a three membership degrees represented in interval form. We call this new number as interval trapezoidal neutrosophic number (ITrNN). The proposed number permits us to deal with more neutrosophic information than SVTrNN. Hence, we need to develop some decision making strategies with the ITrNNs. At present no studies have been reported in the literature for MADM with ITrNNs.

The main objectives of the study are:

- To introduce ITrNN and present some of its operational rules.
- To define normalized Hamming distance measure of ITrNNs.
- To develop a novel strategy for solving MADM problem with ITrNNs.

The remainder of the paper is outlined as follows: Section 2reviews some basics on single valued neutrosophic sets, interval neutrosophic sets. Section 3 introduces ITrNNs and defines some arithmetical operations. Section 4 presents a novel strategyfor solving MADM with interval trapezoidal neutrosophic numbers. Section 5 provides an illustrative example to illustrate the proposed strategy. Finally, Section 6 draws some concluding remarks with future research directions.

2 Preliminaries

Definition 1. [2] Assume that$X$ be a universe of discourse. A single-valued neutrosophic set A in X is given by

$$A = \{x, < T_A(x), I_A(x), F_A(x) > \mid x \in X\} \quad (1)$$

where $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$, with the condition

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X.$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ represent, respectively, the truth membership function, the indeterminacy function and the falsity membership function of the element x to the set A.

Definition 2. [3] Let X be a universe of discourse and $D[0,1]$ be the set of all closed sub-intervals. An interval neutrosophic set $\tilde{A}$ in X is given by

$$\tilde{A} = \{x, < T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x) > \mid x \in X\} \quad (2)$$

where $\tilde{T}_{\tilde{A}}(x): X \to D[0,1], \tilde{I}_{\tilde{A}}(x): X \to D[0,1]$ and $\tilde{F}_{\tilde{A}}(x): X \to D[0,1]$, with the condition

$0 \le sup\tilde{T}_{\tilde{A}}(x) + sup\tilde{I}_{\tilde{A}}(x) + sup\tilde{F}_{\tilde{A}}(x) \le 3$ for all $x \in X.$

The intervals $\tilde{T}_{\tilde{A}}(x)$, $\tilde{I}_{\tilde{A}}(x)$ and $\tilde{F}_{\tilde{A}}(x)$ represent the truth membership degree, the indeterminacy membership degree and the falsity membership degree of the element x to the set $\tilde{A}$, respectively.

3. Interval trapezoidal neutrosophic numbers (ITrNNs)

In this section, we present the concept of interval trapezoidal neutrosophic number and define its basic operations.

Definition 3. Let $\tilde{a}$ is a trapezoidal neutrosophic number in the set of real numbers, its truth membership function is

$$T_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a)t_{\tilde{a}}}{b-a} & a \le x < b; \\ t_{\tilde{a}} & b \le x \le c; \\ \dfrac{(d-x)t_{\tilde{a}}}{d-c} & c < x \le d; \\ 0 & otherwise, \end{cases}$$

Its indeterminacy membership function is

$$I_{\tilde a}(x)=\begin{cases}\dfrac{b-x+(x-a)i_{\tilde a}}{b-a} & a\le x<b;\\ i_{\tilde a} & b\le x\le c;\\ \dfrac{x-c+(d-x)i_{\tilde a}}{d-c} & c<x\le d;\\ 0 & otherwise,\end{cases}$$

and its falsity membership function is

$$F_{\tilde a}(x)=\begin{cases}\dfrac{b-x+(x-a)f_{\tilde a}}{b-a} & a\le x<b;\\ f_{\tilde a} & b\le x\le c;\\ \dfrac{x-c+(d-x)f_{\tilde a}}{d-c} & c<x\le d;\\ 0 & otherwise,\end{cases}$$

where $t_{\tilde a}\subset[0,1]$, $i_{\tilde a}\subset[0,1]$, and $f_{\tilde a}\subset[0,1]$ are interval numbers and $0\le \sup(t_{\tilde a})+\sup(i_{\tilde a})+\sup(f_{\tilde a})\le 3$.

Then $\tilde a$ is called an interval trapezoidal neutrosophic number and it is denoted by $\tilde a=([a,\,b,\,c,\,d];t_{\tilde a},i_{\tilde a},f_{\tilde a})$. For convenience we can take $t_{\tilde a}=[\underline{t},\,\overline{t}]$, $i_{\tilde a}=[\underline{i},\,\overline{i}]$, and $f_{\tilde a}=[\underline{f},\,\overline{f}]$. Then the number $\tilde a$ can be denoted by $\tilde a=([a,\,b,\,c,\,d];[\underline{t},\,\overline{t}],\,[\underline{i},\,\overline{i}],[\underline{f},\,\overline{f}])$.

Definition 4. Let $\tilde a=([a,\,b,\,c,\,d];[\underline{t},\,\overline{t}],\,[\underline{i},\,\overline{i}],[\underline{f},\,\overline{f}])$ be an ITrNN. If $a\ge 0$ and one of the four values $of\ a,\,b,\,c$ and d is not equal to zero, then the ITrNN $\tilde a$ is called positive ITrNN.

3.1. Some arithmetic operations on ITrNNs

Definition 5. Let $\tilde a_1=([a_1,\,b_1,\,c_1,\,d_1];[\underline{t_1},\,\overline{t_1}],\,[\underline{i_1},\,\overline{i_1}],[\underline{f_1},\,\overline{f_1}])$ and

$\tilde a_2=([a_2,\,b_2,\,c_2,\,d_2];[\underline{t_2},\,\overline{t_2}],\,[\underline{i_2},\,\overline{i_2}],[\underline{f_2},\,\overline{f_2}])$ be two INTrNs and $\lambda\ge 0$. Then the following operations are valid.

1. $$\tilde a_1\oplus\tilde a_2=\begin{pmatrix}[a_1+a_2,\,b_1+b_2,\,c_1+c_2,\,d_1+d_2];\\ \left[\underline{t_1}+\underline{t_2}-\underline{t_1}\,\underline{t_2},\ \overline{t_1}+\overline{t_2}-\overline{t_1}\,\overline{t_2}\right],\\ \left[\underline{i_1}\,\underline{i_2},\ \overline{i_1}\,\overline{i_2}\right],\ \left[\underline{f_1}\,\underline{f_2},\ \overline{f_1}\,\overline{f_2}\right]\end{pmatrix}$$

2. $$\tilde a_1\otimes\tilde a_2=\begin{pmatrix}[a_1+a_2,\,b_1+b_2,\,c_1+c_2,\,d_1+d_2];\left[\underline{t_1}\,\underline{t_2},\ \overline{t_1}\,\overline{t_2}\right],\\ \left[\underline{i_1}+\underline{i_2}-\underline{i_1}\,\underline{i_2},\ \overline{i_1}+\overline{i_2}-\overline{i}\,\overline{i_2}\right],\\ \left[\underline{f_1}+\underline{f_2}-\underline{f_1}\,\underline{f_2},\ \overline{f_1}+\overline{f_2}-\overline{f_1}\,\overline{f_2}\right]\end{pmatrix};$$

3. $$\lambda\,\tilde a_1=\begin{pmatrix}[\lambda\,a_1,\,\lambda\,b_1,\,\lambda\,c_1,\,\lambda\,d_1];\\ \left[1-\left(1-\underline{t_1}\right)^{\lambda},1-(1-\overline{t_1})^{\lambda}\right],\\ \left[\left(\underline{i_1}\right)^{\lambda},\,\left(\overline{i_1}\right)^{\lambda}\right],\ \left[\left(\underline{f_1}\right)^{\lambda},\,\left(\overline{f_1}\right)^{\lambda}\right]\end{pmatrix},\lambda>0$$

4. $$(\tilde a_1)^{\lambda}=\begin{pmatrix}[a_1^{\lambda},b_1^{\lambda},c_1^{\lambda},d_1^{\lambda}];\left[\left(\underline{t_1}\right)^{\lambda},\,\left(\overline{t_1}\right)^{\lambda}\right],\\ \left[1-\left(1-\underline{i_1}\right)^{\lambda},1-\left(1-\overline{i_1}\right)^{\lambda}\right],\\ \left[1-\left(1-\underline{f_1}\right)^{\lambda},1-\left(1-\overline{f_1}\right)^{\lambda}\right]\end{pmatrix},\lambda>0.$$

Definition 6. The ideal choice of interval neutrosophic trapezoidal number is

$$\tilde I^{+}=([1,\,1,\,1,\,1];[1,\,1],\,[0,0],[0,0]). \tag{3}$$

3.2. Hamming distance between two ITrNNs.

Let $\tilde a_1=([a_1,\,b_1,\,c_1,\,d_1];[\underline{t_1},\,\overline{t_1}],\,[\underline{i_1},\,\overline{i_1}],[\underline{f_1},\,\overline{f_1}])$ and $\tilde a_2=([a_2,\,b_2,\,c_2,\,d_2];[\underline{t_2},\,\overline{t_2}],\,[\underline{i_2},\,\overline{i_2}],[\underline{f_2},\,\overline{f_2}])$ be any two INTrNs, then the normalized Hamming distance between $\tilde a_1$ and $\tilde a_2$ is defined as follows:

$d(\tilde a_1,\,\tilde a_2)=$

$$\frac{1}{24}\begin{pmatrix}\left|\begin{array}{l}a_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+a_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -a_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-a_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\\ +\left|\begin{array}{l}b_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+b_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -b_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-b_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\\ +\left|\begin{array}{l}c_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+c_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -c_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-c_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\\ +\left|\begin{array}{l}d_1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right)+d_1\left(2+\overline{t_1}-\overline{i_1}-\overline{f_1}\right)\\ -d_2\left(2+\underline{t_2}-\underline{i_2}-\underline{f_2}\right)-d_2\left(2+\overline{t_2}-\overline{i_2}-\overline{f_2}\right)\end{array}\right|\end{pmatrix} \tag{4}$$

Theorem 1. The normalized Hamming distance measure $d(.)$ between $\tilde a_1$ and $\tilde a_2$ obeys the following properties:

i. $d(\tilde a_1,\tilde a_2)\ge 0$,
ii. $d(\tilde a_1,\tilde a_2)=d(\tilde a_2,\tilde a_1)$,
iii. $d(\tilde a_1,\tilde a_3)\le d(\tilde a_1,\tilde a_2)+d(\tilde a_2,\tilde a_3)$, where $\tilde a_3=([a_3,\,b_3,\,c_3,\,d_3];[\underline{t_3},\,\overline{t_3}],\,[\underline{i_3},\,\overline{i_3}],[\underline{f_3},\,\overline{f_3}])$is an ITrNN.

Proof.

i. The distance measure $d(\tilde a_1,\tilde a_2)>0$holds for any two $\tilde a_1$ and $\tilde a_2$. If $\tilde a_1\approx\tilde a_2$ that is for $a_1=a_2, b_1=$

$b_2, c_1 = c_2, d_1 = d_2, \underline{t_1} = \underline{t_2}$, $\bar{t}_1 = \bar{t}_2$, $\underline{i_1} = \underline{i_2}$, $\bar{\iota}_1 = \bar{\iota}_2, \underline{f_1} = \underline{f_2}, \bar{f}_1 = \bar{f}_2$, then we have $d(\tilde{a}_1, \tilde{a}_1) = 0$ and consequently, $d(\tilde{a}_1, \tilde{a}_2) \geq 0$.

ii. The proof is obvious.

iii. The normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_3$ is taken as follows:

$$d(\tilde{a}_1, \tilde{a}_3) = \frac{1}{24}\left(\begin{array}{l} \left|\begin{array}{l} a_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+a_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -a_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-a_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} b_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+b_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -b_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-b_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} c_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+c_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -c_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-c_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} d_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+d_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -d_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-d_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \end{array}\right)$$

$$= \frac{1}{24}\left(\begin{array}{l} \left|\begin{array}{l} a_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+a_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ +a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+a_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-a_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -a_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-a_2(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} b_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+b_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ +b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+b_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-b_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -b_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-b_2(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} c_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+c_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ +c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+c_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-c_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -c_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-c_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} d_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+d_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ +d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+d_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-d_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -d_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-d_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \end{array}\right)$$

$$\leq \frac{1}{24}\left(\begin{array}{l} \left|\begin{array}{l} a_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+a_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-a_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \end{array}\right| \\ +\left|\begin{array}{l} b_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+b_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-b_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \end{array}\right| \\ +\left|\begin{array}{l} c_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+c_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-c_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \end{array}\right| \\ +\left|\begin{array}{l} d_1\left(2+\underline{t}_1-\underline{i}_1-\underline{f}_1\right)+d_1(2+\bar{t}_1-\bar{\iota}_1-\bar{f}_1) \\ -d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)-d_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \end{array}\right| \end{array}\right)$$

$$+\frac{1}{24}\left(\begin{array}{l} \left|\begin{array}{l} a_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+a_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -a_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-a_2(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} b_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+b_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -b_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-b_2(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} c_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+c_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -c_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-c_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \\ +\left|\begin{array}{l} d_2\left(2+\underline{t}_2-\underline{i}_2-\underline{f}_2\right)+d_2(2+\bar{t}_2-\bar{\iota}_2-\bar{f}_2) \\ -d_3\left(2+\underline{t}_3-\underline{i}_3-\underline{f}_3\right)-d_3(2+\bar{t}_3-\bar{\iota}_3-\bar{f}_3) \end{array}\right| \end{array}\right)$$

$\leq d(\tilde{a}_1, \tilde{a}_2) + d(\tilde{a}_2, \tilde{a}_3)$. □

4 MADM strategy with interval trapezoidal neutrosophic numbers

In this section we put forward a framework for determining the attribute weights and the ranking orders for all the alternatives with incomplete weight information under interval trapezoidal neutrosophic number environment.

Consider a MADM problem, where $A = \{A_1, A_2, \ldots, A_m\}$ is a set of m alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ is a set of n attributes.The attribute value of alternative $A_i (i = 1,2, \ldots, m)$ over the attribute $C_j (j = 1,2, \ldots, n)$ is expressed in terms of ITrNNs $\tilde{a}_{ij} = \left([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; \tilde{t}_{ij}, \tilde{\iota}_{ij}, \tilde{f}_{ij}\right)$, where, $0 \leq \tilde{t}_{ij} \leq 1, 0 \leq \tilde{\iota}_{ij} \leq 1$, $0 \leq \tilde{f}_{ij} \leq 1$, and $0 \leq \tilde{t}_{ij} + \tilde{\iota}_{ij} + \tilde{f}_{ij} \leq 3$ for $i = 1,2, \ldots, m$ and $j = 1,2, \ldots, n$. Here, $\tilde{t}_{ij}$ denotes the interval truth membership degree, $\tilde{\iota}_{ij}$ denotes the interval indeterminate membership degree, and $\tilde{f}_{ij}$ denotes the interval falsity membership degree to consider the trapezoidal number $[a_{ij}, b_{ij}, c_{ij}, d_{ij}]$ as the rating values of A_i with respect to the attribute C_j.

We consider an MADM problem in the decision matrix form where each entry represents the rating of alternatives with respect to the corresponding attribute. Thus we obtain the following neutrosophic decision matrix (see Equation 5):

$$D = \left(\tilde{a}_{ij}\right)_{m\times n} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{pmatrix}. \qquad (5)$$

We assume that the attributes have different weights. The weight vector of the attributes is prescribed as $W = (\widetilde{w}_1, \widetilde{w}_2, .., \widetilde{w}_n)$, where $\widetilde{w}_j$ is the weight of the attribute $C_j (j = 1,2, \dots, n)$ and expressed in the form of ITrNNs.

Using the following steps, we present MADM strategy under ITrNN environment.

Step-1. Determine the weight of attributes

Weight measure plays an important role in MADM problems and has a direct relationship with the distance measure between two rating values. To deal with decision information with ITrNNs, we use normalized Hamming distance between two ITrNNs.

We assume that the attribute weight $\widetilde{w}_j$ is expressed by-ITrNNs as:

$\widetilde{w}_j = ([w_j^1, w_j^2, w_j^3, w_j^4]; [\underline{t_j}, \overline{t_j}], [\underline{i_j}, \overline{i_j}], [\underline{f_j}, \overline{f_j}])$ for $j = 1,2, .., n$.

If $\Delta(\widetilde{w}_j, \tilde{I}^+)$ is a distance between weight $\widetilde{w}_j$ and $\tilde{I}^+$, then the distance vector is given by

$$\Lambda(\mathrm{W}) = (\Delta(\widetilde{w}_1, \tilde{I}^+), \Delta(\widetilde{w}_2, \tilde{I}^+), \dots, \Delta(\widetilde{w}_n, \tilde{I}^+)), \qquad (6)$$

where

$$\Delta(\widetilde{w}_j, \tilde{I}^+) = \frac{1}{24}\begin{pmatrix} \left|w_j^1\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right) + w_j^1(2+\overline{t_1}-\overline{i_1}-\overline{f_1}) - 6\right| \\ + \left|w_j^2\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right) + w_j^2(2+\overline{t_1}-\overline{i_1}-\overline{f_1}) - 6\right| \\ + \left|w_j^3\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right) + w_j^3(2+\overline{t_1}-\overline{i_1}-\overline{f_1}) - 6\right| \\ + \left|w_j^4\left(2+\underline{t_1}-\underline{i_1}-\underline{f_1}\right) + w_j^4(2+\overline{t_1}-\overline{i_1}-\overline{f_1}) - 6\right| \end{pmatrix} \qquad (7)$$

The corresponding normalized distance vector is given by

$$\bar{\Lambda} = (\bar{\Delta}(\widetilde{w}_1, \tilde{I}^+), \bar{\Delta}(\widetilde{w}_2, \tilde{I}^+), \dots, \bar{\Delta}(\widetilde{w}_n, \tilde{I}^+)) \qquad (8)$$

where, $\bar{\Delta}(\widetilde{w}_j, \tilde{I}^+) = \left[\frac{\Delta(\widetilde{w}_j, \tilde{I}^+)}{\max\limits_j \Delta(\widetilde{w}_j, \tilde{I}^+)}\right]$ for $j = 1,2, .., n$.

The concept of entropy [21] has been extended in this paper and, the entropy measure of the jth attribute (C_j) for m available alternative can be obtained from

$$e_j = -\frac{1}{In(m)}\left[\frac{\bar{\Delta}(\widetilde{w}_j, \tilde{I}^+)}{\sum_{j=1}^{n}\bar{\Delta}(\widetilde{w}_j, \tilde{I}^+)} In\left(\frac{\bar{\Delta}(\widetilde{w}_j, \tilde{I}^+)}{\sum_{j=1}^{n}\bar{\Delta}(\widetilde{w}_j, \tilde{I}^+)}\right)\right]. \qquad (9)$$

Using Equation (9), we finally obtain the normalized weight of the jth attribute

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)}. \qquad (10)$$

Consequently, we get the weight vector $w = (w_1, w_2, \dots, w_n)$, where $0 \le w_j \le 1$ for $j = 1,2, \dots, n$.

Step 2. Determine the aggregated weighted distances between ideal alternative and each alternative

The Hamming distance measure between the attribute value

$$(\tilde{a}_{ij}) = \left([a_{ij}, b_{ij}, c_{ij}, d_{ij}]; [\underline{t_{ij}}, \overline{t_{ij}}], [\underline{i_{ij}}, \overline{i_{ij}}], [\underline{f_{ij}}, \overline{f_{ij}}]\right)$$

and the ideal value $\tilde{I}^+ = ([1, 1, 1, 1]; [1, 1], [0,0], [0,0])$ is obtained as follows:

$$\Delta(\tilde{a}_{ij}, \tilde{I}^+) = \frac{1}{24}\begin{pmatrix} \left|a_{ij}\left(2+\underline{t_{ij}}-\underline{i_{ij}}-\underline{f_{ij}}\right) + a_{ij}(2+\overline{t_{ij}}-\overline{i_{ij}}-\overline{f_{ij}}) - 6\right| \\ + \left|b_{ij}\left(2+\underline{t_{ij}}-\underline{i_{ij}}-\underline{f_{ij}}\right) + b_{ij}(2+\overline{t_{ij}}-\overline{i_{ij}}-\overline{f_{ij}}) - 6\right| \\ + \left|c_{ij}(2+\overline{t_{ij}}-\overline{i_{ij}}-\overline{f_{ij}}) + c_{ij}(2+\overline{t_{ij}}-\overline{i_{ij}}-\overline{f_{ij}}) - 6\right| \\ + \left|d_{ij}(2+\overline{t_{ij}}-\overline{i_{ij}}-\overline{f_{ij}}) + d_{ij}(2+\overline{t_{ij}}-\overline{i_{ij}}-\overline{f_{ij}}) - 6\right| \end{pmatrix}. \qquad (11)$$

Therefore the distance vector for the alternative $A_i (i = 1,2, \dots, m)$ with respect to ideal value $\tilde{I}^+$ can be set as

$$\Lambda(\mathrm{A}_i) = (\Delta(\tilde{a}_{i1}, \tilde{I}^+), \Delta(\tilde{a}_{i2}, \tilde{I}^+), \dots, \Delta(\tilde{a}_{in}, \tilde{I}^+)) \qquad (12)$$

Using Equation (10), and Equation(11), we calculate the aggregated weighted distance $\Delta^{\mathrm{w}}(A_i, \tilde{I}^+)$ between the ideal point and the alternative A_i for $i = 1,2, \dots, m$ as

$$\Delta^{w}(A_i, \tilde{I}^+) = \sum_{j=1}^{n} w_j\, \Delta(\tilde{a}_{ij}, \tilde{I}^+) \text{ for } i = 1,2, \dots, m; j = 1,2, \dots, n. \qquad (13)$$

Step 3. Determine the rank of alternatives

Finally, the ranking of alternatives is performed using the values of the distances $\Delta^{w}(A_i, \tilde{I}^+)$ for $i = 1,2, \dots, m$. The basic idea of ranking the alternative is – smaller the value of $\Delta^{w}(A_i, \tilde{I}^+)$ better the performance/closeness of an alternative to ideal solution.

The schematic diagram of the proposed strategy is presented in the Figure 1.

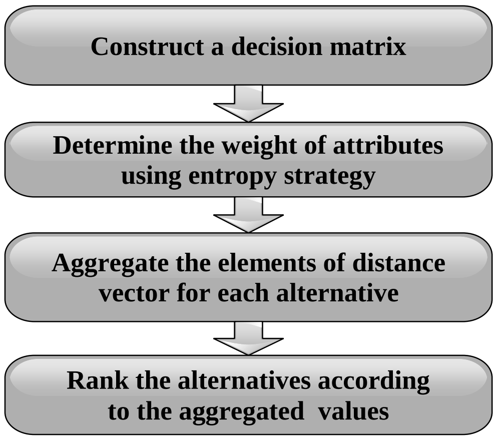

Figure 1. The schematic diagram of the proposed strategy

5 An illustrative numerical example of MADM

In this section, we consider an MADM problem which deals with the supplier selection in supply chain management.

Assume that the MADM problem consists of three suppliers A_1, A_2, A_3 and four attributes C_1, C_2, C_3, C_4.

The four attributes are
1. Product quality(C_1),
2. Service(C_2),
3. Delivery (C_3) and
4. Affordable price(C_4).

We also assume that the alternatives A_1, A_2, A_3 are to be assessed in terms of the interval neutrosophic trapezoidal numbers with respect to the four attributes C_1, C_2, C_3, C_4. The following decision matrix represents the assessment values of alternatives over the attributes:

Table 1. Rating values of alternatives

	C_1	C_2
A_1	$([.3,.4,.5,.6];\ [.6,.7],[.3,.4][.1,.3])$	$([.4,.5,.6,.7];\ [.5,.6],[.4,.5][.2,.3])$
A_2	$([.7,.8,.9,.1.0];\ [.5,.7],[.2,.3][.1,.2])$	$([.6,.7,.8,.9];\ [.4,.6],[.2,.4][.2,.3])$
A_3	$([.2,.3,.5,.6];\ [.5,.6],[.3,.4][.2,.3])$	$([.3,.4,.6,.7];\ [.7,.8],[.1,.2][.1,.2])$
	C_3	C_4
A_1	$([.3,.4,.5,.6];\ [.5,.6],[.3,.4][.2,.3])$	$([.6,.7,.8,.9];\ [.7,.8],[.1,.2][.1,.2])$
A_2	$([.5,.6,.8,.9];\ [.5,.7],[.1,.2][.1,.2])$	$([.6,.7,.8,.9];\ [.6,.8],[.2,.3][.1,.2])$
A_3	$([.6,.7,.8,.9];\ [.5,.6],[.3,.4][.2,.3])$	$([.4,.6,.7,.8];\ [.4,.5],[.2,.3][.1,.2])$

The importance of attributes $C_j (j = 1,2,3,4)$ are given by

$$W = \left\{ \begin{matrix} \langle([.2,.3,.4,.5];\ [.5,.6],[.3,.4][.2,.3])\rangle, \\ \langle([.1,.2,.3,.4];\ [.4,.5],[.1,.2][.1,.2])\rangle \\ \langle([.3,.4,.5,.6];\ [.5,.6],[.3,.4][.2,.3])\rangle, \\ \langle([.4,.5,.6,.7];\ [.3,.5],[.2,.4][.1,.3])\rangle \end{matrix} \right\}$$
$$= \{\widetilde{w}_1, \widetilde{w}_2, \widetilde{w}_3, \widetilde{w}_4\} \quad (12)$$

In order to solve the problem, we consider the following steps:

Step-1. Determine the weights of attributes

Using Equation (7) and Equation (8), we obtain the distance vector with respect to ideal interval neutrosophic trapezoidal number as:

$\Lambda = (0.7725,\ 0.8208,\ 0.7075,\ 0.6517))$.

Utilizing Equation (9) and Equation (10), we obtain the weight vector of the attributes:

$w = \{0.2485,\ 0.2468,\ 0.2509,\ 0.2538\}$.

Step 2. Determine the aggregated weighted distances for each alternative

Using Equation (13) and the weight vector$w$, we obtain the aggregated weighted distances of alternatives:

$\Delta(A_1,\ \tilde{I}^+) = 0.6092, \Delta(A_2,\ \tilde{I}^+) = 0.4512,$ and $\Delta(A_3,\ \tilde{I}^+) = 0.6039.$

Step 3. Rank the alternatives

Smaller value of distance indicates the better alternative. So the ranking of the alternatives appears as:

$$A_2 \succ A_3 \succ A_1.$$

The ranking order reflects that A_2 is the best supplier for the considered problem.

6 Conclusions

In this paper, we have introduced new neutrosophic number called interval neutrosophic trapezoidal number (INTrN) characterized by interval valued truth, indeterminacy, and falsity membership degrees. We have defined some arithmetic operations on INTrNs, and normalized Hamming distance between INTrNs. We have developed a new multi-attribute decision making strategy, where the rating values of alternatives over the attributes and the importance of weight of attributes assume the form of INTrNs. We have used entropy strategy to determine attribute weight and then used it to calculate aggregated weighted distance measure. We have determined ranking order of alternatives with the help of aggregated weighted distance measures. Finally, we have provided an illustrative example to show the feasibility, applicability and effectiveness of the proposed strategy. We hope that the proposed interval neutrosophic trapezoidal number as well as the proposed MADM strategy will be widely applicable in decision making science, especially, in brick selection [22, 23], logistics centre location selection [24, 25], school choice [26], teacher selection [27, 28], weaver selection [29], etc.

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Interval Neutrosophic Tangent Similarity Measure Based MADM strategy and its Application to MADM Problems

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Abstract: In this paper, tangent similarity measure of interval valued neutrosophic sets is proposed and its properties are examined. The concept of interval valued neutrosophic set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. The concept of this tangent similarity measure is based on interval valued neutrosophic information. We present a multi-attribute decision making strategy based on the proposed similarity measure. Using this tangent similarity measure, an application, namely, selection of suitable sector for money investment of a government employee for a financial year is presented. Finally, a comparison of the proposed strategy with the existing strategies has been provided in order to exhibit the effectiveness and practicality of the proposed strategy.

Keywords: Neutrosophic set, interval valued neutrosophic set, tangent function, similarity measure, multi attribute decision making

1 Introduction

Decision making in every real field is a very challenging task for an individual. Decision making is done based on some attributes. In real life situations, attribute information involves indeterminacy, incompleteness and inconsistency. Indeterminacy plays an important role in real world decision-making problems. Neutrosophic set [1] is an important tool to deal with imprecise, indeterminate, and inconsistent data.

The concept of neutrosophic set generalizes the fuzzy set [2], intuitionistic fuzzy set [3]. Wang et al. [4] proposed interval valued neutrosophic sets in which the truth-membership, indeterminacy-membership, and false-membership were extended to interval valued numbers. Realizing the difficulty in applying the neutrosophic sets in realistic problems, Wang et al. [5] introduced the concept of single valued neutrosophic set, a subclass of neutrosophic set. Single valued neutrosophic set can be applied in real scientific and engineering fields. It offers us extra possibility to represent uncertainty, imprecise, incomplete, and inconsistent information.

During the last seven years neutrosophic sets and single valued have been studied and applied in different fields such as medical diagnosis [6, 7], decision making problems [8-12], social problems [13, 14], educational problem [15, 16], image processing [17, 18], conflict resolution [19] , etc.

The concept of similarity is very important for decision making problems. Some strategies [20, 21] have been proposed for measuring the degree of similarity between fuzzy sets. However, these strategies are not capable of dealing with the similarity measures involving indeterminacy, and inconsistency. In the literature, few studies have addressed similarity measures for neutrosophic sets, single-valued neutrosophic sets and interval valued neutrosophic sets [22-28].

Salama and Blowi [29] defined the correlation coefficient on the domain of neutrosophic sets, which is another kind of similarity measure. Broumi and Smarandache [30] extended the Hausdorff distance to neutrosophic sets. After that, a new series of similarity measures has been proposed for neutrosophic set using different approaches. Broumi and Smarandache [31] also proposed the correlation coefficient between interval valued neutrosphic sets. Majumdar and Smanta [32] studied several similarity measures of single valued neutrosophic sets (SVNS) based on distances, a matching function, memebership grades, and entropy measure for a SVNS.

Ye [33] proposed the distance-based similarity measure of SVNSs and applied it to the group decision making problems with single-valued neutrosophic information. Ye [34] also proposed three vector similarity measures for SNSs, an instance of SVNS and interval valued neutrosophic set, including the Jaccard, Dice, and cosine similarity and applied them to multi-attribute decision-making problems with simplified neutrosophic information. Recently, Ye [35] presented similarity measures on interval valued neutrosophic set based on Hamming distance and Euclidean distance and offered a numerical example of its use in decision making problems. Broumi and Smarandache [36] proposed a cosine similarity measure of interval valued neutrosophic sets.

Ye [37] further studied and found that there exsit some disadvantages of existing cosine similarity measures defined in vector space in some situations. Ye [37] mentioned that the defined function may produce absurd result in some real cases. In order to overcome theses disadvantages, Ye [37] proposed improved cosine similarity measures based on cosine function, including single-valued neutrosophic cosine similarity measures and interval valued neutrosophic cosine similarity measures. In his study, Ye [37] proposed medical diagnosis strategy based on the improved cosine similarity measures. Ye and Fu [38] further studied medical diagnosis problem namely, multi-period medical diagnosis using a single-valued neutrosophic similarity measure based on tangent function. Recently, Biswas et al. [39] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. In hybrid environment, Pramanik and Mondal [40] proposed cosine similarity measure of rough neutrosophic sets and provided its application in medical diagnosis. Pramanik and Mondal [41] also proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.

Pramanik and Mondal [42] proposed weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. Pramanik and Mondal [43] also proposed tangent similarity measures between intuitionistic fuzzy sets and studied some of its properties and applied it for medical diagnosis. Mondal and Pramanik [44] also proposed tangent similarity measures between single-valued neutrosophic sets and studied some of its properties and applied in decision making.

Research gap: MADM strategy using similarity measure based on tangent function under interval neutrosophic environment is yet to appear.

Research questions:

- Is it possible to define a new similarity measure between interval neutrosophic sets using tangent function?
- Is it possible to develop a new MADM strategy based on the proposed similarity measure in interval neutrosophic environment?

Having motivated from the above researches on neutrosophic similarity measures, we have extended the concept of neutrosophic tangent similarity measure [44] to interval valued neutrosophic environment. We have defined a new similarity measure called "interval valued tangent similarity measure'' for interval valued neutrosophic sets. The properties of similarity are established. We establish a multi-attribute decision making strategy based on the interval valued tangent similarity measure. The proposed tangent similarity measure based MADM strategy is applied to money investment decision making problem.

The objectives of the paper:

- To define tangent similarity measures for interval valued neutrosophic set environment and prove its basic properties.
- To develop a multi-attribute decision making strategy based on proposed similarity measures.
- To present a numerical example for the effectiveness of the proposed strategy.

Rest of the paper is structured as follows. Section 2 presents neutrosophic preliminaries. In Section 3 we present tangent similarity measure for interval valued neutrosophic sets and prove some of its properties. Section 4 is devoted to presents multi attribute decision-making based on interval valued neutrosophic tangent similarity measure. Section 5 presents the application of the proposed multi attribute decision-making strategy to a problem, namely, money investment of an Indian government employee after a financial year. Section 6 conducts a comparative analysis of the approach to other existing strategies. Section 7 presents the contributions of the paper. Finally, Section 8 presents concluding remarks and scope for future research.

2 Neutrosophic preliminaries

2.1 Neutrosophic sets

Assume that X be an universe of discourse. Then the neutrosophic set [1] P can be presented of the form: $P = \{< x: T_P(x), I_P(x), F_P(x)>, x \in X\}$, where the functions $T, I, F: X \to \,]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set P satisfying the following the condition

$$^-0 \le \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \le 3^+ .$$

For two netrosophic sets (NSs), $P_{NS} = \{<x: T_P(x), I_P(x), F_P(x) > | \ x \in X\}$ and $Q_{NS} = \{< x, T_Q(x), I_Q(x), F_Q(x) > | \ x \in X\}$ the two relations are defined as follows:

(1) $P_{NS} \subseteq Q_{NS}$ if and only if $T_P(x) \le T_Q(x)$, $I_P(x) \ge I_Q(x)$, $F_P(x) \ge F_Q(x)$

(2) $P_{NS} = Q_{NS}$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$

2.2 Single valued neutrosophic sets (SVNS)

Assume that X be a space of points with generic element in X denoted by x. A SVNS [5] P in X is characterized by a truth-membership function $T_P(x)$, an indeterminacy-membership function $I_P(x)$, and a falsity membership function $F_P(x)$, for each point x in X, $T_P(x)$, $I_P(x)$, $F_P(x) \in [0, 1]$. When X is continuous, a SVNS P can be written as follows:

$$P = \int_x \frac{<T_P(x), I_P(x), F_P(x)>}{x} : x \in X$$

When X is discrete, a SVNS P can be written as follows:

$$P = \sum_{i=1}^{n} \frac{<T_P(x_i), I_P(x_i), F_P(x_i)>}{x_i} : x_i \in X$$

For two SVNSs , $P = \{<x: T_P(x), I_P(x), F_P(x)> \mid x \in X\}$ and $Q = \{<x, T_Q(x), I_Q(x), F_Q(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $P \subseteq Q$ if and only if $T_P(x) \le T_Q(x)$, $I_P(x) \ge I_Q(x)$, $F_P(x) \ge F_Q(x)$

(2) $P = Q$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$ for any $x \in X$

2.3 Interval valued neutrosophic sets (IVNS)

Assume that X be a space of points with generic element $x \in X$. An interval valued neutrosophic set [4] A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$, $F_A(x)$ are considered as interval form.

We have, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for all $x \in X$.

Assume that

$A = \{<x, \langle [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] \rangle> \mid x \in X\}$ and

$B = \{<x, \langle [T_B^L(x), T_B^U(x)], [I_B^L(x), I_B^U(x)], [F_B^L(x), F_B^U(x)] \rangle> \mid x \in X\}$ be two IVNS. Then the following relations are defined as follows:

- $A \subseteq B$ if and only if $T_A^L \le T_B^L, T_A^U \le T_B^U$; $I_A^L \ge I_B^L$, $I_A^U \ge I_B^U$; $F_A^L \ge F_B^L$, $F_A^U \ge F_B^U$
- $A = B$ if and only if $T_A^L = T_B^L, T_A^U = T_B^U$; $I_A^L = I_B^L$, $I_A^U = I_B^U$; $F_A^L = F_B^L$, $F_A^U = F_B^U$
  for all $x \in X$

3 Tangent similarity measures for interval valued neutrosophic sets

Definition 1: Assume that

$A = \langle [T_A^L(x_i), T_A^U(x_i)], [I_A^L(x_i), I_A^U(x_i)], [F_A^L(x_i), F_A^U(x_i)] \rangle$ and

$B = \langle [T_B^L(x_i), T_B^U(x_i)], [I_B^L(x_i), I_B^U(x_i)], [F_B^L(x_i), F_B^U(x_i)] \rangle$ be any two interval valued neutrosophic sets. Now, similarity measure based on tangent function between two interval valued neutrosophic sets is defined as follows:

$T_{IVNS}(A,B)=$

$$1 - \frac{1}{n} \sum_{i=1}^{n} \tan\left(\frac{\pi}{12} \left(\begin{array}{l} \left|T_A^\lambda(x_i) - T_B^\lambda(x_i)\right| + \left|I_A^\lambda(x_i) - I_B^\lambda(x_i)\right| \\ + \left|F_A^\lambda(x_i) - F_B^\lambda(x_i)\right| \end{array} \right) \right) \quad (1)$$

Here,

$T_A^\lambda(x_i) = \lambda T_A^L(x_i) + (1-\lambda) T_A^U(x_i)$,

$T_B^\lambda(x_i) = \lambda T_B^L(x_i) + (1-\lambda) T_B^U(x_i)$,

$I_A^\lambda(x_i) = \lambda I_A^L(x_i) + (1-\lambda) I_A^U(x_i)$,

$I_B^\lambda(x_i) = \lambda I_B^L(x_i) + (1-\lambda) I_B^U(x_i)$,

$F_A^\lambda(x_i) = \lambda F_A^L(x_i) + (1-\lambda) F_A^U(x_i)$,

$F_B^\lambda(x_i) = \lambda F_B^L(x_i) + (1-\lambda) F_B^U(x_i)$ and $0 \le \lambda \le 1$.

Theorem 1:

The defined tangent similarity measure $T_{IVNS}(A, B)$ between IVNS A and B satisfies the following properties:

1.1. $0 \le T_{IVNS}(A, B) \le 1$

1.2. $T_{IVNS}(A, B) = 1$ if and only if $A = B$

1.3. $T_{IVNS}(A, B) = T_{IVNS}(B, A)$

1.4. If C is a IVNS in X and $A \subset B \subset C$ then $T_{IVNS}(A, C) \le T_{IVNS}(A, B)$ and $T_{IVNS}(A, C) \le T_{IVNS}(B, C)$.

Proofs:

1.1. Tangent function is monotonic incresing in the interval $[0, \pi/4]$. It also lies in the interval [0, 1]. Therefore, $0 \le T_{IVNS}(A, B) \le 1$.

1.2. For any two IVNS A and B and $0 \le \lambda \le 1$,

$A = B$

$\Rightarrow T_A^\lambda(x_i) = T_B^\lambda(x_i)$, $I_A^\lambda(x_i) = F_B^\lambda(x_i)$, $F_A^\lambda(x_i) = F_B^\lambda(x_i)$

$\Rightarrow \left|T_A^\lambda(x_i) - T_B^\lambda(x_i)\right| = 0$, $\left|I_A^\lambda(x_i) - I_B^\lambda(x_i)\right| = 0$, $\left|F_A^\lambda(x_i) - F_B^\lambda(x_i)\right| = 0$

Therefore, $T_{IVNS}(A, B) = 1$.

Conversely,

$T_{IVNS}(A, B) = 1$

$\Rightarrow \left|T_A^\lambda(x_i) - T_B^\lambda(x_i)\right| = 0$, $\left|I_A^\lambda(x_i) - I_B^\lambda(x_i)\right| = 0$, $\left|F_A^\lambda(x_i) - F_B^\lambda(x_i)\right| = 0$

$\Rightarrow T_A^\lambda(x_i) = T_B^\lambda(x_i)$, $I_A^\lambda(x_i) = F_B^\lambda(x_i)$, $F_A^\lambda(x_i) = F_B^\lambda(x_i)$

Therefore $A = B$.

1.3. $T_{IVNS}(A,B)=$

$$1-\frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|+\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|+\\ \left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$=$

$$1-\frac{1}{n}\sum_{i=1}^{n}\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_B^{\lambda}(x_i)-T_A^{\lambda}(x_i)\right|+\left|I_B^{\lambda}(x_i)-I_A^{\lambda}(x_i)\right|\\ +\left|F_B^{\lambda}(x_i)-F_A^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$= T_{IVNS}(B, A)$

1.4. If $A\subset B\subset C$

then $T_A^{\lambda}(x_i)\le T_B^{\lambda}(x_i)\le T_C^{\lambda}(x_i)$, $I_A^{\lambda}(x_i)\le I_B^{\lambda}(x_i)\le I_C^{\lambda}(x_i)$, $F_A^{\lambda}(x_i)\le F_B^{\lambda}(x_i)\le F_C^{\lambda}(x_i)$,

for $x\in X$. Now, we have the inequalities:

$\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$,

$\left|T_B^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$;

$\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$,

$\left|I_B^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$;

$\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$,

$\left|F_B^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$.

From eqn (1), we can say that $T_{IVNS}(A, C)\le T_{IVNS}(A, B)$ and $T_{IVNS}(A, C)\le T_{IVNS}(B, C)$.

Definition 2: Assume that

$A=\{<x, \left([T_A^L(x), T_A^U(x)],[I_A^L(x), I_A^U(x)],[F_A^L(x), F_A^U(x)]\right)> \mid x\in X\}$ and

$B=\{<x, \left([T_B^L(x), T_B^U(x)],[I_B^L(x), I_B^U(x)],[F_B^L(x), F_B^U(x)]\right)> \mid x\in X\}$ be any two interval valued neutrosophic sets.

Now, weighted similarity measure based on tangent function between two interval valued neutrosophic sets is defined as follows:

$T_{WIVNS}(A,B)=$

$$1-\sum_{i=1}^{n}w_i.\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|+\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\\ +\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\end{array}\right)\right) \quad (2)$$

Here $\sum_{i=1}^{n}w_i=1$.

Theorem 2:

The weighted tangent similarity measure $T_{W\text{-}IVNS}(A,B)$ between IVNS A and B satisfies the following properties:

2.1. $0\le T_{W\text{-}IVNS}(A, B)\le 1$

2.2. $T_{W\text{-}IVNS}(A, B) = 1$ if and only if $A = B$

2.3. $T_{W\text{-}IVNS}(A, B) = T_{W\text{-}IVNS}(B, A)$

2.4. If C is a IVNS in X and $A\subset B\subset C$ then $T_{W\text{-}IVNS}(A, C)\le T_{W\text{-}IVNS}(A, B)$ and $T_{W\text{-}IVNS}(A, C)\le T_{W\text{-}IVNS}(B, C)$.

Proofs:

2.1. Tangent function is monotonic incresing in the interval $[0, \pi/4]$. It also lies in the interval [0, 1] and $\sum_{i=1}^{n}w_i=1$. Therefore, $0\le T_{IVNS}(A, B)\le 1$.

2.2. For any two IVNS A and B,

$A = B$

$\Rightarrow T_A^{\lambda}(x_i)=T_B^{\lambda}(x_i)$, $I_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$, $F_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$

$\Rightarrow \left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|=0$, $\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|=0$, $\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|=0$

Therefore, $T_{W\text{-}IVNS}(A, B) = 1$ for $0\le\lambda\le1$ and $\sum_{i=1}^{n}w_i=1$.

Conversely,

$T_{W\text{-}IVNS}(A, B) = 1$

$\Rightarrow \left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|=0$, $\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|=0$, $\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|=0$

$\Rightarrow T_A^{\lambda}(x_i)=T_B^{\lambda}(x_i)$, $I_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$, $F_A^{\lambda}(x_i)=F_B^{\lambda}(x_i)$

Therefore $A = B$.

2.3. $T_{W\text{-}IVNS}(A, B) =$

$$1-\sum_{i=1}^{n}w_i\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|+\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\\ +\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$=$

$$1-\sum_{i=1}^{n}w_i\tan\left(\frac{\pi}{12}\left(\begin{array}{l}\left|T_B^{\lambda}(x_i)-T_A^{\lambda}(x_i)\right|+\left|I_B^{\lambda}(x_i)-I_A^{\lambda}(x_i)\right|\\ +\left|F_B^{\lambda}(x_i)-F_A^{\lambda}(x_i)\right|\end{array}\right)\right)$$

$= T_{W\text{-}IVNS}(B, A)$

2.4. If $A\subset B\subset C$

then $T_A^{\lambda}(x_i)\le T_B^{\lambda}(x_i)\le T_C^{\lambda}(x_i)$, $I_A^{\lambda}(x_i)\le I_B^{\lambda}(x_i)\le I_C^{\lambda}(x_i)$, $F_A^{\lambda}(x_i)\le F_B^{\lambda}(x_i)\le F_C^{\lambda}(x_i)$, for $x\in X$ and $\sum_{i=1}^{n}w_i=1$. Now, we have the inequalities:

$\left|T_A^{\lambda}(x_i)-T_B^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$,

$\left|T_B^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|\le\left|T_A^{\lambda}(x_i)-T_C^{\lambda}(x_i)\right|$;

$\left|I_A^{\lambda}(x_i)-I_B^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$,

$\left|I_B^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|\le\left|I_A^{\lambda}(x_i)-I_C^{\lambda}(x_i)\right|$;

$\left|F_A^{\lambda}(x_i)-F_B^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$,

$\left|F_B^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|\le\left|F_A^{\lambda}(x_i)-F_C^{\lambda}(x_i)\right|$. From eqn (2), we can say that $T_{W\text{-}IVNS}(A, C)\le T_{W\text{-}IVNS}(A, B)$ and $T_{W\text{-}IVNS}(A, C)\le T_{W\text{-}IVNS}(B, C)$.

The following notations are adopted in the paper.

$P = \{P_1, P_2, ..., P_m\}$ $(m \geq 2)$ is the set of alternatives
$C = \{C_1, C_2, ..., C_n\}$ $(n \geq 2)$ is the set of attributes.
The decision maker provides the ranking of alternatives with respect to each attribute. The ranking presents the performances of alternatives P_i (i = 1, 2,..., m) based on the attributes C_j (j = 1, 2, ..., n). The values associated with the alternatives for multi- attributes decision making problem can be presented in the following decision matrix (see Table 1). The relation between alternatives and attributes in terms of IVNSs are given in the following decision matrix (see Table 1):

Table 1: The decision matrix

	C_1	C_2	...	C_n
P_1	$\left\langle [T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U] \right\rangle$	$\left\langle [T_{12}^L, T_{12}^U], [I_{12}^L, I_{12}^U], [F_{12}^L, F_{12}^U] \right\rangle$	...	$\left\langle [T_{1n}^L, T_{1n}^U], [I_{1n}^L, I_{1n}^U], [F_{1n}^L, F_{1n}^U] \right\rangle$
P_2	$\left\langle [T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U] \right\rangle$	$\left\langle [T_{11}^L, T_{11}^U], [I_{11}^L, I_{11}^U], [F_{11}^L, F_{11}^U] \right\rangle$	...	$\left\langle [T_{2n}^L, T_{2n}^U], [I_{2n}^L, I_{2n}^U], [F_{2n}^L, F_{2n}^U] \right\rangle$
.	...	...	...	...
P_m	$\left\langle [T_{m1}^L, T_{m1}^U], [I_{m1}^L, I_{m1}^U], [F_{m1}^L, F_{m1}^U] \right\rangle$	$\left\langle [T_{m2}^L, T_{m2}^U], [I_{m2}^L, I_{m2}^U], [F_{m2}^L, F_{m2}^U] \right\rangle$	...	$\left\langle [T_{mn}^L, T_{mn}^U], [I_{mn}^L, I_{mn}^U], [F_{mn}^L, F_{mn}^U] \right\rangle$

Here, $\left\langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \right\rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) are interval valued neutrosophic sets. Multi attributes decision making procedure based on tangent similarity measure in interval valued neutrosophic environment is presented using the following steps.

Step 1: Determine the decision matrix in terms of SVNSs

Decision matrix in terms of SVNSs is constructed with the transformation $\Omega_{ij}^{\lambda} = \lambda \Omega_{ij}^{L} + (1-\lambda)\Omega_{ij}^{U}$,

where $\Omega = T, I, F$; i = 1, 2, ..., m; j = 1, 2, ..., n and $0 \leq \lambda \leq 1$.

Table 2: Decision matrix in terms of SVNSs

	C_1	C_2	...	C_n
P_1	$\left\langle T_{11}^{\lambda}, I_{11}^{\lambda}, F_{11}^{\lambda} \right\rangle$	$\left\langle T_{12}^{\lambda}, I_{12}^{\lambda}, F_{12}^{\lambda} \right\rangle$	...	$\left\langle T_{1n}^{\lambda}, I_{1n}^{\lambda}, F_{1n}^{\lambda} \right\rangle$
P_2	$\left\langle T_{21}^{\lambda}, I_{21}^{\lambda}, F_{21}^{\lambda} \right\rangle$	$\left\langle T_{22}^{\lambda}, I_{22}^{\lambda}, F_{22}^{\lambda} \right\rangle$	...	$\left\langle T_{2n}^{\lambda}, I_{2n}^{\lambda}, F_{2n}^{\lambda} \right\rangle$
…	…	…	…	…
…	…	…	…	…
P_m	$\left\langle T_{m1}^{\lambda}, I_{m1}^{\lambda}, F_{m1}^{\lambda} \right\rangle$	$\left\langle T_{m2}^{\lambda}, I_{m2}^{\lambda}, F_{m2}^{\lambda} \right\rangle$	...	$\left\langle T_{mn}^{\lambda}, I_{mn}^{\lambda}, F_{mn}^{\lambda} \right\rangle$

where $\Omega = T, I, F$; i = 1, 2, ..., m; j = 1, 2, ..., n and $0 \leq \lambda \leq 1$.

Step 2: Determine the benefit type attributes and cost type attributes

Generally, the attributes can be categorized into two types: benefit attributes and cost attributes. In the proposed decision making strategy, an ideal alternative can be identified by using a maximum operator for the benefit attributes and a minimum operator for the cost attributes to determine the best value of each attribute among all alternatives. Therefore, we define an ideal alternative as follows:

$P^* = \{C_1^*, C_2^*, \ldots, C_m^*\}$,

Here the benefit attributes is

$$C_j^* = \left\lfloor \max_i T_{C_j}^{\lambda\,(P_i)}, \min_i I_{C_j}^{\lambda\,(P_i)}, \min_i F_{C_j}^{\lambda\,(P_i)} \right\rfloor \tag{3}$$

and the cost attributes is

$$C_j^* = \left[\min_i T_{C_j}^{\lambda\,(P_i)}, \max_i I_{C_j}^{\lambda\,(P_i)}, \max_i F_{C_j}^{\lambda\,(P_i)} \right] \tag{4}$$

Step 3: Calculate of the measure values between ideal alternatives and decision elements

Calculate tangent similarity measures (choosing various values of λ) between ideal alternatives and the decision elements of Table 2 using eqn.(1).

Step 4: Determine the weights of the attributes

The importance of all the attributes may or may not be same in decision making context. The decision maker may use normalized weights or differential weights for attributes based on his/her needs and practical decision making situation. If the attributes are assumed as extremely importance to the decision maker, then the weight of each attribute will be taken as $1/n$ where n is the number of attributes.

Step 5: Determination of the accumulated measure values

To aggregate the similarity measures corresponding to each alternative, we define accumulated measure function (AMF) as follows:

$$D_{AMF}^{i} = \sum_{j=1}^{n} w_j \cdot T_{IVNS}(P_{ij}, P^*) \tag{5}$$

Step 6: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of accumulated measure values. Highest value reflects the best alternative.

Step 7: End

5 Numerical example

Consider the illustrative example, which is very important for Indian government employees after a financial year to select suitable money Investment Company for more tax rebate and more return value after investment span. For a financial year, every government employee desires to invest a sum of money to reduce his/her annual income tax amount and to place the money in more secure investment company. This is the crucial time when most of the government employee gets confused too much and takes a decision which he/she starts to dislike later. Employees often confuse to decide which money Investment Company should choose. If the chosen Investment Company is improper, the employee may encounter a negative impact to his/her future economical condition. It is very important for any employee to choose carefully from various options available to him/her in which he/she is interested. So, it is necessary to utilize a suitable mathematical decision making strategy.

The feature of the proposed strategy is that it includes interval valued truth membership, interval valued indeterminate and interval valued falsity membership function simultaneously. Assume that, a government employee determines to invest a sum of money to a suitable investment sector, namely, Public provident fund (S_1), Postal Life insurance (S_2), Stock Market (S_3). The employee must invest his/her money with respect to the attributes, namely, Growth analysis (C_1), Risk analysis (C_2), Government norms and regulation (C_3). Our solution is to make decision to choose suitable money Investment Company. The values associated with the alternatives for multi- attributes decision-making problem can be presented in the following decision matrix:

Table 3: The decision matrix

$$
\begin{array}{c|ccc}
 & C_1 & C_2 & C_3 \\
\hline
A_1 & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.2,0.4],\\ [0.3,0.5]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.2,0.4],\\ [0.4,0.6],\\ [0.3,0.7]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.4,0.8],\\ [0.5,0.7],\\ [0.4,0.6]\end{array}\right\rangle \\
A_2 & \left\langle \begin{array}{l}[0.4,0.6],\\ [0.3,0.5],\\ [0.5,0.7]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.1,0.3],\\ [0.2,0.6],\\ [0.2,0.6]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.5,0.7],\\ [0.4,0.8],\\ [0.3,0.7]\end{array}\right\rangle \\
A_3 & \left\langle \begin{array}{l}[0.6,0.8],\\ [0.2,0.4],\\ [0.4,0.6]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.3,0.5],\\ [0.1,0.5],\\ [0.3,0.5]\end{array}\right\rangle & \left\langle \begin{array}{l}[0.3,0.7],\\ [0.3,0.5],\\ [0.1,0.5]\end{array}\right\rangle
\end{array}
$$

The decision making calculation is presented using the following steps:

Step 1: Determine the decision matrix in terms of SVNS

Each element of IVNS in Table 3 is transformed to an element of SVNS. This transformation is shown in Table 4.

Table 4: Relation between alternatives and attributes in terms of SVNSs

$$
\begin{array}{c|ccc}
 & C_1 & C_2 & C_3 \\
\hline
A_1 & \left\langle \begin{array}{l}0.6\lambda+0.8(1-\lambda),\\ 0.2\lambda+0.4(1-\lambda),\\ 0.3\lambda+0.5(1-\lambda)\end{array}\right\rangle & \left\langle \begin{array}{l}0.2\lambda+0.4(1-\lambda),\\ 0.4\lambda+0.6(1-\lambda),\\ 0.3\lambda+0.7(1-\lambda)\end{array}\right\rangle & \left\langle \begin{array}{l}0.4\lambda+0.8(1-\lambda),\\ 0.5\lambda+0.7(1-\lambda),\\ 0.4\lambda+0.6(1-\lambda)\end{array}\right\rangle \\
A_2 & \left\langle \begin{array}{l}0.4\lambda+0.6(1-\lambda),\\ 0.3\lambda+0.5(1-\lambda),\\ 0.5\lambda+0.7(1-\lambda)\end{array}\right\rangle & \left\langle \begin{array}{l}0.1\lambda+0.3(1-\lambda),\\ 0.2\lambda+0.6(1-\lambda),\\ 0.2\lambda+0.6(1-\lambda)\end{array}\right\rangle & \left\langle \begin{array}{l}0.5\lambda+0.7(1-\lambda),\\ 0.4\lambda+0.8(1-\lambda),\\ 0.3\lambda+0.7(1-\lambda)\end{array}\right\rangle \\
A_3 & \left\langle \begin{array}{l}0.6\lambda+0.8(1-\lambda),\\ 0.2\lambda+0.4(1-\lambda),\\ 0.4\lambda+0.6(1-\lambda)\end{array}\right\rangle & \left\langle \begin{array}{l}0.3\lambda+0.5(1-\lambda),\\ 0.1\lambda+0.5(1-\lambda),\\ 0.3\lambda+0.5(1-\lambda)\end{array}\right\rangle & \left\langle \begin{array}{l}0.3\lambda+0.7(1-\lambda),\\ 0.3\lambda+0.5(1-\lambda),\\ 0.1\lambda+0.5(1-\lambda)\end{array}\right\rangle
\end{array}
$$

Step 2: Determine the benefit type attributes and cost type attributes

C_1, C_3 are treated as benefit type attributes and C_2 is treated as cost type attributes. Using Table 2, eqn.(3) and eqn.(4), we calculate ideal alternative solutions as follows (Table 5):

Table 5: Ideal alternative solutions

	λ	C_1	C_2	C_3
P^*	0.1	[0.78, 0.38, 0.48]	[0.28, 0.58, 0.66]	[0.76, 0.48, 0.46]
	0.2	[0.76, 0.36, 0.46]	[0.26, 0.56, 0.62]	[0.72, 0.46, 0.42]
	0.3	[0.74, 0.34, 0.44]	[0.24, 0.54, 0.58]	[0.68, 0.44, 0.38]
	0.4	[0.72, 0.32, 0.42]	[0.22, 0.52, 0.54]	[0.62, 0.42, 0.34]
	0.5	[0.70, 0.30, 0.40]	[0.20, 0.50, 0.50]	[0.60, 0.40, 0.30]
	0.6	[0.58, 0.28, 0.38]	[0.18, 0.48, 0.46]	[0.56, 0.38, 0.26]
	0.7	[0.66, 0.26, 0.36]	[0.16, 0.32, 0.42]	[0.56, 0.36, 0.22]
	0.8	[0.64, 0.24, 0.34]	[0.14, 0.44, 0.38]	[0.54, 0.34, 0.18]
	0.9	[0.62, 0.22, 0.32]	[0.12, 0.42, 0.34]	[0.52, 0.32, 0.14]

Step 3: Calculate the measure values between ideal alternatives and decision elements

Using eqn. (1), we calculate tangent similarity measures for different values of λ between ideal alternatives (Table 5) and the decision elements in Table 4 (see Table 6).

Step 4: Determine the weights of the attributes

We take each attribute weight as w_i =1/3 (i = 1, 2, 3).

Step 5: Determine the accumulated measure values

Using eqn. 5, we calculate AMF values as follows (Table 7).

Table 7: Ranking results (with equal attributes weights)

Proposed strategy	λ	Measure values	Ranking orders
	0.1	0.9633; 0.8964; 0.9386	$S_1 \succ S_3 \succ S_2$
	0.2	0.9615; 0.8982; 0.9386	$S_1 \succ S_3 \succ S_2$
$T_{IVNS}(P, P^*)$	0.3	0.9598; 0.9000; 0. 9386	$S_1 \succ S_3 \succ S_2$
	0.4	0.9562; 0.9036; 0.9404	$S_1 \succ S_3 \succ S_2$
	0.5	0.9562; 0.9036, 0.9616	$S_3 \succ S_1 \succ S_2$
	0.6	0.9545; 0.9107; 0.9386	$S_1 \succ S_3 \succ S_2$
	0.7	0.9369;0.9070, 0.9475	$S_3 \succ S_1 \succ S_2$
	0.8	0.9456; 0.9036; 0.9333	$S_1 \succ S_3 \succ S_2$
	0.9	0.9420; 0.9036, 0.9316	$S_1 \succ S_3 \succ S_2$

Step 6: Ranking the alternatives

Ranking of the alternatives is prepared based on the descending order of accumulated measure values. When $\lambda = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 0.9,$ Public provident fund (S_1) is the best alternative to invest money (see Table 7). When $\lambda = 0.5, 0.7,$ Stock market (S_3) is the best alternative to invest money (see Table 7).

6 Comparative analysis

For the sake of validating the flexibility and feasibility of the proposed strategy, a comparative study is conducted. In order to do so, different existing strategies are used to solve the same decision-making problem with the interval valued neutrosophic information. Literature review reflects that Broumi and Smarandache [36] proposed cosine similarity measure of interval valued neutrosophic sets. Ye [35] proposed Similarity measures between interval neutrosophic sets and apply in multicriteria decision-making. Şahin [45] proposed cross-entropy measure on interval valued neutrosophic sets and presenter its applications in multicriteria decision making. Table 8 shows that the ranking results obtained from different strategy differ. Ranking results from proposed strategy with $\lambda = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 0.9$ are similar to the ranking result of cosine similarity measure [36] (Broumi and Smarandache, 2014). Ranking results obtained from proposed strategy with $\lambda = 0.5, 0.7$ are similar to the ranking results of Ye`s strategy (Ye, 2014d) and cross entropy strategy [45].

Table 8: The ranking results of different strategies

strategies	Ranking results
Proposed strategy with $\lambda = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 0.9$	$S_1 \succ S_3 \succ S_2$
Proposed strategy with $\lambda = 0.5, 0.7$	$S_3 \succ S_1 \succ S_2$
Cosine similarity measure (Broumi and Smarandache, [36]	$S_1 \succ S_2 \succ S_3$
Ye [35]	$S_3 \succ S_1 \succ S_2$
Cross entropy strategy [45]	$S_3 \succ S_1 \succ S_2$

7. Contributions of the paper

- We define tangent similarity measures for IVNS. We have also proved their basic properties.
- We developed a decision making strategy based on the proposed weighted tangent similarity measure.
- Steps and calculations of the proposed strategy are easy to use.
- We have solved a numerical example to show the applicability of the proposed strategy.

8. Conclusion

In this paper, we have defined tangent similarity measure and proved its properties in interval valued neutrosophic environment. We also also developed a novel multi attribute decision making strategy based on the proposed tangent similarity measure in interval valued neutrosophic environment. We have presented an application, namely, selection of best investment sector for an Indian government employee. We also presented a comparative analysis with the existing strategies in the literature.The concept presented in this paper can be applied in teacher selection, school choice, medical diagnosis, pattern rcognition, purchasing decision making, commodity recommendation in interval valued neutrosophic environment. It is worth of further study to formulate a multi attribute decision making strategy that considers the priority of attributes.

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Table 6: Tangent similarity measure values

	$\lambda=0.1$			$\lambda=0.2$			$\lambda=0.3$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
S_1	1.0000	0.9738	0.9160	1.0000	0.9738	0.9108	1.0000	0.9738	0.9055
S_2	0.8683	0.9686	0.8523	0.8683	0.9633	0.8630	0.8683	0.9581	0.8737
S_3	0.9738	0.8683	0.9738	0.9738	0.8683	0.9738	0.9738	0.8683	0.9738
	$\lambda=0.4$			$\lambda=0.5$			$\lambda=0.6$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
S_1	1.0000	0.9738	0.8949	1.0000	0.9738	0.8949	1.0000	0.9738	0.8896
S_2	0.8683	0.9528	0.8896	0.8683	0.9476	0.8949	0.8949	0.9423	0.8949
S_3	0.9738	0.8683	0.9790	0.9738	0.9371	0.9738	0.9738	0.8683	0.9738
	$\lambda=0.7$			$\lambda=0.8$			$\lambda=0.9$		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
S_1	1.0000	0.9371	0.8737	1.0000	0.9738	0.8630	1.0000	0.9738	0.8523
S_2	0.8683	0.9738	0.8790	0.8683	0.9318	0.9108	0.8949	0.9266	0.9160
S_3	0.9738	0.9055	0.9633	0.9738	0.8683	0.9580	0.9738	0.8683	0.9528

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VIKOR Based MAGDM strategy under Bipolar Neutrosophic Set Environment

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Abstract. In this paper, we extend the VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) strategy to multiple attribute group decision-making (MAGDM) with bipolar neutrosophic set environment. In this paper, we first define VIKOR strategy in bipolar neutrosophic set environment to handle MAGDM problems, which means we combine the VIKOR with bipolar neutrosophic number to deal with MAGDM. We propose a new strategy for solving MAGDM. Finally, we solve MAGDM problem using our newly proposed VIKOR strategy under bipolar neutrosophic set environment. Further, we present sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives.

Keywords: Bipolar neutrosophic sets, VIKOR strategy, Multi attribute group decision making.

1 Introduction

In 1965, Zadeh [1] first introduced the fuzzy set to deal with the vague, imprecise data in real life specifying the membership degree of an element. Thereafter, in 1986 Atanassov [2] introduced intuitionistic fuzzy set to tackle the uncertainity in data in real life expressing membership degree and non-membership degree of an element as independent component. As a generalization of classical set, fuzzy set and intuitionistic fuzzy set, Smarandache [3] introduced the neutrosophic set by expressing the membership degree (truth membership degree), indeterminacy degree and non-membership degree (falsity membership degree) of an element independently. For real applications of neutrosophic set, Wang et al. [4] introduced the single valued neutrosophic set which is a sub class of neutrosophic set.

Decision making process involves seleting the best alternative from the set of feasible alternatives. There exist many decision making strategies in crisp set environment[5-7], fuzzy [8-12], intuitionistic fuzzy set environment [13-19]. vauge set environment [20, 21]. Theoretical as well as practical applications multi attribute decision making (MADM) of SVNS environment [22-42] and interval neutrosophic set (INS) environment [43-56] have been reported in the literaure. Recently, decision making in hybrid neutrosophic set environment have drawn much attention of the researches such as rough neutrosophic environment [57-73], neutrosophic soft set environment [74-80], neutrosophic soft expert set environment [81-82], neutrosophic hesitant fuzzy set environment [83-87], neutrosophic refined set environment [88-93], neutrosophic cubic set environment [94-104], etc. In 2015, Deli *et al.* [105] proposed bipolar neutrosophic set (BNS) using the concept of bipolar fuzzy sets [106, 107] and neutrosophic sets [3]. A BNS consists of two fully independent parts, which are positive membership degrees $T^+ \rightarrow [0, 1]$, $I^+ \rightarrow [0, 1]$, $F^+ \rightarrow [0, 1]$, and negative membership degrees $T^- \rightarrow [-1, 0]$, $I^- \rightarrow [-1, 0]$, $F^- \rightarrow [-1, 0]$ where the positive membership degrees T^+, I^+, F^+ represent truth membership degree, indeterminacy membership degree and false membership degree respectively of an element and the negative membership degrees T^-, I^-, F^- represent truth membership degree, indeterminacy membership degree and false membership degree respectively of an element to some implicit counter property corresponding to a BNS. Deli *et al.* [105] defined some operations namely, score function, accuracy function, and certainty function to compare BNSs and provided some operators in order to aggregate BNSs. Deli and Subas [108] defined correlation coefficient similarity measure for dealing with MADM problems under bipolar set environment. Şahin *et al*. [109] proposed Jaccard vector similarity measure for MADM problems under bipolar neutrosophic set environment. Uluçay *et al*. [110] presented Dice similarity measure, weighted Dice similarity measure, hybrid vector similarity measure, weighted hybrid vector similarity measure for BNSs and established a MADM strategy by employing the proposed similarity measures. Dey *et al*. [111] established TOPSIS strategy for MADM problems with bipolar neutrosophic information where the weights of the attributes are completely unknown to the decision maker. Pramanik *et al.* [112] defined projection, bidirectional projection and hybrid projection measures for BNSs and proved their basic properties. In the same study, Pramanik *et al*. [112], proposed three new MADM strategies based on the proposed projection, bidirectional projection and hybrid projection measures with bipoar neutrosophic information. Wang *et al.* [113] defined Frank operations of bipolar neutrosophic numbers (BNNs) and proposed Frank bipolar neutrosophic Choquet Bonferroni mean operators by combining Choquet integral operators and Bonferroni mean operators based on Frank operations of BNNs. In the same study, Wang et al. [113] developed MADM strategy based on Frank Choquet Bonferroni operators of BNNs in bipolar neutrosophic environment. Recently, many researcher has given attention to develop various strategies under bipolar neutrosophic set environment in various fields [114-117].

Opricovic [118] proposed the VIKOR strategy for a MCDM problem with conflicting attributes [119-120]. In 2015, Bausys and Zavadskas [121] proposed VIKOR strategy to solve multi criteria decision making problem in interval neutrosophic set environment. Further, Hung et al. [122] proposed VIKOR strategy for interval neutrosophic multi attribute group decision making (MAGDM). Pouresmaeil *et al.* [123] proposed a MAGDM strategy based on TOPSIS and VIKOR strategies in single valued neutrosophic set environment. Liu and Zhang [124] extended VIKOR strategy in neutrosophic hesitant fuzzy set environment. Hu *et al.* [125] proposed interval neutrosophic projection based VIKOR strategy and applied it for doctor selection. Selvakumari et al. [126] proposed VIKOR strategy for decision making problem using octagonal neutrosophic soft matrix.

VIKOR strategy in bipolar neutrosophic set is yet to appear.

Research gap:

VIKOR based MAGDM strategy in BNS environment. This study answers the following research questions:

i. Is it possible to extend VIKOR strategy in BNS environment?

ii. Is it possible to develop a new VIKOR based MAGDM strategy in BNS environment?

Motivation:

The above-mentioned analysis [118-126] describes the motivation behind proposing a novel VIKOR strategy for MAGDM in the BNS environment. This study develops a novel VIKOR strategy for MAGDM that can deal with multiple decision-makers.

The objectives of the paper are:

i. To extend VIKOR strategy in BNS environment.

ii. To develop a new MAGDM strategy based on proposed VIKOR strategy in BNS environment.

To fill the research gap, we propose VIKOR based strategy, which is capable of dealing with MAGDM problem in BNS environment.

The main contributions of this paper are summarized below:

i. We extend VIKOR strategy in bipolar neutrosophic environment.

ii. We introduce a bipolar neutrosophic weighted aggregation operator and prove its basic properties.

iii. We develop a novel VIKOR based MAGDM strategy in bipolar neutrosophic set environment to solve MAGDM problems.

iv. In this paper, we solve a MAGDM problem based on proposed VIKOR strategy.

The remainder of this paper is organized as follows: In the Section 2, we review some basic concepts and operations related to neutrosophic set, single valued neutrosophic set (SVNS), bipolar neutrosophic set. In Section 3, we propose the bipolar neutrosophic number weighted aggregation (BNNWA) operator and prove its basic properties. In section 4, we develop a novel MAGDM strategy based on VIKOR strategy to solve the MADGM problems with bipolar neutrosophic information. In Section 5, we present an example to illustrate the proposed strategy. Then in Section 6, we present the sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives.. In section 7, we present conclusion and future direction of research.

2. Preliminaries

In this section, we describe the basic definitions related to neutrosophic sets, bipolar neutrosophic sets.

Definition 2.1 Neutrosophic set

Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets [3] A in U is characterized by a truth-membership function $T_A(u)$, an indeterminacy-membership function $I_A(u)$ and a falsity-membership function $F_A(u)$,

where, $T_A(u), I_A(u), F_A(u): U \to]^-0, 1^+[$.

Neutrosophic set A can be written as:

$A = \{u, < T_A(u), I_A(u), F_A(u) >: u \in U\}$,

where, $T_A(u), I_A(u), F_A(u) \in]^-0, 1^+[$.

The sum of $T_A(u), I_A(u), F_A(u)$ is

$^-0 \le T_A(u) + I_A(u) + F_A(u) \le 3^+$.

Definition 2.2: Single valued neutrosophic set

Let U be a space of points (objects) with a generic element in U denoted by u. A single valued neutrosophic set [4] J in U is characterized by a truth-membership function $T_J(u)$, an indeterminacy-membership function $I_J(u)$ and a falsity-membership function $F_J(u)$, where,

$T_J(u), I_J(u), F_J(u): U \to [0, 1]$. A single valued neutrosophic set J can be expressed by

$J = \{u, < (T_J(u), I_J(u), F_J(u)) >: u \in U\}$.

Therefore for each $u \in U$, $T_J(u), I_J(u), F_J(u) \in [0, 1]$ the sum of three functions lies between 0 and 1, i.e.

$0 \le T_J(u) + I_J(u) + F_J(u) \le 3$.

Definition 2.3: Bipolar neutrosophic set

Let U be a space of points (objects) with a generic element in U denoted by u. A bipolar neutrosophic set [105] H in U is defined as an object of the form

$H = \{u, < T_H^+(u), I_H^+(u), F_H^+(u), T_H^-(u), T_H^-(u), F_H^-(u) >: u \in U\}$, where,

$T_H^+(u), I_H^+(u), F_H^+(u): U \to [0,1]$ and

$T_H^-(u), I_H^-(u), F_H^-(u): U \to [-1, 0]$.

We denote

$H = \{u, < T_H^+(u), I_H^+(u), F_H^+(u), T_H^-(u), I_H^-(u), F_H^-(u) >: u \in U\}$ simply $H = < T_H^+, I_H^+, F_H^+, T_H^-, I_H^-, F_H^- >$ as a bipolar neutrosophic number (BNN).

Definition 2.4 Containment of two bipolar neutrosophic sets [105]

Let

$H_1 = \{u, < T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u) >: u \in U\}$

and

$H_2 = \{u, < T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u) >: u \in U\}$ be any two bipolar neutrosophic sets in U. Then $H_1 \subseteq H_2$ iff

$T_1^+(u) \le T_2^+(u)$, $I_1^+(u) \ge I_2^+(u)$, $F_1^+(u) \ge F_2^+(u)$ and $T_1^-(u) \ge T_2^-(u)$, $I_1^-(u) \le I_2^-(u)$, $F_1^-(u) \le F_2^-(u)$ for all $u \in U$.

Definition 2.5 Equality of two bipolar neutrosophic sets [103]

Let

$H_1 = \{u, < T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u) >: u \in U\}$

and

$H_2 = \{u, < T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u) >: u \in U\}$

be any two bipolar neutrosophic sets in U. Then, $H_1 = H_2$ iff $T_1^+(u) = T_2^+(u)$, $I_1^+(u) = I_2^+(u)$, $F_1^+(u) = F_2^+(u)$ and $T_1^-(u) = T_2^-(u)$, $I_1^-(u) = I_2^-(u)$, $F_1^-(u) = F_2^-(u)$ for all $u \in U$.

Definition 2.6 Union of any two bipolar neutrosophic sets [105]

Let $H_1 = \{u, < T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u) >: u \in U\}$ and $H_2 = \{u, < T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u) >: u \in U\}$ be any two bipolar neutrosophic sets in U. Then, their union is defined as follows:

$H_3(u) = H_1(u) \cup H_2(u) = \{u, < \max(T_1^+(u), T_2^+(u))$,
$\min(I_1^+(u), I_2^+(u)), \min(F_1^+(u), F_2^+(u))$,
$\min(T_1^-(u), T_2^-(u)), \max(I_1^-(u), I_2^-(u))$,
$\max(F_1^-(u), F_2^-(u)) >: u \in U\}$, for all $u \in U$.

Definition 2.7 Intersection of two bipolar neutrosophic sets

Let $H_1 = \{u, < T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u) >: u \in U\}$ and $H_2 = \{u, < T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u) >: u \in U\}$ be any two bipolar neutrosophic sets in U. Then, their intersection [105] is defined as follows:

$H_4(u) = H_1(u) \cap H_2(u) = \{u, < \min(T_1^+(u), T_2^+(u))$,
$\max(I_1^+(u), I_2^+(u)), \max(F_1^+(u), F_2^+(u))$,
$\max(T_1^-(u), T_2^-(u)), \min(I_1^-(u), I_2^-(u))$,
$\min(F_1^-(u), F_2^-(u)) >: u \in U\}$ for all $u \in U$.

Definition 2.8 Complement of a bipolar neutrosophic set [105]

Let $H_1 = \{u, < T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u) >: u \in U\}$ be a bipolar neutrosophic set in U. Then the complement of H_1 is denoted by H_1^c and is defined by

$H_1^c = \{u, < 1 - T_1^+(u), 1 - I_1^+(u), 1 - F_1^+(u), \{-1\} - T_1^-(u)$,
$\{-1\} - I_1^-(u), \{-1\} - F_1^-(u) >: u \in U\}$
for all $u \in U$.

Definition 2.13 Hamming distance measure between two BNNs [115]

Let $h_1 = < T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^- >$ and $h_2 = < T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^- >$ be any two BNNs in U. Then Hamming distance measure between h_1 and h_2 is denoted by $D(h_1, h_2)$ and defined as follows:

$$D(h_1,h_2) = \frac{1}{6}[|T_1^+ - T_2^+| + |I_1^+ - I_2^+| + |F_1^+ - F_2^+| + |T_1^- - T_2^-| + |I_1^- - I_2^-| + |F_1^- - F_2^-|] \quad (1)$$

Definition 2.14: Normalization procedure

In decision making situation, cost type attribute and benefit type attribute may exist simultaneously. Assume that, h_{ij} be a BNN to express the rating value of i-th alternative with respect to j-th attribute (c_j). If c_j belongs to the cost type attributes, then h_{ij} should be standardized by employing the complement of BNN h_{ij}. When the attribute c_j belongs to benefit type attributes, h_{ij} does not need to be standardized, we use the following formula of normalization as follows:

$$h_{ij}^* = <\{1\} - T_{ij}^+, \{1\} - I_{ij}^+, \{1\} - F_{ij}^+, \{-1\} - T_{ij}^-, \{-1\} - I_{ij}^-, \{-1\} - F_{ij}^- > \quad (2)$$

3. Bipolar neutrosophic number weighted aggregation operator

Let $\{h_{ij}^1, h_{ij}^2, ..., h_{ij}^t\}$ be the set of t bipolar neutrosophic numbers and $\{\beta_1, \beta_2, \beta_3, ..., \beta_t\}$ be the set of corresponding weights of t bipolar neutrosophic numbers with conditions $\beta_p \geq 0$ and $\sum_{p=1}^{t}\beta_p = 1$. Then the bipolar neutrosophic number weighted aggregation (BNNWA) operator is defined as follows:

$$h_{ij} = BNNWA_\beta(\tilde{h}_{ij}^1 \; \tilde{h}_{ij}^2 ... \; \tilde{h}_{ij}^t) =$$
$$(\beta_1\tilde{h}_{ij}^1 \oplus \beta_2\tilde{h}_{ij}^2 \oplus \beta_3\tilde{h}_{ij}^3 \oplus ... \oplus \beta_p\tilde{h}_{ij}^t) =$$
$$< \left(\sum_{p=1}^{t}\beta_p \tilde{T}_{ij}^{+(p)}, \sum_{p=1}^{t}\beta_p \tilde{I}_{ij}^{+(p)}, \sum_{p=1}^{t}\beta_p \tilde{F}_{ij}^{+(p)}, \sum_{p=1}^{t}\beta_p \tilde{T}_{ij}^{-(p)}, \sum_{p=1}^{t}\beta_p \tilde{I}_{ij}^{-(p)}, \sum_{p=1}^{t}\beta_p \tilde{F}_{ij}^{-(p)} \right) > \quad (3)$$

The BNNWA operator satisfies the following properties:
1. Idempotency
2. Monotoncity
3. Boundedness

Property: 1. Idempotency

If all $h_{ij}^1, \; h_{ij}^2, ... \;, h_{ij}^t = h$ are equal, then

$$h_{ij} = BNNWA_\beta(h_{ij}^1, \; h_{ij}^2, ... \;, h_{ij}^t) = h$$

Proof:

Since $h_{ij}^1 = h_{ij}^2 = ... \; = h_{ij}^t = h$, based on the Equation (3) and with conditions, $\beta_p \geq 0$ and $\sum_{p=1}^{t}\beta_p = 1$, we obtain

$$h_{ij} = BNNWA_\beta(h_{ij}^1, h_{ij}^2, ... \;, h_{ij}^t) =$$
$$(\beta_1 h_{ij}^1 \oplus \beta_2 h_{ij}^2 \oplus \beta_3 h_{ij}^3 \oplus ... \oplus \beta_t h_{ij}^t) =$$
$$(\beta_1 h \oplus \beta_2 h \oplus \beta_3 h \oplus ... \oplus \beta_t h) =$$
$$< \left([T^+ \sum_{p=1}^{t}\beta_p, I^+ \sum_{p=1}^{t}\beta_p, F^+ \sum_{p=1}^{t}\beta_p, T^- \sum_{p=1}^{t}\beta_p, I^- \sum_{p=1}^{t}\beta_p, F^- \sum_{p=1}^{t}\beta_p] \right) >$$
$$= < \left(T^+, I^+, F^+, T^-, I^-, F^- \right) > = h.$$

Property: 3. Monotonicity

Assume that $\{h_{ij}^1, h_{ij}^2, .. \;, h_{ij}^t\}$ and $\{h_{ij}^{*1}, h_{ij}^{*2}, .. \;, h_{ij}^{*t}\}$ be any two set of collections of t bipolar neutrosophic nubers with the condition $t_{ij}^p \leq t_{ij}^{*p}$ (p = 1, 2, ..., t), then

$$BNNWA_\beta(h_{ij}^1, h_{ij}^2, ..., h_{ij}^t) \leq BNNWA_\beta(h_{ij}^{*1}, h_{ij}^{*2}, ..., h_{ij}^{*t}).$$

Proof:

From the given condition $T_{ij}^{+(p)} \leq T_{ij}^{+*(p)}$, we have

$$\beta_p T_{ij}^{+(p)} \leq \beta_p T_{ij}^{+*(p)}$$
$$\Rightarrow \sum_{p=1}^{t}\beta_p T_{ij}^{+(p)} \leq \sum_{p=1}^{t}\beta_p T_{ij}^{+*(p)}.$$

From the given condition $I_{ij}^{+(p)} \geq I_{ij}^{+*(p)}$, we have

$$\beta_p I_{ij}^{+(p)} \geq \beta_p I_{ij}^{+*(p)}$$
$$\Rightarrow \sum_{p=1}^{t}\beta_p I_{ij}^{+(p)} \geq \sum_{p=1}^{t}\beta_p I_{ij}^{+*(p)}.$$

From the given condition $F_{ij}^{+(p)} \geq F_{ij}^{+*(p)}$, we have

$$\beta_p F_{ij}^{+(p)} \geq \beta_p F_{ij}^{+*(p)}$$
$$\Rightarrow \sum_{p=1}^{t}\beta_p F_{ij}^{+(p)} \geq \sum_{p=1}^{t}\beta_p F_{ij}^{+*(p)}.$$

From the given condition $T_{ij}^{-(p)} \geq T_{ij}^{-*(p)}$, we have

$$\beta_p T_{ij}^{-(p)} \geq \beta_p T_{ij}^{-*(p)}$$
$$\Rightarrow \sum_{p=1}^{t}\beta_p T_{ij}^{-(p)} \geq \sum_{p=1}^{t}\beta_p T_{ij}^{-*(p)}.$$

From the given condition $I_{ij}^{-(p)} \leq I_{ij}^{-*(p)}$, we have

$$\beta_p I_{ij}^{-(p)} \leq \beta_p I_{ij}^{-*(p)}$$
$$\Rightarrow \sum_{p=1}^{t}\beta_p I_{ij}^{-(p)} \leq \sum_{p=1}^{t}\beta_p I_{ij}^{-*(p)}.$$

From the given condition $F_{ij}^{-(p)} \leq F_{ij}^{-*(p)}$, we have

$$\beta_p F_{ij}^{-(p)} \leq \beta_p F_{ij}^{-*(p)}$$
$$\Rightarrow \sum_{p=1}^{t}\beta_p F_{ij}^{-(p)} \leq \sum_{p=1}^{t}\beta_p F_{ij}^{-*(p)}.$$

From the above relations, we obtain

$$BNNWA_\beta(h_{ij}^1, h_{ij}^2, ..., h_{ij}^t) \leq BNNWA_\beta(h_{ij}^{*1}, h_{ij}^{*2}, ..., h_{ij}^{*t}).$$

$$\mathrm{NCNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le \mathrm{BNNWA}_{\beta}(h_{ij}^{*1}, h_{ij}^{*2}, ..., h_{ij}^{*t}).$$

Property: 2. Boundedness

Let $\{h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}\}$ be any collection of t bipolar neutrosophic numbers.

If

$$h^{+} = < \max_{p}\{T_{ij}^{+(p)}\}, \min_{p}\{I_{ij}^{+(p)}\}, \min_{p}\{F_{ij}^{+(p)}\}, \min_{p}\{T_{ij}^{-(p)}\},$$
$$\max_{p}\{I_{ij}^{-(p)}\}, \max_{p}\{F_{ij}^{-(p)}\} >$$
$$h^{-} = < \min_{p}\{T_{ij}^{+(p)}\}, \max_{p}\{I_{ij}^{+(p)}\}, \max_{p}\{F_{ij}^{+(p)}\}, \max_{p}\{T_{ij}^{-(p)}\},$$
$$\min_{p}\{I_{ij}^{-(p)}\}, \min_{p}\{F_{ij}^{-(p)}\} > \text{(p= 1, 2, 3, ...,t).}$$

Then, $h^{-} \le \mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le h^{+}$.

Proof:

From Property 1 and Property 2, we obtain

$$\mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \ge \mathrm{BNNWA}_{\beta}(h^{-}, h^{-}, ..., h^{-}) = h^{-}$$

and

$$\mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le \mathrm{BNNWA}_{\beta}(h^{+}, h^{+}, ..., h^{+}) = h^{+}.$$

So, we have

$$h^{-} \le \mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le h^{+}.$$

4. VIKOR strategy for solving MAGDM problem under bipolar neutrosophic environment

In this section, we propose a MAGDM strategy under bipolar neutrosophic set environment. Assume that, $A = \{A_1, A_2, A_3, ..., A_r\}$ be a set of r alternatives and $C = \{c_1, c_2, c_3, ..., c_s\}$ be a set of s attributes. Assume that, $\alpha = \{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_s\}$ be the weight vector of the attributes, where $\alpha_k \ge 0$ and $\sum_{k=1}^{s}\alpha_k = 1$. Let $DM = \{DM_1, DM_2, DM_3, ..., DM_t\}$ be the set of t decision makers and $\beta = \{\beta_1, \beta_2, \beta_3, ..., \beta_t\}$ be the set of weight vector of decision makers, where $\beta_p \ge 0$ and $\sum_{p=1}^{t}\beta_p = 1$.

In this section, we describe the VIKOR based MAGDM strategy under bipolar neutrosophic set environment. The proposed strategy consists of the following steps (see Figure 1):

Step: 1. Construction of the decision matrix

Let $M^{p} = (h_{ij}^{p})_{r\times s}$ (p = 1, 2, 3, …, t) be the p-th decision matrix, where information about the alternative A_i is provided by the decision maker DM_p with respect to attribute c_j (j = 1, 2, 3, …, s). The p-th decision matrix denoted by M^{p} (See eq. (4)) is constructed as follows:

$$M^{p} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ A_r \end{array}\begin{pmatrix} c_1 & c_2 & \dots & c_s \\ h_{11}^{p} & h_{12}^{p} & \dots & h_{1s}^{p} \\ h_{21}^{p} & h_{22}^{p} & & h_{2s}^{p} \\ . & & \dots & . \\ h_{r1}^{p} & h_{r2}^{p} & \dots & h_{rs}^{p} \end{pmatrix} \quad (4)$$

Here p = 1, 2, 3,…, t; i = 1, 2, 3,…, r; j = 1, 2, 3,…, s.

Step: 2. Normalization of the decision matrix

Cost type attributes and benefit type attributes are generally existed in decision making process. Therefore the considered attribute values need to be normalized to aviod different physical dimensional unit. To normalize we can use the following equation:

$$h_{ij}^{*} = < \{1\} - T_{ij}^{+}, \{1\} - I_{ij}^{+}, \{1\} - F_{ij}^{+},$$
$$\{-1\} - T_{ij}^{-}, \{-1\} - I_{ij}^{-}, \{-1\} - F_{ij}^{-} > .$$

Using the normalized method, we obtain the following normalized decision matrix (See eq. (5)):

$$M^{p} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ A_r \end{array}\begin{pmatrix} c_1 & c_2 & \dots & c_s \\ \tilde{h}_{11}^{p} & \tilde{h}_{12}^{p} & \dots & \tilde{h}_{1s}^{p} \\ \tilde{h}_{21}^{p} & \tilde{h}_{22}^{p} & & \tilde{h}_{2s}^{p} \\ . & & \dots & . \\ \tilde{h}_{r1}^{p} & \tilde{h}_{r2}^{p} & \dots & \tilde{h}_{rs}^{p} \end{pmatrix} \quad (5)$$

Where,

$$\tilde{h}_{ij}^{p} = \begin{cases} h_{ij}^{p} & \text{if } c_j \text{ is benefit type attribute.} \\ (h_{ij}^{*})^{p} & \text{if } c_j \text{ is cost type attribute.} \end{cases}$$

Step: 3. Aggregation of the decision matrices

Using BNNWA operator in eq. (3), we obtain the aggregated decision matrix as follows:

$$M = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ A_r \end{array}\begin{pmatrix} c_1 & c_2 & \dots & c_s \\ h_{11} & h_{12} & \dots & h_{1s} \\ h_{21} & h_{22} & & h_{2s} \\ . & & \dots & . \\ h_{r1} & h_{r2} & \dots & h_{rs} \end{pmatrix} \quad (6)$$

where, i = 1, 2, 3, …, r; j = 1, 2, 3, …, s; *p=1, 2, ….t.*

Step: 4. Define the positive ideal solution and negative ideal solution

$$h_{ij}^{+} = < \max_{i} T_{ij}^{+}, \min_{i} I_{ij}^{+}, \min_{i} F_{ij}^{+}, \min_{i} T_{ij}^{-}, \max_{i} I_{ij}^{-}, \max_{i} F_{ij}^{-} > \quad (7)$$

$$h_{ij}^{-} = < \min_{i} T_{ij}^{+}, \max_{i} I_{ij}^{+}, \max_{i} F_{ij}^{+}, \max_{i} T_{ij}^{-}, \min_{i} I_{ij}^{-}, \min_{i} F_{ij}^{-} >$$

(8)

Step: 5. Define and compute the value of Γ_i and Z_i (i = 1, 2, 3, ..., r)

Γ_i and Z_i represent the average and worst group scores for the alternative A_i respectively, with the relations

$$\Gamma_i = \sum_{j=1}^{s} \frac{\alpha_j \times D(h_{ij}^{+}, \tilde{h}_{ij})}{D(h_{ij}^{+}, h_{ij}^{-})} \quad (9)$$

$$Z_i = \max_j \left\{ \frac{\alpha_j \times D(h_{ij}^{+}, \tilde{h}_{ij})}{D(h_{ij}^{+}, h_{ij}^{-})} \right\} \quad (10)$$

Here, α_j is the weight of c_j.

The smaller values of Γ_i and Z_i correspond to the better average and worse group scores for alternative A_i, respectively.

Step: 6. Calculate the values of index VIKOR ϕ_i (i = 1, 2, 3, …, r) by the relation

$$\phi_i = \gamma \frac{(\Gamma_i - \Gamma^{-})}{(\Gamma^{+} - \Gamma^{-})} + (1-\gamma) \frac{(Z_i - Z^{-})}{(Z^{+} - Z^{-})} \quad (11)$$

Here, $\Gamma_i^{-} = \min_i \Gamma_i$, $\Gamma_i^{+} = \max_i \Gamma_i$,
$Z_i^{-} = \min_i Z_i$, $Z_i^{+} = \max_i Z_i$ (12)

and γ depicts the decision making mechanism coefficient. If $\gamma > 0.5$, it is for "the maximum group utility"; if $\gamma < 0.5$, it is " the minimum regret"; it has been inferred that the decision making mechanism coefficient is mostly taken as $v = 0.5$.

Step: 7. Rank the priority of alternatives

We rank the alternatives by ϕ_i, Γ_i, and Z_i according to the rule of traditional VIKOR strategy. The smaller value indicates the better alternative.

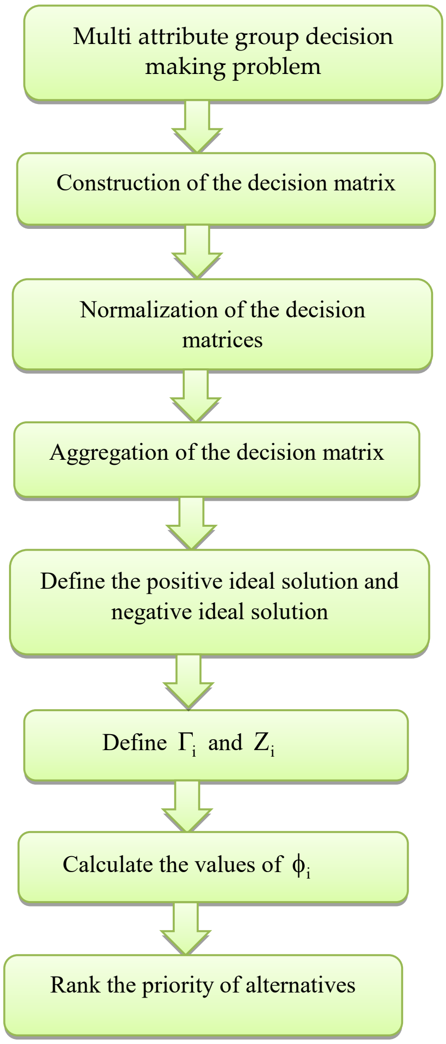

Figure 1. Decision making procedure of proposed MAGDM strategy.

5. Illustrative example

To demonstrate the applicability and fesibility of the proposed strategy, we solve a MAGDM problem adapted from [45]. We assume that an investment company wants to invest a sum of money in the best option. The investment company forms a decision making board involving of three members (DM_1, DM_2, DM_3) who evaluate the four alternatives to invest money. The alternatives are Car company (A_1), Food company (A_2), Computer company (A_3) and Arm company (A_4). Decision makers take decision to evaluate alternatives based on the criteria namely, risk factor (c_1), growth factor (c_2), environment impact (c_3). We consider three criteria as benefit type based on Zhang et al. [127]. Assume that the weight vector of attributes is

$\alpha = (0.37, 0.33, 0.3)^T$ and weight vector of decision makers is $\beta = (0.38, 0.32, 0.3)^T$. Now, we apply the proposed MAGDM strategy to solve the problem using the following steps.

Step: 1. Construction of the decision matrix

We construct the decision matrix information provided by the decision makers in terms of BNNs with respect to the criteria as follows:

Decision matrix for DM_1

$$M^1 = \begin{array}{cc} & \begin{array}{ccc} c_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \left(\begin{array}{ccc} (.5,.6,.7,-.3,-.6,-.3) & (.8,.5,.6,-.4,-.6,-.3) & (.9,.4,.6,-.1,-.6,-.5) \\ (.6,.2,.2,-.4,-.5,-.3) & (.6,.3,.7,-.4,-.3,-.5) & (.7,.5,.3,-.4,-.3,-.3) \\ (.8,.3,.5,-.6,-.4,-.5) & (.5,.2,.4,-.1,-.5,-.3) & (.4,.2,.8,-.5,-.3,-.2) \\ (.7,.5,.3,-.6,-.3,-.3) & (.8,.7,.2,-.8,-.6,-.1) & (.6,.3,.4,-.3,-.4,-.7) \end{array}\right) \end{array}$$

Decision matrix for DM_2

$$M^2 = \begin{array}{cc} & \begin{array}{ccc} c_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \left(\begin{array}{ccc} (.6,.3,.4,-.5,-.3,-.7) & (.5,.3,.4,-.3,-.3,-.4) & (.1,.5,.7,-.5,-.2,-.6) \\ (.7,.4,.5,-.3,-.2,-.1) & (.8,.4,.5,-.7,-.3,-.2) & (.6,.2,.7,-.5,-.2,-.9) \\ (.8,.3,.2,-.5,-.2,-.6) & (.3,.2,.1,-.6,-.3,-.4) & (.7,.5,.4,-.4,-.3,-.2) \\ (.3,.5,.2,-.5,-.5,-.2) & (.5,.6,.4,-.3,-.6,-.7) & (.4,.3,.8,-.5,-.6,-.5) \end{array}\right) \end{array}$$

Decision matrix for DM_3

$$M^3 = \begin{array}{cc} & \begin{array}{ccc} c_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \left(\begin{array}{ccc} (.9,.6,.4,-.7,-.3,-.2) & (.7,.5,.3,-.6,-.2,-.5) & (.4,.2,.3,-.2,-.5,-.7) \\ (.5,.3,.2,-.6,-.4,-.1) & (.5,.2,.7,-.3,-.2,-.5) & (.6,.3,.2,-.7,-.6,-.3) \\ (.2,.5,.6,-.4,-.5,-.7) & (.3,.2,.7,-.2,-.3,-.5) & (.8,.2,.4,-.2,-.3,-.6) \\ (.8,.5,.5,-.4,-.6,-.3) & (.9,.3,.4,-.5,-.6,-.7) & (.7,.4,.3,-.2,-.5,-.7) \end{array}\right) \end{array}$$

Step: 2. Normalization of the decision matrix

Since all the criteria are considered as benefit type, we do not need to normalize the decision matrices (M^1, M^2, M^3).

Step: 3. Aggregated decision matrix

Using eq. (3), the aggregated decision matrix is presented as follows:

$$M = \begin{array}{cc} & \begin{array}{ccc} c_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \left(\begin{array}{ccc} (.22,.17,.17,-.16,-.14,-.13) & (.22,.14,.15,-.14,-.13,-.13) & (.16,.12,.18,-.10,-.10,-.20) \\ (.20,.10,.10,-.14,-.12,-.10) & (.21,.10,.21,-.15,-.10,-.13) & (.21,.11,.13,-.17,-.12,-.16) \\ (.21,.12,.16,-.17,-.12,-.20) & (.13,.10,.13,-.10,-.12,-.13) & (.21,.10,.18,-.13,-.10,-.11) \\ (.20,.17,.11,-.17,-.15,-.10) & (.24,.18,.11,-.19,-.20,-.16) & (.19,.11,.17,-.11,-.16,-.21) \end{array}\right) \end{array}$$

Step: 4. Define the positive ideal solution and negative ideal solution

The positive ideal solution h_{ij}^{+} =

$$\begin{array}{ccc} c_1 & C_2 & C_3 \\ \end{array}$$
$$\left(\begin{array}{ccc} (.22,.10,.10,-.14,-.12,-.10) & (.24,.10,.11,-.19,-.10,-.13) & (.21,.10,.13,-.17,-.10,-.11) \end{array}\right)$$

and the negative ideal solution

h_{ij}^{-} =

$$\begin{array}{ccc} c_1 & C_2 & C_3 \\ \end{array}$$
$$\left(\begin{array}{ccc} (.20,.17,.17,-.14,-.15,-.20) & (.13,.18,.21,-.10,-.20,-.16) & (.16,.12,.18,-.10,-.16,-.11) \end{array}\right)$$

Step: 5. Compute Γ_i and Z_i

We have computed the values of Γ_i by eq. (9) and the values of Z_i by eq. (10), the values are presented as follows:

$\Gamma_1 = 0.75$, $\Gamma_2 = 0.38$, $\Gamma_3 = 0.60$, $\Gamma_4 = 0.75$ and $Z_1 = 0.34$, $Z_2 = 0.16$, $Z_3 = 0.33$, $Z_4 = 0.34$

Step: 6. Calculate the values of ϕ_i

Using $\gamma = 0.5$, and eq. (11) and eq. (12), we obtain

$\phi_1 = 1$, $\phi_2 = \mathbf{0}$, $\phi_3 = 0.77$, $\phi_4 = 1$

Step: 7. Rank the priority of alternatives

The preference order of the alternatives based on the traditional rules of the VIKOR strategy is $A_2 \succ A_3 \succ A_4 \approx A_1$.

6. The influence of parameter γ

In this section, we present sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives Figure 2 represents the graphical representation of alternatives (A_i) versus (i = 1, 2, 3, 4) for different values of γ.

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Table 1 shows that the ranking order of alternatives (A_i) with the value of γ changing from 0.1 to 0.9.

Values of γ	Values of ϕ_i	Preference order of alternatives

$\gamma = 0.1$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.915,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.2$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.880,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.3$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.845,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.4$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.810,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.5$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.770,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.6$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.740,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.7$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.700,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.8$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.670,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.9$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.640,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.

Table 1. Values of $\phi_i\ (i = 1, 2, 3, 4)$ and ranking of alternatives for different values of $\gamma\,\gamma$.

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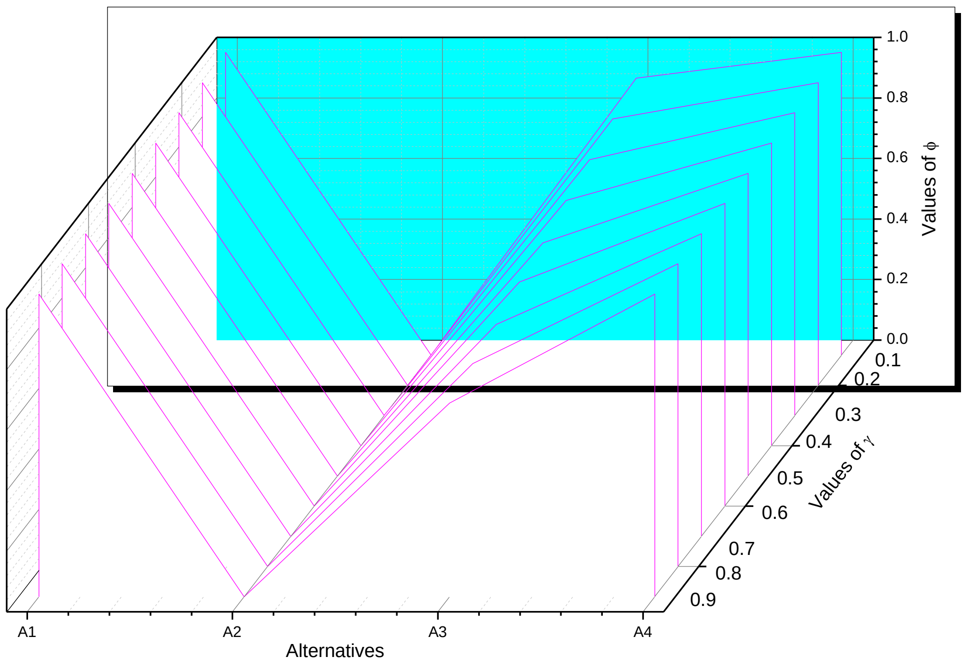

Fig 2. Graphical representation of ranking order of alternatives for different values of γ .

7. Conclusion

In this paper, we have extended the VIKOR strategy to MAGDM with bipolar neutrosophic environment. We have introduced bipolar neutrosophic numbers weighted aggregation operator and applied it to aggregate the individual opinion to one group opinion. We have developed a VIKOR based MAGDM strategy with bipolar neutrosophic set. Finally, we have solved a MAGDM problem to show the feasibility and efficiency of the proposed MAGDM strategy. We have presented a sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives. The proposed VIKOR based MAGDM strategy can be employed to solve a variety of problems such as logistics center selection [128], teacher selection [19, 129], renewable energy selection [131], fault diagnosis [132], weaver selection [14, 54], brick selection [13], school choice [130] etc.

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Correlation Coefficient Measures of Interval Bipolar Neutrosophic Sets for Solving Multi-Attribute Decision Making Problems

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Abstract. Interval bipolar neutrosophic set is a significant extension of interval neutrosophic set where every element of the set comprises of three independent positive membership functions and three independent negative membership functions. In this study, we first define correlation coefficient, and weighted correlation coefficient measures of interval bipolar neutrosophic sets and prove their basic properties. Then, we develop a new multi-attribute decision making strategy based on the proposed weighted correlation coefficient measure. Finally, we solve an investment problem with interval bipolar neutrosophic information and comparison is given to demonstrate the applicability and effectiveness of the proposed strategy.

Keywords: Interval bipolar neutrosophic set, multi-attribute decision making, correlation coefficient measure.

1 Introduction

Correlation coefficient is an important decision making apparatus in statistics to evaluate the relation between two sets. In neutrosophic environment [1], Hanafy et al. [2] derived a formula for correlation coefficient between two neutrosophic sets (NSs). Hanafy et al. [3] obtained the correlation coefficient of NSs by using centroid strategy which lies in [-1, 1]. The correlation coefficient obtained from [3] provides the information about the degree of the relationship between two NSs and also informs us whether the NSs are positive or negatively related. In 2013, Ye [4] defined correlation, correlation coefficient, weighted correlation coefficient in single valued neutrosophic set (SVNS) [5] environment and established a multi-criteria decision making (MCDM) based on the proposed weighted correlation coefficient measure. Broumi and Smarandache [6] introduced the concept of correlation coefficient and weighted correlation coefficient between two interval neutrosophic sets (INSs) [7] and established some of their basic properties. Hanafy et al. [8] studied the notion of correlation and correlation coefficient of neutrosophic data under probability spaces. Ye [9] suggested an improved correlation coefficient between two SVNSs in order to overcome the drawbacks of the correlation coefficient discussed in [4] and investigated its properties. In the same study, Ye [9] extended the concept of correlation coefficient measure of SVNS to correlation coefficient measure of INS environment. Furthermore, Ye [9] developed strategies for solving multi-attribute decision making (MADM) problems with single valued neutrosophic and interval neutrosophic environments based on the proposed correlation coefficient measures. Broumi and Deli [10] defined correlation measure of two neutrosophic refined (multi) sets [11] by extending the correlation measure of two intuitionistic fuzzy multi-sets proposed by Rajarajeswari and Uma [12] and proved some of its basis properties. Zhang et al. [13] defined an improved weighted correlation coefficient on the basis of integrated weight for INSs and a decision making strategy is developed. Karaaslan [14] proposed a strategy to compute correlation coefficient between possibility neutrosophic soft sets and presented several properties related to the proposed strategy. Karaaslan [15] defined a new mathematical structure called single-valued neutrosophic refined soft sets (SNRSSs) and presented its set theoretical operations such as union, intersection and complement and proved some of their basic properties. In the same study [15], two formulas to determine correlation coefficient between two SNRSSs are proposed and the developed strategy is used to solve a clustering analysis problem. Şahin and Liu [16] defined single valued neutrosophic hesitant fuzzy sets (SVNHFSs) and established some basic properties and finally proposed a decision making strategy. Liu and Luo [17] defined correlation coefficient and weighted correlation coefficient for interval-valued neutrosophic hesitant fuzzy sets (INHFSs) due to Liu and Shi [18] and studied their properties. Then, Liu and Luo [17] developed a MADM strategy within the framework of INHFSs based on weighted correlation coefficient. Ye [19] suggested a dynamic single valued neutrosophic multiset (DSVNM) based on dynamic information obtained from different time intervals in several practical situations in order to express dynamical data and operational relations of DSVNMs. In the same study [19], correlation coefficient and weighted correlation coefficient measures between DSVNMs are proposed and a MADM strategy is developed on the basis of the proposed weighted correlation coefficient under DSVNM setting. Recently, Ye [20] proposed two correlation coefficient between normal neutrosophic sets (NNSs) based on the score functions of normal neutrosophic numbers and investigated their essential properties. In the same study, Ye [20] formulated a MADM strategy by employing correlation coefficient of NNSs in normal neutrosophic environment. Pramanik et al. [21] defined correlation coefficient and weighted correlation coefficient between two rough neutrosophic sets and proved their basic properties. In the same study, Pramanik et al. [21] developed a multi-criteria decision making strategy based on the proposed correlation coefficient measure and solved an illustrative example in medical diagnosis.

In 2015, Deli et al. [22] introduced a novel concept called bipolar neutrosophic sets (BNSs) by generalizing the concepts of bipolar fuzzy sets [23, 24] and bipolar intuitionistic fuzzy sets [25]. In the same study, Deli et al. [22] defined score, accuracy and certainty functions to compare BNSs and formulated a MCDM approach based on the score, accuracy and certainty functions and bipolar neutrosophic weighted average operator (A_w) and bipolar neutrosophic weighted geometric operator (G_w). In bipolar neutrosophic environment, Dey et al. [26] developed a MADM approach based on technique for order of preference by similarity to ideal solution (TOPSIS) strategy. Deli and Subas [27] and Şahin et al. [28] developed MCDM strategies based on correlation coefficient and Jaccard similarity measures, respectively in BNS environment. Uluçay et al. [29] defined Dice, weighted Dice similarity measures, hybrid and weighted hybrid similarity measures for MCDM problems with bipolar neutrosophic information. Pramanik et al. [30] defined projection, bidirectional projection and hybrid projection measures between BNSs and proved their basic properties and then, three new MADM models are developed based on proposed measures.

Mahmood et al. [31] and Deli et al. [32] incorporated the notion of interval bipolar neutrosophic sets (IBNSs) and defined some operations and operators for IBNSs. Recently, Pramanik et al. [33] defined new cross entropy and weighted cross entropy measures in BNS and IBNS environment and discussed some of their essential properties. In the same study, Pramanik et al. [33] developed two novel MADM strategies on the basis of the proposed weighted cross entropy measures.

Research gap:

MADM strategy based on correlation coefficient under IBNSs environment.

This paper answers the following research questions:

i. Is it possible to introduce a novel correlation coefficient measure for IBNSs?
ii. Is it possible to introduce a novel weighted correlation coefficient measure for IBNSs?
iii. Is it feasible to formulate a novel MADM strategy based on the proposed correlation coefficient measure in IBNS environment?
iv. Is it feasible to formulate a novel MADM strategy based on the proposed weighted correlation coefficient measure in IBNS environment?

Motivation:

The aforementioned analysis presents the motivation behind developing correlation coefficient -based strategy for handling MADM problems with IBNS information.

The objectives of the paper are as follows:

1. To define a new correlation coefficient measure and a new weighted correlation coefficient measure in IBNS environment and prove their basic properties.
2. To develop a new MADM strategy based on weighted correlation coefficient measure in IBNS environment.

In order to fill the research gap, we propose correlation coefficient-based MADM strategy in IBNS environment.

Rest of the article is organized as follows. Section 2 provides the preliminaries of bipolar fuzzy sets, bipolar intuitionistic fuzzy sets, BNSs and IBNSs. Section 3 defines the correlation coefficient and weighted correlation coefficient measures in IBNS environment and establishes their basic properties. In section 4, a new MADM strategy based on the proposed weighted correlation coefficient measure is developed. In section 5, we solve a numerical example and comparison analysis is given. Finally, in the last section, conclusions are presented.

2 Preliminaries

2.1 Bipolar fuzzy sets

A bipolar fuzzy set [23, 24] B in X is characterized by a positive membership function $\alpha_B^+(x)$ and a negative membership function $\alpha_B^-(x)$. A bipolar fuzzy set B is expressed in the following way.

$B = \{x, \langle \alpha_B^+(x), \alpha_B^-(x) \rangle \mid x \in X\}$

where $\alpha_B^+(x)$: $X \to [0, 1]$ and $\alpha_B^-(x)$: $X \to [-1, 0]$ for each point $x \in X$.

2.2 Bipolar intuitionistic fuzzy sets

Consider X be a non-empty set, then a BIFS [25] E is expressed in the following way.

$E = \{x, \langle \alpha_E^+(x), \alpha_E^-(x), \beta_E^+(x), \beta_E^-(x) \rangle \mid x \in X\}$

where $\alpha_E^+(x), \beta_E^+(x)$: $X \to [0, 1]$ and $\alpha_E^-(x), \beta_E^-(x)$: $X \to [-1, 0]$ for each point $x \in X$ such that $0 \le \alpha_E^+(x) + \beta_E^+(x) \le 1$ and $-1 \le \alpha_E^-(x) + \beta_E^-(x) \le 0$.

2.3 Bipolar neutrosophic sets

A BNS [22]M in X is presented as follows:

$M = \{x, \langle \alpha_M^+(x), \beta_M^+(x), \gamma_M^+(x), \alpha_M^-(x), \beta_M^-(x), \gamma_M^-(x) \rangle \mid x \in X\}$

where $\alpha_M^+(x)$, $\beta_M^+(x)$, $\gamma_M^+(x)$: $X \to [0, 1]$ and $\alpha_M^-(x)$, $\beta_M^-(x)$, $\gamma_M^-(x)$: $X \to [-1, 0]$.The positive membership degrees $\alpha_M^+(x)$, $\beta_M^+(x)$, $\gamma_M^+(x)$ denote the truth membership, indeterminate membership, and false membership functions of an object $x \in X$ corresponding to a BNS M and the negative membership degrees $\alpha_M^-(x)$, $\beta_M^-(x)$, $\gamma_M^-(x)$ denote the truth membership, indeterminate membership, and false membership of an object $x \in X$ to several implicit counter property associated with a BNS M.

Definition 2.3.1

Let, $M_1 = \{x, \langle \alpha_{M_1}^+(x), \beta_{M_1}^+(x), \gamma_{M_1}^+(x), \alpha_{M_1}^-(x), \beta_{M_1}^-(x), \gamma_{M_1}^-(x) \rangle \mid x \in X\}$ and $M_2 = \{x, \langle \alpha_{M_2}^+(x), \beta_{M_2}^+(x), \gamma_{M_2}^+(x), \alpha_{M_2}^-(x), \beta_{M_2}^-(x), \gamma_{M_2}^-(x) \rangle \mid x \in X\}$ be any two BNSs. Then, a BNS M_1 is contained in another BNS M_2, represented by $M_1 \subseteq M_2$ if and only if

$\alpha_{M_1}^+(x) \le \alpha_{M_2}^+(x)$, $\beta_{M_1}^+(x) \ge \beta_{M_2}^+(x)$, $\gamma_{M_1}^+(x) \ge \gamma_{M_2}^+(x)$;

$\alpha_{M_1}^-(x) \ge \alpha_{M_2}^-(x)$, $\beta_{M_1}^-(x) \le \beta_{M_2}^-(x)$, $\gamma_{M_1}^-(x) \le \gamma_{M_2}^-(x)$ for all $x \in X$.

Definition 2.3.2

Let, $M_1 = \{x, \langle \alpha_{M_1}^+(x), \beta_{M_1}^+(x), \gamma_{M_1}^+(x), \alpha_{M_1}^-(x), \beta_{M_1}^-(x), \gamma_{M_1}^-(x) \rangle \mid x \in X\}$ and $M_2 = \{x, \langle \alpha_{M_2}^+(x), \beta_{M_2}^+(x), \gamma_{M_2}^+(x), \alpha_{M_2}^-(x), \beta_{M_2}^-(x), \gamma_{M_2}^-(x) \rangle \mid x \in X\}$ be any two BNSs [22] , then $M_1 = M_2$ if and only if

$\alpha_{M_1}^+(x) = \alpha_{M_2}^+(x)$, $\beta_{M_1}^+(x) = \beta_{M_2}^+(x)$, $\gamma_{M_1}^+(x) = \gamma_{M_2}^+(x)$,

$\alpha_{M_1}^-(x) = \alpha_{M_2}^-(x)$, $\beta_{M_1}^-(x) = \beta_{M_2}^-(x)$, $\gamma_{M_1}^-(x) = \gamma_{M_2}^-(x)$ for all $x \in X$.

Definition 2.3.3

The complement of a BNS [33] M is M^c == $\{x, \langle \alpha_{M^c}^+(x), \beta_{M^c}^+(x), \gamma_{M^c}^+(x), \alpha_{M^c}^-(x), \beta_{M^c}^-(x), \gamma_{M^c}^-(x) \rangle \mid x \in X\}$

where

$\alpha_{M^c}^+(x) = \gamma_M^+(x)$, $\beta_{M^c}^+(x) = 1 - \beta_M^+(x)$, $\gamma_{M^c}^+(x) = \alpha_M^+(x)$;

$\alpha_{M^c}^-(x) = \gamma_M^-(x)$, $\beta_{M^c}^-(x) = -1 - \beta_M^-(x)$, $\gamma_{M^c}^-(x) = \alpha_M^-(x)$.

Definition 2.3.4

The union [30]of two BNSs M_1 and M_2 represented by $M_1 \cup M_2$ is defined as follows:

$M_1 \cup M_2 = \{\text{Max}\,(T_{M_1}^+(x), T_{M_2}^+(x)), \text{Min}\,(I_{M_1}^+(x), I_{M_2}^+(x)), \text{Min}\,(F_{M_1}^+(x), F_{M_2}^+(x)), \text{Min}\,(T_{M_1}^-(x), T_{M_2}^-(x)), \text{Max}\,(I_{M_1}^-(x), I_{M_2}^-(x)), \text{Max}\,(F_{M_1}^-(x), F_{M_2}^-(x))\}, \forall\ x \in X.$

Definition 2.3.5

The intersection [30] of two BNSs M_1 and M_2 denoted by $M_1 \cap M_2$ is defined as follows:

$M_1 \cap M_2 = \{\text{Min}\,(T_{M_1}^+(x), T_{M_2}^+(x)), \text{Max}\,(I_{M_1}^+(x), I_{M_2}^+(x)), \text{Max}\,(F_{M_1}^+(x), F_{M_2}^+(x)), \text{Max}\,(T_{M_1}^-(x), T_{M_2}^-(x)), \text{Min}\,(I_{M_1}^-(x), I_{M_2}^-(x)), \text{Min}\,(F_{M_1}^-(x), F_{M_2}^-(x))\}, \forall\ x \in X.$

2.4 Interval bipolar neutrosophic sets

Consider X be the space of objects, then an IBNS [31, 32] L in X is is represented as follows:

$$L= \{x, \left\langle \begin{matrix} [\inf \alpha_L^+(x), \sup \alpha_L^+(x)], [\inf \beta_L^+(x), \sup \beta_L^+(x)], \\ [\inf \gamma_L^+(x), \sup \gamma_L^+(x)], [\inf \alpha_L^-(x), \sup \alpha_L^-(x)], \\ [\inf \beta_L^-(x), \sup \beta_L^-(x)], [\inf \gamma_L^-(x), \sup \gamma_L^-(x)] \end{matrix} \right\rangle \mid x \in X\}$$

where L is characterized by positive and negative truth-membership $\alpha_L^+(x)$, $\alpha_L^-(x)$; inderterminacy-membership $\beta_L^+(x)$, $\beta_L^-(x)$; falsity-membership $\gamma_L^+(x)$, $\gamma_L^-(x)$ functions respectively. Here, $\alpha_L^+(x)$, $\beta_L^+(x)$, $\gamma_L^+(x) \subseteq [0,1]$; $\alpha_L^-(x)$, $\beta_L^-(x)$, $\gamma_L^-(x) \subseteq [-1, 0]$ for all $x \in X$ with the conditions $0 \le \sup \alpha_L^+(x) + \sup \beta_L^+(x) + \sup \gamma_L^+(x) \le 3$, and $-3 \le \sup \alpha_L^-(x) + \sup \beta_L^-(x) + \sup \gamma_L^-(x) \le 0$.

Definition 2.4.1 : Let $L_1 = \{x, < [\inf \alpha_{L_1}^+(x), \sup \alpha_{L_1}^+(x)]; [\inf \beta_{L_1}^+(x), \sup \beta_{L_1}^+(x)]; [\inf \gamma_{L_1}^+(x), \sup \gamma_{L_1}^+(x)]; [\inf \alpha_{L_1}^-(x), \sup \alpha_{L_1}^-(x)]; [\inf \beta_{L_1}^-(x), \sup \beta_{L_1}^-(x)]; [\inf \gamma_{L_1}^-(x), \sup \gamma_{L_1}^-(x)] > \mid x \in X\}$ and $L_2 == \{x, < [\inf \alpha_{L_2}^+(x), \sup \alpha_{L_2}^+(x)]; [\inf \beta_{L_2}^+(x), \sup \beta_{L_2}^+(x)]; [\inf \gamma_{L_2}^+(x), \sup \gamma_{L_2}^+(x)]; [\inf \alpha_{L_2}^-(x), \sup \alpha_{L_2}^-(x)]; [\inf \beta_{L_2}^-(x), \sup \beta_{L_2}^-(x)]; [\inf \gamma_{L_2}^-(x), \sup \gamma_{L_2}^-(x)] > \mid x \in X\}$ be two IBNSs [31] . Then $L_1 \subseteq L_2$ if and only if

$\inf \alpha_{L_1}^+(x) \le \inf \alpha_{L_2}^+(x)$, $\sup \alpha_{L_1}^+(x) \le \sup \alpha_{L_2}^+(x)$, $\inf \beta_{L_1}^+(x) \ge \inf \beta_{L_2}^+(x)$, $\sup \beta_{L_1}^+(x) \ge \sup \beta_{L_2}^+(x)$, $\inf \gamma_{L_1}^+(x) \ge \inf \gamma_{L_2}^+(x)$, $\sup \gamma_{L_1}^+(x) \ge \sup \gamma_{L_2}^+(x)$, $\inf \alpha_{L_1}^-(x) \ge \inf \alpha_{L_2}^-(x)$, $\sup \alpha_{L_1}^-(x) \ge \sup \alpha_{L_2}^-(x)$, $\inf \beta_{L_1}^-(x) \le \inf \beta_{L_2}^-(x)$, $\sup \beta_{L_1}^-(x) \le \sup \beta_{L_2}^-(x)$, $\inf \gamma_{L_1}^-(x) \le \inf \gamma_{L_2}^-(x)$, $\sup \gamma_{L_1}^-(x) \le \sup \gamma_{L_2}^-(x)$, for all $x \in X$.

Definition 2.4.2: Consider $L_1 = \{x, < [\inf \alpha_{L_1}^+(x), \sup \alpha_{L_1}^+(x)]; [\inf \beta_{L_1}^+(x), \sup \beta_{L_1}^+(x)]; [\inf \gamma_{L_1}^+(x), \sup \gamma_{L_1}^+(x)]; [\inf \alpha_{L_1}^-(x), \sup \alpha_{L_1}^-(x)]; [\inf \beta_{L_1}^-(x), \sup \beta_{L_1}^-(x)]; [\inf \gamma_{L_1}^-(x), \sup \gamma_{L_1}^-(x)] > \mid x \in X\}$ and $L_2 = \{x, < [\inf \alpha_{L_2}^+(x), \sup \alpha_{L_2}^+(x)]; [\inf \beta_{L_2}^+(x), \sup \beta_{L_2}^+(x)]; [\inf \gamma_{L_2}^+(x), \sup \gamma_{L_2}^+(x)]; [\inf \alpha_{L_2}^-(x), \sup \alpha_{L_2}^-(x)]; [\inf \beta_{L_2}^-(x), \sup \beta_{L_2}^-(x)]; [\inf \gamma_{L_2}^-(x), \sup \gamma_{L_2}^-(x)] > \mid x \in X\}$ be two IBNSs [31] . Then $L_1 = L_2$ if and only if

$\inf \alpha_{L_1}^+(x) = \inf \alpha_{L_2}^+(x)$, $\sup \alpha_{L_1}^+(x) = \sup \alpha_{L_2}^+(x)$, $\inf \beta_{L_1}^+(x) = \inf \beta_{L_2}^+(x)$, $\sup \beta_{L_1}^+(x) = \sup \beta_{L_2}^+(x)$, $\inf \gamma_{L_1}^+(x) = \inf \gamma_{L_2}^+(x)$, $\sup \gamma_{L_1}^+(x) = \sup \gamma_{L_2}^+(x)$, $\inf \alpha_{L_1}^-(x) = \inf \alpha_{L_2}^-(x)$, $\sup \alpha_{L_1}^-(x) = \sup \alpha_{L_2}^-(x)$, $\inf \beta_{L_1}^-(x) = \inf \beta_{L_2}^-(x)$, $\sup \beta_{L_1}^-(x) = \sup \beta_{L_2}^-(x)$, $\inf \gamma_{L_1}^-(x) = \inf \gamma_{L_2}^-(x)$, $\sup \gamma_{L_1}^-(x) = \sup \gamma_{L_2}^-(x)$, for all $x \in X$.

Definition 2.4.3: The complement [33]of $L = \{x, < [\inf \alpha_L^+(x), \sup \alpha_L^+(x)]; [\inf \beta_L^+(x), \sup \beta_L^+(x)]; [\inf \gamma_L^+(x), \sup \gamma_L^+(x)]; [\inf \alpha_L^-(x), \sup \alpha_L^-(x)]; [\inf \beta_L^-(x), \sup \beta_L^-(x)]; [\inf \gamma_L^-(x), \sup \gamma_L^-(x)] > \mid x \in X\}$ is defined as $L^C = \{x, < [\inf \alpha_{L^C}^+(x), \sup \alpha_{L^C}^+(x)]; [\inf \beta_{L^C}^+(x), \sup \beta_{L^C}^+(x)]; [\inf \gamma_{L^C}^+(x), \sup \gamma_{L^C}^+(x)]; [\inf \alpha_{L^C}^-(x), \sup \alpha_{L^C}^-(x)]; [\inf \beta_{L^C}^-(x), \sup \beta_{L^C}^-(x)]; [\inf \gamma_{L^C}^-(x), \sup \gamma_{L^C}^-(x)] > \mid x \in X\}$ where

$\inf \alpha_{L^C}^+(x) = \inf \gamma_L^+(x)$, $\sup \alpha_{L^C}^+(x) = \sup \gamma_L^+(x)$, $\inf \beta_{L^C}^+(x) = 1 - \sup \beta_L^+(x)$, $\sup \beta_{L^C}^+(x) = 1 - \inf \beta_L^+(x)$, $\inf \gamma_{L^C}^+(x) = \inf \alpha_L^+$, $\sup \gamma_{L^C}^+(x) = \sup \alpha_L^+$, $\inf \alpha_{L^C}^-(x) = \inf \gamma_L^-$, $\sup \alpha_{L^C}^-(x) = \sup \gamma_L^-$, $\inf \beta_{L^C}^-(x) = -1 - \sup \beta_L^-(x)$, $\sup \beta_{L^C}^-(x) = -1 - \inf \beta_L^-(x)$, $\inf \gamma_{L^C}^-(x) = \inf \alpha_L^-(x)$, $\sup \gamma_{L^C}^-(x) = \sup \alpha_L^-(x)$ for all $x \in X$.

3 Correlation coefficient measures under IBNSs setting

Definition 3.1: Let L_1 and L_2 be two IBNSs in $X = \{x_1, x_2, \ldots, x_n\}$, then the correlation between L_1 and L_2 is defined as follows:

$$R(L_1, L_2) = \sum_{i=1}^{n} \left(\begin{matrix} \inf \alpha_{L_1}^+(x_i).\inf \alpha_{L_2}^+(x_i) + \sup \alpha_{L_1}^+(x_i).\sup \alpha_{L_2}^+(x_i) + \\ \inf \beta_{L_1}^+(x_i).\inf \beta_{L_2}^+(x_i) + \sup \beta_{L_1}^+(x_i).\sup \beta_{L_2}^+(x_i) + \\ \inf \gamma_{L_1}^+(x_i).\inf \gamma_{L_2}^+(x_i) + \sup \gamma_{L_1}^+(x_i).\sup \gamma_{L_2}^+(x_i) + \\ \inf \alpha_{L_1}^-(x_i).\inf \alpha_{L_2}^-(x_i) + \sup \alpha_{L_1}^-(x_i).\sup \alpha_{L_2}^-(x_i) + \\ \inf \beta_{L_1}^-(x_i).\inf \beta_{L_2}^-(x_i) + \sup \beta_{L_1}^-(x_i).\sup \beta_{L_2}^-(x_i) + \\ \inf \gamma_{L_1}^-(x_i).\inf \gamma_{L_2}^-(x_i) + \sup \gamma_{L_1}^-(x_i).\sup \gamma_{L_2}^-(x_i) \end{matrix} \right)$$

Definition 3.2: Consider L_1 and L_2 be two IBNSs in $X = \{x_1, x_2, \ldots, x_n\}$, then the correlation coefficient between L_1 and L_2 is defined as follows:

$$Cor\,(L_1, L_2) = \frac{R(L_1, L_2)}{[R(L_1, L_1).R(L_2, L_2)]^{1/2}} \quad (1)$$

where

$$R(L_1, L_2) = \sum_{i=1}^{n} \begin{pmatrix} \inf \alpha^+_{L_1}(x_i).\inf \alpha^+_{L_2}(x_i) + \sup \alpha^+_{L_1}(x_i).\sup \alpha^+_{L_2}(x_i) + \\ \inf \beta^+_{L_1}(x_i).\inf \beta^+_{L_2}(x_i) + \sup \beta^+_{L_1}(x_i).\sup \beta^+_{L_2}(x_i) + \\ \inf \gamma^+_{L_1}(x_i).\inf \gamma^+_{L_2}(x_i) + \sup \gamma^+_{L_1}(x_i).\sup \gamma^+_{L_2}(x_i) + \\ \inf \alpha^-_{L_1}(x_i).\inf \alpha^-_{L_2}(x_i) + \sup \alpha^-_{L_1}(x_i).\sup \alpha^-_{L_2}(x_i) + \\ \inf \beta^-_{L_1}(x_i).\inf \beta^-_{L_2}(x_i) + \sup \beta^-_{L_1}(x_i).\sup \beta^-_{L_2}(x_i) + \\ \inf \gamma^-_{L_1}(x_i).\inf \gamma^-_{L_2}(x_i) + \sup \gamma^-_{L_1}(x_i).\sup \gamma^-_{L_2}(x_i) \end{pmatrix},$$

$$R(L_1, L_1) = \sum_{i=1}^{n} \begin{pmatrix} (\inf \alpha^+_{L_1}(x_i))^2 + (\sup \alpha^+_{L_1}(x_i))^2 + (\inf \beta^+_{L_1}(x_i))^2 + \\ (\sup \beta^+_{L_1}(x_i))^2 + (\inf \gamma^+_{L_1}(x_i))^2 + (\sup \gamma^+_{L_1}(x_i))^2 + \\ (\inf \alpha^-_{L_1}(x_i))^2 + (\sup \alpha^-_{L_1}(x_i))^2 + (\inf \beta^-_{L_1}(x_i))^2 + \\ (\sup \beta^-_{L_1}(x_i))^2 + (\inf \gamma^-_{L_1}(x_i))^2 + (\sup \gamma^-_{L_1}(x_i))^2 \end{pmatrix}$$

$$R(L_2, L_2) = \sum_{i=1}^{n} \begin{pmatrix} (\inf \alpha^+_{L_2}(x_i))^2 + (\sup \alpha^+_{L_2}(x_i))^2 + (\inf \beta^+_{L_2}(x_i))^2 + \\ (\sup \beta^+_{L_2}(x_i))^2 + (\inf \gamma^+_{L_2}(x_i))^2 + (\sup \gamma^+_{L_2}(x_i))^2 + \\ (\inf \alpha^-_{L_2}(x_i))^2 + (\sup \alpha^-_{L_2}(x_i))^2 + (\inf \beta^-_{L_2}(x_i))^2 + \\ (\sup \beta^-_{L_2}(x_i))^2 + (\inf \gamma^-_{L_2}(x_i))^2 + (\sup \gamma^-_{L_2}(x_i))^2 \end{pmatrix}$$

Theorem 1. The correlation coefficient measure *Cor* (L_1, L_2) between two IBNSs L_1, L_2 satisfies the following properties:

(C1) *Cor* (L_1, L_2) = *Cor* (L_2, L_1) ;

(C2) $0 \le$ *Cor* (L_1, L_2) ≤ 1;

(C3) *Cor* (L_1, L_2) = 1, if $L_1 = L_2$.

Proof:

(1) $Cor\,(L_1, L_2) = \dfrac{R(L_1, L_2)}{[R(L_1, L_1) \times R(L_2, L_2)]^{1/2}}$

$= \dfrac{R(L_2, L_1)}{[R(L_2, L_2) \times R(L_1, L_1)]^{1/2}} = Cor\,(L_2, L_1)$.

(2) Since, R (L_1, L_2) ≥ 0, R (L_1, L_1) ≥ 0, R (L_2, L_2) ≥ 0 and using Cauchy-Schwarz inequality we can easily prove that *Cor* (L_1, L_2) ≤ 1, therefore, $0 \le$ *Cor* (L_1, L_2) ≤ 1.

(3) If $L_1 = L_2$, then inf $\alpha^+_{L_1}(x) = \inf \alpha^+_{L_2}(x)$, sup $\alpha^+_{L_1}(x) = \sup \alpha^+_{L_2}(x)$, inf $\beta^+_{L_1}(x) = \inf \beta^+_{L_2}(x)$, sup $\beta^+_{L_1}(x) = \sup \beta^+_{L_2}(x)$, $\inf \gamma^+_{L_1}(x) = \inf \gamma^+_{L_2}(x)$, $\sup \gamma^+_{L_1}(x) = \sup \gamma^+_{L_2}(x)$, $\inf \alpha^-_{L_1}(x) = \inf \alpha^-_{L_2}(x)$, $\sup \alpha^-_{L_1}(x) = \sup \alpha^-_{L_2}(x)$, $\inf \beta^-_{L_1}(x) = \inf \beta^-_{L_2}(x)$, $\sup \beta^-_{L_1}(x) = \sup \beta^-_{L_2}(x)$, inf $\alpha^-_{L_1}(x)$ = inf $\alpha^-_{L_2}(x)$, $\sup \gamma^-_{L_1}(x) = \sup \gamma^-_{L_2}(x)$ for any $x \in X$ and therefore, *Cor* (L_1, L_2) = 1.

Definition 3.3: Let $w_i = (w_1, w_2, ..., w_n) \in [0, 1]$ be the weight vector of the elements x_j (j = 1, 2, ..., n), the weighted correlation coefficient between two IBNSs L_1, L_2 can be defined by the following formula

$$Cor_w\,(L_1, L_2) = \frac{R_w\,(L_1, L_2)}{[R_w\,(L_1, L_1).R_w\,(L_2, L_2)]^{1/2}} \quad (2)$$

where

$$R_w(L_1, L_2) = \sum_{i=1}^{n} w_i \begin{pmatrix} \inf \alpha^+_{L_1}(x_i).\inf \alpha^+_{L_2}(x_i) + \sup \alpha^+_{L_1}(x_i).\sup \alpha^+_{L_2}(x_i) + \\ \inf \beta^+_{L_1}(x_i).\inf \beta^+_{L_2}(x_i) + \sup \beta^+_{L_1}(x_i).\sup \beta^+_{L_2}(x_i) + \\ \inf \gamma^+_{L_1}(x_i).\inf \gamma^+_{L_2}(x_i) + \sup \gamma^+_{L_1}(x_i).\sup \gamma^+_{L_2}(x_i) + \\ \inf \alpha^-_{L_1}(x_i) \inf \alpha^-_{L_2}(x_i) + \sup \alpha^-_{L_1}(x_i).\sup \alpha^-_{L_2}(x_i) + \\ \inf \beta^-_{L_1}(x_i).\inf \beta^-_{L_2}(x_i) + \sup \beta^-_{L_1}(x_i).\sup \beta^-_{L_2}(x_i) + \\ \inf \gamma^-_{L_1}(x_i).\inf \gamma^-_{L_2}(x_i) + \sup \gamma^-_{L_1}(x_i).\sup \gamma^-_{L_2}(x_i) \end{pmatrix},$$

$$R_w(L_1, L_1) = \sum_{i=1}^{n} w_i \begin{pmatrix} (\inf \alpha^+_{L_1}(x_i))^2 + (\sup \alpha^+_{L_1}(x_i))^2 + (\inf \beta^+_{L_1}(x_i))^2 + \\ (\sup \beta^+_{L_1}(x_i))^2 + (\inf \gamma^+_{L_1}(x_i))^2 + (\sup \gamma^+_{L_1}(x_i))^2 + \\ (\inf \alpha^-_{L_1}(x_i))^2 + (\sup \alpha^-_{L_1}(x_i))^2 + (\inf \beta^-_{L_1}(x_i))^2 + \\ (\sup \beta^-_{L_1}(x_i))^2 + (\inf \gamma^-_{L_1}(x_i))^2 + (\sup \gamma^-_{L_1}(x_i))^2 \end{pmatrix}$$

$$R(L_2, L_2) = \sum_{i=1}^{n} w_i \begin{pmatrix} (\inf \alpha^+_{L_2}(x_i))^2 + (\sup \alpha^+_{L_2}(x_i))^2 + (\inf \beta^+_{L_2}(x_i))^2 + \\ (\sup \beta^+_{L_2}(x_i))^2 + (\inf \gamma^+_{L_2}(x_i))^2 + (\sup \gamma^+_{L_2}(x_i))^2 + \\ (\inf \alpha^-_{L_2}(x_i))^2 + (\sup \alpha^-_{L_2}(x_i))^2 + (\inf \beta^-_{L_2}(x_i))^2 + \\ (\sup \beta^-_{L_2}(x_i))^2 + (\inf \gamma^-_{L_2}(x_i))^2 + (\sup \gamma^-_{L_2}(x_i))^2 \end{pmatrix}$$

If $w = (1/n, 1/n, ..., 1/n)^T$, the Eq. (2) is reduced to Eq. (1).

Theorem 2. The weighted correlation coefficient measure Cor_w (L_1, L_2) between two IBNSs L_1, L_2 *also* satisfies the following properties:

(C1) Cor_w (L_1, L_2) = Cor_w (L_2, L_1);

(C2) $0 \le Cor_w$ (L_1, L_2) ≤ 1;

(C3) Cor_w (L_1, L_2) = 1, if $L_1 = L_2$.

Proof:

(1) $Cor_w\,(L_1, L_2) = \dfrac{R_w\,(L_1, L_2)}{[R_w\,(L_1, L_1).R_w\,(L_2, L_2)]^{1/2}}$

$= \dfrac{R_w\,(L_2, L_1)}{[R_w\,(L_2, L_2).R_w\,(L_1, L_1)]^{1/2}} = Cor_w\,(L_2, L_1)$.

(2) Since, R_w (L_1, L_2) ≥ 0, R_w (L_1, L_1) ≥ 0, R_w (L_2, L_2) ≥ 0 and using Cauchy-Schwarz inequality we can easily prove that Cor_w (L_1, L_2) ≤ 1, so, $0 \le Cor_w$ (L_1, L_2) ≤ 1.

(3) If $L_1 = L_2$, then $\inf \alpha^+_{L_1}(x) = \inf \alpha^+_{L_2}(x)$, $\sup \alpha^+_{L_1}(x) = \sup \alpha^+_{L_2}(x)$, $\inf \beta^+_{L_1}(x) = \inf \beta^+_{L_2}(x)$, $\sup \beta^+_{L_1}(x) = \sup \beta^+_{L_2}(x)$, $\inf \gamma^+_{L_1}(x) = \inf \gamma^+_{L_2}(x)$, $\sup \gamma^+_{L_1}(x) = \sup \gamma^+_{L_2}(x)$, $\inf \alpha^-_{L_1}(x) = \inf \alpha^-_{L_2}(x)$, $\sup \alpha^-_{L_1}(x) = \sup \alpha^-_{L_2}(x)$, $\inf \beta^-_{L_1}(x) = \inf \beta^-_{L_2}(x)$, $\sup \beta^-_{L_1}(x) = \sup \beta^-_{L_2}(x)$, $\inf \alpha^-_{L_1}(x) = \inf \alpha^-_{L_2}(x)$, $\sup \gamma^-_{L_1}(x) = \sup \gamma^-_{L_2}(x)$ for any $x \in X$ and hence, $Cor_w(L_1, L_2) = 1$.

Example 1. Suppose that L_1 = < [0.3, 0.7], [0.3, 0.8], [0.5, 0.9], [-0.9, -0.3], [-0.6, -0.2], [-0.8, -0.4] > and L_2 = < [0.1, 0.6], [0.2, 0.7], [0.3, 0.5], [-0.8, -0.2], [-0.8, -0.3], [-0.7, -0.4] > be two IBNSs, then correlation coefficient between L_1 and L_2 is obtain using Eq. (1) as follows:

$Cor(L_1, L_2) == 0.4870391$.

Example 2. If $w = 0.4$, then the weighted correlation coefficient between L_1 = < [0.3, 0.7], [0.3, 0.8], [0.5, 0.9], [-0.9, -0.3], [-0.6, -0.2], [-0.8, -0.4] > and L_2 = < [0.1, 0.6], [0.2, 0.7], [0.3, 0.5], [-0.8, -0.2], [-0.8, -0.3], [-0.7, -0.4] > is calculated by using Eq. (2) as follows.

$Cor_w(L_1, L_2) = 0.5689123$.

4. MADM strategy based on weighted correlation coefficient measure in IBNS environment

In this section, we have developed a novel MADM strategy based on weighted correlation coefficient measure in interval bipolar neutrosophic environment. Let, $F = \{F_1, F_2, \ldots, F_m\}$, $(m \geq 2)$ be a discrete set of m feasible alternatives, $G = \{G_1, G_2, \ldots, G_n\}$, $(n \geq 2)$ be a set of n predefined attributes and w_j be the weight vector of the attributes such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. The steps for solving MADM problems in IBNS environment are presented as follows.

Step 1. The evaluation of the performance value of alternative F_i (i = 1, 2, …, m) with regard to the predefined attribute G_j (j = 1, 2, …, n) provided by the decision maker or expert can be presented in terms of interval bipolar neutrosophic values q_{ij} = < [$\inf \alpha^+_{ij}$, $\sup \alpha^+_{ij}$], [$\inf \beta^+_{ij}$, $\sup \beta^+_{ij}$], [$\inf \gamma^+_{ij}$, $\sup \gamma^+_{ij}$], [$\inf \alpha^-_{ij}$, $\sup \alpha^-_{ij}$], [$\inf \beta^-_{ij}$, $\sup \beta^-_{ij}$], [$\inf \gamma^-_{ij}$, $\sup \gamma^-_{ij}$] > = < c_{ij}, d_{ij}, e_{ij}, f_{ij}, g_{ij}, h_{ij}, r_{ij}, s_{ij}, t_{ij}, u_{ij}, v_{ij}, w_{ij}>, i = 1, 2, ..., m; j = 1, 2, ..., n. The interval bipolar neutrosophic decision matrix $[\widetilde{R}_{ij}]_{m\times n}$ is presented as given below.

$$[\widetilde{R}_{ij}]_{m\times n} = \begin{array}{c} \\ F_1 \\ F_2 \\ . \\ . \\ F_m \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & \cdots & G_n \end{array} \\ \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ q_{m1} & q_{m2} & \cdots & q_{mn} \end{pmatrix} \end{array}$$

Step 2. The interval bipolar neutrosophic positive ideal solution (IBN-PIS) can be defined as follows: $Q^* = < c^+_j, d^+_j, e^+_j, f^+_j, g^+_j, h^+_j, r^+_j, s^+_j, t^+_j, u^+_j, v^+_j, w^+_j > = <$ [{ $\underset{i}{\mathrm{Max}}(c_{ij})$ | $j \in J^+$; $\underset{i}{\mathrm{Min}}(c_{ij})$ | $j \in J^-$}, { $\underset{i}{\mathrm{Max}}(d_{ij})$ | $j \in J^+$}; $\underset{i}{\mathrm{Min}}(d_{ij})$ | $j \in J^-$}], [{ $\underset{i}{\mathrm{Min}}(e_{ij})$ | $j \in J^+$; $\underset{i}{\mathrm{Max}}(e_{ij})$ | $j \in J^-$}, { $\underset{i}{\mathrm{Min}}(f_{ij})$ | $j \in J^+$}; $\underset{i}{\mathrm{Max}}(f_{ij})$ | $j \in J^-$}], [{ $\underset{i}{\mathrm{Min}}(g_{ij})$ | $j \in J^+$; $\underset{i}{\mathrm{Max}}(g_{ij})$ | $j \in J^-$}, { $\underset{i}{\mathrm{Min}}(h_{ij})$ | $j \in J^+$}; $\underset{i}{\mathrm{Max}}(h_{ij})$ | $j \in J^-$}], [{ $\underset{i}{\mathrm{Min}}(r_{ij})$ | $j \in J^+$; $\underset{i}{\mathrm{Max}}(r_{ij})$ | $j \in J^-$}, { $\underset{i}{\mathrm{Min}}(s_{ij})$ | $j \in J^+$; $\underset{i}{\mathrm{Max}}(s_{ij})$ | $j \in J^-$}], [{ $\mathrm{Max}(t_{ij})$ | $j \in J^+$; { $\underset{i}{\mathrm{Min}}(t_{ij})$ | $j \in J^-$}, { $\underset{i}{\mathrm{Max}}(u_{ij})$ | $j \in J^+$}; $\underset{i}{\mathrm{Min}}(u_{ij})$ | $j \in J^-$}], [{ $\underset{i}{\mathrm{Max}}(v_{ij})$ | $j \in J^+$; { $\underset{i}{\mathrm{Min}}(v_{ij})$ | $j \in J^-$}, { $\underset{i}{\mathrm{Max}}(w_{ij})$ | $j \in J^+$}; $\underset{i}{\mathrm{Min}}(w_{ij})$ | $j \in J^-$}] >, j = 1, 2, …, n, where J^+, J^- denote the benefit and cost type attributes, respectively.

Step 3. The weighted correlation coefficient of IBNS between alternative F_i (i = 1, 2, ..., m) and the ideal alternative Q^* can be derived as follows:

$$Cor_w(F_i, Q^*) = \frac{R_w(F_i, Q^*)}{[R_w(F_i, F_i).\, R_w(Q^*, Q^*)]^{1/2}}$$

where,

$$R_w(F_i, Q^*) = \sum_{j}^{n} w_j[c_{ij}.c^+_j + d_{ij}.d^+_j + e_{ij}.e^+_j + f_{ij}.f^+_j + g_{ij}.g^+_j + h_{ij}.h^+_j + r_{ij}.r^+_j + s_{ij}.s^+_j + t_{ij}.t^+_j + u_{ij}.u^+_j + v_{ij}.v^+_j + w_{ij}.w^+_j]$$

$$R_w(F_i, F_i) = \sum_{j=1}^{n} w_j[(c_{ij})^2 + (d_{ij})^2 + (e_{ij})^2 + (f_{ij})^2 + (g_{ij})^2 + (h_{ij})^2 + (r_{ij})^2 + (s_{ij})^2 + (t_{ij})^2 + (u_{ij})^2 + (v_{ij})^2 + (w_{ij})^2]$$

$$R_w(Q^*, Q^*) = \sum_{i=1}^{n} w_j[(c^+_j)^2 + (d^+_j)^2 + (e^+_j)^2 + (f^+_j)^2 + (g^+_j)^2 + (h^+_j)^2 + (r^+_j)^2 + (s^+_j)^2 + (t^+_j)^2 + (u^+_j)^2 + (v^+_j)^2 + (w^+_j)^2].$$

Step 4: The biggest value of $Cor_w(F_i, Q^*)$, i = 1, 2, ..., m implies F_i, (i = 1, 2, ..., m) is the better alternative.

In Fig 1. we represent the steps for solving MADM problems based on weighted correlation coefficient measure in IBNS environment.

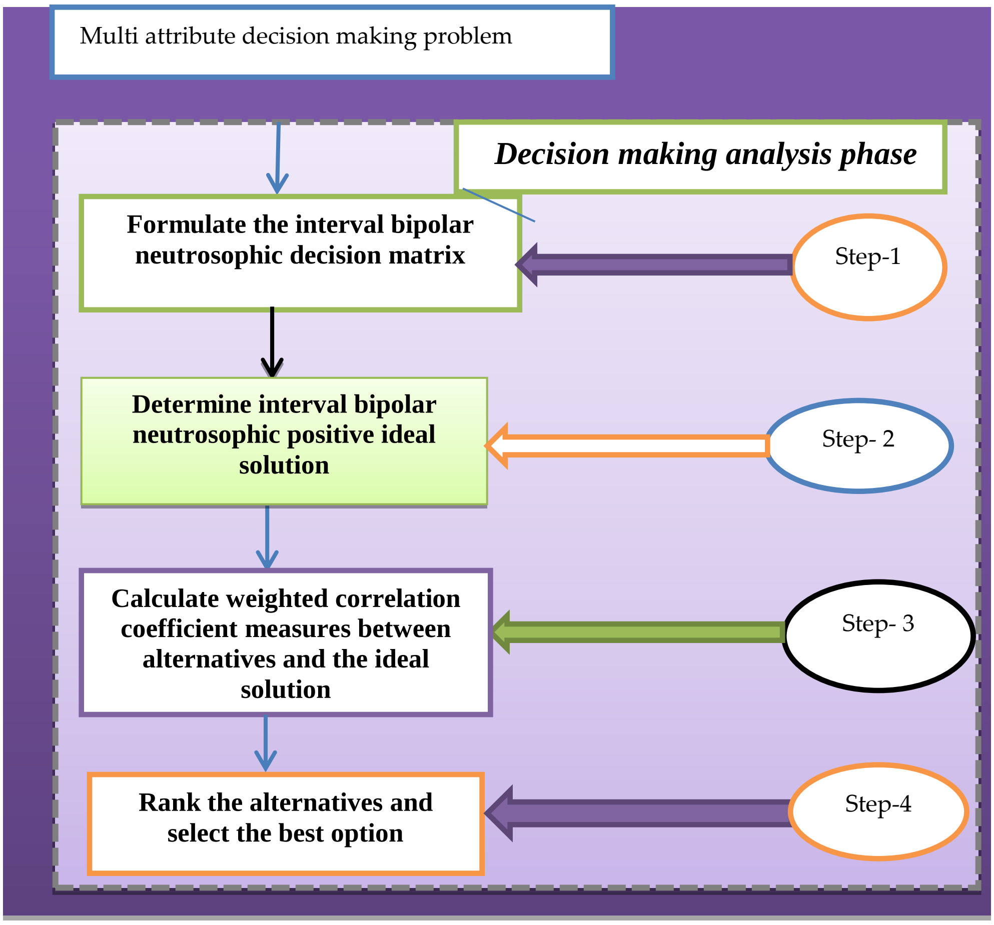

Figure.1 Decision making procedure of proposed MADM strategy

5. Numerical example

In this section, an illustrative numerical problem is solved to illustrate the proposed strategy. We consider an MADM studied in [31, 33] where there are four possible alternatives to invest money namely, a food company (F_1), a car company (F_2), a arm company (F_3), and a computer company (F_4). The investment company must take a decision based on the three predefined attributes namely growth analysis (G_1), risk analysis (G_2), and environment analysis (G_3) where G_1, G_2 are the benefit type and G_3 is the cost type attribute [34] and the weight vector of G_1, G_2, and G_3 is given by $w = (w_1, w_2, w_3) = (0.35, 0.25, 0.4)$ [31].

The proposed strategy consisting of the following steps:

Step 1. The evaluation of performance value of the alternatives with respect to the attributes provided by the decision maker can be expressed by interval bipolar neutrosophic values and the decision matrix is presented as follows:

Interval bipolar neutrosophic decision matrix

$$G_1$$

$$\begin{pmatrix} F_1 & [[0.4,0.5],[0.2,0.3],[0.3,0.4],[-0.3,-0.2],[-0.4,-0.3],[-0.5,-0.4]] \\ F_2 & [[0.6,0.7],[0.1,0.2],[0.2,0.3],[-0.2,-0.1],[-0.3,-0.2],[-0.7,-0.6]] \\ F_3 & [[0.3,0.6],[0.2,0.3],[0.3,0.4],[-0.3,-0.2],[-0.4,-0.3],[-0.6,-0.3]] \\ F_4 & [[0.7,0.8],[0.0,0.1],[0.1,0.2],[-0.1,-0.0],[-0.2,-0.1],[-0.8,-0.7]] \end{pmatrix}$$

$$G_2$$

$$\begin{pmatrix} F_1 & [[0.4,0.6],[0.1,0.3],[0.2,0.4],[-0.3,-0.1],[-0.4,-0.2],[-0.6,-0.4]] \\ F_2 & [[0.6,0.7],[0.1,0.2],[0.2,0.3],[-0.2,-0.1],[-0.3,-0.2],[-0.7,-0.6]] \\ F_3 & [[0.5,0.6],[0.2,0.3],[0.3,0.4],[-0.3,-0.2],[-0.4,-0.3],[-0.6,-0.5]] \\ F_4 & [[0.6,0.7],[0.1,0.2],[0.1,0.3],[-0.2-0.1],[-0.3,-0.1],[-0.7,-0.6]] \end{pmatrix}$$

$$G_3$$

$$\begin{pmatrix} F_1 & [[0.7,0.9],[0.2,0.3],[0.4,0.5],[-0.3,-0.2],[-0.5,-0.4],[-0.9,-0.7]] \\ F_2 & [[0.3,0.6],[0.3,0.5],[0.8,0.9],[-0.5,-0.3],[-0.9,-0.8],[-0.6,-0.3]] \\ F_3 & [[0.4,0.5],[0.2,0.4],[0.7,0.9],[-0.4,-0.2],[-0.9,-0.7],[-0.5,-0.4]] \\ F_4 & [[0.6,0.7],[0.3,0.4],[0.8,0.9],[-0.4,-0.3],[-0.9,-0.8],[-0.7,-0.6]] \end{pmatrix}$$

.

Step 2. Determine the IBN-PIS (Q^*) from interval bipolar neutrosophic decision matrix as follows:

$\left\langle [c_1^+, d_1^+],[e_1^+, f_1^+],[g_1^+, h_1^+],[r_1^-, s_1^-],[t_1^-, u_1^-],[v_1^-, w_1^-] \right\rangle =$ < [0.7, 0.8], [0.0, 0.1], [0.1, 0.2], [-0.3, -0.2], [-0.2, -0.1], [-0.5, -0.3];

$\left\langle [c_2^+, d_2^+],[e_2^+, f_2^+],[g_2^+, h_2^+],[r_2^-, s_2^-],[t_2^-, u_2^-],[v_2^-, w_2^-] \right\rangle =$ < [0.6, 0.7], [0.1, 0.2], [0.1, 0.3], [-0.3, -0.2], [-0.3, -0.1], [-0.6, -0.4];

$\left\langle [c_3^+, d_3^+],[e_3^+, f_3^+],[g_3^+, h_3^+],[r_3^-, s_3^-],[t_3^-, u_3^-],[v_3^-, w_3^-] \right\rangle =$ < [0.3, 0.5], [0.3, 0.5], [0.8, 0.9], [-0.3, -0.2], [-0.9, -0.8], [-0.9, -0.7].

Step 3. The weighted correlation coefficient $Cor_w(F_i, Q^*)$ between alternative F_i (i = 1, 2, ..., m) and IBN-PIS Q^* is obtained as given below.

$R_w(F_1, Q^*) = 2.4465$, $R_w(F_1, F_1) = 2.585351$, $R_w(Q^*, Q^*) = 2.850693$, $Cor_w(F_1, Q^*) = 0.331952$,

$R_w(F_2, Q^*) = 2.9205$, $R_w(F_2, F_2) = 2.905408$, $Cor_w(F_2, Q^*) = 0.3526141$,

$R_w(F_3, Q^*) = 2.6625$, $Q_w(F_3, F_3) = 2.701919$, $Cor_w(F_3, Q^*) = 0.3456741$,

$R_w(F_4, Q^*) = 3.098$, $Q_w(F_4, F_4) = 3.048081$, $Cor_w(F_4, Q^*) = 0.3565369$.

We observe that $Cor_w(F_4, Q^*) > Cor_w(F_2, Q^*) > Cor_w(F_3, Q^*) > Cor_w(F_1, Q^*)$.

Step 4. According to the weighted correlation coefficient values, the ranking order of the companies is presented as: $F_4 > F_2 > F_3 > F_1$.

Hence, the most desirable investment company is F_4.

In Fig 2. we represent the graphical representation of alternatives versus weighted correlation coefficient values.

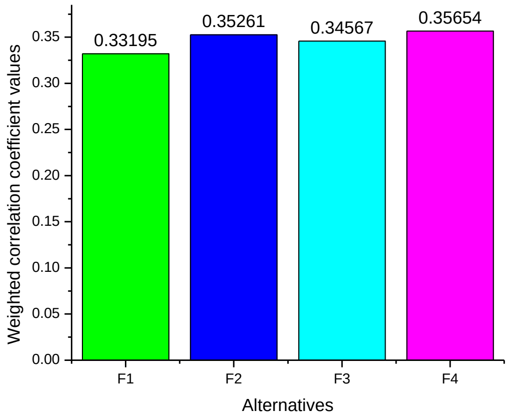

Fig 2. Graphical representation of alternatives versus weighted correlation coefficient values.

Next, we compare the obtained results with the results of Mahmood et al. [31] and Pramanik et al. [33] in Table 1 where the weight vector of the attributes is w = (0.35, 0.25, 0.4) [31]. We see that ranking orders of alternatives derived by the proposed strategy and the strategies discussed by Mahmood et al. [31] and Pramanik et al. [33] are different. We also observe that F_4 is the best option obtained by the proposed strategy as well as the strategy discussed by Mahmood et al. [31] . However, Pramanik et al. [33] found that F_2 is the most desirable alternative based on weighted cross entropy measure.

Table 1. The results derived from different strategies

strategy	Ranking results	Best choice
The proposed weighted correlation coefficient strategy	$F_4 \succ F_2 \succ F_3 \succ F_1$	F_4
Mahmood et al.'s strategy [31]	$F_4 \succ F_1 \succ F_3 \succ F_2$	F_4
Weighted cross entropy measure [33]	$F_1 \prec F_3 \prec F_4 \prec F_2$	F_2

6 Conclusion

In the study, we have defined correlation coefficient and weighted correlation coefficient measures in interval bipolar neutrosophic environments and prove their basic properties. Using the proposed weighted correlation coefficient measure, we have developed a novel MADM strategy in interval bipolar neutrosophic environment. We have solved an investment problem with interval bipolar neutrosophic information. Comparison analysis with other existing strategies is presented to demonstrate the feasibility and applicability of the proposed strategy. We hope that the proposed correlation coefficient measures can be employed to tackle realistic multi attribute decision making problems such as clustering analysis [15], medical diagnosis [21], weaver selection [35-37], fault diagnosis [38], brick selection [39- 40], data mining [41], logistic centre location selection [42- 43], school selection [44], teacher selection [45-47], image processing, information fusion, etc. in interval bipolar neutrosophic environment. Using aggregation operators, the proposed strategy can be extended to multi attribute group decision making problem in interval bipolar neutrosophic set environment.

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Neutrosophic Goal Geometric Programming Problem based on Geometric Mean Method and its Application

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Abstract: This paper describes neutrosophic goal geometric programming method, a new concept to solve multi-objective non-linear optimization problem under uncertainty. The proposed method is described here as an extension of fuzzy and intuitionistic fuzzy goal geometric programming technique in which the degree of acceptance , degree of indeterminacy and degree of rejection is simultaneously considered. A bridge network complex model is presented here to demonstrate the applicability and efficiency of the proposed method. The method is numerically illustrated and the result shows that the neutrosophic goal geometric programming is very efficient to find the best optimal solution than compare to other existing methods.

Keywords: Neutrosophic set, Goal programming, Geometric programming, Bridge network, Reliability optimization.

INTRODUCTION:

In real life situations, most of the time it is unable to find deterministic optimization problems which are well defined because of imprecise information and unknown data. Thus to handle this type of uncertainty and imprecise nature , fuzzy set theory was first introduced by Zadeh [1] in 1965. Fuzzy optimization problems are more realistic and allow to find solutions which are more acceptable to the real problems . In recent time, fuzzy set theory has been widely developed and there are various modification and generalizations has appeared, intuitionistic fuzzy sets (IFS) is one of them. In 1986, Atanassov [2] developed the idea of IFS , which is characterized by the membership degree as well as non-membership degree such that the sum of these two values is less than one. Intuitionistic fuzzy sets can handle the incomplete information but unable to deal with the indeterminate information. Thus further generalization of it is required. To overcome this, neutrosophy [3] was first introduced by Samarandache in 1995, by adding another independent membership function named as indeterminacy membership along with truth membership and falsity membership function.

Goal programming (GP) is one of the most effective and efficient methods among various kinds of existing methods to solve a particular type of non-linear multi-objective decision making problems. In 1977, Charns and Copper [4] first introduced goal programming problem for a linear model. In a standard GP problem, goals and constraints are not always well defined and it is not possible to find the exact value due to vague nature of the coefficients and parameters. Fuzzy and intuitionistic fuzzy approach can handle this type of situations. Many authors use fuzzy goal programming technique to solve various types of multi-objective linear programming problems [7] ,[8] . M.Zangiabadi [18] applied goal programming approach to solve multi-objective transportation problem in fuzzy environment. B.B.Pal [5] described a goal programming procedure for multi-objective linear programming problem. Since geometric programming gives better result to solve non-linear goal programming problem compare to the other non-linear programming methods, P.Ghosh and T.K.Roy [12] ,[13] described the fuzzy goal geometric programming method in intuitionistic environment. Paramanik and Roy [6] introduced intuitionistic fuzzy goal programming approach in vector optimization problem. Sometimes goal of the system and conditions include some vague and undetermined situations. Hence we cannot handle this type of situations by the concept of fuzzy set and intuitionistic fuzzy set theory. Mathematically, to express the decision maker's unclear target levels for the goals and to optimize all goals at the same level, we have to go through a complicated calculations. Here we introduced neutrosophic approach for goal programming to solve this kind of unclear difficulties. Many researchers applied goal programming for solving multi-objective problems in neutrosophic environment [9],[10],[11] . But it is very first when neutrosophic goal geometric programming method is applied to multi-objective non-linear programming problem.

The present study investigates computational algorithm for solving multi-objective goal geometric programming problem by single valued NGGPP technique . The motivation of this paper is to apply an efficient and modified optimization technique to find a pareto optimal solution of the proposed bridge network reliability model to produce highly reliable system with minimum system cost than the other existing methods. An illustrative example is given to show the utility of NGGPP on the reliability model and also the result of the proposed approach is compared with fuzzy goal geometric programming (FGGP) and intuitionistic fuzzy goal geometric programming (IFGGP) approach at the end of this paper.

The structure of the paper is as follows: In Section 2, some basic definitions and Neutrosophic goal geometric programming problem (NGGPP) method is introduced; In section 3, a bridge network reliability model is introduced and provide NGGPP method for solving the proposed model. In Section 4, numerical examples are solved and compared with the existing method .Finally the conclusions are drawn in section 4.

2. Neutrosophic goal geometric programming problem (NGGPP):

Definition 2.1. Let X be a space of points and $x \in$ X. A neutrosophic set (NS) $\tilde{A}^N$ in X having the form

$\tilde{A}^N = \{< x \quad \mu_A(x), \nu_A(x) \,,\, \sigma_A(x) > \mid x \in X\}$, where $\mu_A(x), \nu_A(x)$ and $\sigma_A(x)$ denote the truth membership degree , falsity membership degree and indeterminacy membership degree of x respectively and they are real standard or non-standard subsets of $]\,0^-,1^+[$ i.e.

$$\mu_A(x): X \to \,]\,0^-,1^+[$$

$$\nu_A(x): X \to \,]\,0^-,1^+[$$

and $$\sigma_A(x): X \to \,]\,0^-,1^+[$$

There is no restriction on the sum of $\mu_A(x), \nu_A(x)$ *and* $\sigma_A(x)$. So,

$$0^- \leq \sup\, \mu_A(x) + \sup\, \nu_A(x) + \sup \sigma_A(x) \leq 3^+.$$

Ye [14] ,[15] reduced NSs of non-standards intervals into a kind of simplified neutrosophic sets of standard intervals that will preserve the operations of NSs.

Definition 2.2. [17] Let X be a space of points with a generic element x in X. A single-valued neutrosophic set (SVNS) $\tilde{A}^N$ in X is characterized by $\mu_A(x), \nu_A(x)$ and $\sigma_A(x)$, and of the form

$$\tilde{A}^N = \{< x : \mu_A(x), \nu_A(x), \sigma_A(x) > | \ x \in X\}$$

Where $\mu_A(x): X \to [0,1]$

$\nu_A(x): X \to [0,1]$

and $\sigma_A(x): X \to [0,1]$ with $0 \le \mu_A(x) + \nu_A(x) + \sigma_A(x) \le 3$ for all $x \in X$.

Here we consider neutrosophic goal geometric problem as an extension of intuitionistic fuzzy goal geometric programming problem. In NGGPP , degree of indeterminacy is also taken into consideration for neutrosophic goal programming objectives together with the degree of acceptance and degree of rejection.

A multi-objective non-linear neutrosophic goal geometric programming problem with k objective functions can be taken as follows-

Find $X = (x_1, x_2, \dots x_m)$ so as to

$\widetilde{Minimize}$ $(Z_{01}(x) = \sum_{p=1}^{N_{01}} C_{0ip} \prod_{j=1}^{m} x_j^{\alpha_{0ipj}}$ satisfying target goal achievement value C_{01} with acceptance tolerance $t_{01}{}^{acc}$, rejection tolerance $t_{01}{}^{rej}$ and indeterminacy tolerance $t_{01}{}^{ind}$.

$\widetilde{Minimize}$ $(Z_{02}(x) = \sum_{p=1}^{N_{02}} C_{0ip} \prod_{j=1}^{m} x_j^{\alpha_{0ipj}}$ satisfying target goal achievement value C_{02} with acceptance tolerance $t_{02}{}^{acc}$, rejection tolerance $t_{02}{}^{rej}$ and indeterminacy tolerance $t_{02}{}^{ind}$.

: :

$\widetilde{Minimize}$ $(Z_{0k}(x) = \sum_{p=1}^{N_{0k}} C_{0ip} \prod_{j=1}^{m} x_j^{\alpha_{0ipj}}$ satisfying target goal achievement value C_{0k} with acceptance tolerance $t_{0k}{}^{acc}$, rejection tolerance $t_{0k}{}^{rej}$ and indeterminacy tolerance $t_{0k}{}^{ind}$.

Subject to,

$$Z_r(x) = \sum_{p=1+T_{(r-1)k}}^{N_{rk}} C_{rp} \prod_{j=1}^{m} x_j^{\alpha_{rpj}},$$

$$r = 1,2,\dots,l \ , \quad x = (x_1, x_2, \dots\dots, x_m) > 0.$$

Where we have …(2.1)

$C_{0ip} > 0$, (for p = 1,2,3 , .. , N_{0i} ; $i = 1,2,\dots,k$),

$C_{rp} > 0$, (for k = $1 + N_{0k}, \dots, N_{1k}, N_{1k} + 1, \dots, T_{lk}$; $r = 1,2,\dots,l$) ,

α_{0ipj} (p = 1,2, .. , N_{0i}; $i = 1,2,\dots,p$; j = 1,2, …,m)

and α_{rpj} (k = $1 + N_{0k}, \dots, N_{1k}, N_{1k} + 1, \dots, N_{lk}$; j = 1,2, …,m) are real numbers.

Now using the concept of neutrosophic sets, construct the truth membership function $\mu_i(Z_{oi}(x))$, indeterminacy membership function $\sigma_i(Z_{oi}(x))$ and falsity membership function $\vartheta_i(Z_{oi}(x))$ of NGP objectives are given by –

$$\mu_i(Z_{oi}(x)) = \begin{cases} 1 & , \ Z_{oi} \le C_{0i}; \\ 1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{acc}} & , C_{0i} \le Z_{oi} \le C_{0i} + t_{0i}{}^{acc} \\ 0 & , Z_{oi} \ge C_{0i} + t_{0i}{}^{acc}; \end{cases} \quad \dots(2.2)$$

$$\vartheta_i(Z_{oi}(x)) = \begin{cases} 0 & , \ Z_{oi} \le C_{0i}; \\ \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{rej}} & , \ C_{0i} \le Z_{oi} \le C_{0i} + t_{0i}{}^{rej} \\ 1 & , \ Z_{oi} \ge C_{0i} + t_{0i}{}^{rej}; \end{cases} \quad \dots(2.3)$$

and

$$\sigma_i(Z_{oi}(x)) = \begin{cases} 1 & , \ Z_{oi} \le C_{0i}; \\ 1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{ind}} & , C_{0i} \le Z_{oi} \le C_{0i} + t_{0i}{}^{ind} \\ 0 & , \ Z_{oi} \ge C_{0i} + t_{0i}{}^{ind}; \end{cases} \quad \dots(2.4)$$

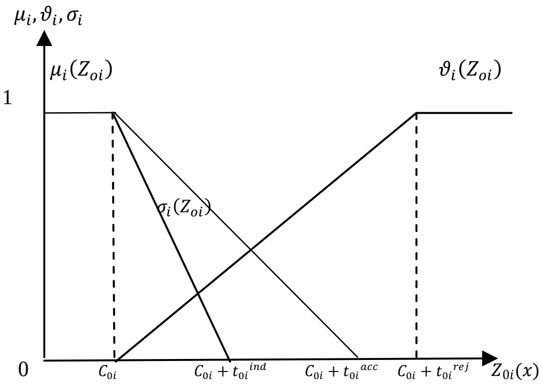

Fig (1) : truth membership function, indeterminacy membership function and falsity membership function for the objective functions $Z_{0i}(x)$.

Now the above NGP model (3.1) can be reduced to a crisp model by maximizing the degree of acceptance, degree of indeterminacy as well as minimizing the degree of falsity of NGP objective functions. Hence we have

Maximize $\mu_i\big(Z_{oi}(x)\big)$ for i = 1,2,…,k

Minimize $\vartheta_i\big(Z_{oi}(x)\big)$ for i = 1,2,…,k

Maximize $\sigma_i\big(Z_{oi}(x)\big)$ for i = 1,2,…,k

Subject to, $Z_r(x) \le b_r \;; r = 1,2,\dots,l$

$$0 \le \mu_i(Z_{oi}) + \vartheta_i(Z_{oi}) + \sigma_i(Z_{oi}) \le 3,$$

$$\vartheta_i(Z_{oi}) \ge 0,$$

$$\mu_i(Z_{oi}) \ge \vartheta_i(Z_{oi}),$$

$\mu_i(Z_{oi}) \ge \sigma_i(Z_{oi})$, for i = 1,2,…,p

and X $= (x_1, x_2, \dots\dots, x_m) > 0.$ … (2.5)

Now (2.5) is equivalent to-

Maximize α Minimize β Maximize γ

Subject to, $\mu_i\big(Z_{oi}(x)\big) \ge \alpha$

$$\vartheta_i\big(Z_{oi}(x)\big) \le \beta$$

$\sigma_i\big(Z_{oi}(x)\big) \ge \gamma$, for i = 1,2,…,

$Z_r(x) \le b_r$; $r = 1,2,\dots,l$

$0 \le \alpha + \beta + \gamma \le 3$, $\alpha \ge \beta$, $\alpha \ge \gamma$,
$\alpha, \beta, \gamma \in [0,3]$,

and X= $(x_1, x_2, \dots\dots, x_m) > 0.$ ….(2.6)

Now by geometric mean method , the above model (2.6) can be written as –

Minimize $\beta(1-\alpha)(1-\gamma)$

Subject to,

$Z_{oi}(x) \le C_{0i} + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind}\left(\frac{\beta(1-\alpha)(1-\gamma)}{3}\right)$,

(for i = 1,2,….,k.)

$\frac{1}{b_r} Z_r(x) \le 1, \quad r = 1,2,\dots,l.$

$0 \le \alpha + \beta + \gamma \le 3$, $\alpha \ge \beta$, $\alpha \ge \gamma$,
$\alpha, \beta, \gamma \in [0,3]$,

and X= $(x_1, x_2, \dots\dots, x_m) > 0.$ …(2.7)

Let , $\beta(1-\alpha)(1-\gamma) = w > 0$, then the above model becomes-

Minimize w

Subject to , $\frac{Z_{oi}(x)}{C_{0i} + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind} \times w} \le 1$,

(for i = 1,2,….,k);

$$\frac{1}{b_r} Z_r(x) \le 1, \quad r = 1,2,\dots,l.$$

X= $(x_1, x_2, \dots\dots, x_m) > 0.$ …(2.8)

From (3.8) we construct the dual programming model as –

Maximize

$$\left(\frac{w}{\delta_{00}}\right)^{\delta_{00}} \prod_{i=1}^{k} \prod_{p=1}^{N_{0i}} \left[\frac{C_{0ip}}{\left(C_{0i} + a_{0i}{}^{acc} \times a_{0i}{}^{rej} \times a_{0i}{}^{ind} \times w\right)\delta_{0ip}}\right]^{\delta_{0ip}}$$

$$\times \prod_{r=1}^{l} \prod_{q=1+T_{(r-1)k}}^{N_{rk}} \left[\frac{C_{rq}}{C_r\,\delta_{orq}}\right]^{\delta_{orq}} \left(\sum_{k=1}^{N_{0i}} \delta_{0ip}\right)^{(\sum_{k=1}^{N_{0i}} \delta_{0ip})}$$

$$\times \left(\sum_{q=1+T_{(r-1)k}}^{N_{rk}} \delta_{0rq}\right)^{\sum_{q=1+T_{(r-1)k}}^{T_{rk}} \delta_{orq}}$$

$$\left.\begin{array}{l}\text{Subject to, } \delta_{00} = 1 \\ \sum_{p=1}^{N_{0i}} \delta_{0ip} = 1 \\ \sum_{q=1+T_{(r-1)k}}^{N_{rk}} \delta_{0rq} = 1,\end{array}\right\} \text{Normality Condition} \quad \text{(for i =1,2,…,k; } r = 1,2,\dots,l)$$

$$\sum_{i=1}^{k}\sum_{p=1}^{N_{0i}} \alpha_{0ipj}\delta_{0ip} + \sum_{r=1}^{l}\sum_{q=1+(r-1)k}^{N_{rk}} \alpha_{rpj}\,\delta_{0rq} = 0.$$

(for $j = 1,2,\dots,m.$) Orthogonality condition

where
$\delta_{0ip} > 0$ (for $p = 1,2,\dots, N_{0i}$; $i = 1,2,\dots,k$)
$\delta_{0rq} > 0$ (for $q = 1 + N_{(r-1)k},\dots, T_{rk}$; $r = 1,2,\dots,l$)
(Positivity conditions) … (2.9)

Let there are total T number of terms in the above primal problem. Then the degree of difficulty (DD) of the single objective geometric programming problem is T – (m+1).

Case I : for T > (m+1) , a solution vector exists for the dual variables.

Case II : T < (m+1) , generally no solution vectors exist for the dual variables, but we can get the approximate solution for this system using different methods.

Now to find out the solution of the geometric programming model (2.8) , firstly we have to find out the optimal solution of the dual problem (2.9) .Hence from the primal-dual relationship, the corresponding values of the primal variable vector x can be easily obtained. The LINGO-16.0 software is used here to find optimal dual variables from the equations of (2.9).

Lemma 3.1: The ranges of truth , indeterminacy and falsity membership function of neutrosophic goal geometric programming problem will satisfy if ${t_{0i}}^{rej} > 2{t_{0i}}^{ind}$ and ${t_{0i}}^{acc} > {t_{0i}}^{ind}$, where ${t_{0i}}^{acc}, {t_{0i}}^{rej}$ and ${t_{0i}}^{ind}$ are acceptance tolerance, rejection tolerance and indeterminacy tolerance respectively of the NGP objective functions.

Proof: From the equations (3.5) we have –

$$\mu_i(Z_{oi}) \geq \sigma_i(Z_{oi})$$

implies $$1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \geq 1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}}$$

or, $$(Z_{oi}(x) - C_{0i})\left(\frac{1}{{t_{0i}}^{ind}} - \frac{1}{{t_{0i}}^{acc}}\right) \geq 0 \qquad \text{(i)}$$

In the above mentioned neutrosophic goal programming problem , we consider each objective functions $Z_{oi}(x)$ satisfying target achievement value C_{0i} and also from the relation – $\vartheta_i(Z_{oi}) \geq 0$

or, $$\frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{rej}} \geq 0$$

or, $$(Z_{oi}(x) - C_{0i}) \geq 0 \qquad \text{(ii)}$$

Thus the relation (i) is true if

$$\left(\frac{1}{{t_{0i}}^{ind}} - \frac{1}{{t_{0i}}^{acc}}\right) \geq 0$$

i.e. $${t_{0i}}^{acc} > {t_{0i}}^{ind} \qquad \text{(iii)}$$

Hence from relation (iii), we have in neutrosophic goal geometric programming problem, acceptance tolerance ${t_{0i}}^{acc}$ should be greater than indeterminacy tolerance ${t_{0i}}^{ind}$.

Again from the relation $\mu_i(Z_{oi}) \geq \vartheta_i(Z_{oi})$ and $\mu_i(Z_{oi}) \geq \sigma_i(Z_{oi})$

we have, $$1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \geq \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{rej}} \qquad \text{(iv)}$$

and $$1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \geq 1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{ind}} \qquad \text{(v)}$$

Adding the above inequalities (iv) and (v) , we get-

$$1 - \frac{Z_{oi}(x) - C_{0i}}{{t_{0i}}^{acc}} \geq \frac{1}{2} + \frac{(Z_{oi}(x) - C_{0i})}{2}\left(\frac{1}{{t_{0i}}^{rej}} - \frac{1}{{t_{0i}}^{ind}}\right) \qquad \text{(vi)}$$

Now from (3.5) using the relation

$\mu_i(Z_{oi}) \geq \vartheta_i(Z_{oi}) \geq 0$ and

$\mu_i(Z_{oi}) + \vartheta_i(Z_{oi}) + \sigma_i(Z_{oi}) \leq 3$

we get , $\sigma_i(Z_{oi}) \leq 3$

or, $1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{ind}} \leq 3$

or, $Z_{oi}(x) - C_{0i} \geq -2t_{0i}{}^{ind}$

or $\frac{1}{Z_{oi}(x) - C_{0i}} \leq -\frac{1}{2t_{0i}{}^{ind}}$ (vii)

Hence from $\mu_i(Z_{oi}) + \vartheta_i(Z_{oi}) + \sigma_i(Z_{oi}) \leq 3$ using (vi) and (vii) –

$\frac{1}{2} + \frac{(Z_{oi}(x) - C_{0i})}{2}\left(\frac{1}{t_{0i}{}^{rej}} - \frac{1}{t_{0i}{}^{ind}}\right) + \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{rej}} + 1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{ind}} \leq 3$ gives $t_{0i}{}^{rej} > 2t_{0i}{}^{ind}$.

Thus from the above relation it is clear that in neutrosophic goal geometric programming problem half of the rejection tolerance $t_{0i}{}^{rej}$ should be greater than the indeterminacy tolerance $t_{0i}{}^{ind}$.

Theorem 3.1: x^* is a pareto optimal solution to NGGPP (3.1) iff x^* is a pareto optimal solution to fuzzy goal geometric programming problem (FGGPP) which is of the form

Minimize $(Z_{01}(x), Z_{02}(x), \ldots, Z_{0k}(x))$

Subject to, $Z_r(x) \leq b_r$, $r = 1,2,\ldots,l$

$$X = (x_1, x_2, \ldots \ldots, x_m) > 0. \qquad \ldots.(2.10)$$

Proof:

Definition: x^* is said to be a pareto optimal solution to the neutrosophic goal geometric programming problem (2.1) iff there does not exist another x such that $\mu_i(Z_{oi}(x)) \geq \mu_i(Z_{oi}(x^*))$, $\vartheta_i(Z_{oi}(x)) \leq \vartheta_i(Z_{oi}(x^*))$ and $\sigma_i(Z_{oi}(x)) \geq \sigma_i(Z_{oi}(x^*))$ for all i = 1,2, ..., k with strict inequality holds for at least one i.

If x^* be a pareto optimal solution of the FGGPP (2.10) then there does not exist any x such that $Z_{oi}(x) \leq Z_{oi}(x^*)$ for all i=1,2,…,k. and $Z_{oi}(x^*) \neq Z_{oi}(x)$ for at least one i.

Then we have for all $X = (x_1, x_2, \ldots \ldots, x_m)$

$$Z_{oi}(x) \leq Z_{oi}(x^*) \qquad \ldots. (A)$$

with strict inequality hold for at least one i.

i.e. $Z_{oi}(x) - C_{0i} \leq Z_{oi}(x^*) - C_{0i}$

or, $\frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{acc}} \leq \frac{Z_{oi}(x^*) - C_{0i}}{t_{0i}{}^{acc}}$

or, $1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{acc}} \geq 1 - \frac{Z_{oi}(x^*) - C_{0i}}{t_{0i}{}^{acc}}$ implies $\mu_i(Z_{oi}(x)) \geq \mu_i(Z_{oi}(x^*))$.

Similarly from (A) we have

$\frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{rej}} \leq \frac{Z_{oi}(x^*) - C_{0i}}{t_{0i}{}^{rej}}$ which implies $\vartheta_i(Z_{oi}(x)) \leq \vartheta_i(Z_{oi}(x^*))$

and also $\frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{ind}} \leq \frac{Z_{oi}(x^*) - C_{0i}}{t_{0i}{}^{ind}}$

or, $1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{ind}} \geq 1 - \frac{Z_{oi}(x^*) - C_{0i}}{t_{0i}{}^{ind}}$

or, $\sigma_i(Z_{oi}(x)) \geq \sigma_i(Z_{oi}(x^*))$. Hence from the definition of pereto optimal solution to the NGGPP , we have x^* is the pareto optimal solution of (2.1).

Conversely, let x^* is a pareto optimal solution to NGGPP (2.1), then from the expression of membership function given in (2.2) we get

$$1 - \frac{Z_{oi}(x) - C_{0i}}{t_{0i}{}^{acc}} \geq 1 - \frac{Z_{oi}(x^*) - C_{0i}}{t_{0i}{}^{acc}}$$

i.e. $Z_{oi}(x) \leq Z_{oi}(x^*)$.

Again using (3.3) we have

$\frac{Z_{oi}(x)-C_{0i}}{t_{0i}{}^{ind}} \leq \frac{Z_{oi}(x^*)-C_{0i}}{t_{0i}{}^{ind}}$ which implies $Z_{oi}(x) \leq Z_{oi}(x^*)$.

Similarly, using (3.4) ,

$1-\frac{Z_{oi}(x)-C_{0i}}{t_{0i}{}^{ind}} \geq 1-\frac{Z_{oi}(x^*)-C_{0i}}{t_{0i}{}^{ind}}$ gives

$Z_{oi}(x) \leq Z_{oi}(x^*)$.

Thus we have $Z_{oi}(x) \leq Z_{oi}(x^*)$ with strict inequality hold for at least one i , $i \in \{1,2,\ldots,k\}$ and which shows that x^* is a pareto optimal solution of (2.10).

3. Numerical Example:

3.1. Bridge network Model [16] :

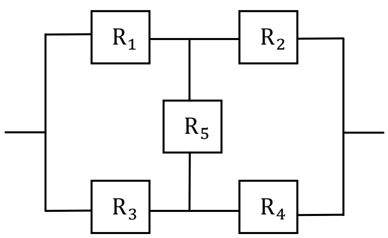

Fig (2) : A five-component complex bridge network system

Here a bridge network system as shown in the figure(3) has been considered, each having a component reliability R_j , j = 1,2,...,5 .

Based on the simple probability theorem

$$\Pr(X \cup Y) = \Pr(X) + \Pr(Y) - \Pr(X \cap Y) \qquad (3.1)$$

the system reliability $R_S(R)$ of the bridge network system is given by as follows:

Now to use equation (3.1) , it is required to found all possible paths from the input node to output node. The system will operate if the components in any one the following sets

$\{R_1, R_2\}$, $\{R_3, R_4\}$, $\{R_1, R_5, R_4\}$ and $\{R_3, R_5, R_2\}$ operate.

Thus the system reliability is given by

$R_S(R) = \Pr(\{R_1, R_2\} \cup \{R_3, R_4\} \cup \{R_1, R_5, R_4\} \cup \{R_3, R_5, R_2\})$

Since all the components operate independently , thus-

$\Pr(\{R_1, R_2\}) = R_1R_2$,

$\Pr(\{R_3, R_4\}) = R_3R_4$,

$\Pr(\{R_1, R_5, R_4\}) = R_1R_5R_4$,

$\Pr(\{R_3, R_5, R_2\}) = R_3R_5R_2$.

Now using equation (3.1) ,

$\Pr(\{R_1, R_2\} \cup \{R_3, R_4\})$

$= \Pr(\{R_1, R_2\}) + \Pr(\{R_3, R_4\}) - \Pr(\{1,2\} \cap \{3,4\})$

$= R_1R_2 + R_3R_4 - R_1R_2R_3R_4$

Similarly

$\Pr(\{R_1, R_2\} \cup \{R_3, R_4\} \cup \{R_1, R_5, R_4\}) =$

$R_1R_2 + R_3R_4 - R_1R_2R_3R_4 + R_1R_5R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 + R_1R_2R_3R_5R_4$.

$\Pr(\{R_1, R_2\} \cup \{R_3, R_4\} \cup \{R_1, R_5, R_4\} \cup \{R_3, R_5, R_2\})$
$= R_1R_2 + R_3R_4 + R_1R_5R_4 + R_3R_5R_2 - R_1R_2R_3R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 - R_1R_3R_5R_2 - R_2R_3R_5R_4 + 2R_1R_2R_3R_5R_4$

Thus the multi- objective reliability optimization model becomes

Maximize $R_S(R) = R_1R_2 + R_3R_4 + R_1R_5R_4 + R_3R_5R_2 - R_1R_2R_3R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 - R_1R_3R_5R_2 - R_2R_3R_5R_4 + 2R_1R_2R_3R_5R_4$

Minimize $C_S(R) = \sum_{i=1}^{n} C_i R_i^{a_i}$

$0 < R_j \leq 1$, $0 \leq R_S \leq 1$, j =1,2, … ,5. (3.2)

Where $C_S(R)$ denote the cost of the system and C_{lim} is the available cost of the system.

3.1. Application of Neutrosophic Goal Geometric Programming on Bridge Network Reliability Model:

To solve the above multi-objective problem using geometric programming approach , the problem should be in minimization form. Thus , the suitable form of optimization model is taken as

Minimize $R_S{}'(R) = -R_1R_2 - R_3R_4 - R_1R_5R_4 - R_3R_5R_2 + R_1R_2R_3R_4 + R_2R_5R_4 + R_1R_3R_5R_4 + R_1R_3R_5R_2 + R_2R_3R_5R_4 - 2R_1R_2R_3R_5R_4$

satisfying target achievement value R_0 with acceptance tolerance $t_R{}^{acc}$, rejection tolerance $t_R{}^{rej}$ and indeterminacy tolerance $t_R{}^{ind}$.

Also , we Minimize $C_S(R) = \sum_{i=1}^{n} C_i R_i{}^{a_i}$ satisfying target achievement value C_0 with acceptance tolerance $t_C{}^{acc}$, rejection tolerance $t_C{}^{rej}$ and indeterminacy tolerance $t_C{}^{ind}$. Now , construct the truth membership function , falsity membership function and indeterminacy membership function as follows –

$$\mu_{R_S{}'}(R) = \begin{cases} 1 & , \quad R_S{}'(R) \le R_0; \\ 1 - \frac{R_S{}' - R_0}{t_R{}^{acc};} & , R_0 \le R_S{}'(R) \le R_0 + t_R{}^{acc}; \\ 0 & , \quad R_S{}'(R) \ge R_0 + t_R{}^{acc}; \end{cases}$$

$$\vartheta_{R_S{}'}(R) = \begin{cases} 0 & , \quad R_S{}'(R) \le R_0; \\ \frac{R_S{}' - R_0}{t_R{}^{rej}} & , \quad R_0 \le R_S{}'(R) \le R_0 + t_R{}^{rej} \\ 1 & , \quad R_S{}'(R) \ge R_0 + t_R{}^{rej}; \end{cases}$$

$$\sigma_{R_S{}'}(R) = \begin{cases} 1 & , \quad R_S{}'(R) \le R_0; \\ 1 - \frac{R_S{}' - R_0}{t_R{}^{ind}} & , R_0 \le R_S{}'(R) \le R_0 + t_R{}^{ind} \\ 0 & , \quad R_S{}'(R) \ge R_0 + t_R{}^{ind}; \end{cases}$$

$$\mu_{C_S}(R) = \begin{cases} 1 & , \quad C_S(R) \le C_0; \\ 1 - \frac{C_S - C_0}{t_C{}^{acc}} & , C_0 \le C_S(R) \le C_0 + t_C{}^{acc} \\ 0 & , \quad C_S(R) \ge C_0 + t_C{}^{acc} \end{cases}$$

$$\vartheta_{C_S}(R) = \begin{cases} 0 & , \quad C_S(R) \le C_0; \\ \frac{C_S - C_0}{t_C{}^{rej}} & , C_0 \le C_S(R) \le C_0 + t_C{}^{rej} \\ 1 & , \quad C_S(R) \ge C_0 + t_C{}^{rej}; \end{cases}$$

$$\sigma_{C_S}(R) = \begin{cases} 1 & , \quad C_S(R) \le C_0; \\ 1 - \frac{C_S - C_0}{t_C{}^{ind}} & , \quad C_0 \le C_S(R) \le C_0 + t_C{}^{ind} \\ 0 & , \quad C_S(R) \ge C_0 + t_C{}^{ind}; \end{cases}$$

Now using (2.5), the above model (3.2) reduces to the following form –

Maximize $\mu_{R_S{}'}$ Maximize μ_{C_S}

Maximize $\sigma_{R_S{}'}$ Maximize σ_{C_S}

Minimize $\vartheta_{R_S{}'}$ Minimize ϑ_{C_S}

Subject to , $0 \le \mu_{R_S{}'} + \vartheta_{R_S{}'} + \sigma_{R_S{}'} \le 3$,

$0 \le \mu_{C_S} + \sigma_{C_S} + \vartheta_{C_S} \le 3$,

$\vartheta_{R_S{}'} \ge 0$, $\vartheta_{C_S} \ge 0$

$\mu_{R_S{}'} \ge \vartheta_{R_S{}'}$, $\mu_{C_S} \ge \vartheta_{C_S}$,

$\mu_{R_S{}'} \ge \sigma_{R_S{}'}$, $\mu_{C_S} \ge \sigma_{C_S}$,

$0 < R_i \le 1$; i=1,2,…,n; …..(3.1.1)

The above model (3.1.1) is equivalent to

Maximize α, Minimize β, Maximize γ

Subject to, $\mu_{R_S'} \geq \alpha$, $\mu_{C_S} \geq \alpha$,

$\vartheta_{R_S'} \leq \beta$, $\vartheta_{C_S} \leq \beta$,

$\sigma_{R_S'} \geq \gamma$, $\sigma_{C_S} \geq \gamma$,

$0 \leq \alpha + \beta + \gamma \leq 3$, $\alpha \geq \beta$, $\alpha \geq \gamma$,

$0 \leq \alpha, \beta, \gamma \leq 1$ …..(3.1.2)

Using geometric mean method (4.1.8) becomes-

Minimize w

Subject to ,

$$\frac{-R_1R_2 - R_3R_4 - R_1R_5R_4 - R_3R_5R_2 + R_1R_2R_3R_4 + R_2R_5R_4 + R_1R_3R_5R_4 + R_1R_3R_5R_2 + R_2R_3R_5R_4 - 2R_1R_2R_3R_5R_4}{R_0 + {a_{0i}}^{acc} \times {a_{0i}}^{rej} \times {a_{0i}}^{ind} \times w} \leq 1 \quad ;$$

$$\frac{\sum_{i=1}^{5} C_i {R_i}^{a_i}}{C_0 + {a_{0i}}^{acc} \times {a_{0i}}^{rej} \times {a_{0i}}^{ind} \times w} \leq 1;$$

$0 < R_i \leq 1$; i=1,2,…,5; ..(3.1.3)

where we take $w = \beta(1 - \alpha)(1 - \gamma) > 0$ as a parameter. The degree of difficulty (D.D) of (4.1.9) is $(5 + 2) - (5 + 1) = 1 \ (> 0)$.

Now using (2.9) , the above model (3.1.3) can be solved by geometric programming technique after finding its dual.

4 Numerical Example

Here we consider the bridge network reliability optimization model for the numerical exposure. Thus the model (4.1) becomes-

Maximize $R_S(R) = R_1R_2 + R_3R_4 + R_1R_5R_4 + R_3R_5R_2 - R_1R_2R_3R_4 - R_1R_2R_5R_4 - R_1R_3R_5R_4 - R_1R_3R_5R_2 - R_2R_3R_5R_4 + 2R_1R_2R_3R_5R_4$

Minimize $C_S(R) = \sum_{i=1}^{n} C_i {R_i}^{a_i}$

$0 < R_j \leq 1$, $0 \leq R_S \leq 1$, j =1,2, … ,5. …(4.1)

Table (1) : The input data for the neutrosophic goal geometric programming problem (5.1) is given as follows –

C_1	C_2	C_3	C_4	C_5	C_0	${t_C}^{acc}$	${t_C}^{rej}$	${t_C}^{ind}$	${t_R}^{acc}$	${t_R}^{rej}$	${t_R}^{ind}$	$a_i, \forall i$	R_0
12	10	15	18	16	100	8	14	6	0.3	0.52	0.25	0.15	0.2

Table (2): Comparison of optimal solutions of (4.1) by NGGPP method with fuzzy goal geometric programming problem (FGGPP) approach and intuitionistic fuzzy goal geometric programming (IFGGPP) approach:

Method	R_1	R_2	R_3	R_4	R_5	$R_S(R)$	$C_S(R)$
FGGPP	0.905917	0.905923	0.896927	0.796213	0.948311	0.970147	69.702
IFGGPP	0.812514	0.992162	0.992359	0.992842	0.892531	0.998364	70.313
NGGPP	0.967124	0.992981	0.993162	0.965927	0.985742	0.999519	70.786

The above table describes the comparison of results of objective functions for primal problem of the proposed neutrosophic goal geometric programming approach with the FGGPP and IFGGPP approach. It is clear from the above table (2) that NGGPP approach gives better result than the IFGGPP approach in perspective of system reliability. But in view of system cost the proposed approach gives a little bit higher value than the IFGGPP and FGGPP method.

5. **Conclusion and future work:**

A new concept to non-linear multi-objective optimization problem in neutrosophic environment is discussed in this paper. In this work we have introduced NGGPP technique to find the best optimal solution of the multi-objective bridge network reliability model in which system reliability and system cost are chosen as two objective function. Finally an illustrative numerical example is provided by comparing the result obtained in NGGPP technique with IFGGPP and FGGPP approach to demonstrate the efficiency of the proposed method. Thus the proposed method is an efficient and modified optimization technique and can construct a highly reliable system than the other existing method. The method presented here is quite general and can be applied to the typical problems in other areas of Operation Research and Engineering Sciences, like Transportation problems, Inventory problems, Structural optimization, etc.

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The Ingenuity of Neutrosophic Topology via *N*-Topology

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Abstract: In this paper we desire to extend the neutrosophic topological spaces into N-neutrosophic topological spaces. Also we show that this theory can be deduced to N-intuitionistic and N-fuzzy topological spaces etc. Further we develop not only the concept of classical generalized closed sets into N-neutrosophic topological spaces but also obtain its basic properties. Finally we investigate its continuous function and generalized continuous function.

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Keywords: N_k-topology, $N_kint(A)$, $N_kcl(A)$, N_k-generalized closed set, N_k-continuous function, N_k-generalized continuous function.

1 Introduction

Set theory is the fundamental concept in mathematics developed by a Russian Mathematician George Cantor in 1877. He showed that the points on two dimensional square has a one to one correspondent with points on different line segment leading to the development of dimensional theory. Frechet and Hausdorff along with others studied general topology. Hausdorff, the German mathematician, following the footsteps of Cantor developed set theory. Set theory enabled us to study various precise concepts in mathematics. But in real life situation we do come across many imprecise concepts or uncertain situation. If a class has fifty students say, to distinguish the taller/stronger students we are left with some short of uncertainty or vagueness. We can overcome the vagueness by fixing the percentage of membership namely the percentage of membership enables us to find out the level of inexactness. This theory is known as fuzzy theory.

The concepts of fuzzy set was established by Zadeh.A [12]. This is an essential tool to analyse imprecise mathematical information. Since 1965, this theory has been greatly acknowledged by the community of mathematicians, scientists, engineers and social scientists [4,9,10,11]. The idea of fuzzy topological space was introduced by Chang.C.L [3]. Atanassov.K introduced the seed of intuitionistic fuzzy set [1] and his colleagues [2] developed it further. Smarandache extended it to a neutrosophic set[7,8]. The notion of neutrosophic crisp sets and topological spaces were the contribution of Salama.A.A and Alblowi.S.A [6]. The geometric existence of N-topology was given by Lellis Thivagar et al. [5] which is a nonempty set equipped with N-arbitrary topologies.

In this paper, we explore the possibility of expanding the classical neutrosophic topological spaces into N-neutrosophic topological spaces and also try to deduce N-intuitionistic and N-fuzzy topological spaces etc. Further we develop the concept of classical generalized closed sets into N-neutrosophic topological spaces and verify its properties. Finally, we investigate the related continuous function and generalized continuous function.

2 Preliminaries

In this section, we discuss some basic definitions and properties of N-topological spaces as well as fuzzy, intuitionistic and neutrosophic topological spaces which are useful in sequel.

Definition 2.1 [5] Let X be a non empty set, then $\tau_1, \tau_2, \dots, \tau_N$ be N-arbitrary topologies defined on X and the collection $N\tau = \{S \subseteq X : S = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i),\ A_i, B_i \in \tau_i\}$ is called a N-topology on X if the following axioms are satisfied:

(i) $X, \emptyset \in N\tau$.

(ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{\infty} \in N\tau$.

(iii) $\bigcap_{i=1}^{n} S_i \in N\tau$ for all $\{S_i\}_{i=1}^{n} \in N\tau$.

Then $(X, N\tau)$ is called a N-topological space on X. The elements of $N\tau$ are known as $N\tau$-open sets on X and its complement is called as $N\tau$-closed on X.

Definition 2.2 [12] Let X be a non empty set. A fuzzy set A is an object having the form $A = \{(x, \mu_A(x)) : x \in X\}$, where $0 \leq \mu_A(x) \leq 1$ represents the degree of membership of each $x \in X$ to the set A.

Definition 2.3 [1,2] Let X be a non empty set. An intuitionistic set A is of the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x)$ and $\gamma_A(x)$ represent the degree of membership and non membership function respectively of each $x \in X$ to the set A and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

Definition 2.4 [7] Let X be a non empty set. A neutrosophic set A having the form $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) respectively of each $x \in X$ to the set A. Also $^{-}0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^{+}$ for all $x \in X$.

Remark 2.5 The following definitions can be deduced into fuzzy if the percentages of indeterminacy and non membership are not taken into consideration so also for intuitionistic case the percentage of indeterminacy is not considered.

Definition 2.6 [7] Let X be a non empty neutrosophic set. if $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \sigma_B(x), \gamma_B(x)) : x \in X\}$ are two neutrosophic sets in X, then the following statements hold:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$.

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $A^c = \{(x, \gamma_A(x), \sigma_A(x), \mu_A(x)) : x \in X\}$ [Complement of A].

(iv) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \min\{\sigma_A(x), \sigma_B(x)\}, \max\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

(v) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \max\{\sigma_A(x), \sigma_B(x)\}, \min\{\gamma_A(x), \gamma_B(x)\}) : x \in X\}$.

Remark 2.7 Let X be a non empty neutrosophic set. We consider the neutrosophic empty set 0 as $0 = \{(x, 0, 0, 1) : x \in X\}$ and the neutrosophic whole set 1 as $1 = \{(x, 1, 1, 0) : x \in X\}$.

Remark 2.8 By the notion k-set we mean any one of the following sets: fuzzy set, intuitionistic set, neutrosophic set.

Definition 2.9 [6,7] Let X be a non empty set. A k-topology on X is a family ${}_k\tau$ of k-sets in X satisfying the following axioms:

(i) the sets 1 and 0 belong to the family ${}_k\tau$.

(ii) an arbitrary union of sets of the family ${}_k\tau$ belong to ${}_k\tau$.

(iii) the finite intersection of sets of the family ${}_k\tau$ belong to ${}_k\tau$.

Then the ordered pair $(X, {}_k\tau)$ (simply X) is called k-topological space on X. The elements of ${}_k\tau$ are known as k-open sets on X and its complement is called as k-closed on X.

Definition 2.10 [6] The interior and closure of a k-set A of a k-topological space $(X, {}_k\tau)$ are respectively defined as

(i) ${}_k int(A) = \cup\{G : G \subseteq A$ and G is k-open in $X\}$.

(ii) ${}_k cl(A) = \cap\{F : A \subseteq F$ and F is k-closed in $X\}$.

Corollary 2.11 [7] If A, B, C and D are k-sets in X, then the followings are true:

(i) $A \subseteq B$ and $C \subseteq D \Rightarrow A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.

(ii) If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$. If $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

(iii) If $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$.

(iv) $(A \cap B)^c = A^c \cup B^c$, $(A \cup B)^c = A^c \cap B^c$ and $(A^c)^c = A$. If $A \subseteq B \Rightarrow B^c \subseteq A^c$.

(v) $1^c = 0$ and $0^c = 1$.

Now, we introduce the notions of image and pre-image of neutrosophic sets. Let us consider X and Y as two non empty sets and $f : X \to Y$ be a function.

Definition 2.12 [6] Let X and Y be two non empty sets, $A = \{(x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X\}$ be a neutrosophic set in X and $B = \{(y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y\}$ be a neutrosophic set in Y. Then

(i) the pre-image of B under f, denoted by $f^{-1}(B)$, is the neutrosophic set in X defined by $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\sigma_B)(x), f^{-1}(\gamma_B)(x)) : x \in X\}$.

(ii) the image of A under f, denoted by $f(A)$, is the neutrosophic set in Y defined by $f(A) = \{(y, f(\mu_A)(y), f(\sigma_A)(y), (1 - f(1 - \gamma_A))(y)) : y \in Y\}$, where

$$f(\mu_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\sigma_A)(y) = \begin{cases} sup_{x \in f^{-1}(y)} \sigma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases}$$

For the sake of simplicity, let us use the symbol $f_-(\gamma_A)$ for $(1 - f(1 - \gamma_A))$.

Corollary 2.13 [6] Let $A_{i \in J}$, $B_{i \in J}$ be k-sets in X and Y respectively and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$.

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(c) $A_{i \in J} \subseteq f^{-1}(f(A_{i \in J}))$ { If f is injective, then $A_{i \in J} = f^{-1}(f(A_{i \in J}))$}.

(d) $f(f^{-1}(B_{i \in J})) \subseteq B_{i \in J}$ { If f is surjective, then $f(f^{-1}(B_{i \in J})) = B_{i \in J}$}.

(e) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$.

(f) $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$.

(g) $f(\cup A_i) = \cup f(A_i)$.

(h) $f(\cap A_i) \subseteq f(A_i)$ { If f is injective, then $f(\cap A_i) = \cap f(A_i)$}.

(i) $f^{-1}(1) = 1$.

(j) $f^{-1}(0) = 0$.

(k) $f(1) = 1$, if f is surjective.

(l) $f(0) = 0$.

(m) $(f(A_{i\in J}))^c \subseteq f(A^c_{i\in J})$, if f is surjective.

(n) $(f^{-1}(B_{i\in J}))^c = f^{-1}(B^c_{i\in J})$.

3 N_k-Topological Spaces

In this section, we introduce N-fuzzy, N-intuitionistic and N-neutrosophic topological spaces and discuss their properties. Henceforth in this paper by the notion $N_k\tau$ we mean N-fuzzy topology (if $k = f$), N-intuitionistic topology (if $k = i$) and N-neutrosophic topology (if $k = n$).

Definition 3.1 Let X be a non empty set, then ${}_k\tau_1, {}_k\tau_2, \dots, {}_k\tau_N$ be N-arbitrary k topologies defined on X and the collection $N_k\tau = \{G \subseteq X : G = (\bigcup_{i=1}^{N} A_i) \cup (\bigcap_{i=1}^{N} B_i),\ A_i, B_i \in {}_k\tau_i\}$ is called N_k-topology on X if the following axioms are satisfied:

(i) $1, 0 \in N_k\tau$.

(ii) $\bigcup_{i=1}^{\infty} G_i \in N_k\tau$ for all $\{G_i\}_{i=1}^{\infty} \in N_k\tau$.

(iii) $\bigcap_{i=1}^{n} G_i \in N_k\tau$ for all $\{G_i\}_{i=1}^{n} \in N_k\tau$.

Then $(X, N_k\tau)$ is called N_k-topological space on X. The elements of $N_k\tau$ are known as N_k-open sets on X and its complement is called N_k-closed sets on X.

Example 3.2 Let $N = 3$, $X = \{a, b, c\}$. Define the neutrosophic sets $A = \{(x, (\frac{a}{1}, \frac{b}{1}, \frac{c}{1}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}))\}$ and $B = \{(x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}))\}$ in X. Then $A \cup B = \{(x, (\frac{a}{1}, \frac{b}{1}, \frac{c}{1}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}))\}$, $A \cap B = \{(x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0}, \frac{b}{0}, \frac{c}{0}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}))\}$. Considering ${}_n\tau_1 O(X) = \{0, 1, A\}$, ${}_n\tau_2 O(X) = \{0, 1, B\}$ and ${}_n\tau_3 O(X) = \{0, 1\}$, we get $3_n\tau O(X) = \{0, 1, A, B, A \cup B, A \cap B\}$ which is a tri-neutrosophic topology on X. The pair $(X, 3_n\tau)$ is called a tri-neutrosophic topological space on X.

Remark 3.3 Considering $N = 2$ in definition 3.1 we get the required definition of bi-neutrosophic topology on X. The pair $(X, 2_n\tau)$ is called a bi-neutrosophic topological space on X.

Example 3.4 Let $N = 2$, $X = \{a, b, c\}$. Define the neutrosophic set $A = \{(x, (\frac{a}{0.4}, \frac{b}{0.3}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.6}), (\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.3}))\}$ in X. If ${}_n\tau_1 O(X) = \{0, 1, A\}$ and ${}_n\tau_2 O(X) = \{0, 1\}$ are two neutrosophic topologies then we get $2_n\tau O(X) = \{0, 1, A\}$ which is a bi-neutrosophic topology on X.

Definition 3.5 Let $(X, N_k\tau)$ be a N_k-topological space on X and A be a k-set on X then the $N_kint(A)$ and $N_kcl(A)$ are respectively defined as

(i) $N_kint(A) = \cup\{G : G \subseteq A$ and G is a N_k-open set in $X\}$.

(ii) $N_kcl(A) = \cap\{F : A \subseteq F$ and F is a N_k-closed set in $X\}$.

Proposition 3.6 Let $(X, N_k\tau)$ be any N_k-topological space. If A and B are any two k-sets in $(X, N_k\tau)$, then the N_k-closure operator satisfy the following properties:

(i) $A \subseteq N_kcl(A)$.

(ii) $N_kint(A) \subseteq A$.

(iii) $A \subseteq B \Rightarrow N_kcl(A) \subseteq N_kcl(B)$.

(iv) $A \subseteq B \Rightarrow N_kint(A) \subseteq N_kint(B)$.

(v) $N_kcl(A \cup B) = N_kcl(A) \cup N_kcl(B)$.

(vi) $N_kint(A \cap B) = N_kint(A) \cap N_kint(B)$.

(vii) $(N_kcl(A))^c = N_kint(A)^c$.

(viii) $(N_kint(A))^c = N_kcl(A)^c$.

Proof

(i) $N_kcl(A) = \cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\}$. Thus, $A \subseteq N_kcl(A)$.

(ii) $N_kint(A) = \cup\{G : G$ is a N_k-open set in X and $G \subseteq A\}$. Thus, $N_kint(A) \subseteq A$.

(iii) $N_kcl(B) = \cap\{G : G$ is a N_k-closed set in X and $B \subseteq G\} \supseteq \cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\} \supseteq N_kcl(A)$. Thus, $N_kcl(A) \subseteq N_kcl(B)$.

(iv) $N_kint(B) = \cup\{G : G$ is a N_k-open set in X and $B \supseteq G\} \supseteq \cup\{G : G$ is a N_k-open set in X and $A \supseteq G\} \supseteq N_kint(A)$. Thus, $N_kint(A) \subseteq N_kint(B)$.

(v) $N_kcl(A \cup B) = \cap\{G : G$ is a N_k-closed set in X and $A \cup B \subseteq G\} = (\cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\}) \cup (\cap\{G : G$ is a N_k-closed set in X and $B \subseteq G\}) = N_kcl(A) \cup N_kcl(B)$. Thus, $N_kcl(A \cup B) = N_kcl(A) \cup N_kcl(B)$.

(vi) $N_kint(A \cap B) = \cup\{G : G$ is a N_k-open set in X and $A \cap B \supseteq G\} = (\cup\{G : G$ is a N_k-open set in X and $A \supseteq G\}) \cap (\cup\{G : G$ is a N_k-open set in X and $B \supseteq G\}) = N_kint(A) \cap N_kint(B)$. Thus, $N_kint(A \cap B) = N_kint(A) \cap N_kint(B)$.

(vii) $N_kcl(A) = \cap\{G : G$ is a N_k-closed set in X and $A \subseteq G\}, (N_kcl(A))^c = \cup\{G^c : G^c$ is a N_k-open set in X and $A^c \supseteq G^c\} = N_kint(A)^c$. Thus, $(N_kcl(A))^c = N_kint(A)^c$.

(viii) $N_kint(A) = \cup\{G : G$ is a N_k-open set in X and $A \supseteq G\}, (N_kint(A))^c = \cap\{G^c : G^c$ is a N_k-closed set in X and $A^c \supseteq G^c\} = N_kcl(A)^c$. Thus, $(N_kint(A))^c = N_kcl(A)^c$.

4 Generalized Closed Sets in N_k-topology

We introduce here the generalized closed sets in N_k-topological spaces and investigate their properties.

Definition 4.1 Let $(X, N_k\tau)$ be a N_k-topological space. A k-set A in $(X, N_k\tau)$ is said to be a N_k-generalized closed set if $N_kcl(A) \subseteq G$, whenever $A \subseteq G$ and G is a N_k-open set. The complement of a N_k-generalized closed set is called a N_k-generalized open set.

Definition 4.2 Let $(X, N_k\tau)$ be a N_k-topological space and A be a k-set in X. Then the N_k-generalized closure and N_k-generalized interior of A are defined as:

(i) $N_kGcl(A) = \cap\{G : G$ is a N_k-generalized closed set in X and $A \subseteq G\}$

(ii) $N_kGint(A) = \cup\{G : G$ is a N_k-generalized open set in X and $G \subseteq A\}$.

Proposition 4.3 Let $(X, N_k\tau)$ be any N_k-topological space. If A and B are any two k-sets in $(X, N_k\tau)$, then the N_k-generalized closure operator satisfies the following properties:

(i) $A \subseteq N_kGcl(A)$.

(ii) $N_kGint(A) \subseteq A$.

(iii) $A \subseteq B \Rightarrow N_kGcl(A) \subseteq N_kGcl(B)$.

(iv) $A \subseteq B \Rightarrow N_kGint(A) \subseteq N_kGint(B)$.

(v) $N_kGcl(A \cup B) = N_kGcl(A) \cup N_kGcl(B)$.

(vi) $N_kGint(A \cap B) = N_kGint(A) \cap N_kGint(B)$.

(vii) $(N_kGcl(A))^c = N_kGint(A)^c$.

(viii) $(N_kGint(A))^c = N_kGcl(A)^c$.

Proof The proof is analogous to Proposition 3.6.

Proposition 4.4 Let $(X, N_k\tau)$ be a N_k-topological space. If B is a N_k-generalized closed set and $B \subseteq A \subseteq N_kcl(B)$, then A is a N_k- generalized closed set.

Proof. Let G be a N_k-open set in $(X, N_k\tau)$ such that $A \subseteq G$. Since $B \subseteq A, B \subseteq G$. Now, B is a N_k-generalized closed set and $N_kcl(B) \subseteq G$. But $N_kcl(A) \subseteq N_kcl(B)$. Since $N_kcl(A) \subseteq N_kcl(B) \subseteq G, N_kcl(A) \subseteq G$. Hence, A is a N_k-generalized closed set.

Proposition 4.5 Let $(X, N_k\tau)$ be a N_k-topological space. Then A is a N_k-generalized open set if and only if $B \subseteq N_kint(A)$, whenever B is an N_k-closed set and $B \subseteq A$.

Proof. Let A be a N_k-generalized open set and B a N_k-closed set such that $B \subseteq A$. Now, $B \subseteq A \Rightarrow A^c \subseteq B^c$ and since A^c is a N_k-generalized closed set, then $N_kcl(A^c) \subseteq B^c$. This means that $B = (B^c)^c \subseteq (N_kcl(A^c))^c$. But $(N_kcl(A^c))^c = N_kint(A)$. Hence, $B \subseteq N_kint(A)$. Conversely, suppose that A is a k-set such that $B \subseteq N_kint(A)$, whenever B is a N_k-closed set and $B \subseteq A$. Now, $A^c \subseteq B \Rightarrow B^c \subseteq A$. Hence by assumption, $B^c \subseteq N_kint(A)$. That is, $(N_kint(A))^c \subseteq B$. But $(N_kint(A))^c = N_kcl(A)^c$. Hence, $N_kcl(A)^c \subseteq B$. This means that A is a N_k-generalized closed set. Therefore, A is a N_k-generalized open set.

Proposition 4.6 If $N_kint(A) \subseteq B \subseteq A$ and A is a N_k-generalized open set, then B is also a N_k-generalized open set.
Proof. Now, $A^c \subseteq B^c \subseteq (N_kint(A))^c = N_kcl(A)^c$. Since A is a N_k-generalized open set, then A^c is a N_k-generalized closed set. By Proposition 3.6, B^c is a N_k-generalized closed set. That is, B is a N_k-generalized open set.

5 Continuous Functions in N_k-Topology

In this section, we generalize continuous functions in N-neutrosophic topological spaces and also establish its relationship with other existing continuous functions.

Definition 5.1 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. A map $f : (X, N_k\tau) \to (Y, N_k\sigma)$ is said to be N_k-continuous if the inverse image of every N_k-closed set in $(Y, N_k\sigma)$ is a N_k-closed set in $(X, N_k\tau)$. Equivalently if the inverse image of every N_k-open set in $(Y, N_k\sigma)$ is a N_k-open set in $(X, N_k\tau)$.

Remark 5.2 By considering $N = 2$ in definition 5.1 we obtain bi-neutrosophic continuous function.

The following properties can be extended to N-fuzzy and N-intuitionistic topological spaces too.

Proposition 5.3 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k- continuous function. Then for every k-set A in X, $f(N_kcl(A)) \subseteq N_kcl(f(A))$.

Proof. Let A be a k-set in $(X, N_k\tau)$. Since $N_kcl(f(A))$ is a N_k-closed set and f is a N_k-continuous function, $f^{-1}(N_kcl(f(A))$ is a N_k-closed set and $f^{-1}(N_kcl(f(A))) \supseteq A$. Now, $N_kcl(A) \subseteq f^{-1}(N_kcl(f(A)))$. Therefore, $f(N_kcl(A)) \subseteq N_kcl(f(A))$.

Proposition 5.4 Let $(X, N_k\tau)$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k-continuous function. Then for every N_k-set A in Y, $N_kcl(f^{-1}(A)) \subseteq f^{-1}(N_kcl(A))$.

Proof. Let A be a N_k-set in $(Y, N_k\sigma)$. Let $B = f^{-1}(A)$. Then, $f(B) = f(f^{-1}(A)) \subseteq A$. By Proposition 5.3, $f(N_kcl(f^{-1}(A))) \subseteq N_kcl(f(f^{-1}(A)))$. Thus, $N_kcl(f^{-1}(A)) \subseteq f^{-1}(N_kcl(A))$.

Definition 5.5 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. A map $f : (X, N_k\tau) \to (Y, N_k\sigma)$ is said to be N_k-generalized continuous if the inverse image of every N_k-closed set in $(Y, N_k\sigma)$ is a N_k-generalized closed set in $(X, N_k\tau)$. Equivalently if the inverse image of every N_k-open set in $(Y, N_k\sigma)$ is a N_k-generalized open set in $(X, N_k\tau)$.

Remark 5.6 For $N = 2$ in the above definition we aquire the needed definition of bi-generalized neutrosophic continuous function.

Proposition 5.7 Let $(X, N_k\tau))$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k-generalized continuous function. Then for every N_k-set A in $X, f(N_kGcl(A)) \subseteq N_kcl(f(A))$.

Proof. Let A be a N_k-set in $(X, N_k\tau)$. Since $N_kcl(f(A))$ is a N_k-closed set and f is a N_k-continuous function, $f^{-1}(N_kcl(f(A))$ is a N_k-generalized closed set and $f^{-1}(N_kcl(f(A))) \supseteq A$. Now, $N_kGcl(A) \subseteq f^{-1}(N_kcl(f(A)))$. Therefore, $f(N_kGcl(A)) \subseteq N_kcl(f(A))$.

Proposition 5.8 Let $(X, N_k\tau)$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. Let $f : (X, N_k\tau) \to (Y, N_k\sigma)$ be a N_k-generalized continuous function. Then for every N_k-set A in Y, $N_kGcl(f^{-1}(A)) \subseteq f^{-1}(N_kcl(A))$.

Proof. Let A be a N_k-set in $(Y, N_k\sigma)$. Let $B = f^{-1}(A)$. Then, $f(B) = f(f^{-1}(A)) \subseteq A$. By Proposition 5.7, $f(N_kGcl(f^{-1}(A))) \subseteq N_kcl(f(f^{-1}(A)))$. Thus, $N_kGcl(f^{-1}(A)) \subseteq f^{-1}(N_kcl(A))$.

Proposition 5.9 Let $(X, N_k\tau)$ and $(Y, N_k\sigma)$ be any two N_k-topological spaces. If $f : (X, N_k\tau) \to (Y, N_k\sigma)$ is a N_k-continuous function, then it is a N_k-generalized continuous function.

Proof. Let A be a N_k-open set in $(Y, N_k\sigma)$. Since f is a N_k-continuous function, $f^{-1}(A)$ is a N_k-open set in $(X, N_k\tau)$. Every N_k-open set is a N_k-generalized open set. Now, $f^{-1}(A)$ is a N_k-generalized open set in $(X, N_k\tau)$. Hence, f is a N_k-generalized continuous function.
The converse of Proposition 5.9 need not be true as it is shown in the following example.

Example 5.10 Let $N = 2$, $X = \{a, b, c\}$ and $Y = \{p, q, r\}$. Define the neutrosophic sets $A = \{(x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.3}))\}$ in X and $B = \{(y, (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}), (\frac{p}{0.4}, \frac{q}{0.5}, \frac{r}{0.6}), (\frac{p}{0.3}, \frac{q}{0.2}, \frac{r}{0.3}))\}$ in Y. Considering ${}_n\tau_1O(X) = \{0, 1, A\}$ and ${}_n\tau_2O(X) = \{0, 1\}$ we get $2_n\tau O(X) = \{0, 1, A\}$. Also by considering ${}_n\sigma_1O(Y) = \{0, 1\}$ and ${}_n\sigma_2O(Y) = \{0, 1, B\}$ we get $2_n\sigma O(Y) = \{0, 1, B\}$. Thus, $(X, 2_n\tau)$ and $(Y, 2_n\sigma)$ are bi-neutrosophic topological space on X and Y, respectively. Define $f : X \to Y$ as $f(a) = q, f(b) = p, f(c) = r$. Then f is bi-generalized neutrosophic continuous but not bi-neutrosophic continuous.

Conclusion

Neutrosophic topology is well equipped to deal with imprecise data. By employing neutrosophic set in spacial data models, we can express the vagueness of the object as expected. This paper has gone a step forward in extending the theory to N-neutrosophic topology that can be used to determine the uncertain situation effectively. Further we also extended the same to N-Fuzzy and N-Intuitionistic topologies and discussed not only the relations but also its properties.

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Multi Attribute Decision Making Strategy on Projection and Bidirectional Projection Measures of Interval Rough Neutrosophic Sets

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Abstract. In this paper, we define projection and bidirectional projection measures between interval rough neutrosophic sets and prove their basic properties. Then two new multi attribute decision making strategies are proposed based on interval rough neutrosophic projection and bidirectional projection measures respectively. Then the proposed methods are applied for solving multi attribute decision making problems. Finally, a numerical example is solved to show the feasibility, applicability and effectiveness of the proposed strategies.

Keywords: Projection measure, Bidirectional projection measure, Interval rough neutrosophic set, MADM problem.

1 Introduction

The concept of neutrosophic set[1, 2, 3, 4, 5] introduced by Smarandache is a generalization of crisp set[6], fuzzy set[7] and intuitionistic fuzzy set[8]. To use neutrosophic set in real fields, Wang et al. extended it to single valued neutrosophic set[9].

Broumi et al. introduced rough neutrosophic set[10, 11] by combining the concept of rough set[12] and neutrosophic set.

Broumi and Smarandache defined interval rough neutrosophic set[13] by combining the concept of rough set and interval neutrosophic set theory[14].

Projection measure is a very useful for solving decision making problems because it takes into account the distance as well as the included angle between points. Yue [15] studied projection based MADM problem in crisp environment.Yue also[16] presented a projection method to obtain weights of the experts in a group decision making problem. Xu and Da [17] and Xu [18] studied projection method for decision making in uncertain environment with preference information. Yang et al. [19] develop projection method for material selection in fuzzy environment. Xu and Hu [20] developed two projection based models for MADM in intuitionistic fuzzy and interval valued intuitionistic fuzzy environment. Zeng et al. [21] provided weighted projection algorithm for intuitionistic fuzzy MADM problems and interval-valued intuitionistic fuzzy MADM problems. Chen and Ye [22] developed the projection based model for solving MADM problem and applied it to select clay-bricks in construction field.

To overcome the shortcomings of the general projection measure Ye [23] introduced a bidirectional projection measure between single valued neutrosophic numbers and developed MADM method for selecting problems of mechanical design schemes under a single valued neutrosophic environment. Ye [24] also presented the bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Dey et al. [25] defined weighted projection measure with interval neutrosophic environment and applied it to solve MADM problems with interval valued neutrosophic information. Yue [26] proposed a projection based approach for partner selection in a group decision making problem with linguistic value and intuitionistic fuzzy information.Dey et al. [27] defined projection, bidirectional projection and hybrid projection measures between bipolar neutrosophic sets and presented bipolar neutrosophic projection based models for MADM problems. Pramanik et al. [28] defined projection and bidirectional projection measure between rough neutrosophic sets and proposed the decision making methods based on them.

Research gap MADM strategy using projection and bidirectional projection measures under interval rough neutrosophic environment.

Research questions

(i) Is it possible to define two new projection and bidirectional projection measure between interval rough neutrosophic sets?

(ii) Is it possible to develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment?

The objectives of the paper are

(i) To define two new projection and bidirectional projection measure between interval rough neutrosophic sets.

(ii) To develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

Contributions

(i) In this paper, we propose projection and bidirectional projection measures under interval rough neutrosophic environment.

(ii) In this paper, we develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

(iii) We also present numerical example to show the effectiveness and applicability of the proposed measures.

Rest of the paper is organized as follows: Section 2 describes preliminaries of neutrosophic number, SVNS, RNS and IRNS. Section 3 presents definitions and properties of proposed projection and bidirectional projection measure between IRNSs. Section 4 describes the MADM methods based on projection and bidirectional projection measures of IRNSs. In section 5 we describe a numerical example. Finally, section 6 presents the conclusion.

2 Preliminaries

In this Section, we provide some basic definitions regarding SVNSs, IRNSs which are useful in the paper.

2.1 Neutrosophic set:

In 1999, Smarandache gave the following definition of neutrosophic set(NS) [1].

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x. A NS A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A , I_A and F_A are real standard or non-standard subsets of $(^-0,1^+)$ that is $T_A:X\rightarrow (^-0, 1^+)$, $I_A:X\rightarrow (^-0, 1^+)$ and $F_A:X\rightarrow (^-0, 1^+)$. It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^-0 \le T_A(X)+I_A(X)+F_A(X)\le 3^+$

Definition 2.1.2: (complement)

The complement of a neutrosophic set A is denoted by C(A) and is defined by $T_{c(A)}(x) = \{1^+\}-T_A(x), I_{c(A)}(x)=\{1^+\}-I_A(x), F_{c(A)}(x)=\{1^+\}-F_A(x)$.

Definition 2.1.3: (Containment)

A neutrosophic set A is contained in the other neutrosophic set B, denoted by $A\subseteq B$ iff

$$\inf T_A(x)\le \inf T_B(x), \sup T_A(x)\le \sup T_B(x),$$
$$\inf I_A(x)\ge \inf I_B(x), \sup I_A(x)\ge \sup I_B(x),$$
$$\inf F_A(x)\ge \inf F_B(x), \sup F_A(x)\ge \sup F_B(x)\ \forall x\in X$$

Definition 2.1.4: (Single-valued neutrosophic set).

Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function $T_A(x)$, a falsity membership function $F_A(x)$ and indeterminacy function $I_A(x)$ with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0,1]\ \forall x \text{ in } X$$

When X is continuous, a SNVS S can be written as follows

$$A=\int_x < T_A(x), F_A(x), I_A(x) > /x \forall x\in X$$

and when X is discrete, a SVNS S can be written as follows

$$A=\sum < T_A(x), F_A(x), I_A(x) > /x \forall x\in X$$

For a SVNS S, $0\le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

Definition2.1.5:

The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1-I_A(x)$, $F_{c(A)}(x) = T_A(x)$.

Definition 2.1.6: A SVNS A is contained in the other SVNS B, denoted as $A\subseteq B$ iff,

$T_A(x)\le T_B(x), I_A(x)\ge I_B(x)$

and $F_A(x)\ge F_B(x) \forall x\in X$.

2.2Rough neutrosophic set

Rough neutrosophic sets [10, 11] are the generalization of rough fuzzy sets [29, 30] and rough intuitionistic fuzzy sets [31].

Definition 2.2.1:

Let Y be a non-null set and R be an equivalence relation on Y. Let P be neutrosophic set in Y with the membership function T_P , indeterminacy function I_P and non-membership function F_P . The lower and the upper approximations of P in the approximation (Y, R) denoted by are respectively defined as:

$\underline{N(P)} =<< x, T_{\underline{N(P)}}(x), I_{\underline{N(P)}}(x), F_{\underline{N(P)}}(x) > /$
$y \in [x]_R, x \in Y >$

and

$\overline{N(P)} =<< x, T_{\overline{N(P)}}(x), I_{\overline{N(P)}}(x), F_{\overline{N(P)}}(x) > /$
$y \in [x]_R, x \in Y >$

where,

$T_{\underline{N(P)}}(x) = \wedge z \in [x]_R T_P(Y),$
$I_{\underline{N(P)}}(x) = \wedge z \in [x]_R I_P(Y),$
$F_{\underline{N(P)}}(x) = \wedge z \in [x]_R F_P(Y)$

and

$T_{\overline{N(P)}}(x) = \vee z \in [x]_R T_P(Y),$
$I_{\overline{N(P)}}(x) = \vee z \in [x]_R I_P(Y),$.
$F_{\overline{N(P)}}(x) = \vee z \in [x]_R F_P(Y)$

So,

$0 \le T_{\underline{N(P)}}(x) + I_{\underline{N(P)}}(x) + F_{\underline{N(P)}}(x) \le 3$

and

$0 \le T_{\overline{N(P)}}(x) + I_{\overline{N(P)}}(x) + F_{\overline{N(P)}}(x) \le 3$

Here $\vee$ and $\wedge$ denote "max" and "min" operators respectively, $T_P(y)$, $I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of Y with respect to P.

Thus NS mapping ,

$\underline{N}, \overline{N} : N(Y) \to N(Y)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N(P)}, \overline{N(P)})$ is called the rough neutrosophic set in (Y, R).

Definition 2.2.2 If $N(P) = (\underline{N(P)}, \overline{N(P)})$ is a rough neutrosophic set in (Y, R) , the rough complement of N(P) is the rough neutrosophic set denoted by

$\sim N(P) = ((\overline{N(P)})^C, (\underline{N(P)})^C)$

,where

$(\underline{N(P)})^C$ and $(\overline{N(P)})^C$

are the complements of neutrosophic sets $\underline{N}(P)$ and $\overline{N(P)}$ respectively.

2.3 Interval rough neutrosophic set

Interval neutrosophic rough set is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache interval neutrosophic roughset is the generalizations of interval valued intuitionistic fuzzy rough set.

Definition 2.3.1

Let R be an equivalence relation on the universal set U.Then the pair (U, R) is called a Pawlak approximationspace. An equivalence class of R containing x will bedenoted by $[x]_R$ for X $\in$ U, the lower and upper approximationof X with respect to (U, R) are denoted by respectively,

$\underline{R}X$ and $\overline{R}X$ and are defined by

$\underline{R}X = \{x \in U : [x]_R \subseteq X\},$

$\overline{R}X = \{x \in U : [x]_R \cap X \neq \emptyset\}.$

Now if $\underline{R}X = \overline{R}X$, then X is called definable; otherwise Xis called a rough set.

Definition 2.3.2

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function $\mu A: U \to [0, 1]$ and non-membership functionν_A: $U \to [0, 1]$ such that $\mu_A(\underline{R}X)=1$and $\nu_A(\underline{R}X) = 0$

ie, $[\mu_A(x), \nu_A(x)]=[1,0]$ if $x \in (\underline{R}X)$ and $\mu_A(U-\overline{R}X) = 0$, $\nu_A(U-\overline{R}X)=1$

ie,

$0 \le \mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X) \le 1$

Definition 2.3.3

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$A = \{<x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]> : x \in U\}$

The lower approximation $\underline{A}_R$ and the upper approximation $\overline{A}_R$ of A in the Pawlak approximation space (U, R) are expressed as follows:

$\underline{A}_R = \{<x, [\wedge_{y\in[x]_R}\{T_A^L(y)\},\ {}_{y\ [x]}\{T_A(y)\}],$
$[\vee_{y\in[x]_R}\{I_A^L(y)\},\ {}_{y\ [x]}\{I_A(y)\}],$
$[\vee_{y\in[x]_R}\{F_A^L(y)\}, \vee_{y\ [x]}\{F_A(y)\}]> : x \in U\}$

$\overline{A}_R = \{<x, [\vee_{y\in[x]_R}\{T_A^L(y)\},\ {}_{y\ [x]}\{T_A(y)\}],$
$[\wedge_{y\in[x]_R}\{I_A^L(y)\},\ {}_{y\ [x]}\{I_A(y)\}],$
$[\wedge_{y\in[x]_R}\{F_A^L(y)\}, \wedge_{y\ [x]}\{F_A(y)\}]> : x \in U\}$

The symbols $\wedge$ and $\vee$ indicate "min" and "max" operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here $[x]_R$ is the equivalence class of the element x. It is obvious that

$[\wedge_{y\in[x]_R}\{T_A(y)\}, \wedge_{y\ [x]}\{T_A^L(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R}\{I_A^U(y)\}, \vee_{y\ [x]}\{I_A(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R}\{F_A^U(y)\}, \vee_{y\ [x]}\{F_A(y)\}] \subset [0,1].$

and $0 \le \wedge_{y\in[x]_R}\{T_A^U(y)\} + \vee_{y\in[x]_R}\{I_A^U(y)\} + \vee_{y\in[x]_R}\{F_A^U(y)\} \le 3$

Then $\underline{A}_R$ is an interval neutrosophic set (INS)

Similarly, we have

$$[\vee_{y\in[x]_R}\{T_A^L(y)\}, \vee_{y\in[x]_R}\{T_A^U(y)\}] \subset [0,1],$$
$$[\wedge_{y\in[x]_R}\{I_A^L(y)\}, \wedge_{y\in[x]_R}\{I_A^U(y)\}] \subset [0,1],$$
$$[\wedge_{y\in[x]_R}\{F_A^L(y)\}, \wedge_{y\in[x]_R}\{F_A^U(y)\}] \subset [0,1]$$

and

$$0 \le \vee_{y\in[x]_R}\{T_A^U(y)\} + \wedge_{y\in[x]_R}\{I_A^U(y)\} + \wedge_{y\in[x]_R}\{F_A^U(y)\}] \le 3$$

Then $\overline{A}_R$ is an interval neutrosophic set.

If $\underline{A}_R = \overline{A}_R$ then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U,R) respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$ respectively.

3 Projection and Bidirectional projection measure of interval rough neutrosophic sets :

Existing projection and bidirectional projection measure does not deal with interval rough neutrosophic set(IRNS)s. Therefore, a new projection and bidirectional projection measure between IRNSs is proposed.

Assume that there are two IRNSs

$$\underline{M} = \{<\underline{x_i}, (\underline{[T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+]}, [\underline{F_{iM}^-}, \underline{F_{iM}^+}],$$
$$[T_{iM}^-, T_{iM}^+], [I_{iM}^-, I_{iM}^+], [F_{iM}^-, F_{iM}^+] >: i = 1, 2, ..., n\}$$

and

$$\underline{N} = \{<\underline{x_i}, \underline{[T_{iN}^-, T_{iN}^+]}, \underline{[I_{iN}^-, I_{iN}^+]}, [\underline{F_{iN}^-}, \underline{F_{iN}^+}],$$
$$[T_{iN}^-, T_{iN}^+], [I_{iN}^-, I_{iN}^+], [F_{iN}^-, F_{iN}^+] >: i = 1, 2, ..., n\}$$

Then the inner product of M and N denoted by M.N can be defined as

$$M.N = \sum_{i=1}^{n} [\underline{T_{iM}^-}.\underline{T_{iN}^-} + \underline{T_{iM}^+}\,\underline{T_{iN}^+} + \underline{I_{iM}^-}\,\underline{I_{iN}^-} + \underline{I_{iM}^+}\,\underline{I_{iN}^+}$$
$$+ \underline{F_{iM}^-}\,\underline{F_{iN}^-} + \underline{F_{iM}^+}\,\underline{F_{iN}^+} + \overline{T_{iM}^-}\,\overline{T_{iN}^-} + \overline{T_{iM}^+}\,\overline{T_{iN}^+}$$
$$+ I_{iM}^- I_{iN}^- + I_{iM}^+ I_{iN}^+ + F_{iM}^- F_{iN}^- + F_{iM}^+ F_{iN}^+]$$

The modulus of M can be defined as

$$\|M\| = \sqrt{\sum_{i=1}^{n}\begin{bmatrix} (\underline{T_{iM}^-})^2 + (\underline{T_{iM}^+})^2 + (\underline{I_{iM}^-})^2 + (\underline{I_{iM}^+})^2 \\ + (\underline{F_{iM}^-})^2 + (\underline{F_{iM}^+})^2 + (\overline{T_{iM}^-})^2 + (\overline{T_{iM}^+})^2 \\ + (I_{iM}^-)^2 + (I_{iM}^+)^2 + (F_{iM}^-)^2 + (F_{iM}^+) \end{bmatrix}}$$

and the modulus of N can be defined as

$$\|N\| = \sqrt{\sum_{i=1}^{n}\begin{bmatrix} (\underline{T_{iN}^-})^2 + (\underline{T_{iN}^+})^2 + (\underline{I_{iN}^-})^2 + (\underline{I_{iN}^+})^2 \\ + (\underline{F_{iN}^-})^2 + (\underline{F_{iN}^+})^2 + (\overline{T_{iN}^-})^2 + (\overline{T_{iN}^+})^2 \\ + (I_{iN}^-)^2 + (I_{iN}^+)^2 + (F_{iN}^-)^2 + (F_{iN}^+) \end{bmatrix}}$$

Definition4.1.The projection of M on N can be defined as

$$\mathrm{Proj}(M)_N = \frac{1}{\|N\|} M.N.$$

Definition4.2.The bidirectional projection measure between the RNSs M and N is defined as

$$\mathrm{BProj}(M, N) = \frac{1}{1 + \big|\|M\| - \|N\|\big| M.N}$$
$$= \frac{\|M\|\|N\|}{\|M\|\|N\| + \big|\|M\| - \|N\|\big| M.N}$$

Here also the bidirectional projection measure satisfies the following properties :

(1) BProj(M,N) = BProj(N,M);

(2) $0 \le BProj(M,N) \le 1$;

(3) BProj(M,N) = 1, iff M = N.

Proof:

(i)

$$\mathrm{BProj}(M, N)$$
$$= \frac{1}{1 + \big|\|M\| - \|N\|\big| M.N}$$
$$= \frac{1}{1 + \big|\|N\| - \|M\|\big| N.M}$$
$$= \mathrm{BProj}(N, M)$$

(ii) As

$$\frac{1}{1 + \big|\|M\| - \|N\|\big| M.N} \ge 0$$

and

$$\frac{1}{1 + \big|\|M\| - \|N\|\big| M.N} \le 1$$

so, $0 \le \mathrm{BProj}(M, N) \le 1$;

(iii) If M=N then

$$\mathrm{BProj}(M, N)$$
$$= \mathrm{BProj}(M, M)$$
$$= \frac{1}{1 + \big|\|M\| - \|M\|\big| M.M}$$
$$= 1$$

4. Projection And Bidirectional Projection Based Decision Making Methods For MADM Problems With Interval Rough Neutrosophic Information

In this section, we develop projection and bidirectional projection based decision making models to solve MADM problems with interval rough neutrosophic information. Consider C={C_1, ...,C_m} be the set of attributes and A={A_1,……, A_n} be a set of alternatives. Now we provide two algorithms for MADM problems involving interval rough neutrosophic information.

4.1. Algorithm 1.(see Fig 1)

Step 1. The value of alternative A_i(i=1,…..,n) for the attribute C_j(j=1,……,m) is evaluated by the decision maker in terms of IRNSs and the interval rough neutrosophic decision matrix is constructed as:

$$D = \langle z_{ij} \rangle_{n \times m} = \begin{pmatrix} z_{11}\ z_{12}\ \ldots\ldots\ldots z_{1m} \\ z_{21}\ z_{22}\ \ldots\ldots\ldots z_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ z_{n1}\ z_{n2}\ \ldots\ldots\ldots z_{nm} \end{pmatrix}$$

where

$$\underline{z_{ij}} = <(\underline{[T^-_{iM}, T^+_{iM}]}, \underline{[I^-_{iM}, I^+_{iM}]}, \underline{[F^-_{iM}, F^+_{iM}]}),$$
$$[T^-_{iM}, T^+_{iM}], [I^-_{iM}, I^+_{iM}], [F^-_{iM}, F^+_{iM}])>$$

with

$$0 \le \vee_{y \in [x]_R} \{T^U_A(y)\} + \wedge_{y \in [x]_R} \{I^U_A(y)\} + \wedge_{y \in [x]_R} \{F^U_A(y)\}] \le 3$$

Step 2. Calculate the weighted alternative decision matrix

For the attribute C_j (j=1,……,m) the weight vector of attribute is considered as : $W = (w_1, w_2, \ldots, w_m)$ with

$$w_j \ge 0 \quad \text{and} \quad \sum_{i=1}^{n} w_j = 1$$

On calculating

$$s_{ij} = <([w_j \underline{T^-_{iM}}, w_j \underline{T^+_{iM}}], [w_j \underline{I^-_{iM}}, w_j \underline{I^+_{iM}}],$$
$$[w_j \underline{F^-_{iM}}, w_j \underline{F^+_{iM}}], [w_j T^-_{iM}, w_j T^+_{iM}],$$
$$[w_j I^-_{iM}, w_j I^+_{iM}], [w_j F^-_{iM}, w_j F^+_{iM}])>$$

for i=1, 2, … , n and j=1, 2, … , m , we obtain the weighted alternative decision matrix

$$S = \langle s_{ij} \rangle_{n \times m} = \begin{pmatrix} s_{11}\ s_{12}\ \ldots\ldots\ldots s_{1m} \\ s_{21}\ s_{22}\ \ldots\ldots\ldots s_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ s_{n1}\ s_{n2}\ \ldots\ldots\ldots s_{nm} \end{pmatrix}$$

Step 3. Determine the ideal solution S^*.

For benefit type attribute,

$$S^* = \{(\min_i \underline{T_{ij}}, \max_i \underline{I_{ij}}, \max_i \underline{F_{ij}}), (\max_i \overline{T_{ij}}, \min_i \overline{I_{ij}}, \min_i \overline{F_{ij}})\}$$

For cost type attribute,

$$S^* = \{(\max_i \underline{T_{ij}}, \min_i \underline{I_{ij}}, \min_i \underline{F_{ij}}), (\min_i \overline{T_{ij}}, \max_i \overline{I_{ij}}, \max_i \overline{F_{ij}})\}$$

Step 4. Compute the projection measure between S^* and $Z_i = <Z_{ij}>_{nxm}$ for all i = 1, ….., n and j = 1, ….., m.

Step 5. Ranking of alternatives is prepared based on the values of projection measure. The highest value reflects the best alternatives.

Step 6. End.

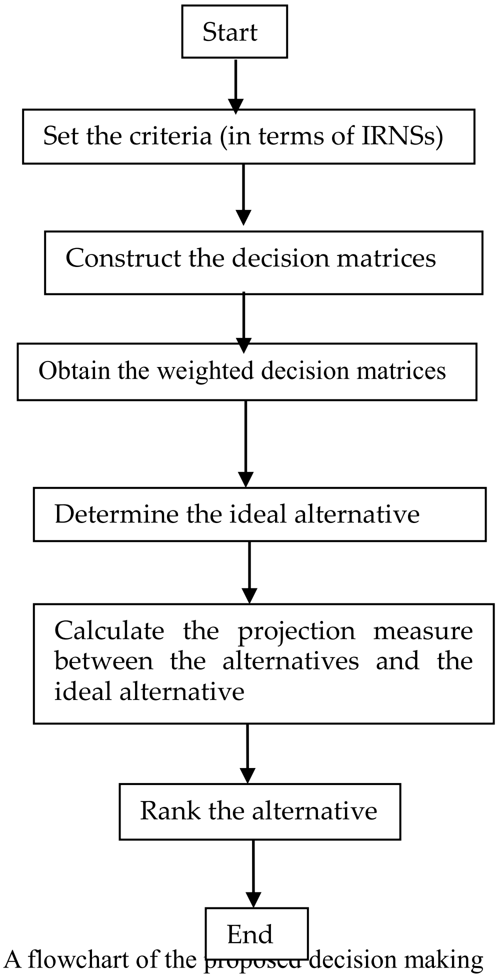

Fig 1. A flowchart of the proposed decision making method

4.2. Algorithm 2.(see Fig 2)

Step 1. The value of alternative A_i(i=1,…..,n) for the attribute C_j(j=1,……,m) is evaluated by the decision maker in terms of IRNSs and the interval rough neutrosophic decision matrix is constructed as:

$$D = \langle z_{ij} \rangle_{n \times m} = \begin{pmatrix} z_{11}\ z_{12}\ \ldots\ldots\ldots z_{1m} \\ z_{21}\ z_{22}\ \ldots\ldots\ldots z_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ z_{n1}\ z_{n2}\ \ldots\ldots\ldots z_{nm} \end{pmatrix}$$

where

$$\underline{z_{ij}} = <(\underline{[T^-_{iM}, T^+_{iM}]}, \underline{[I^-_{iM}, I^+_{iM}]}, \underline{[F^-_{iM}, F^+_{iM}]}),$$
$$[T^-_{iM}, T^+_{iM}], [I^-_{iM}, I^+_{iM}], [F^-_{iM}, F^+_{iM}])>$$

with

$$0 \le \vee_{y\in[x]_R} \{T_A^U(y)\} + \wedge_{y\in[x]_R} \{I_A^U(y)\} + \wedge_{y\in[x]_R} \{F_A^U(y)\}] \le 3$$

Step 2. Calculate the weighted alternative decision matrix
For the attribute C_j (j=1,……,m) the weight vector of attribute is considered as : W = (w_1, w_2, … , w_m) with

$$w_j \ge 0 \quad \text{and} \quad \sum_{i=1}^{n} w_j = 1$$

On calculating

$$s_{ij} = <([w_j \underline{T}^-_{iM}, w_j \underline{T}^+_{iM}], [w_j \underline{I}^-_{iM}, w_j \underline{I}^+_{iM}], [w_j \underline{F}^-_{iM}, w_j \underline{F}^+_{iM}], [w_j \overline{T}^-_{iM}, w_j \overline{T}^+_{iM}], [w_j \overline{I}^-_{iM}, w_j \overline{I}^+_{iM}], [w_j \overline{F}^-_{iM}, w_j \overline{F}^+_{iM}])>$$

for i=1, 2, … , n and j=1, 2, … , m , we obtain the weighted alternative decision matrix

$$S = < s_{ij} >_{n\times m} = \begin{pmatrix} s_{11}\ s_{12}\ \cdots\cdots\ s_{1m} \\ s_{21}\ s_{22}\ \cdots\cdots\ s_{2m} \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ s_{n1}\ s_{n2}\ \cdots\cdots\ s_{nm} \end{pmatrix}$$

Step 3. Determine the ideal solution S^*.
For benefit type attribute,

$$S^* = \{(\min_i \underline{T}_{ij}, \max_i \underline{I}_{ij}, \max_i \underline{F}_{ij}), (\max_i \overline{T}_{ij}, \min_i \overline{I}_{ij}, \min_i \overline{F}_{ij})\}$$

For cost type attribute,

$$S^* = \{(\max_i \underline{T}_{ij}, \min_i \underline{I}_{ij}, \min_i \underline{F}_{ij}), (\min_i \overline{T}_{ij}, \max_i \overline{I}_{ij}, \max_i \overline{F}_{ij})\}$$

.
Step 4. Compute the bidirectional projection measure between S^* and $Z_i = <Z_{ij}>_{nxm}$ for all i = 1, ….., n and j = 1, ….., m.
Step 5. Ranking of alternatives is prepared based on the values of bidirectional projection measure. The highest value reflects the best alternatives.
Step 6. End.

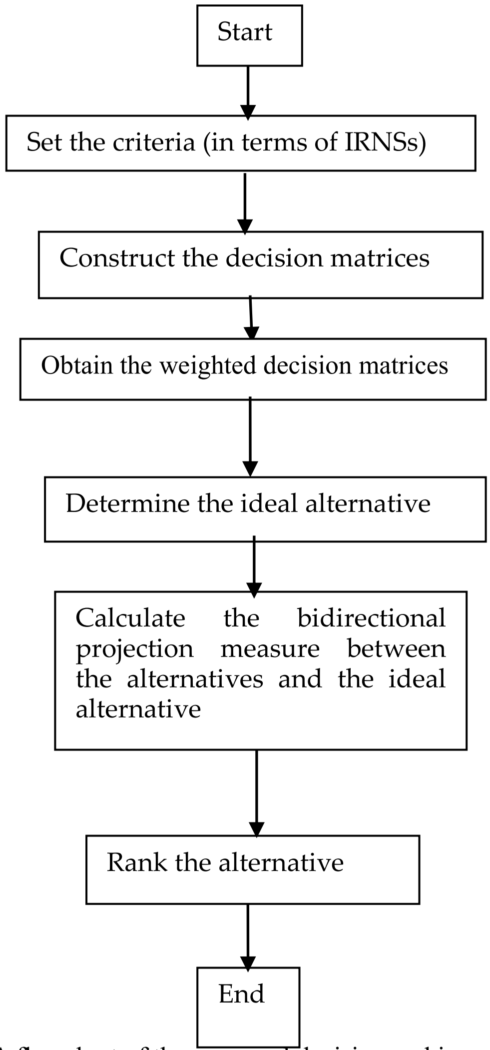

Fig 2. A flowchart of the proposed decision making method

5. A Numerical Example:

Assume that a decision maker intends to select the most suitable laptop for random use from the three initially chosen laptops (A_1, A_2, A_3) by considering four attributes namely: features C_1, reasonable price C_2, customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 5, the considered problem is solved by the following steps:
Step1: Construct the decision matrix with interval rough neutrosophic number
The decision maker construct the decision matrix with respect to the three alternatives and four attributes in terms of interval rough neutrosophic number.

	C_1	C_2	C_3	C_4
$\underline{A_1}$	<([.6, .7], [.3, .5], [.3, .4]), ([.8, .9], [.1, .3], [.1, .2])>	<([.5, .7], [.3, .4], [.1, .2]), ([.7, .9], [.3, .5], [.3, .4])>	<([.5, .6], [.4, .5], [.4, .6]), ([.7, .8], [.2, .4], [.3, .4])>	<([.8, .9], [.3, .4], [.5, .6]), ([.7, .8], [.3, .5], [.3, .5])>
$\underline{A_2}$	<([.7, .8], [.2, .3], [.0, .2]), ([.7, .9], [.1, .2], [.1, .2])>	<([.6, .7], [.1, .2], [.0, .2]), ([.6, .7], [.1, .3], [.1, .3])>	<([.5, .7], [.2, .3], [.1, .2]), ([.6, .9], [.3, .5], [.2 .4])>	<([.7, .8], [.3, .5], [.1, .3]), ([.5, .7], [.5, .6], [.2, .3])>
$\underline{A_3}$	<([.6, .7], [.3, .4], [.0, .3]), ([.6, .9], [.1, .2], [.1, .2])>	<([.5, .7], [.2, .4], [.2, .4]), ([.6, .8], [.1, .3], [.1, .2])>	<([.6, .8], [.2, .4], [.3, .4]), ([.6, .8], [.2, .5], [.3, .5])>	<([.4, .7], [.2, .4], [.4, .5]), ([.5, .8], [.2, .5], [.0, .2])>

Step 2: The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively. The weighted decision matrix is:

	C_1	C_2	C_3	C_4
S_1	<([0.21,0.245], [0.105,0.175], [0.105,0.14]), ([0.28,0.315], [0.035,0.105], [0.035,0.07])>	<([0.125,0.175], [0.075,0.1], [0.025,0.05]), ([0.175,0.225], [0.075,0.125], [0.075,0.1])>	<([0.125,0.15], [0.1,0.125], [0.1,0.15]), ([0.175,0.2], [0.05,0.1], [0.075,0.1])>	<([0.12,0.135], [0.045,0.06], [0.075,0.09]), ([0.105,0.12], [0.045,0.075], [0.045,0.75])>
S_2	<([0.245,0.28], [0.07,0.105], [0.0,0.07]), ([0.245,0.315], [0.035,0.07], [0.035,0.07])>	<([0.15,0.175], [0.025,0.05], [0.0,0.05]), ([0.15,0.175], [0.025,0.075], [0.025,0.075])>	<([0.125,0.175], [0.05,0.075], [0.025,0.05]), ([0.15,0.225], [0.075,0.125], [0.05,0.1])>	<([0.105,0.12], [0.045,0.75], [0.015,0.045]), ([0.075,0.105], [0.075,0.09], [0.03,0.045])>
S_3	<([0.21,0.245], [0.105,0.14], [0.0,0.105]), ([0.21,0.315], [0.035,0.7], [0.035,0.7])>	<([0.125,0.175], [0.05,0.1], [0.05,0.1]), ([0.15,0.2], [0.025,0.075], [0.025,0.05])>	<([0.15,0.2], [0.05,0.1], [0.075,0.1]), ([0.15,0.2], [0.05,0.125], [0.075,0.125])>	<([0.06,0.105], [0.03,0.06], [0.06,0.075]), ([0.075,0.12], [0.03,0.075], [0.0,0.03])>

Step3: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C_1, C_2, C_3 and one cost type attribute C_4. We calculate the ideal alternative as follows:

$$S^* = \{<([.21,.245],[.07,.175],[.105,.14]), ([.28,.315],[.035,.07],[.035,.07])>,$$
$$<([.15,.175],[.075,.1],[.05,.1]), ([.175,.225],[.025,.075],[.025,.05])>,$$
$$<([.15,.15],[.1,.1],[.1,.1]), ([.175,.225],[.075,.125],[.075,.125])>,$$
$$<([.12,.135],[.03,.06],[.015,.045]), ([.075,.105],[.075,.09],[.045,.075])>)>\}$$

Step4:Calculate the projection and bidirectional projection measure of the alternatives

$$\|S_1\| = 0.918273,$$
$$\|S_2\| = 0.829533,$$
$$\|S_3\| = 0.832331.$$
$$\|S^*\| = 0.818175.$$
$$S_1.S^* = 0.815425,$$
$$S_2.S^* = 0.563137,$$
$$S_3.S^* = 0.7337.$$
$$\Pr oj(S_1)_{S^*} = 0.99663886,$$
$$\Pr oj(S_2)_{S^*} = 0.68828490,$$
$$\Pr oj(S_3)_{S^*} = 0.89675192.$$
$$\Rightarrow \Pr oj(S_1)_{S^*} > \Pr oj(S_3)_{S^*} > \Pr oj(S_2)_{S^*}.$$
$$B\Pr oj(S_1, S^*) = 0.92453705,$$
$$B\Pr oj(S_2, S^*) = 0.99364454,$$
$$B\Pr oj(S_3, S^*) = 0.98972051.$$
$$\Rightarrow B\Pr oj(S_2, S^*) > B\Pr oj(S_3, S^*) > B\Pr oj(S_1, S^*).$$

Step5: Rank the alternatives

Ranking of alternatives is prepared based on the descending order of projection and bidirectional measures. The highest value reflects the best alternatives.

Hence, according to the projection measure, the laptop A_1 is the best alternative and according to the bidirectional projection measure, the laptop A_2 is the best alternative. As bidirectional projection measure gives better result than projection measure, so A_2 is the best laptop for random use.

6. Comparative study and discussions:

Mondal and Pramanik study the MADM method in interval rough neutrosophic environment using cosine, dice and Jaccard similarity measure [32]. We take the same problem and solve the problem using projection and bidirectional projection measure based decision making method. In the existing methods, S_2 is the best alternatives. But in new method S_1 is the best alternative.

7. Conclusion:

In this paper, we have defined projection measure, weighted projection measure, bidirectional projection measure, weighted bidirectional projection measure between interval rough neutrosophic sets. We have also proved their basic properties. We have developed two new MADM strategies based on the proposed projection and bidirectional projection measures respectively. Finally, we have solved a numerical example to demonstrate the feasiblity, applicability and effectiveness of the proposed strategies. The proposed strategies can be applied to solve different MADM problems such as teacher selection [33, 34, 35], school selection [36], weaver selection [37, 38, 39], brick field selection [40, 41], logistics center location selection [42, 43], data mining [44] etc. The proposed strategies can also be extended for MAGDM in interval rough neutrosophic environment.

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Multi-Attribute Decision Making Based on Several Trigonometric Hamming Similarity Measures under Interval Rough Neutrosophic Environment

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Abstract. In this paper, the sine, cosine and cotangent similarity measures of interval rough neutrosophic sets is proposed. Some properties of the proposed measures are discussed. We have proposed multi attribute decision making approaches based on proposed similarity measures. To demonstrate the applicability, a numerical example is solved.

Keywords: sine hamming similarity measure, cosine hamming similarity measure, cotangent hamming similarity measure, interval rough neutrosophic set.

1 Introduction

The basic concept of neutrosophic set grounded by Smarandache [1, 2, 3, 4, 5] is a generalization of classical set or crisp set [6], fuzzy set [7], intuitionistic fuzzy set [8]. Wang et al.[9] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs). Broumi et al. [10, 11] proposed new hybrid intelligent structure namely, rough neutrosophic set combing the concept of rough set theory [12] and the concept of neutrosophic set theory to deal with uncertainty and incomplete information. Rough neutrosophic set is the generalization of rough fuzzy sets [13, 14] and rough intuitionistic fuzzy sets [15]. Several studies of rough neutrosophic sets have been reported in the literature. Mondal and Pramanik [16] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis. Pramanik and Mondal [17] presented cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [18] also proposed some rough neutrosophic similarity measures namely Dice and Jaccard similarity measures of rough neutrosophic environment. Mondal and Pramanik [19] proposed rough neutrosophic multi attribute decision making based on rough score accuracy function. Pramanik and Mondal [20] presented cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Pramanik and Mondal [21] presented trigonometric Hamming similarity measure of rough neutrosophic sets. Pramanik et al. [22] proposed rough neutrosophic multi attribute decision making based on correlation coefficient. Pramanik et al. [23] also proposed rough neutrosophic projection and bidirectional projection measures. Mondal et al. [24] presented multi attribute decision making based on rough neutrosophic variational coefficient similarity measures. Mondal at al. [25] also presented rough neutrosophic TOPSIS for multi attribute group decision making. Mondal and Pramanik [26] presented tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. In 2015, Broumi and Smarandache [27] combined the concept of rough set theory [12] and interval neutrosophic set theory [28] and defined interval rough neutrosophic set. Pramanik et al. [29] presented multi attribute decision making based on projection and bidirectional projection measures under interval rough neutrosophic environment.

Multi-attribute decision making using trigonometric Hamming similarity measures under interval rough neutrosophic environment is not addressed in the literature.

Research gap MADM strategy using sine, cosine and cotangent similarity measures under interval rough neutrosophic environment.

Research questions

(i) Is it possible to define sine, cosine and cotangent similarity measures between interval rough neutrosophic sets?

(ii) Is it possible to develop new MADM strategies based on the proposed measures in interval rough neutrosophic environment?

The objectives of the paper are

i. to define sine, cosine and cotangent similarity measures between interval rough neutrosophic sets.

ii. to prove the basic properties of sine, cosine and cotangent similarity measures of interval rough neutrosophic sets.

iii. to develop new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

Contributions

(i) In this paper, we propose sine, cosine and cotangent similarity measures under interval rough neutrosophic environment.

(ii) We develop new MADM strategy based on the proposed measures in interval rough neutrosophic environment.

(iii) We also present numerical example to show the feasibility and applicability of the proposed measures.

Rest of the paper is organized in the following way. Section 2 describes preliminaries of neutrosophic sets and rough neutrosophic sets and interval rough neutrosophic sets. Section 3, Section 4 and Section 5 presents definitions and propositions of the proposed measures. Section 6 presents multi attribute decision-making strategies based on the similarity measures. Section 7 provides a numerical example. Section 8 presents the conclusion and future scopes of research.

2 Preliminaries

In this Section, we provide some basic definitions regarding SVNSs, IRNSs which are useful in the paper.

In 1999, Smarandache presented the following definition of neutrosophic set (NS) [1].

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x. A NS A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A , I_A and F_A are real standard or non-standard subsets of $(^-0,1^+)$ that is $T_A:X \to (^-0, 1^+)$, $I_A:X \to (^-0, 1^+)$ and $F_A:X \to (^-0, 1^+)$. It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^-0 \le T_A(X) + I_A(X) + F_A(X) \le 1^+$

Definition 2.1.2: (Single-valued neutrosophic set) [9]. Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function $T_A(x)$, a falsity membership function $F_A(x)$ and indeterminacy function $I_A(x)$ with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0,1] \ \forall \text{ x in X}$$

When X is continuous, a SNVS S can be written as follows

$$A = \int_x < T_A(x), F_A(x), I_A(x) > / \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows

$$A = \sum < T_A(x), F_A(x), I_A(x) > / \forall x \in X$$

For a SVNS S, $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

2.2 Rough neutrosophic set

Rough neutrosophic sets [10, 11] are the generalization of rough fuzzy sets [13, 14] and rough intuitionistic fuzzy sets [15].

Definition 2.2.1: Let Y be a non-null set and R be an equivalence relation on Y. Let P be neutrosophic set in Y with the membership function T_P , indeterminacy function I_P and non-membership function F_P . The lower and the upper approximations of P in the approximation (Y, R) denoted by are respectively defined as:

$$\underline{N(P)} =<< x, T_{\underline{N(P)}}(x), I_{\underline{N(P)}}(x), F_{\underline{N(P)}}(x) > / y \in [x]_R, \ x \in Y >$$

and

$$\overline{N(P)} =<< x, T_{\overline{N(P)}}(x), I_{\overline{N(P)}}(x), F_{\overline{N(P)}}(x) > / y \in [x]_R, \ x \in Y >$$

where,

$T_{\underline{N(P)}}(x) = \wedge z \in [x]_R T_P(Y), I_{\underline{N(P)}}(x) = \wedge z \in [x]_R I_P(Y),$
$F_{\underline{N(P)}}(x) = \wedge z \in [x]_R F_P(Y)$

and

$T_{\overline{N(P)}}(x) = \vee z \in [x]_R T_P(Y), I_{\overline{N(P)}}(x) = \vee z \in [x]_R I_P(Y),$
$F_{\overline{N(P)}}(x) = \vee z \in [x]_R F_P(Y)$.

So,

$0 \le T_{\underline{N(P)}}(x) + I_{\underline{N(P)}}(x) + F_{\underline{N(P)}}(x) \le 3$

and

$0 \le T_{\overline{N(P)}}(x) + I_{\overline{N(P)}}(x) + F_{\overline{N(P)}}(x) \le 3$

Here $\vee$ and $\wedge$ denote "max" and "min" operators respectively, $T_P(y)$, $I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of Y with respect to P.

Thus NS mapping,

$\underline{N}, \overline{N} : N(Y) \to N(Y)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N(P)}, \overline{N(P)})$ is called the rough neutrosophic set in (Y, R).

2.3 Interval rough neutrosophic set

Interval rough neutrosophic set (IRNS) [22] is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache, IRNS is the generalizations of interval valued intuitionistic fuzzy rough set.

Definition 2.3.1

Let R be an equivalence relation on the universal set U.Then the pair (U, R) is called a Pawlak approximationspace. An equivalence class of R containing x will bedenoted by $[x]_R$ for X $\in$ U, the lower and upper approximationof X with respect to (U, R) are denoted by respectively

$\underline{R}X$ and $\overline{R}X$ and are defined by

$\underline{R}X = \{x \in U : [x]_R \subseteq X\}$,

$\overline{R}X = \{x \in U : [x]_R \cap X \neq \emptyset\}$.

Now if $\underline{R}X = \overline{R}X$, then X is called definable; otherwise Xis called a rough set.

Definition 2.3.2

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function μA:U→ [0, 1] and non-membership functionν_A: U→ [0, 1] such that $\mu_A(\underline{R}X)$=1and $\nu_A(\underline{R}X) = 0$

ie, $[\mu_A(x), \nu_A(x)]=[1,0]$ if $x \in (\underline{R}X)$ and $\mu_A(U-\overline{R}X)= 0$, $\nu_A(U-\overline{R}X)=1$

ie,

$0 \le \mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X) \le 1$

Definition 2.3.3

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$A = \{<x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]> : x \in U\}$

The lower approximation $\underline{A}_R$ and the upper approximation $\overline{A}_R$ of A in the Pawlak approximation space (U, R) are expressed as follows:

$\underline{A}_R = \{<x, [\wedge_{y\in[x]_R} \{T_A^L(y)\}, \ _{y\ [x]} \{T_A(y)\}],$
$[\vee_{y\in[x]_R} \{I_A^L(y)\}, \ _{y\ [x]} \{I_A(y)\}],$
$[\vee_{y\in[x]_R} \{F_A^L(y)\}, \vee_{y\ [x]} \{F_A(y)\}]> : x \in U\}$

$\overline{A}_R = \{<x, [\vee_{y\in[x]_R} \{T_A^L(y)\}, \ _{y\ [x]} \{T_A(y)\}],$
$[\wedge_{y\in[x]_R} \{I_A^L(y)\}, \ _{y\ [x]} \{I_A(y)\}],$
$[\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\ [x]} \{F_A(y)\}]> : x \in U\}$

The symbols $\wedge$ and $\vee$ indicate "min" and "max" operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here $[x]_R$ is the equivalence class of the element x. It is obvious that

$[\wedge_{y\in[x]_R} \{T_A^U(y)\}, \wedge_{y\ [x]} \{T_A(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R} \{I_A(y)\}, \vee_{y\ [x]} \{I_A^L(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R} \{F_A(y)\}, \vee_{y\ [x]} \{F_A^L(y)\}] \subset [0,1].$
and $0 \le \wedge_{y\in[x]_R} \{T_A(y)\} + \vee_{y\ [x]} \{I_A^U(y)\} +$
$\vee_{y\in[x]_R} \{F_A^U(y)\} \quad 3$

Then $\underline{A}_R$ is an interval neutrosophic set (INS)

Similarly, we have

$[\vee_{y\in[x]_R} \{T_A^L(y)\}, \vee_{y\ [x]} \{T_A(y)\}] \subset [0,1],$
$[\wedge_{y\in[x]_R} \{I_A^L(y)\}, \wedge_{y\ [x]} \{I_A(y)\}] \subset [0,1],$
$[\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\ [x]} \{F_A(y)\}] \subset [0,1]$ and
$0 \le \vee_{y\in[x]_R} \{T_A^U(y)\} + \wedge_{y\ [x]} \{I_A(y)\} +$
$\wedge_{y\in[x]_R} \{F_A^U(y)\}] \quad 3$

Then $\underline{A}_R$ is an interval neutrosophic set.

If $\underline{A}_R = \overline{A}_R$ then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U,R) respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$ respectively.

2.4 Hamming distance

Hamming distance between two neutrosophic sets $M = (T_M(x), I_M(x), F_M(x))$ and $N = (T_N(x), I_N(x), F_N(x))$ is defined as

$$H(M,N) = \frac{1}{3}\sum_{i=1}^{n}(|T_M(x_i) - T_N(x_i)| + |I_M(x_i) - I_N(x_i)| + |F_M(x_i) - F_N(x_i)|).$$

3. Cosine Hamming Similarity Measure of IRNS

Assume that

$$M = \{< x_i, ([\underline{T}_{iM}^-, \underline{T}_{iM}^+], [\underline{I}_{iM}^-, \underline{I}_{iM}^+], [\underline{F}_{iM}^-, \underline{F}_{iM}^+], [\overline{T}_{iM}^-, \overline{T}_{iM}^+], [\overline{I}_{iM}^-, \overline{I}_{iM}^+], [\overline{F}_{iM}^-, \overline{F}_{iM}^+] >: i = 1, 2, ..., n\}$$

and

$$N = \{< x_i, [\underline{T}_{iN}^-, \underline{T}_{iN}^+], [\underline{I}_{iN}^-, \underline{I}_{iN}^+], [\underline{F}_{iN}^-, \underline{F}_{iN}^+], [\overline{T}_{iN}^-, \overline{T}_{iN}^+], [\overline{I}_{iN}^-, \overline{I}_{iN}^+], [\overline{F}_{iN}^-, \overline{F}_{iN}^+] >: i = 1, 2, ..., n\}$$

in X = {x1, x2, …, xn}be any two IRNSs. A cosine Hamming similarity operator between IRNS M and N is defined as follows:

$$\cos(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{\Pi}{6}(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|)).$$

$$\Delta T_M(x_i) = \frac{(\underline{T}_{iM}^- + \underline{T}_{iM}^+ + \overline{T}_{iM}^- + \overline{T}_{iM}^+)}{4},$$

$$\Delta I_M(x_i) = \frac{(\underline{I}_{iM}^- + \underline{I}_{iM}^+ + \overline{I}_{iM}^- + \overline{I}_{iM}^+)}{4},$$

$$\Delta F_M(x_i) = \frac{(\underline{F}_{iM}^- + \underline{F}_{iM}^+ + \overline{F}_{iM}^- + \overline{F}_{iM}^+)}{4},$$

$$\Delta T_N(x_i) = \frac{(\underline{T}_{iN}^- + \underline{T}_{iN} + \overline{T}_{iN} + \overline{T}_{iN})}{4},$$

$$\Delta I_N(x_i) = \frac{(\underline{I}_{iN}^- + \underline{I}_{iN}^+ + \overline{I}_{iN}^- + \overline{I}_{iN}^+)}{4},$$

$$\Delta F_N(x_i) = \frac{(\underline{F}_{iN}^- + \underline{F}_{iN}^+ + \overline{F}_{iN}^- + \overline{F}_{iN}^+)}{4}.$$

Properties 3.1

The defined rough neutrosophic cosine hamming similarity operator cos(M, N) between IRNSs M and N satisfies the following properties:

1. $0 \le \cos(M,N) \le 1$.
2. cos(M,N) = 1 if and only if M = N.
3. cos(M,N) = cos(N,M).

Proof:

1. Since the functions $\Delta T_M(x), \Delta I_M(x), \Delta F_M(x), \Delta T_N(x), \Delta I_N(x)$ and $\Delta F_N(x)$ the value of the cosine function are within [0, 1], the similarity measure based on interval rough neutrosophic cosine Hamming similarity function also lies within [0, 1]. Hence $0 \le \cos(M,N) \le 1$.

This completes the proof.

2. For any two RNSs M and N, if M = N, then the following relations hold

$$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i), \Delta F_M(x_i) = \Delta F_N(x_i).$$

Hence,

$$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0, |\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$$

Thus cos(M,N) = 1

Conversely,

If cos(M,N) = 1, then

$$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0, |\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$$

Since cos(0) = 1. So we can write

$$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i), \Delta F_M(x_i) = \Delta F_N(x_i).$$

Hence M = N**.**

3. As

$$\cos(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{\Pi}{6}(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|))$$
$$= \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{\Pi}{6}(|\Delta T_N(x_i) - \Delta T_M(x_i)| + |\Delta I_N(x_i) - \Delta I_M(x_i)| + |\Delta F_N(x_i) - \Delta F_M(x_i)|))$$
$$= \cos(N,M)$$

This completes the proof.

4. Sine Hamming Similarity Measure of IRNS

Assume that

$$M = \{< x_i, ([\underline{T}_{iM}^-, \underline{T}_{iM}^+], [\underline{I}_{iM}^-, \underline{I}_{iM}^+], [\underline{F}_{iM}^-, \underline{F}_{iM}^+], [\overline{T}_{iM}^-, \overline{T}_{iM}^+], [\overline{I}_{iM}^-, \overline{I}_{iM}^+], [\overline{F}_{iM}^-, \overline{F}_{iM}^+] >: i = 1, 2, ..., n\}$$

and

$$N = \{< x_i, [\underline{T}_{iN}^-, \underline{T}_{iN}^+], [\underline{I}_{iN}^-, \underline{I}_{iN}^+], [\underline{F}_{iN}^-, \underline{F}_{iN}^+], [\overline{T}_{iN}^-, \overline{T}_{iN}^+], [\overline{I}_{iN}^-, \overline{I}_{iN}^+], [\overline{F}_{iN}^-, \overline{F}_{iN}^+] >: i = 1, 2, ..., n\}$$

in $X = \{x_1, x_2, \ldots, x_n\}$ be any two IRNSs. A sine Hamming similarity operator between IRNS M and N is defined as follows:

$$\sin(M,N) = 1 - [\frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\Pi}{}(\left|\Delta T_M(x_i) - \Delta T_N(x_i)\right| + \left|\Delta I_M(x_i) - \Delta I_N(x_i)\right| + \left|\Delta F_M(x_i) - \Delta F_N(x_i)\right|))].$$

Here,

$$\Delta T_M(x_i) = \frac{(\underline{T_{iM}^-} + \underline{T_{iM}^+} + \overline{T_{iM}^-} + \overline{T_{iM}^+})}{4},$$

$$\Delta I_M(x_i) = \frac{(\underline{I_{iM}^-} + \underline{I_{iM}^+} + \overline{I_{iM}^-} + \overline{I_{iM}^+})}{4},$$

$$\Delta F_M(x_i) = \frac{(\underline{F_{iM}^-} + \underline{F_{iM}^+} + \overline{F_{iM}^-} + \overline{F_{iM}^+})}{4},$$

$$\Delta T_N(x_i) = \frac{(\underline{T_{iN}^-} + \underline{T_{iN}^+} + \overline{T_{iN}^-} + \overline{T_{iN}^+})}{4},$$

$$\Delta I_N(x_i) = \frac{(\underline{I_{iN}^-} + \underline{I_{iN}^+} + \overline{I_{iN}^-} + \overline{I_{iN}^+})}{4},$$

$$\Delta F_N(x_i) = \frac{(\underline{F_{iN}^-} + \underline{F_{iN}^+} + \overline{F_{iN}^-} + \overline{F_{iN}^+})}{4}.$$

Properties 4.1

The defined rough neutrosophic sine hamming similarity operator sin (M, *N*) between IRNSs M and N satisfies the following properties:

1. $0 \le \sin(M,N) \le 1$.
2. sin (M,N) = 1 if and only if M = N.
3. sin (M,N) = sin (N,M).

Proof:

1.Since the functions $\Delta T_M(x), \Delta I_M(x), \Delta F_M(x), \Delta T_N(x), \Delta I_N(x)$ and $\Delta F_N(x)$ the value of the sine function are within [0,1], the similarity measure based on interval rough neutrosophic cosine Hamming similarity function also lies within [0,1]. Hence $0 \le \sin(M,N) \le 1$.

This completes the proved.

2.For any two RNSs M and N, if M = N, then the following relations hold

$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i),$

$\Delta F_M(x_i) = \Delta F_N(x_i).$

Hence,

$\left|\Delta T_M(x_i) - \Delta T_N(x_i)\right| = 0, \left|\Delta I_M(x_i) - \Delta I_N(x_i)\right| = 0,$

$\left|\Delta F_M(x_i) - \Delta F_N(x_i)\right| = 0.$

Thus sin(M,N) = 1

Conversely,

If sin(M,N) = 1, then

$\left|\Delta T_M(x_i) - \Delta T_N(x_i)\right| = 0, \left|\Delta I_M(x_i) - \Delta I_N(x_i)\right| = 0,$

$\left|\Delta F_M(x_i) - \Delta F_N(x_i)\right| = 0.$

Since sin(0) = 1. So we can write

$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i),$

$\Delta F_M(x_i) = \Delta F_N(x_i).$

Hence M = N**.**

3. As

$$\sin(M,N) = 1 - [\frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\Pi}{}(\left|\Delta T_M(x_i) - \Delta T_N(x_i)\right| + \left|\Delta I_M(x_i) - \Delta I_N(x_i)\right| + \left|\Delta F_M(x_i) - \Delta F_N(x_i)\right|))]$$
$$= 1 - [\frac{1}{n}\sum_{i=1}^{n}\sin(\frac{\Pi}{}(\left|\Delta T_N(x_i) - \Delta T_M(x_i)\right| + \left|\Delta I_N(x_i) - \Delta I_M(x_i)\right| + \left|\Delta F_N(x_i) - \Delta F_M(x_i)\right|))]$$
$$= \sin(N,M).$$

This completes the proof.

5. Cotangent Hamming Similarity Measure of IRNS

Assume that

$$M = \{< x_i, ([\underline{T_{iM}^-}, \underline{T_{iM}^+}], [\underline{I_{iM}^-}, \underline{I_{iM}^+}], [\underline{F_{iM}^-}, \underline{F_{iM}^+}], [\overline{T_{iM}^-}, \overline{T_{iM}^+}], [\overline{I_{iM}^-}, \overline{I_{iM}^+}], [\overline{F_{iM}^-}, \overline{F_{iM}^+}] >: i = 1, 2, \ldots, n\}$$

and

$$N = \{< x_i, [\underline{T_{iN}^-}, \underline{T_{iN}^+}], [\underline{I_{iN}^-}, \underline{I_{iN}^+}], [\underline{F_{iN}^-}, \underline{F_{iN}^+}], [\overline{T_{iN}^-}, \overline{T_{iN}^+}], [\overline{I_{iN}^-}, \overline{I_{iN}^+}], [\overline{F_{iN}^-}, \overline{F_{iN}^+}] >: i = 1, 2, \ldots, n\}$$

in $X = \{x_1, x_2, \ldots, x_n\}$ be any two IRNSs. A cosine Hamming similarity operator between IRNS M and N is defined as follows:

$$\cot(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cot(\frac{\Pi}{4} + \frac{\Pi}{12}(\left|\Delta T_M(x_i) - \Delta T_N(x_i)\right| + \left|\Delta I_M(x_i) - \Delta I_N(x_i)\right| + \left|\Delta F_M(x_i) - \Delta F_N(x_i)\right|)).$$

Here,

$$\Delta T_M(x_i) \quad \frac{(\underline{T_{iM}^-} + \underline{T_{iM}} \quad \overline{T_{iM}} \quad \overline{T_{iM}})}{4},$$

$$\Delta I_M(x_i) \quad \frac{(\underline{I_{iM}^-} + \underline{I_{iM}} \quad \overline{I_{iM}} \quad \overline{I_{iM}})}{4},$$

$$\Delta F_M(x_i) = \frac{(\underline{F_{iM}^-} + \underline{F_{iM}^+} + \overline{F_{iM}^-} + \overline{F_{iM}^+})}{4},$$

$$\Delta T_N(x_i) = \frac{(\underline{T_{iN}^-} + \underline{T_{iN}^+} + \overline{T_{iN}^-} + \overline{T_{iN}^+})}{4},$$

$$\Delta I_N(x_i) = \frac{(\underline{I_{iN}^-} + \underline{I_{iN}^+} + \overline{I_{iN}^-} + \overline{I_{iN}^+})}{4},$$

$$\Delta F_N(x_i) = \frac{(\underline{F_{iN}^-} + \underline{F_{iN}^+} + \overline{F_{iN}^-} + \overline{F_{iN}^+})}{4}.$$

Properties 5.1

The defined rough neutrosophic cosine hamming similarity operator cot(M, *N*) between IRNSs M and N satisfies the following properties:

1. cot(M, N) = 1 if and only if M = N.
2. cot(M, N) = cot(N, M).

Proof:

1.For any two RNSs M and N, if M = N, then the following relations hold

$$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i), \Delta F_M(x_i) = \Delta F_N(x_i).$$

Hence,

$$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0,$$

$$|\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$$

Thus cot(M,N) = 1

Conversely,

If cot(M,N) = 1, then

$$|\Delta T_M(x_i) - \Delta T_N(x_i)| = 0, |\Delta I_M(x_i) - \Delta I_N(x_i)| = 0,$$

$$|\Delta F_M(x_i) - \Delta F_N(x_i)| = 0.$$

Since cot($\frac{\Pi}{4}$) = 1. So we can write

$$\Delta T_M(x_i) = \Delta T_N(x_i), \Delta I_M(x_i) = \Delta I_N(x_i),$$

$$\Delta F_M(x_i) = \Delta F_N(x_i).$$

Hence M = N**.**

2. As,

$$\cot(M,N) = \frac{1}{n}\sum_{i=1}^{n}\cot\left(\frac{\Pi}{4} + \frac{\Pi}{12}\left(|\Delta T_M(x_i) - \Delta T_N(x_i)| + |\Delta I_M(x_i) - \Delta I_N(x_i)| + |\Delta F_M(x_i) - \Delta F_N(x_i)|\right)\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}\cot\left(\frac{\Pi}{4} + \frac{\Pi}{12}\left(|\Delta T_N(x_i) - \Delta T_M(x_i)| + |\Delta I_N(x_i) - \Delta I_M(x_i)| + |\Delta F_N(x_i) - \Delta F_M(x_i)|\right)\right)$$

$$= \cot(N, M).$$

This completes the proof.

6. Decision making under trigonometric interval rough neutrosophic Hamming similarity measures

In this section, we apply interval rough cosine, sine and cotangent Hamming similarity measures between IRNSs to the multi-attribute decision making problem. Consider C={C_1,C_2, ... ,C_m} be the set of attributes and A={A_1,A_2, ... , A_n} be a set of alternatives. Now we provide an algorithm for MADM problems involving interval rough neutrosophic information.

Algorithm 1. (see Fig 1)

Step 1: Construction of the decision matrix with interval rough neutrosophic number

Decision maker considers the decision matrix with respect to m alternatives and n attributes in terms of interval rough neutrosophic numbers as follows:

Table1: Interval Rough neutrosophic decision matrix

$$D = \langle Z_{ij} \rangle_{n\times m} = \begin{pmatrix} Z_{11} & Z_{12} & \dots\dots\dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots\dots\dots & Z_{2m} \\ \dots & \dots & \dots\dots\dots & \dots \\ \dots & \dots & \dots\dots\dots & \dots \\ \dots & \dots & \dots\dots\dots & \dots \\ Z_{n1} & Z_{n2} & \dots\dots\dots & Z_{nm} \end{pmatrix}$$

Where

Zij = <([$\underline{T_{iM}^-}$, $\underline{T_{iM}^+}$],[$\underline{I_{iM}^-}$, $\underline{I_{iM}^+}$],[$\underline{F_{iM}^-}$, $\underline{F_{iM}^+}$], with [$\overline{T_{iM}^-}$, $\overline{T_{iM}^+}$],[$\overline{I_{iM}^-}$, $\overline{I_{iM}^+}$],[$\overline{F_{iM}^-}$, $\overline{F_{iM}^+}$])>

$$0 \le \vee_{y\in[x]_R}\{T_A^U(y)\} + \wedge_{y\in[x]_R}\{I_A^U(y)\} + \wedge_{y\in[x]_R}\{F_A^U(y)\}] \le 3$$

Step 2: Determination of the ideal alternative

Generally, the evaluation attribute can be categorized into two types: benefit type attribute and cost type attribute. We define an ideal alternative S^* .

For benefit type attribute,

S^*=

$$\{(\min_i \underline{T_{ij}}, \max_i \underline{I_{ij}}, \max_i \underline{F_{ij}}), (\max_i \overline{T_{ij}}, \min_i \overline{I_{ij}}, \min_i \overline{F_{ij}})\}.$$

For cost type attribute,

S^*=

$$\{(\max_i \underline{T_{ij}}, \min_i \underline{I_{ij}}, \min_i \underline{F_{ij}}), (\min_i \overline{T_{ij}}, \max_i \overline{I_{ij}}, \max_i \overline{F_{ij}})\}.$$

Step 3: Determination of the interval rough trigonometric neutrosophic Hamming similarity function of the alternatives

We compute interval rough trigonometric neutrosophic similarity measure between the ideal alternative S^* and each alternative $Z_i = \langle Z_{ij} \rangle_{n\times m}$ for all i = 1, ….., n and j = 1, ….., m.

Step 4: Ranking the alternatives

Using the interval rough trigonometric neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative is selected with the highest similarity value.

Step 5: End.

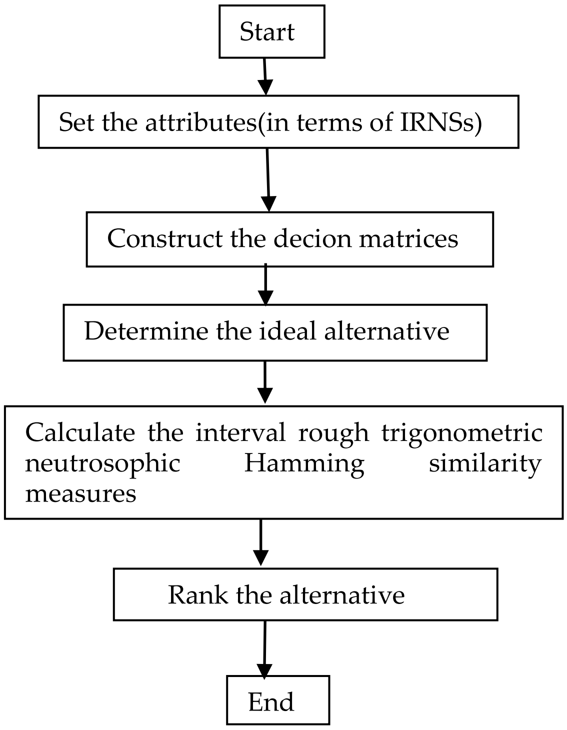

Fig 1. A flowchart of the proposed decision making method

7. Numerical example

Assume that a decision maker intends to select the most suitable laptop for random use from the three initially chosen laptops (S_1, S_2, S_3) by considering four attributes namely: features C_1, reasonable price C_2, customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 5, the considered problem is solved by the following steps:

Step1: Construct the decision matrix with interval rough neutrosophic number

The decision maker construct the decision matrix with respect to the three alternatives and four attributes in terms of interval rough neutrosophic number.

	C_1	**C_2**	**C_3**	**C_4**
S_1	<([.6, .7], [.3, .5], [.3, .4]), ([.8, .9], [.1, .3], [.1, .2])>	<([.5, .7], [.3, .4], [.1, .2]), ([.7, .9], [.3, .5], [.3, .4])>	<([.5, .6], [.4, .5], [.4, .6]), ([.7, .8], [.2, .4], [.3, .4])>	<([.8, .9], [.3, .4], [.5, .6]), ([.7, .8], [.3, .5], [.3, .5])>
S_2	<([.7, .8], [.2, .3], [.0, .2]), ([.7, .9], [.1, .2], [.1, .2])>	<([.6, .7], [.1, .2], [.0, .2]), ([.6, .7], [.1, .3], [.1, .3])>	<([.5, .7], [.2, .3], [.1, .2]), ([.6, .9], [.3, .5], [.2 .4])>	<([.7, .8], [.3, .5], [.1,.3]), ([.5, .7], [.5, .6], [.2, .3])>
S_3	<([.6, .7], [.3, .4], [.0, .3]), ([.6, .9], [.1, .2], [.1, .2])>	<([.5, .7], [.2, .4], [.2, .4]), ([.6, .8], [.1, .3], [.1, .2])>	<([.6, .8], [.2, .4], [.3, .4]), ([.6, .8], [.2, .5], [.3, .5])>	<([.4, .7], [.2, .4], [.4, .5]), ([.5, .8], [.2, .5], [.0, .2])>

Step 2: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C_1, C_2, C_3 and one cost type attribute C_4. We calculate the ideal alternative as follows:

$$S^* =$$
$$\{< ([.6,.7],[.3,.5],[.3,.4]),([.8,.9],[.1,.2],[.1,.2]) >,$$
$$< ([.5,.7],[.3,.4],[.2,.4]),([.7,.9],[.1,.3],[.1,.2]) >,$$
$$< ([.5,.6],[.4,.5],[.4,.6]),([.7,.9],[.2,.4],[.2,.4]) >,$$
$$< ([.8,.9],[.2,.4],[.1,.3]),([.5,.7],[.5,.6],[.3,.5]) >) >\}$$

Step3: Calculate the interval rough trigonometric neutrosophic Hamming similarity measure of the alternatives

$$\cos(S_1,S^*) = 0.999998923,$$
$$\cos(S_1,S^*) = 0.999997135,$$
$$\cos(S_1,S^*) = 0.999998505,$$

$$\sin(S_1,S^*) = 0.999531651,$$
$$\sin(S_1,S^*) = 0.997658256,$$

$\sin(S_1, S^*) = 0.998343644,$
$\cot(S_1, S^*) = 70.25049621,$
$\cot(S_1, S^*) = 67.22363275,$
$\cot(S_1, S^*) = 68.81008448.$

Step 4: Rank the alternatives

Ranking of alternatives is prepared based on the descending order of similarity measures. The highest value reflects the best alternatives.
Here,
$\cos(S_1, S^*) > \cos(S_3, S^*) > \cos(S_2, S^*)$.
$\sin(S_1, S^*) > \sin(S_3, S^*) > \sin(S_2, S^*)$.
$\cot(S_1, S^*) > \cot(S_3, S^*) > \cot(S_2, S^*)$.

Hence, the laptop S_1 is the best alternative for random use.

8. Conclusions

In this paper, we have proposed interval rough trigonometric Hamming similarity measures and proved their properties. We have developed three MADM strategies base on sine, cosine and cotangent similarity measures under interval rough neutrosophic environment. Then we solved an illustrative numerical example to demonstrate the feasibility, applicability of the developed strategies. The concept presented in this paper can be applied other multiple attribute decision making problems such as teacher selection [30, 31, 32], school selection [33], weaver selection [34, 35, 36], brick field selection [37, 38], logistics center location selection [39, 40], data mining [41] etc. under interval rough neutrosophic environment.

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Entropy based Single Valued Neutrosophic Digraph and its applications

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Abstract: This paper introduces the single valued neutrosophic (i.e. SVN) digraph. The basic terminologies and operations of SVN digraphs have been defined. Later certain types of SVN digraphs are shown and some of the important properties of SVN digraphs are investigated. Finally SVN digraphs are applied in solving a multi-criterion decision making problems.

Keywords: SVN set, SVN digraph, Entropy, Similarity, Decision making.

1 Introduction

In 1995, neutrosophic logic and set theory was introduced by Smarandache [22, 23]. The neutrosophic sets are characterized by a truth membership function(t), a falsity membership function (f) and an indeterminacy membership function(i) respectively, which lies between the nonstandard unit interval $[0, 1]^*$. Unlike intuitionistic fuzzy sets, here the uncertainties present i.e. the indeterminacy factor, is independent of truth and falsity values. Hence Neutrosophic sets are more general than intuitionistic fuzzy set [6] and draw a special attraction to the researchers. Later on Wang et al. [25] introduced a special type of neutrosophic set say single valued neutrosophic set (SVNS). They also introduced the interval valued neutrosophic set (IVNS) in [26]. The SVN set is a generalization of classical set, fuzzy set [27], intuitionistic fuzzy set [6] etc. To see the practical application of the neutrosophic sets and SVN sets, one may see [1, 2, 3, 4, 7, 8] etc.

On the other hand, nowadays graphs and digraphs are widely used by the researchers to solve many pratical problems. The graphs are used as a tool for solving combinatorial problems in algebra, analysis, geometry etc. Many works on fuzzy graph theory, fuzzy digraph theory, intuitionistic fuzzy graphs, soft digraphs etc. are carried out by a number of researchers [12, 13, 15, 16, 17, 21]. Four main categories of neutrosophic graphs have been defined by Samarandache in the paper [24]. However the concept of single valued neutrosophic graphs was introduced by Broumi et al. [9, 10, 11].

In this paper we have introduced the notion of SVN digraphs for the first time. In section 2, some preliminaries regarding neutrosophic sets, graph theory, SVN sets etc. are discussed. In section 3, we have defined the SVN digraph and some terminologies regarding SVN digraphs with examples. We have solved a real life problem by using SVN digraph in Section 4. In Section 5, we have defined the volume of a SVN digraph and also the similarity measure between two SVN digraphs by using the volume of each SVN digraph. Finally in this section, we have computed the similarity measure of the digraphs of Section 4 and compared the results. Section 6 concludes the paper.

2 Preliminaries

In this section, we will discuss some definitions and terminologies regarding neutrosophic sets which will be used in the rest of the paper. However, for details on the neutrosophic sets, one can see [20].

Definition 1 *[20] Let X be a universal set. A neutrosophic set A on X is characterized by a truth membership function t_A, an indeterminacy membership function i_A and a falsity membership function f_A, where $t_A, i_A, f_A : X \to [0, 1]$, are functions and $\forall\ x \in X$, $x = x(t_A(x), i_A(x), f_A(x)) \in A$ is a single valued neutrosophic element of A.*

A single valued neutrosophic set (SVNS) A over a finite universe $X = \{x_1, x_2, \ldots, x_n\}$ is represented as below:

$$A = \sum_{i=1}^{n} \frac{x_i}{\langle t_A(x_i), i_A(x_i), f_A(x_i)\rangle}$$

Definition 2 *[20] The complement of a SVNS A is denoted by A^c and is defined by $t_{A^c}(x) = f_A(x), i_{A^c}(x) = 1-i_A(x), f_{A^c}(x) = t_A(x)\ \forall\ x \in X$.*

Definition 3 *[20] A SVNS A is contained in the other SVNS B, denoted as $A \subseteq B$, if and only if $t_A(x) \leq t_B(x), i_A(x) \leq i_B(x)$ and $f_A(x) \geq f_B(x)\ \forall x \in X$. Two sets will be equal if $A \subseteq B$ and $B \subseteq A$.*

Definition 4 *[20] Suppose $N(X)$ be the collection of all SVN sets on X and $A, B \in N(X)$. A similarity measure between two SVN sets A and B is a function $S : N(X) \times N(X) \to [0, 1]$ which satisfies the following condition:*

(i) $0 \leq S(A, B) \leq 1$,

(ii) $S(A, B) = 1$ *if and only if* $A = B$.

iii) $S(A, B) = S(B, A)$

(iv) *If $A \subseteq B \subseteq C$, then $S(A, C) \leq S(A, B)$ and $S(A, C) \leq S(B, C)$ for all $A, B, C \in N(X)$.*

Note that here (i)-(iii) are essential for any similarity measure and (iv) is a desirable property although not mandatory.

Definition 5 *[20] The entropy of SVNS A is defined as a function $E : N(X) \to [0, 1]$ which satisfies the following axioms:*

(i) *$E(A) = 0$ if A is a crisp set.*

(ii) *$E(A) = 1$ if $(t_A(x), i_A(x), f_A(x)) = (0.5, 0.5, 0.5)$ $\forall x \in X$.*

(iii) *$E(A) \geq E(B)$ if A is more uncertain than B i.e. $t_A(x) + f_A(x) \leq t_B(x) + f_B(x)$ and $|i_A(x) - i_{A^c}(x)| \leq |i_B(x) - i_{B^c}(x)|$ $\forall$ $x \in X$ $A, B \in X$.*

(iv) *$E(A) = E(A^c)$ $\forall A \in N(X)$, where $N(X)$ is the collection of all SVNS over X.*

Example 6 *An entropy measure of an element x_1 of a SVNS A can be calculated as follows:*

$$E_1(x_1) = 1 - (t_A(x_1) + f_A(x_1)) \times |i_A(x_1) - i_{A^c}(x_1)|.$$

Graph and digraphs played an important role in many applications of mathematics like Chinese post-man problems, shortest path problems etc. For graph theoretic terminologies, one can see any standard reference, e.g. [14] or [19].

3 SVN Digraph

In this section, we will define SVN digraph D for the first time corresponding to a SVNS $V_D = \{(v_i, \langle t_{V_D}(v_i), i_{V_D}(v_i), f_{V_D}(v_i)\rangle), i = 1, \dots, n\}$ over a finite universal set X. For sake of implicity henceforth we will denote V_D by $V_D = \{v_1, v_2, \dots, v_n\}$ in the rest of paper.

Definition 7 *A SVN digraph D is of the form $D = (V_D, A_D)$ where,*

(i) *$V_D = \{v_1, v_2, v_3, \dots, v_n\}$ such that the functions $t_{V_D} : V_D \to [0, 1]$, $i_{V_D} : V_D \to [0, 1]$, $f_{V_D} : V_D \to [0, 1]$ denote the truth-membership function, a indeterminacy-membership function and falsity-membership function of the element $v_i \in V_D$ respectively and $0 \leq t_{V_D}(v_i) + i_{V_D}(v_i) + f_{V_D}(v_i) \leq 3$, $\forall v_i \in V_D, i = 1, 2, \dots, n$.*

(ii) *$A_D = \{(v_i, v_j); (v_i, v_j) \in V_D \times V_D\}$ provided $0 < E(v_i) - E(v_j) \leq 0.5$ and the functions $t_{A_D} : A_D \to [0, 1], i_{A_D} : A_D \to [0, 1], f_{A_D} : A_D \to [0, 1]$ are defined by*

$$t_{A_D}(\{v_i, v_j\}) \leq min[t_{V_D}(v_i), t_{V_D}(v_j)],$$
$$i_{A_D}(\{v_i, v_j\}) \geq max[i_{V_D}(v_i), i_{V_D}(v_j)],$$
$$f_{A_D}(\{v_i, v_j\}) \geq max[f_{V_D}(v_i), f_{V_D}(v_j)]$$

where $t_{A_D}, i_{A_D}, f_{A_D}$ denotes the truth-membership function, a indeterminacy-membership function and falsity-membership function of the arc $(v_i, v_j) \in A_D$ respectively where $0 \leq t_{A_D}(v_i, v_j) + i_{A_D}(v_i, v_j) + f_{A_D}(v_i, v_j) \leq 3$, $\forall (v_i, v_j) \in A_D,\ i, j \in \{1, 2, \dots n\}$.

We call V_D as the vertex set of D, A_D as the arc set of D where $E(v)$ is the entropy of the vertex v. Please note that if $E(v_i) = E(v_j)$, then $\{(v_i, v_j), (v_j, v_i)\} \in A_D$. Since for a vertex $v \in V_D$ of a SVN digraph D we have $E(v) = E(v)$, thus every vertex of a SVN digraph D contains a loop (v, v) at v. On the other hand, if $E(v_i) - E(v_j) > 0.5$, we define that there exists no arc between the vertices v_i and v_j. A SVN digraph $D = (V_D, A_D)$ is said to be symmetric if $(u, v) \in A_D$ implies $(v, u) \in A_D$. On the other hand, D is asymmetric if $(u, v) \in A_D$ implies $(v, u) \notin A_D$.

Remark 8 *Here, we are trying to represent a SVN set by a SVN digraph. For this reason, we have taken a SVN set V_D and have considered the set V_D as the vertex set of the SVN Digraph D. Thus we are only considering the entropy of the vertex set V_D of the SVN digraph D. However we have seen that the arc set A_D of D forms a new SVN set and we have the following corollary.*

Corollary 9 *The arc set A_D of a SVN digraph $D = (V_D, A_D)$ forms a neutrosophic set on $X \times X$.*

Example 10 *Consider the SVN digraph $D_1 = (V_{D_1}, A_{D_1})$ in Figure 1 with vertex set $V_{D_1} = \{v_1, v_2, v_3, v_4\}$ and arc set $A_{D_1} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_4, v_1)\}$ with one loop at each vertex as follows:*

$$\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 \\ t_{V_D} & 0.4 & 0.4 & 0.5 & 0.2 \\ i_{V_D} & 0.1 & 0.3 & 0.2 & 0.5 \\ f_{V_D} & 0.2 & 0.1 & 0.5 & 0.3 \\ E & 0.52 & 0.8 & 0.4 & 1 \end{bmatrix},$$

$$\begin{bmatrix} & (v_2, v_1) & (v_2, v_3) & (v_4, v_1) & (v_1, v_3) & (v_4, v_2) \\ t_{A_D} & 0.3 & 0.2 & 0.1 & 0.4 & 0.2 \\ i_{A_D} & 0.4 & 0.3 & 0.5 & 0.3 & 0.5 \\ f_{A_D} & 0.2 & 0.6 & 0.3 & 0.4 & 0.4 \end{bmatrix}.$$

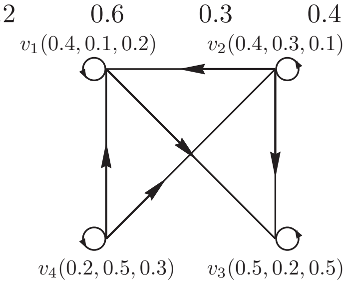

Figure 1: The SVN Digraph D_1

Remark 11 (i) In a SVN digraph D, if $x = (v_i, v_j)$ is an arc, we say that x is *incident* with v_i and v_j; v_i is *adjacent to*

v_j; and v_j is *adjacent from* v_i. It is customary to represent a digraph by a diagram with nodes representing the vertices and directed line segments (arcs) representing the arcs of the digraph. Every vertex of any SVN digraph contains loops by definition. For the sake of simplicity, we define $(t_{A_D}(v,v), i_{A_D}(v,v), f_{A_D}(v,v)) = (0,1,1)$ for each arc $(v,v) \in A_D$ of a SVN digraph D.

(ii) The *order* of D, denoted by $|D|$, is the number of vertices of D. The *size* of a SVN digraph D, is the number of arcs of D i.e. $|A_D|$. For example, the order and size of the digraph D_1 in Figure 1 is 4 and 9 respectively.

Definition 12 *A SVN-subdigraph $H = (V_H, A_H)$ of a SVN-digraph $D = (V_D, A_D)$ is a SVN-digraph such that*

(i) $V_H \subseteq V_D$ *where* $t_{V_H}(v_i) \leq t_{V_D}(v_i)$, $i_{V_H}(v_i) \leq i_{V_D}(v_i)$, $f_{V_H}(v_i) \geq f_{V_D}(v_i) \quad \forall v_i \in V_H$.

(ii) $A_H \subseteq A_D$ *where* $t_{A_H}(v_i,v_j) \leq t_{A_D}(v_i,v_j)$, $i_{A_H}(v_i,v_j) \leq i_{A_D}(v_i,v_j)$, $f_{A_H}(v_i,v_j) \geq f_{A_D}(v_i,v_j) \quad \forall (v_i,v_j) \in A_H$.

Example 13 *Consider the SVN digraph $D_2 = (V_{D_2}, A_{D_2})$ in Figure 2 with vertex set $V_{D_2} = \{v_1, v_2, v_3\}$ and arc set $A_{D_2} = \{(v_2,v_1), (v_1,v_3), (v_2,v_3)\}$ with one loop at each vertex as follows:*

$$\left[\begin{array}{cccc} & v_1 & v_2 & v_3 \\ t_{V_D} & 0.4 & 0.4 & 0.5 \\ i_{V_D} & 0.1 & 0.3 & 0.2 \\ f_{V_D} & 0.2 & 0.1 & 0.5 \\ E & 0.52 & 0.8 & 0.4 \end{array}\right]$$

,

$$\left[\begin{array}{cccc} & (v_2,v_1) & (v_2,v_3) & (v_1,v_3) \\ t_{A_D} & 0.3 & 0.2 & 0.4 \\ i_{A_D} & 0.4 & 0.3 & 0.3 \\ f_{A_D} & 0.2 & 0.6 & 0.4 \end{array}\right].$$

It is clear that the SVN digraph D_2 in Figure 2 is a SVN subdigraph of D_1 in Figure 1.

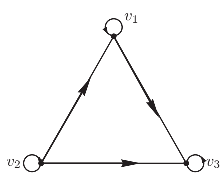

Figure 2: The SVN Digraph D_2

Definition 14 *A SVN-digraph $K = (V_K, A_K)$ is a spanning SVN-subdigraph of a SVN-digraph $D = (V_D, A_D)$ if*

(i) $V_K = V_D$ *where* $t_{V_K}(v_i) = t_{V_D}(v_i)$, $i_{V_K}(v_i) = i_{V_D}(v_i)$, $f_{V_K}(v_i) = f_{V_D}(v_i) \quad \forall v_i \in V_K$.

(ii) $A_K \subseteq A_D$ *where* $t_{A_K}(v_i,v_j) = t_{A_D}(v_i,v_j), i_{A_K}(v_i,v_j) = i_{A_D}(v_i,v_j), f_{A_K}(v_i,v_j) = f_{A_D}(v_i,v_j) \quad \forall (v_i,v_j) \in A_K$.

Definition 15 *For vertices u, v in a SVN digraph D, a u-v SVN path $P = (V_P, A_P)$ is a SVN subdigraph of D whose distinct vertices and arcs can be written in an alternating sequence:*

$$v_1(v_1,v_2)v_2(v_2,v_3)v_3 \cdots v_{k-1}(v_{k-1},v_k)v_k,$$

where $v_1 = u, v_k = v$ and $t_{A_D}(v_i,v_{i+1}) > 0, i_A(v_i,v_{i+1}) > 0, f_A(v_i,v_{i+1}) > 0, 0 < E(v_i) - E(v_{i+1}) \leq 0.5, \ \forall 1 \leq i \leq k$. Further, if (v,u) is an arc in D, then the subdigraph P together with (v,u) is a SVN cycle of length k or a k-SVN cycle in D. For convenience, we denote the cycle as $C = [v_1, v_2, \ldots v_k]$. A SVN digraph having no cycle of length greater than 1 is said to be acyclic. A 1-cycle consists of a vertex v and a loop at v.

Definition 16 *A SVN digraph D is said to be connected if the simple SVN graph G associated to D (i.e., the graph with vertex set V_D and edge set $\{\{u,v\} : (u,v) \in A_D, u \neq v\}$) is connected. The SVN digraph D is said to be strongly connected if for every pair (u,v) of vertices, D contains a u-v SVN path and a v-u SVN path both. The maximal connected (resp. strongly connected) SVN subdigraphs of D are called components (resp. strong components) of SVN D.*

Definition 17 *Let $D = (V_D, A_D)$ be a SVN digraph. Then the outdegree (resp. indegree) of any vertex v is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those arcs which are adjacent from (resp. to) vertex v denoted by $O_d(v) = O(d_t(v), d_i(v), d_f(v))$ and $I_d(v) = I(d_t(v), d_i(v), d_f(v))$, where,*

$$\begin{aligned} d_t(v) &= \frac{1}{n}\sum t_{A_D}(u,v) \text{ denotes the degree of truth membership} \\ d_i(v) &= \frac{1}{n}\sum i_{A_D}(u,v) \text{ denotes the degree of indeterminacy members} \\ d_f(v) &= \frac{1}{n}\sum f_{A_D}(u,v) \text{ denotes the degree of falsity membership,} \end{aligned}$$

where n is the number of arcs adjacent from (resp. to) vertex v.

Example 18 *Consider the SVN digraph D_1 in Figure 1. Here we have, the outdegree and indegree of each vertex as follows:*

$$\begin{aligned} O_d(v_1) &= (0.8, 0.86, 0.88), O_d(v_2) = (0.05, 0.73, 0.7), \\ O_d(v_3) &= (0, 1, 1), O_d(v_4) = (0.05, 0.83, 0.78), \\ I_d(v_1) &= (0.07, 0.82, 0.75), I_d(v_2) = (0.04, 0.90, 0.88), \\ I_d(v_3) &= (0.1, 0.76, 0.83), I_d(v_4) = (0, 1, 1). \end{aligned}$$

Remark 19 *The set of in-degrees (out-degrees) of the vertices of an SVN digraph forms a neutrosophic set on V_D (i.e. on X).*

Definition 20 *A SVN digraph $D = (V_D, A_D)$ is called k-regular SVN digraph if the sum of outdegree and indegree of each vertex v is k. That is, $d(v) = \sum\{O_d(v) + I_d(v)\} = (k,k,k)$ for all $v \in V_D$.*

Definition 21 *A SVN digraph $D = (V_D, A_D)$ is called strong SVN digraph if*

$$t_{A_D}(v_i, v_j) = min[t_{V_D}(v_i), t_{V_D}(v_j)],$$
$$i_{A_D}(v_i, v_j) = max[i_{V_D}(v_i), i_{V_D}(v_j)],$$
$$f_{A_D}(v_i, v_j) = max[f_{V_D}(v_i), f_{V_D}(v_j)]$$

for all $(v_i, v_j) \in A_D$

Example 22 *Consider the SVN digraph $D_3 = (V_{D_3}, A_{D_3})$ in Figure 3 with vertex set $V_{D_3} = \{v_1, v_2, v_3, v_4\}$ and arc set $A_{D_3} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_4), (v_4, v_1)\}$ with one loop at each vertex as follows:*

$$\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 \\ t_{V_D} & 0.9 & 0.1 & 0.5 & 0.2 \\ i_{V_D} & 0.3 & 0.6 & 0.1 & 0.6 \\ f_{V_D} & 0.8 & 0.1 & 0.5 & 0.1 \\ E & 0.3 & 0.6 & 0.2 & 0.4 \end{bmatrix}$$

,

$$\begin{bmatrix} (v_2, v_1) & (v_2, v_3) & (v_4, v_3) & (v_4, v_1) & (v_1, v_3) & (v_2, v_4) \\ 0.1 & 0.1 & 0.2 & 0.2 & 0.5 & 0.1 \\ 0.6 & 0.6 & 0.6 & 0.6 & 0.3 & 0.6 \\ 0.8 & 0.5 & 0.5 & 0.8 & 0.8 & 0.1 \end{bmatrix}.$$

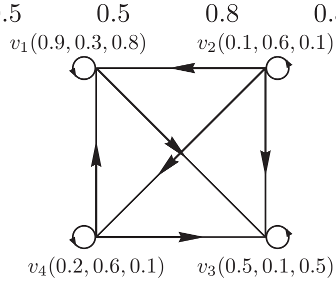

Figure 3: The SVN Digraph D_3

Definition 23 *A SVN digraph $D = (V_D, A_D)$ corresponding to a SVNS V_D is called complete if the following holds:*

(i) $V_D = \{v_1, v_2, v_3, \ldots, v_n\}$.

(ii) $A_D = \{(v_i, v_j); v_i, v_j \in V_D\}$, *provided $E(v_i) = E(v_j)\ \forall v_i \in V_D$.*

Example 24 *Consider the SVN digraph $D_4 = (V_{D_4}, A_{D_4})$ in Figure 4 with vertex set $V_{D_4} = \{v_1, v_2, v_3\}$ and arc set $A_{D_4} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3), (v_1, v_2), (v_3, v_1), (v_3, v_2)\}$ with one loop at each vertex as follows:*

$$\begin{bmatrix} & v_1 & v_2 & v_3 \\ t_{V_D} & 0.5 & 0.4 & 0.7 \\ i_{V_D} & 0.2 & 0.2 & 0.2 \\ f_{V_D} & 0.4 & 0.5 & 0.5 \\ E(v_i) & 0.46 & 0.46 & 0.46 \end{bmatrix}.$$

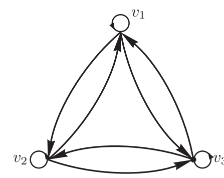

Figure 4: The SVN Digraph D_4

In D_4, we consider that each non-loop arc has neutrosophic value as $(0.4, 0.2, 0.5)$. It is clear that the SVN digraph D_4 is a complete digraph and also a k-regular digraph.

Remark 25 (i) *There does not exist any asymmetric SVN digraph with a cycle of length ≥ 3. Suppose an asymmetric cyclic SVN digraph $D = (V_D, A_D)$ has vertex set $V_D = \{v_1, v_2, v_3, \ldots, v_n\}$. Without loss of generality, let D has a cycle of length 3 say $\langle v_1, v_2, v_3 \rangle$. Then we have $E(v_1) > E(v_2) > E(v_3) > E(v_1)$- which is impossible. Hence D does not have a cycle of length ≥ 3.*

(ii) *Every SVN digraph D is self-complementary. For any SVN digraph having vertex set V_D and its complement set V_D^C, each vertex have same entropy. Hence the result follows.*

Definition 26 *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H| = n$ corresponding to the SVNS V_D and V_H over an universal set X. Then the union of two SVN digraphs D and H is defined as a SVN digraph $C = (V_C, A_C)$ in which the following holds;*

(i) $V_C = V_D \cup V_H$,

(ii) $t_{V_C}(v) = max(t_{V_D}(v), t_{V_H}(v))$;
$i_{V_C}(v) = max(i_{V_D}(v), i_{V_H}(v))$;
$f_{V_C}(v) = min(f_{V_D}(v), f_{V_H}(v)); \forall\ v \in V_D \cup V_H$ *and,*

(iii) $A_C = \{(v_i, v_j); (v_i, v_j) \in V_C \times V_C\}$ *provided* $0 < E(v_i) - E(v_j) \leq 0.5$,

Definition 27 *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H|$ corresponding to the SVNS V_D and V_H over an universal set X. Then the intersection of two SVN digraphs D and H is defined as a SVN digraph $C = (V_C, A_C)$ in which the following holds:*

(i) $V_C = V_D \cap V_H$,

(ii) $t_{V_C}(v) = min(t_{V_D}(v), t_{V_H}(v))$;
$i_{V_C}(v) = min(i_{V_D}(v), i_{V_H}(v))$;
$f_{V_C}(v) = max(f_{V_D}(v), f_{V_H}(v)); \forall\ v \in V_D \cap V_H$ *and,*

(iii) $A_C = \{(v_i, v_j); (v_i, v_j) \in V_C \times V_C\}$ *provided* $0 < E(v_i) - E(v_j) \leq 0.5$,

Definition 28 *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H|$ corresponding to the SVNS V_D and V_H over an universal set X. Consider $V_D = V_H =$*

$\{v_1, v_2, v_3, \ldots, v_n\}$. *The similarity measure between the neutrosophic digraphs D and H can be evaluated by the function,*

$$S(D, H) = \frac{|A_D \cap A_H|}{|A_D \cup A_H|},$$

where $|A_D \cap A_H|, |A_D \cup A_H|$ *denotes the number of arcs in* $A_D \cap A_H$ *and* $A_D \cup A_H$ *respectively with a set theoretic point of view.*

It is clear that $S(D, H)$ satisfies the properties of the Definition 4.

4 An application using SVN Digraph

In this section, we have applied our SVN digraph to solve a multi-criteria decision-making problem. To find out the best alternative decision set, we will use the idea of model set. For this purpose now, we define a model set $M = \{(.1, 0, 0.05), (.1, 0, 0.04), (.1, 0, .03), (.1, 0, 0.02)\}$.

The problem is based on a similar problem discussed in [13]. Now we assume that there exists a set of suppliers $S = \{S_1, S_2, S_3, S_4\}$ whose performances are examined w.r.t the following criteria (T_1, T_2, T_3, T_4), where T_1 the adaptation of new technology , T_2 performance in supply , T_3 the ability of controlling man-power and T_4 quality of service. We will use our proposed decision making technique to select the best supplier. The evaluation of an supplier $S_i, i = 1, 2, 3, 4$ with respect to a criterion $T_j; j = 1, 2, 3, 4$, it based on the knowledge of a domain expert. For example, the opinion of an expert about a supplier S_1 with respect to a criterion T_1 , is as follows: the statement is good is 0.4 and the statement is poor is 0.3 and the degree of not sure is 0.4. For a neutrosophic point of view, it can be expressed as neutrosophic element $v_{11} = \{0.4, 0.3, 0.4\}$. Thus for the alternative S_1, the neutrosophic set is $M_1 = \{(0.4, 0.3, 0.40), (0.6, 0.3, 0.30), (0.2, 0.2, 0.5), (0.5, 0.3, 0.2)\}$. Similarly the other neutrosophic sets for the alternatives S_2, S_3, S_4 respectively are

$$\begin{aligned} M_2 = \{&(0.4, 0.4, 0.2), (0.5, 0.4, 0.30), (0.5, 0.20, 0.40), \\ &(0.5, 0.3, 0.1)\}, \\ M_3 = \{&(0.6, 0.2, 0.2), (0.6, 0.3, 0.40), (0.5, 0.4, 0.10), \\ &(0.3, 0.2, 0.40)\}, \\ M_4 = \{&(0.6, 0.2, 0.20), (0.1, 0.4, 0.50), (0.4, 0.2, 0.60), \\ &(0.4, 0.1, 0.3). \end{aligned}$$

Now we first draw the SVN digraph D_M for the model set M as follows:

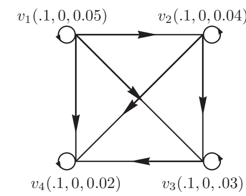

Figure 5: The SVN Digraph D_M

Now we draw simultaneously the digraphs $D_{M_1}, D_{M_2}, D_{M_3}, D_{M_4}$ in the Figure 6, Figure 7 respectively. In each case we now calculate the similarity measure of each of the digraphs $D_{M_i};\ i = 1, 2, 3, 4$.

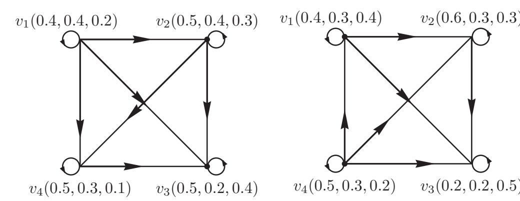

Figure 6: The SVN Digraphs $D_{M_1},\ D_{M_2}$

(i) $S_1(D_M, D_{M_1}) = \frac{7}{13} = 0.538$

(ii) $S_2(D_M, D_{M_2}) = \frac{9}{11} = 0.81$

(iii) $S_3(D_M, D_{M_3}) = \frac{6}{14} = 0.42$

(iv) $S_4(D_M, D_{M_4}) = \frac{8}{12} = 0.666$

Therefore as per the similarity measures, the ranking order of the four suppliers is $S_2 > S_4 > S_1 > S_3$. Hence, the best supplier is S_2. From the above example, we can observe that the proposed single valued neutrosophic multi-criteria decision-making method can be handled easily with the help of SVN digraphs. Ashraf et al. [5] have studied SVN graph where as we have tried SVN Digraph. In SVN graph there is no edge direction. Thus it is quite familiar that between any two vertices of SVN graph there is always an arc satisfying some required condition. In SVN digraph theory, between any two vertices there may not be an arc. Here arcs are present depending on the entropy difference of the vertices. Thus these two notions of SVN graphs and digraphs are completely different. Also Ashraf et al. [5] have studied regular SVN graphs which has equal degree of each vertices. But in SVN digraphs all vertices may or may not have same degrees. In future one may study the SVN regular digraphs which may be a completely new idea.

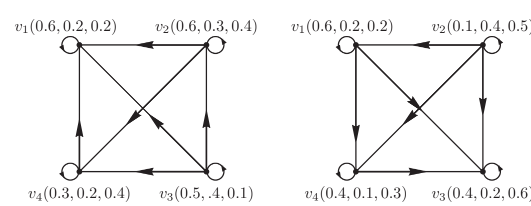

Figure 7: The SVN Digraphs $D_{M_3},\ D_{M_4}$

5 Similarity Measure using the Volume of a SVN Digraph

In this section we introduce some new definition which are given below:

Definition 29 *Suppose A and B be two SVNS over X. Then A is said to be more uncertain than B, denoted by $A \leq B$,*

(i) $t_A(x) + f_A(x) \leq t_B(x) + f_B(x)$

(ii) $|i_A(x) - i_{A^C}(x)| \leq |i_B(x) - i_{B^C}(x)|, \forall \in X$.

Example 30 *Suppose*

$$A = \{(0.4, 0.3, 0.3), (0.3, 0.5, 0.2), (0.5, 0.4, 0.1), (0.2, 0.2, 0.3)\},$$

$$B = \{(0.5, 0.1, 0.6), (0.4, 0.3, 0.4), (0.4, 0.2, 0.6), (0.4, 0.1, 0.5)\}$$

be two SVNS over an universal set $X = \{x_1, \ldots, x_4\}$. Then for all $x_i \in X$, the above definition $A \leq B$ is satisfied.

Remark 31 *From Definition 5 and Example 6, it follows that $E(A) \geq E(B)$ for any two SVNS $A, B \in X$ with $A \leq B$.*

Definition 32 *Suppose A and B be two SVNS over X. Then their maximum sum $C = A \bigoplus B$, denoted by $Max(A, B)$ is again an SVNS on X which is defined as follows:*

(i) $t_c(x) = max\{t_A(x), t_B(x)\}$,

(ii) $f_c(x) = max\{f_A(x), f_B(x)\}$,

(iii) $i_c(x) = max\{|i_A(x) - i_{A^C}(x)|, |i_B(x) - i_{B^C}(x)|\}$, $\forall x \in X$.

Example 33 *From Example 30, it follows that the SVNS*

$$C = \{(0.5, 0.8, 0.6), (0.4, 0.4, 0.4), (0.5, 0.6, 0.6), (0.4, 0.8, 0.5)\}$$

is the maximum sum of two SVNS A, B over X.

Remark 34 *It can be easily proved that $A, B \leq A \bigoplus B$. So, $E(A) \geq E(A \bigoplus B)$ and $E(B) \geq E(A \bigoplus B)$. Therefore*

$$2E(A \bigoplus B) \leq E(A) + E(B).$$

Definition 35 *Suppose V_D and V_H are two SVNS over an universal set X such that $V_D = V_H = \{v_1, v_2, \ldots, v_n\}$. Consider $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraph corresponding to the SVN set D and H respectively. Then the maximum sum digraph C of two digraphs D, H is a SVN digraph $C = (V_C, A_C)$ where $V_C = V_D \bigoplus V_H$ and $A_C = \{(v_i, v_j); (v_i, v_j) \in V_C \times V_C\}$ provided $0 < E(v_i) - E(v_j) \leq 0.5$.*

Definition 36 *Suppose $D_1 = (V_{D_1}, A_{D_1})$ be a SVN digraph over a set X. Then the volume of a SVN digraph D_1 is defined as*

$$Vol(G) = \sum_{v \in V_{D_1}} E(v) = E(V_{D_1}).$$

Example 37 *Consider the SVN digraph D_M in Figure 5 in Section 4. Then the volume of the digraph D_M is*

$$Vol(D_M) = \sum\{E(v_1) + E(v_2) + E(v_3) + E(v_4)\}$$

$= 0.85 + 0.84 + 0.83 + 0.82 = 3.34.$

Definition 38 *Now we define the similarity measure between two SVN digraph D and H as follows:*

$$S(V, H) = \frac{2Vol(V_D \bigoplus V_H)}{Vol(V_D) + Vol(V_H)}.$$

It can be noted that the Definition 38 satisfies the first three postulates of the Definition 4. Now we consider the SVN digraphs $D_M, D_{M_i}, i = 1 \ldots, 4$ in Figure 5,6,7 in Section 4 and calculate the similarity measure between them by using the Definition 38. Here we the following result obtained by the Definition 38:

(i) $S_1(D_M, D_{M_1}) = \frac{1.8}{5.96} = 0.30$

(ii) $S_2(D_M, D_{M_2}) = \frac{2.2}{6.46} = 0.34$

(iii) $S_3(D_M, D_{M_3}) = \frac{1.8}{5.68} = 0.31$

(iv) $S_4(D_M, D_{M_4}) = \frac{1.8}{5.55} = 0.32$

According to the similarity measure followed by the Definition 38 we have obtained the order $S_2 > S_4 > S_3 > S_1$. Hence the best supplier is S_2.

Thus it can be seen that using both the techniques described in Section 4 and 5, we have got a similar result using SVN digraphs.

6 Conclusion

Although Fuzzy digraph theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all sorts of uncertainties prevailing in different real physical problems such as problems involving incomplete information. Hence further generalizations of fuzzy and intuitionistic fuzzy digraphs are required. So there are also scopes of evolution of new theories which will have more powers of handling different kinds of uncertainties. Unlike in intuitionistic fuzzy digraphs, where the incorporated uncertainty is dependent of the degree of belongingness and degree of non-belongingness, here the uncertainty present, i.e. indeterminacy factor, is independent of truth and falsity values. Single valued neutrosophic digraphs were motivated from the practical point of view and that can be used in real scientific and engineering applications. The single valued neutrosophic digraph theory is a generalization of fuzzy digraph theory, intuitionistic digraph theory.

SVN digraph theory is based on the entropy differences of the vertices of SVN digraph. It represents a SVN neutrosophic Set which is used as the vertices of the SVN digraph. In fuzzy digraph theory or Intuitionistic Fuzzy digraph theory, arcs of these digraphs arises depending on the binary relations of the vertices. In SVN digraphs, every vertices have loop it it where as in other theories it may not be possible. Thus the recently proposed notion of SVN digraph theory is a general formal framework for studying uncertainties arising due to 'indeterminacy' factors. Also single valued neutrosophic digraph theory can be used in modeling real scientific and engineering problems. It is also possible to combine neutrosophic digraphs with other digraphs such as soft digraphs etc. to generate different hybrid graphical structure. Therefore the study of neutrosophic digraph theory and its properties have a considerable significance in the sense of applications as well as in understanding the fundamentals of uncertainty. This new topic is very sophisticated and it has immense possibilities which are to be explored.

Smarandache gave the idea of a neutrosophic set to deal with uncertain, incomplete, and inconsistent information that exist in real world. It has been seen that the neutrosophic set draws a special attraction to the researchers than classical set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, interval valued intuitionistic fuzzy set etc. simply because all these sets can be obtained from a neutrosophic set as special cases. In this paper we have developed the SVN digraph theory and studied some of its important properties and shown its application in solving multi-criteria decision making problem. In future, one may further study the deeper properties of SVN digraphs and may apply it in solving many real life problems which involves uncertainty.

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(α, β, γ)-Equalities of single valued neutrosophic sets

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Abstract: The single valued neutrosophic set (SVNS) is a subclass of neutrosophic set, which can describe and handle indeterminate information and inconsistent information. Since a SVNS is characterized independently by three functions: a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. This paper introduces (α, β, γ)-equalities of SVNS, which contains three parameters corresponding to three characteristic functions of SVNS. Then we show how various operations of single valued neutrosophic sets affect these three parameters.

Keywords: Neutrosophic set, Single valued neutrosophic set, (α, β, γ)-equality.

1 Introduction

Neutrosophic sets introduced by Smarandache [17] are the generalization of fuzzy sets [23] and intuitionistic fuzzy sets [3]. A neutrosophic set is characterized independently by three functions: a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. However, since these three functions are real standard or non-standard subsets of $]^{-}0, 1^{+}[$, it will be difficult to apply in real engineering fields [18]. Thus, Wang et.al [18] introduced the concept of single valued neutrosophic set (SVNS), which membership functions are the normal standard subsets of real unit interval $[0, 1]$. SVNS can deal with indeterminate and inconsistent information and therefore have been applied to many domains [9, 13, 14, 19, 20, 21].

Pappis [16] studied the value approximation of fuzzy systems variables. As a generalization of the work of Pappis, Hong and Hwang [10] discussed the value similarity of fuzzy system variables. Further, Cai introduced the so-called δ-equalities of fuzzy sets and applied them to discuss robustness of fuzzy reasoning. Georgescu [7, 8] generalized δ-equalities of fuzzy sets to (δ, H)-equality of fuzzy sets based on triangular norms. Dai et al. [6] and Jin et al. [11] discussed robustness of fuzzy reasoning based on (δ, H)-equality of fuzzy sets. Zhang et al. [22] studied the δ-equalities of complex fuzzy sets and applied the new concept in a signal processing application. Ngan and Ali [15] studied the δ-equalities of intuitionistic fuzzy sets and applied the new concept the application of medical diagnosis. Ali et al. [2] studied the δ-equalities of neutrosophic sets. Moreover, Ali and Smarandache [1] studied the δ-equalities of complex neutrosophic sets.

However, the concepts in [4, 5, 15, 22, 1, 2] are based on distance measures. Only one parameter is used to measure the degree of equality of fuzzy sets and their extensions. As we know, a SVNS is characterized independently by three functions. For example, from [2] we have $A = (0.2)B$ and $A = (0.2)C$ for $A \equiv (1, 0, 0)$, $B \equiv (1, 0, 0.8)$ and $C \equiv (0.2, 0.8, 0.8)$, i.e., B and C satisfy the same δ-equality with respect to A for $\delta = 0.2$. But B and C are quite different. Based on the above analysis, we find out that the only parameter given in [2] is a little rough to some extent. In view of this, it is more suitable to use three parameters to measure the degree of equality in these three functions respectively.

This paper investigates the concept of (α, β, γ)-equalities between single valued neutrosophic sets by following the work of Smarandache [17], Wang et.al [18] and Cai [4, 5]. Different from the distance based concepts in [1, 4, 5, 22], the new concept uses three parameters to measure the equality degree of three characteristic functions independently.

The rest of this paper is organized as follows: In section 2 ,we first briefly recall the concept of single valued neutrosophic set and its operations. In section 3, we introduce the concept of (α, β, γ)-equalities of single valued neutrosophic sets and its basic properties. Section 4 discusses (α, β, γ)-equalities with respect to operations of single valued neutrosophic sets. Finally, conclusions are stated in section 6.

2 Preliminaries

Definition 1. *[18] Suppose X is a universe containing all related objects. A SVNS A in X is characterized by three functions, i.e., a truth-membership function $T_A : X \to [0, 1]$, an indeterminacy-membership function $I_A : X \to [0, 1]$, and a falsity-membership function $F_A : X \to [0, 1]$. Then, a SVNS A can be defined as follows*

$$A = \{x, T_A(x), I_A(x), F_A(x)|x \in X\},$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ for each $x \in X$.

We use the notation $SVN(X)$ to denote the set of all single valued neutrosophic sets of X.

Suppose A and B are two single valued neutrosophic sets of X, then the following relations and operations are defined as follows [18, 21].

(i) $A \subseteq B$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x), \forall x \in X$.

(ii) $A = B$ if and only if $A \subseteq B, B \subseteq A$.

(iii) $A^c = \{x, F_A(x), 1 - I_A(x), T_A(x) | x \in X\}$.

(iv) $A \cup B = \{x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) | x \in X\}$.

(v) $A \cap B = \{x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) | x \in X\}$.

(vi) $A + B = \{x, T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) | x \in X\}$.

(vii) $A \times B = \{x, T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) | x \in X\}$.

(viii) $\lambda A = \{x, 1 - (1 - T_A(x))^\lambda, I_A^\lambda(x), F_A^\lambda(x) | x \in X\}$, $\lambda > 0$.

(ix) $A^\lambda = \{x, T_A^\lambda(x), 1 - (1 - I_A(x))^\lambda, 1 - (1 - F_A(x))^\lambda | x \in X\}$, $\lambda > 0$.

To facilitate future discussion, we review the following two lemmas.

Lemma 2. *[10] Let f and g be bounded, real valued functions on a set X. Then*

(i) $|\bigvee_{x\in X} f(x) - \bigvee_{x\in X} g(x)| \le \bigvee_{x\in X} |f(x) - g(x)|$,

(ii) $|\bigwedge_{x\in X} f(x) - \bigwedge_{x\in X} g(x)| \le \bigvee_{x\in X} |f(x) - g(x)|$.

Lemma 3. *[12] Let $a, b \in [0,1]$ and $\lambda > 0$. Then*

(i) *If $0 < \lambda \le 1$, then $|a^\lambda - b^\lambda| \le |a - b|^\lambda$;*

(ii) *If $\lambda \ge 1$, then $|a^\lambda - b^\lambda| \ge |a - b|^\lambda$.*

3 (α, β, γ)-equalities of single valued neutrosophic sets

Definition 4. *[2] Suppose A and B are two neutrosophic sets and $\delta \in [0,1]$, then A and B are said to be δ-equal, if and only if, the following properties hold*

$$\bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| \le 1 - \delta,$$
$$\bigvee_{x\in X} \big|I_A(x) - I_B(x)\big| \le 1 - \delta,$$
$$\bigvee_{x\in X} \big|F_A(x) - F_B(x)\big| \le 1 - \delta.$$

It is denoted by $A = (\delta)B$.

Definition 5. *Suppose A and B are two single valued neutrosophic sets and $\alpha, \beta, \gamma \in [0,1]$, then A and B are said to be (α, β, γ)-equal, if and only if, the following properties hold*

$$\bigvee_{x\in X} \big|T_A(x) - T_B(x)\big| \le 1 - \alpha, \tag{1}$$
$$\bigvee_{x\in X} \big|I_A(x) - I_B(x)\big| \le 1 - \beta, \tag{2}$$
$$\bigvee_{x\in X} \big|F_A(x) - F_B(x)\big| \le 1 - \gamma. \tag{3}$$

It is denoted by $A = (\alpha, \beta, \gamma)B$.

Remark 6.

(i) *In Definition 4, if two single valued neutrosophic sets A and B are 1-equal, then $A = B$ holds and vice versa, i.e., $A = (1)B$ iff $A = B$. However, when we consider the case $A = (\delta)B$ for $\delta \ne 1$. See the example in the Introduction section, let $A \equiv (1, 0, 0)$, $B \equiv (1, 0, 0.8)$ and $C \equiv (0.2, 0.8, 0.8)$, then it follows from [2] that B and C satisfy the same δ-equality with respect to A for $\delta = 0.2$. Note that B and C are quite different. Using Definition 5, we have $A = (1, 1, 0.2)B$, and $A = (0.2, 0.2, 0.2)C$. These are consistent with the fact that B is close to A while C is far from A.*

(ii) *The new concept is a generalization of the existing concepts in [2, 4, 15]. We note that $A = (\alpha, \beta, \gamma)B \Rightarrow A = (\delta)B$, where $\delta = \min(\alpha, \beta, \gamma)$. When A and B are two intuitionistic fuzzy sets, i.e, $T_A(x) + I_A(x) + F_A(x) = 1$ and $T_B(x) + I_B(x) + F_B(x) = 1$ for all $x \in X$, then it follows from [15] that A and B are δ-equal for $\delta = \min(\alpha, \gamma)$. When A and B are two fuzzy sets, i.e, $T_A(x) + F_A(x) = 1$ and $T_B(x) + F_B(x) = 1$ for all $x \in X$, then we have $\alpha = \gamma, \beta = 1$ from $A = (\alpha, \beta, \gamma)B$, it follows from [4] that A and B are δ-equal for $\delta = \alpha$.*

Example 7. *Let $X = \{x_1, x_2\}$ and two single valued neutrosophic sets defined as*

$$A = \{ < x_1, 0.1, 0.2, 0.9 >, < x_2, 0.1, 0.2, 1.0 > \},$$
$$B = \{ < x_1, 0.2, 0.2, 0.1 >, < x_2, 0.1, 0.1, 0.1 > \}.$$

It is easy to know that $A = (0.9, 0.9, 0.1)B$.

If we consider the degree of equality based on the single valued neutrosophic distance measure, we only obtain one value for the degree of equality between single valued neutrosophic sets. For instance, if we use the following distance of single valued neutrosophic sets

$$d(A, B) = \max\Big\{ \bigvee_{x\in X} |T_A(x) - T_B(x)|, \bigvee_{x\in X} |I_A(x) - I_B(x)|, \bigvee_{x\in X} |F_A(x) - F_B(x)| \Big\} \tag{4}$$

then we have $d(A,B) = 0.9 = 1 - 0.1$. However, 0.1 is not a rational estimation of degree of equality for truth-membership function and indeterminacy-membership function in this example. We note that $A = (\delta)B \Leftrightarrow d(A,B) \leq 1-\delta$. Based on an overall consideration of three characteristic functions, there parameters have been used accordingly.

And it is easy to know that $A = (\alpha,\beta,\gamma)B$ implies $d(A,B) \leq 1-\alpha\wedge\beta\wedge\gamma$.

Theorem 8. *Suppose A, B and C are single valued neutrosophic sets, then the following hold*

(i) $A = (0,0,0)B$;

(ii) $A = (1,1,1)B$ *if and only if* $A = B$;

(iii) $A = (\alpha,\beta,\gamma)B$ *if and only if* $B = (\alpha,\beta,\gamma)A$;

(iv) $A = (\alpha_1,\beta_1,\gamma_1)B$ *and* $\alpha_2 \leq \alpha_1$, $\beta_2 \leq \beta_1$ *and* $\gamma_2 \leq \gamma_1$, *then* $A = (\alpha_2,\beta_2,\gamma_2)B$;

(v) *If* $A = (\alpha_1,\beta_1,\gamma_1)B$ *and* $B = (\alpha_2,\beta_2,\gamma_2)C$, *then* $A = (\alpha_1 * \alpha_2, \beta_1 * \beta_2, \gamma_1 * \gamma_2)C$,

where $a * b = (a+b-1)\vee 0$ *for any* $a,b \in [0,1]$

Proof. Properties (i)(iv) can be proved easily. We only prove (v).

Since $A = (\alpha_1,\beta_1,\gamma_1)B$, then

$$\bigvee_{x\in X} \left|T_A(x) - T_B(x)\right| \leq 1-\alpha_1, \tag{5}$$

$$\bigvee_{x\in X} \left|I_A(x) - I_B(x)\right| \leq 1-\beta_1, \tag{6}$$

$$\bigvee_{x\in X} \left|F_A(x) - F_B(x)\right| \leq 1-\gamma_1. \tag{7}$$

From $B = (\alpha_2,\beta_2,\gamma_2)C$, we obtain

$$\bigvee_{x\in X} \left|T_B(x) - T_C(x)\right| \leq 1-\alpha_2, \tag{8}$$

$$\bigvee_{x\in X} \left|I_B(x) - I_C(x)\right| \leq 1-\beta_2, \tag{9}$$

$$\bigvee_{x\in X} \left|F_B(x) - F_C(x)\right| \leq 1-\gamma_2. \tag{10}$$

Then from (5) and (8),

$$\begin{aligned}
&\bigvee_{x\in X} \left|T_A(x) - T_C(x)\right| \\
=\ &\bigvee_{x\in X} \left|T_A(x) - T_B(x) + T_B(x) - T_C(x)\right| \\
\leq\ &\bigvee_{x\in X} \left|T_A(x) - T_B(x)\right| + \bigvee_{x\in X} \left|T_B(x) - T_C(x)\right| \\
\leq\ & 1-\alpha_1+1-\alpha_2 \\
=\ & 1-(\alpha_1+\alpha_2-1).
\end{aligned}$$

And from the definition 1, $1-(\alpha_1+\alpha_2-1) \in [0,1]$. Thus, $\bigvee_{x\in X} |T_A(x) - T_C(x)| \leq 1-\alpha_1 * \alpha_2$.

Similarly, we can get $\bigvee_{x\in X} \left|I_A(x) - I_C(x)\right| \leq 1-\beta_1 * \beta_2$ from (6) and (9), and $\bigvee_{x\in X} \left|F_A(x) - F_C(x)\right| \leq 1-\gamma_1 * \gamma_2$ from (7) and (10). Thus, $A = (\alpha_1 * \alpha_2, \beta_1 * \beta_2, \gamma_1 * \gamma_2)C$. □

4 (α,β,γ)-equalities with respect to operations

Theorem 9. *If $A = (\alpha,\beta,\gamma)B$, then $A^c = (\gamma,\beta,\alpha)B^c$.*

Proof. Since

$$\bigvee_{x\in X} \left|T_{A^c}(x) - T_{B^c}(x)\right| = \bigvee_{x\in X} \left|F_A(x) - F_B(x)\right| \leq 1-\gamma,$$
$$\bigvee_{x\in X} \left|F_{A^c}(x) - F_{B^c}(x)\right| = \bigvee_{x\in X} \left|T_A(x) - T_B(x)\right| \leq 1-\alpha,$$

and

$$\begin{aligned}
\bigvee_{x\in X} \left|I_{A^c}(x) - I_{B^c}(x)\right| &= \bigvee_{x\in X} \left|1 - I_A(x) - (1 - I_B(x))\right| \\
&= \bigvee_{x\in X} \left|I_A(x) - I_B(x)\right| \\
&\leq 1-\beta.
\end{aligned}$$

Then, $A^c = (\gamma,\beta,\alpha)B^c$. □

Remark 10. *In [2], we have $A = (\delta)B \Leftrightarrow A^c = (\delta)B^c$. However, by using Definition 5 we have $A = (\alpha,\beta,\gamma)B \Leftrightarrow A^c = (\gamma,\beta,\alpha)B^c$, where $(\alpha,\beta,\gamma) \neq (\gamma,\beta,\alpha)$. It is consistent with the fact that $A(x) = \big(T_A(x), I_A(x), F_A(x)\big) \Rightarrow A^c(x) = \big(F_A(x), I_A(x), T_A(x)\big)$.*

Example 11. *Let A, B be two single valued neutrosophic sets defined in Example 1, then*

$$A^c = \big\{ < x_1, 0.9, 0.8, 0.1 >, < x_2, 1.0, 0.8, 0.1 > \big\},$$
$$B^c = \big\{ < x_1, 0.1, 0.8, 0.2 >, < x_2, 0.1, 0.9, 0.1 > \big\}.$$

It is easy to know that $A^c = (0.1, 0.9, 0.9)B^c$, whereas $A = (0.9, 0.9, 0.1)B$.

However, if we use the distance defined in (4), we obtain $d(A^c, B^c) = d(A,B) = 0.9 = 1-0.1$. Thus we have $A = (0.1)B$ and $A^c = (0.1)B^c$ from Definition 4. This is difficult to know the changes of single valued neutrosophic sets by using the complement operation.

Theorem 12. *If $A_1 = (\alpha_1,\beta_1,\gamma_1)B_1$ and $A_2 = (\alpha_2,\beta_2,\gamma_2)B_2$, then*

$$A_1 \cup A_2 = (\alpha_1\wedge\alpha_2, \beta_1\wedge\beta_2, \gamma_1\wedge\gamma_2)B_1 \cup B_2, \tag{11}$$
$$A_1 \cap A_2 = (\alpha_1\wedge\alpha_2, \beta_1\wedge\beta_2, \gamma_1\wedge\gamma_2)B_1 \cap B_2, \tag{12}$$
$$A_1 + A_2 = (\alpha_1 * \alpha_2, \beta_1 * \beta_2, \gamma_1 * \gamma_2)B_1 + B_2, \tag{13}$$
$$A_1 \times A_2 = (\alpha_1 * \alpha_2, \beta_1 * \beta_2, \gamma_1 * \gamma_2)B_1 \times B_2. \tag{14}$$

Proof. We only give the proof of (11). From lemma 1, we obtain

$$\begin{aligned} & \bigvee_{x\in X} \big|T_{A_1\cup A_2}(x) - T_{B_1\cup B_2}(x)\big| \\ = \ & \bigvee_{x\in X} \big|T_{A_1}(x) \vee T_{A_2}(x) - T_{B_1}(x) \vee T_{B_2}(x)\big|, \\ \leq \ & \max\big\{ \bigvee_{x\in X} \big|T_{A_1}(x) - T_{B_1}(x)\big|, \bigvee_{x\in X} \big|T_{A_2}(x) - T_{B_2}(x)\big|\big\} \\ \leq \ & (1-\alpha_1) \vee (1-\alpha_2) \\ \leq \ & 1 - \alpha_1 \wedge \alpha_2. \end{aligned}$$

$$\begin{aligned} & \bigvee_{x\in X} \big|I_{A_1\cup A_2}(x) - I_{B_1\cup B_2}(x)\big| \\ = \ & \bigvee_{x\in X} \big|I_{A_1}(x) \wedge I_{A_2}(x) - I_{B_1}(x) \wedge I_{B_2}(x)\big| \\ \leq \ & \max\big\{ \bigvee_{x\in X} \big|I_{A_1}(x) - I_{B_1}(x)\big|, \bigvee_{x\in X} \big|I_{A_2}(x) - I_{B_2}(x)\big|\big\} \\ \leq \ & (1-\beta_1) \vee (1-\beta_2) \\ \leq \ & 1 - \beta_1 \wedge \beta_2. \end{aligned}$$

and

$$\begin{aligned} & \bigvee_{x\in X} \big|F_{A_1\cup A_2}(x) - F_{B_1\cup B_2}(x)\big| \\ = \ & \bigvee_{x\in X} \big|F_{A_1}(x) \wedge F_{A_2}(x) - F_{B_1}(x) \wedge F_{B_2}(x)\big| \\ \leq \ & \max\big\{ \bigvee_{x\in X} \big|F_{A_1}(x) - F_{B_1}(x)\big|, \bigvee_{x\in X} \big|F_{A_2}(x) - F_{B_2}(x)\big|\big\} \\ \leq \ & (1-\gamma_1) \vee (1-\gamma_2) \\ \leq \ & 1 - \gamma_1 \wedge \gamma_2. \end{aligned}$$

Thus, $A_1 \cup A_2 = (\alpha_1 \wedge \alpha_2, \beta_1 \wedge \beta_2, \gamma_1 \wedge \gamma_2)B_1 \cup B_2$. □

Corollary 13. *If $A_k = (\alpha_k, \beta_k, \gamma_k)B_k$ and $k = 1, 2, ..., n$, then*

$$\bigcup_{k=1}^{n} A_k = (\alpha, \beta, \gamma) \bigcup_{k=1}^{n} B_k, \tag{15}$$

$$\bigcap_{k=1}^{n} A_k = (\alpha, \beta, \gamma) \bigcap_{k=1}^{n} B_k, \tag{16}$$

$$\sum_{k=1}^{n} A_k = (\alpha', \beta', \gamma') \sum_{k=1}^{n} B_k, \tag{17}$$

$$\prod_{k=1}^{n} A_k = (\alpha', \beta', \gamma') \prod_{k=1}^{n} B_k, \tag{18}$$

where $\alpha = \bigwedge_{k-1}^{n} \alpha_k$, $\beta = \bigwedge_{k-1}^{n} \beta_k$, $\gamma = \bigwedge_{k-1}^{n} \gamma_k$, $\alpha' = \alpha_1 * \alpha_2 * \cdots * \alpha_n$, $\beta' = \beta_1 * \beta_2 * \cdots * \beta_n$ and $\gamma' = \gamma_1 * \gamma_2 * \cdots * \gamma_n$.

Proof. It can be proven from Theorem 12. □

Theorem 14. *Let A, B be two single valued neutrosophic sets, the following properties hold*

(i) *If $A = (\alpha, \beta, \gamma)B$ and $0 < \lambda \leq 1$, then*

$$\lambda A = (\alpha', \beta', \gamma')\lambda B, \tag{19}$$

$$A^{\lambda} = (\alpha', \beta', \gamma')B^{\lambda}, \tag{20}$$

where $\alpha' = 1-(1-\alpha)^{\lambda}$, $\beta' = 1-(1-\beta)^{\lambda}$ and $\gamma' = 1-(1-\gamma)^{\lambda}$.

(ii) *If $\lambda A = (\alpha, \beta, \gamma)\lambda B$ for some $\lambda \geq 1$, then*

$$A = (\alpha', \beta', \gamma')B, \tag{21}$$

where $\alpha' = 1 - (1-\alpha)^{1/\lambda}$, $\beta' = 1 - (1-\beta)^{1/\lambda}$ and $\gamma' = 1 - (1-\gamma)^{1/\lambda}$.

(iii) *If $A^{\lambda} = (\alpha, \beta, \gamma)B^{\lambda}$ for some $\lambda \geq 1$, then*

$$A = (\alpha', \beta', \gamma')B, \tag{22}$$

where $\alpha' = 1 - (1-\alpha)^{1/\lambda}$, $\beta' = 1 - (1-\beta)^{1/\lambda}$ and $\gamma' = 1 - (1-\gamma)^{1/\lambda}$.

Proof. We only give the proof of (i). From lemma 1 and lemma 2(i), we obtain

$$\begin{aligned} & \bigvee_{x\in X} \big|T_{\lambda A}(x) - T_{\lambda B}(x)\big| \\ = \ & \bigvee_{x\in X} \big|1 - (1-T_A(x))^{\lambda} - (1-(1-T_B(x))^{\lambda})\big| \\ = \ & \bigvee_{x\in X} \big|(1-T_A(x))^{\lambda} - (1-T_B(x))^{\lambda}\big| \\ \leq \ & \bigvee_{x\in X} \big|(1-T_A(x)) - (1-T_B(x))\big|^{\lambda} \\ = \ & \bigvee_{x\in X} \big|T_A(x) - T_B(x)\big|^{\lambda} \\ \leq \ & (1-\alpha)^{\lambda} = 1 - (1-(1-\alpha)^{\lambda}). \end{aligned}$$

$$\begin{aligned} & \bigvee_{x\in X} \big|I_{\lambda A}(x) - I_{\lambda B}(x)\big| \\ = \ & \bigvee_{x\in X} \big|I_A(x)^{\lambda} - I_B(x)^{\lambda}\big| \\ \leq \ & \bigvee_{x\in X} \big|I_A(x) - I_B(x)\big|^{\lambda} \\ \leq \ & (1-\beta)^{\lambda} = 1 - (1-(1-\beta)^{\lambda}). \end{aligned}$$

and

$$\begin{aligned}
& \bigvee_{x\in X} \left|F_{\lambda A}(x) - F_{\lambda B}(x)\right| \\
= & \bigvee_{x\in X} \left|F_A(x)^\lambda - F_B(x)^\lambda\right| \\
\leq & \bigvee_{x\in X} \left|F_A(x) - F_B(x)\right|^\lambda \\
\leq & (1-\gamma)^\lambda = 1 - (1 - (1-\gamma)^\lambda).
\end{aligned}$$

Thus, $\lambda A = (\alpha', \beta', \gamma')\lambda B$, where $\alpha' = 1 - (1-\alpha)^\lambda$, $\beta' = 1 - (1-\beta)^\lambda$ and $\gamma' = 1 - (1-\gamma)^\lambda$. □

5 Conclusions

Since a SVNS is characterized by three functions independently, this paper introduced (α, β, γ)-equalities corresponding to characteristic functions of SVNS. The new concept is more comprehensive than the traditional method based distance measure. Firstly, three parameters in the new concept can measure the degree of equality for different characteristic functions (See Example 1). Secondly, the new concept describe the changes of degree of equality with respect to operations more accurate and detailed (See Example 2). Thirdly, since $A = (\alpha, \beta, \gamma)B$ implies $d(A, B) \leq 1 - \alpha \wedge \beta \wedge \gamma$, we can obtain the traditional distance-based parameter by $\delta = \alpha \wedge \beta \wedge \gamma$.

As future work , we can consider the soundness of neutrosophic logic systems and the reliability of neutrosophic fault diagnosis.

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Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality

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Abstract The concept of neutrosophic can provide a generalization of fuzzy set and intuitionistic fuzzy set that make it is the best fit in representing indeterminacy and uncertainty. Single Valued Triangular Numbers (SVTrN-numbers) is a special case of neutrosophic set that can handle ill-known quantity very difficult problems. This work intended to introduce a framework with two types of ranking methods. The results indicated that each ranking method has its own advantage. In this perspective, the weighted value and ambiguity based method gives more attention to uncertainty in ranking and evaluating ISQ as well as it takes into account cut sets of SVTrN numbers that can reflect the information on Truth-membership-membership degree, false membership-membership degree and Indeterminacy-membership degree. The value index and ambiguity index method can reflect the decision maker's subjectivity attitude to the SVTrN- numbers.

Key words: Single Valued Triangular Neutrosophic Number (SVTrN), Single-Valued Trapezoidal Neutrosophic Number (SVTN number), Information Systems Quality (ISQ), Multi-Criteria Decision Making (MCDM).

1. Introduction

The neutrosophic concept became a key research topic. Neutrosophic theory involves philosophy viewpoint which addresses nature and scope of neutralities, as well as their interactions with different ideational spectra [9]. Neutrosophic includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic that it can be applied in many fields in order to solve problems related to indeterminacy [26, 23]. Neutrosophic not only considers the truth-membership and falsity- membership but also indeterminacy. Neutrosophic can provide is a generalization of classical set, fuzzy set and intuitionistic fuzzy set [22, 25, 23]. The neutrosophic set can handle many applications in information systems and decision support systems such as relational database systems, semantic web services, and financial data set detection [28]. Neutrosophic sets can represent inconsistent and incomplete information about real world problems [27, 24]. The neutrosophic set theory can be used to handle the uncertainty that related to ambiguity in a manner analogous to human thought [22]. In the neutrosophic set, the membership function independently indicates: Truth-membership-membership degree, false membership-membership degree, and Indeterminacy-membership degree. According to [24] neutrosophic set can exemplify ambiguous and conflicting information about real world. SVTrN-number is a special case of neutrosophic set that can handle ill-known quantity very difficult problem in Multi-Criteria Decision Making (MCDM) MCDM involves a process of solving the problem and achieving goals under asset of constraints, and it can be very difficult in some cases because of incomplete and imprecise information [1]. Also, in a MCDM problem the process of ranking alternatives with neutrosophic numbers is very difficult because neutrosophic numbers are not ranked by ordinary methods as real numbers. However, it is possible with score functions, aggregation operators, distance measures, and so on. Ye [14] introduced the notations of simplified neutrosophic sets and developed a ranking method. Then, he introduced some aggregation operators. Biswas et al. [35] developed a new approach for multi-attribute group decision making problems by extending the technique for order preference by similarity to ideal solution under single-valued neutrosophic environment. In [32] introduced combination of a neutrosophic set and a soft set that can be applied to problems that contain uncertainty. In [38] a new cross entropy measure under interval neutrosophic set (INS) environment was defined and can call IN-cross entropy measure and prove its basic properties. De and Das [20] developed a ranking method for trapezoidal intuitionistic fuzzy numbers and presented the values and ambiguities of the membership degree and the non-membership degree. Pramanik et al. [37] developed a new multi attribute group decision making (MAGDM) strategy for ranking of the alternatives based on the weighted SN-cross entropy measure between each alternative and the ideal alternative. Mitchell [2] proposed a ranking method to order triangular intuitionistic fuzzy numbers by accepting a statistical viewpoint and interpreting each

IFN as ensemble of ordinary fuzzy numbers. In [33] the notion of the interval valued neutrosophic soft set (ivn-soft sets) and generalized the concept of the soft set, fuzzy soft set, interval valued fuzzy soft set, intuitionistic fuzzy soft set, interval valued intuitionistic fuzzy soft set and neutrosophic soft set. Prakash et al [21] introduced a ranking method for both trapezoidal intuitionistic fuzzy numbers and triangular intuitionistic fuzzy numbers using the centroid concept and showed the proposed method is flexible and effective. Pramanik et al. [39] introduced new vector similarity measures of single valued and interval neutrosophic sets by hybridizing the concepts of Dice and cosine similarity measures and presented their applications in multi attribute decision making under neutrosophic environment. Peng et al [13] introduced the concept of multi-valued neutrosophic set, gave two multi-valued neutrosophic power aggregation operators. In [11, 29] the score based method can provide a simple method to rank the Single-Valued Trapezoidal Neutrosophic Number (SVTN number). Li [4] provides ratio ranking method for TIFNs and cut sets of intuitionistic trapezoidal fuzzy numbers. The existing methods of ranking fuzzy numbers and intuitionistic fuzzy number may be extended to SVN-numbers [10]. In [34] triangular fuzzy number neutrosophic weighted arithmetic averaging operator and triangular fuzzy number neutrosophic weighted geometric averaging operator are defined to aggregate triangular fuzzy number neutrosophic sets. Li et al. [5] introduced a ranking method of triangular intuitionistic fuzzy numbers and defined the notation of cut sets of intuitionistic fuzzy numbers and their values and ambiguities of membership and non-membership functions. The main advantage of this method that it pays more attention to the impact of uncertainty and takes into account θ-weighted value of intuitionistic fuzzy numbers by using the concepts of cut sets of intuitionistic fuzzy numbers. Biswas et al. [36] developed a ranking method based on value and ambiguity index based of single-valued trapezoidal neutrosophic numbers. According to [3] there are many ranking methods. However, there is no unique best method exists. This paper intended to introduce a framework with two types of ranking methods. This paper is organized as the follows: the first section presents the introduction for this work; the second section provides basic definitions; the third section describes the proposed framework with two ranking methods of SVTrN-numbers with the scale based approach for evaluating ISQ; the fourth section describes a case study; the fifth section gives conclusion and future work; the final section provides references.

2. Basic Definitions

Fuzzy theory is an important and interesting research topic in decision-making theory and science. However, fuzzy set is characterized only by its membership function between 0 and 1, but not a non-membership function [12]. To overcome the insufficient of fuzzy set, Atanassov [19] extended fuzzy set and introduced intuitionistic fuzzy set by adding an additional non-membership degree, which may express more flexible information as compared with the fuzzy set. Intuitionistic fuzzy set can be defined as the follows:

Definition 1. According to [18], let E be a universe. An intuitionistic fuzzy set K over E is defined by: K = {<x, $\mu_k(x)$, $\gamma_k(x)$ >: x ∈ E} where μ_k: E [0, 1] and γ_k : E [0, 1] such that 0≤, $\mu_k(x) + \gamma_k(x) \geq 1$ for any x ∈ E. For each x ∈ E, the values, $\mu_k(x)$ and $\gamma_k(x)$ are degree of membership function and non-membership function of x, respectively.

Smarandache [7] introduced the concept of neutrosophic set, which is differentiated by truth-membership function, indeterminacy-membership function and falsity membership function. The concept of neutrosophic set came from a philosophical point of view to express indeterminate and inconsistent information Neutrosophic set can be defined as the follows:

Definition 2. . According to [8], let E be a universe. Neutrosophic sets A over E is defined by: A = {<x, ($T_A(x)$, $I_A(x)$, $F_A(x)$) >: x∈ E} where $T_A(x)$, $I_A(x)$, and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity membership function, respectively. They are respectively defined by T_A: E]-0, 1+[, I_A : E]-0, 1+[, F_A : E]-0, 1+[Such that. 0≤- ($T_A(x) + I_A(x) + F_A(x)$) ≥3+

2.1. Single Valued Triangular Neutrosophic Numbers

Single valued triangular neutrosophic numbers (SVTrN-numbers) is a special case of neutrosophic set that can handle ill-known quantity very difficult problem in multi-attribute decision making and ranking. SVTrN-numbers is suitable for the expression of incomplete, indeterminate, and inconsistent information in actual applications. Specially, it has been widely applied in many areas [16]. According to [31] the SVTrN-number ā can be defined as the follows:

Definition 3. As [31] [10] pointed out, Let ā = ((a, b, c); $w_{\bar{a}}$, $u_{\bar{a}}$,$y_{\bar{a}}$) where is ā SVTrN-number whose truth-membership, indeterminacy-membership and falsity-membership functions can be respectively defined by :

$$\mu_{\bar{a}}(x) = \begin{cases} \frac{(x-a)w_{\bar{a}}}{b-a}, & a \leq x < b \\ \frac{(c-x)w_{\bar{a}}}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

$$\upsilon_{\bar{a}}(x) = \begin{cases} \frac{(b-x+u_{\bar{a}}(x-a))}{b-a}, & a \leq x < b \\ \frac{(x-b+u_{\bar{a}}(c-x))}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

$$\lambda_{\bar{a}}(x) = \begin{cases} \frac{(b-x+y_{\bar{a}}\ (x-a))}{b-a}, & a \le x < b \\ \frac{(x-b+y_{\bar{a}}\ (c-x))}{c-b}, & b \le x \le c \\ 0, & \text{otherwise} \end{cases} \qquad (2.3)$$

If $a \geq 0$ and at least $c>0$, then $\bar{a} = ((a, b, c); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$ is called a positive SVTrN-number, denoted by $\bar{a}>0$. Likewise, If $a \leqslant 0$ and at least $c<0$, $\bar{a} = ((a, b, c); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$ is called a negative SVTrN-number, denoted by $\bar{a}<0$.

Definition 4. According to [31] let $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$, $\bar{e} = ((a_2, b_2, c_2); w_{\bar{e}}, u_{\bar{e}}, y_{\bar{e}})$ be two SVTrN-numbers and $\gamma \neq 0$ 0 be any real number, then

$\bar{a} + \bar{e} = ((a_1+ a_2, b_1+ b_2, c_1+ c_2); \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\})$ (2.4)

$\bar{a}\bar{e}=$

$$\begin{cases} ((a_1a_2, b_1b_2, c_1c_2), \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) & (c_1 > 0, c_2 > 0) \\ ((a_1c_2, b_1b_2, c_1a_2), \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) & (c_1 < 0, c_2 > 0) \\ ((c_1c_2, b_1b_2, a_1a_2), \min\{w_{\bar{a}}, w_{\bar{e}}\}, \max\{u_{\bar{a}}, u_{\bar{e}}\}, \max\{y_{\bar{a}}, y_{\bar{e}}\}) & (c_1 < 0, c_2 < 0) \end{cases}$$

(2.5)

$$\gamma\bar{a} = \begin{cases} ((\gamma a_1, \gamma\, b_1, \gamma c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}}) & (\gamma > 0) \\ ((\gamma c_1, \gamma\, b_1, \gamma a_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}}) & (\gamma < 0) \end{cases}$$

(2.6)

3.1.1 Concepts of Values and Ambiguities for SVTrN-Numbers

Concept of cut (or level) sets, values, ambiguities, weighted values and weighted ambiguities of SVTrN-numbers have desired properties and can reflect information on membership degrees and non-membership degrees.

Definition 5. As [10] [4] pointed out, let $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$ is an arbitrary SVTrN-number. Then,

(1) α-cut set of the SVTrN-number $\bar{a}$ for truth-membership is calculated as:

$[L_{\bar{a}}(\alpha), R_{\bar{a}}(\alpha)] = [((w_{\bar{a}} - \alpha) a+ \alpha b)/w_{\bar{a}}, ((w_{\bar{a}}-\alpha) c + \alpha b)/w_{\bar{a}}]$

If $f(\alpha)=\alpha$, where $f(\alpha) \in [0, 1]$ and $f(\alpha)$ is monotonic and non-decreasing of $\alpha \in [0, w_{\bar{a}}]$, the value and ambiguity of the SVTrN-number $\bar{a}$ can be calculated as:

$$V_\mu(\bar{a}) = \int_0^{w_{\bar{a}}} [(a+c) + \frac{(2b-a-c)\alpha}{w_{\bar{a}}}]\,\alpha\, d\alpha$$

$$=[\frac{(a+c)]\alpha^2}{2} + \frac{(2b-a-c)\alpha^3}{3w_{\bar{a}}}]|_0^{w_{\bar{a}}}$$

$$= \frac{(a+4b+c)\,(w_{\bar{a}})^2}{6} \qquad (2.7)$$

And

$$A_\mu(\bar{a}) = \int_0^{w_{\bar{a}}} [(c-a) - \frac{(c-a)\alpha}{w_{\bar{a}}}]\,\alpha\, d\alpha$$

$$= [\frac{(c-a)\alpha^2}{2} - \frac{(c-a)\alpha^3}{3w_{\bar{a}}}]$$

$$= \frac{(c-a)\,(w_{\bar{a}})^2}{6} \qquad (2.8)$$

(2) β-cut set of the SVTrN-number $\bar{a}$ for indeterminacy membership is calculated as;

$[\acute{L}_{\bar{a}}(\beta), \acute{R}_{\bar{a}}(\beta)] = [((1-\beta)b + (\beta-u_{\bar{a}})a)/(1-u_{\bar{a}}), ((1-\beta)b+(\beta-u_{\bar{a}})c)/(1-u_{\bar{a}})]$

If $g(\beta)=1-\beta$, where $g(\beta) \in [0, 1]$ and $g(\beta)$ is monotonic and non-increasing of $\beta \in [u_{\bar{a}},1]$, the value and ambiguity of the SVTrN-number $\bar{a}$ can be calculated, respectively, as the follows:

$$V_\upsilon(\bar{a}) = \int_{u_{\bar{a}}}^1 [(a+c) + \frac{(2b-a-c)(1-\beta)}{1-u_{\bar{a}}}](1-\beta)\, d\beta$$

$$= [-\frac{(a+c)\,(1-\beta)^2}{2} \quad \frac{(2b-a-c)(1-\beta)^3}{3(1-u_{\bar{a}})}]\,|_{u_{\bar{a}}}^1$$

$$= \frac{(a+4b+c)\,(1-u_{\bar{a}})^2}{6} \qquad (2.9)$$

And

$$A_\upsilon(\bar{a}) = \int_{u_{\bar{a}}}^1 [(c-a) - \frac{(c-a)(1-\beta)}{1-u_{\bar{a}}}](1-\beta)\, d\beta$$

$$= [-\frac{(c-a)(1-\beta)^2}{2} + \frac{(c-a)(1-\beta)^3}{3(1-u_{\bar{a}})}]|_{u_{\bar{a}}}^1$$

$$= \frac{(c-a)\,(1-u_{\bar{a}})^2}{6} \qquad (2.10)$$

(3) γ-cut set of the SVTrN-number a for falsity-membership is calculated as:

$[\acute{L}'_{\bar{a}}(\gamma), \acute{R}'_{\bar{a}}(\gamma)] = [((1-\gamma)b+(\gamma-y_{\bar{a}})a)/(1-y_{\bar{a}})),((1-\gamma)b+(\gamma-y_{\bar{a}})c)/(1-y_{\bar{a}})]$

If $h(\gamma)=1-\gamma$, where $h(\gamma) \in [0, 1]$ and $h(\gamma)$ is monotonic and non-increasing of $\gamma \in [y_{\bar{a}},1]$, the value and ambiguity of the SVTrN-number $\bar{a}$, respectively, as;

$$V_\lambda(\bar{a}) = \int_{y_{\bar{a}}}^1 [(a+c) + \frac{(2b-a-c)(1-\gamma)}{1-y_{\bar{a}}}](1-\gamma)\, d\gamma$$

$$= [-\frac{(a+c)\,(1-\gamma)^2}{2} - \frac{(2b-a-c)(1-\gamma)^3}{3(1-y_{\bar{a}})})]\,|_{y_{\bar{a}}}^1$$

$$= \frac{(a+4b+c)\,(1-y_{\bar{a}})^2}{6} \qquad (2.11)$$

And

$$A_\lambda(\bar{a}) = \int_{y_{\bar{a}}}^1 [(c-a) - \frac{(c-a)(1-\gamma)}{(1-y_{\bar{a}})})](1-\gamma)\, d\gamma$$

$$= [-\frac{(c-a)(1-\gamma)^2}{2} - \frac{(c-a)(1-\gamma)^3}{3(1-y_{\bar{a}})}]$$

$$= \frac{(c-a)\ (1-y_{\bar{a}})^2}{6} \quad (2.12)$$

The function f(α) gives different weights to elements at different α -cut sets and these cut sets come from values of $\mu_{\bar{a}}(x)$ which have a considerable amount of uncertainty. Therefore, $V_\mu(\bar{a})$ can reflect the information on membership degrees. Also, g(β) can lessen the contribution of the higher β -cut sets come from values of $\upsilon_{\bar{a}}(x)$ which have a considerable amount of uncertainty. Therefore, $V_\upsilon(\bar{a})$ can reflect the information on non-membership degrees. Likewise, $V_\lambda(\bar{a})$ can reflect the information on non-membership degrees.

3.1.2 The Weighted Values and Ambiguities of the SVTrN-numbers

The weighted values of the SVTrN-numbers can be calculated as follows:

Definition 6. According to [10] let $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$ be a SVTrN-number. Then, for θ∈ [0, 1], the θ -weighted value of the SVTrN-number ā can be defined as:

$$V_\theta(\bar{a}) = (a + 4b + c)/6\ [\theta w_{\bar{a}}^2 + (1-\theta)(1-u_{\bar{a}})^2 + (1-\theta)(1-y_{\bar{a}})^2] \quad (2.13)$$

The θ - weighted ambiguity of SVTrN-number a are defined as:

$$A_\theta(\bar{a}) = (c-a)/6\ [\theta w_{\bar{a}}^2 + (1-\theta)(1-u_{\bar{a}})^2 + (1-\theta)(1-y_{\bar{a}})^2] \quad (2.14)$$

Definition 7. Let $\bar{a} = ((a_1, b_1, c_1); w_{\bar{a}}, u_{\bar{a}}, y_{\bar{a}})$ be a SVTrN-number. Based on [10]; [20] [4] the values index and ambiguities index can generalized to the SVTrN-numbers and they can be respectively calculated for $\lambda \in [0, 1]$ as follows:

$$V(\bar{a}, \lambda) = (a+4b+c)/6\ [\lambda w_{\bar{a}}^2 + (1-\lambda)(1-u_{\bar{a}})^2 + (1-\lambda)(1-y_{\bar{a}})^2] \quad (2.15)$$

$$= V_\mu(\bar{a})\lambda + V_\upsilon(\bar{a})(1-\lambda) + V_\lambda(\bar{a})(1-\lambda) \quad (2.16)$$

And

$$A(\bar{a}, \lambda) = (c-a)/6\ [\lambda w_{\bar{a}}^2 + (1-\lambda)(1-u_{\bar{a}})^2 + (1-\lambda)(1-y_{\bar{a}})^2] \quad (2.17)$$

$$= A_\mu(\bar{a})\lambda + A_\upsilon(\bar{a})(1-\lambda) + A_\lambda(\bar{a})(1-\lambda) \quad (2.18)$$

Where λ ∈ [0, 1] and λ is a weight which represents the decision maker's preference information. λ ∈ [0,1/2] shows that the decision maker prefers pessimistic or negative feeling; λ ∈ [1/2,1] shows that the decision maker prefers optimistic or positive feeling; λ = 1/2 shows that the decision maker is indifferent between positive feeling and negative feeling.

$$V(\bar{a}, 1/2) = V_\mu(\bar{a})\ 1/2 + V_\upsilon(\bar{a})\ (1-1/2) + V_\lambda(\bar{a})\ (1-1/2)$$

$$= V_\mu(\bar{a})\ 1/2 + V_\upsilon(\bar{a})\ 1/2 + V_\lambda(\bar{a})\ 1/2$$

$$= ½(V_\mu(\bar{a}) + V_\upsilon(\bar{a}) + V_\lambda(\bar{a})) \quad (2.19)$$

And

$$A(\bar{a}, 1/2) = A_\mu(\bar{a})\ 1/2 + A_\upsilon(\bar{a})\ )(1-1/2\ ) + A_\lambda(\bar{a})(1-1/2\ )$$

$$= A_\mu(\bar{a})\ 1/2 + A_\upsilon(\bar{a})\ 1/2 + A_\lambda(\bar{a})\ 1/2$$

$$= ½\ (A_\mu(\bar{a}) + A_\upsilon(\bar{a}) + A_\lambda(\bar{a})) \quad (2.20)$$

Definition 8. Let ā and ē be two SVTrN-numbers and θ∈ [0, 1]. For weighted values and ambiguities of the SVTrN-numbers ā and ē, the ranking order of ā and ē can be defined as;

(1) If $V_\theta(\bar{a}) > V_\theta(\bar{e})$, then ā is bigger than ē
(2) If $V_\theta(\bar{a}) < V_\theta(\bar{e})$, then ā is smaller than ē
(3) If $V_\theta(\bar{a}) = V_\theta(\bar{e})$, then
 (i) If $A_\theta(\bar{a}) = A_\theta(\bar{e})$, then then ā is equal to ē
 (ii) If $A_\theta(\bar{a}) > A_\theta(\bar{e})$, then ā is bigger than ē
 (iii) If $A_\theta(\bar{a}) < A_\theta(\bar{e})$, then ā is smaller than ē

3. The Proposed Framework with Two Ranking Methods for Evaluating Information Systems Quality

The proposed framework aims to introduce the scale based approach with SVTrN-numbers for evaluating ISQ. The proposed framework consists of four phases as the follows:

Phase 1: Using Single Valued Triangular Neutrosophic Numbers with scale based approach

The first phase aims to enable the IS evaluator to give every quality attribute one of the scale categories. The scale ranging is designed from 0 to 1 on which the value of every attribute needs to be marked. The scale is divided into categories: Low, Not low, Very low, Completely low, More or less low, Fairly low, Essentially low, Neither low nor high, High, Not high, Very high, Completely high, More or less high, Fairly high, Essentially high, having corresponding values ((4.6; 5.5; 8.6); 0.4; 0.7; 0.2), ((4.7; 6.9; 8.5); 0.7; 0.2; 0.6), ((6.2; 7.6; 8.2); 0.4; 0.1; 0.3), ((7.1; 7.7; 8.3); 0.5; 0.2; 0.4), ((5.8; 6.9; 8.5); 0.6; 0.2; 0.3), ((5.5; 6.2; 7.3); 0.8; 0.1; 0.2), ((5.3; 6.7; 9.9); 0.3; 0.5; 0.2), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((4.4; 5.9; 7.2); 0.7; 0.2; 0.3), ((6.6; 8.8; 10); 0.6; 0.2; 0.2), ((6.3; 7.5; 8.9); 0.7; 0.4; 0.6), ((5.3; 7.3; 8.7); 0.7; 0.2; 0.8), ((6.5; 6.9; 8.5); 0.6; 0.8; 0.1), ((7.5; 7.9; 8.5); 0.8; 0.5; 0.4). The user according to his/her evaluation of every quality attribute (in table 1) gives them one of the 15 defined values.

Phase 2: Construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker

The second phase aims to construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker as the follows: Let Q= (q_1, q_2… q_n) a set of information systems. C= (c_1, c_2… c_m) be ISQ criteria, and let $[A_{ij}] = ((a_{ij}, b_{ij}, c_{ij}); w_{\bar{a}ij}$

$,u_{\bar{a}ij}$ $,y_{\bar{a}ij})$ ($i \in I_m$ for ISQ criteria , $j \in I_n$ information systems) be a SVTrN-number. Then decision matrix can be identified as the follows:

$$[A_{ij}]_{m*n} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{pmatrix}$$

Phase 3: Calculate the Comprehensive Values

At the first, Compute the normalized decision-making matrix $R= [r_{ij}]_{m*n}$ and compute $U= [u_{ij}]_{m*n}$ as the follows:

- Compute the normalized decision-making matrix $R= [r_{ij}]_{m*n}$ where

$R_{ij}= ((a_{ij}/\bar{a}^+, b_{ij}/\bar{a}^+, c_{ij}/\bar{a}^+); w_{\bar{a}ij}, u_{\bar{a}ij}, y_{\bar{a}ij})$

Such that $\bar{a}^+= \max \{c_{ij}.\ i \in I_m, j \in I_n\}$

- Compute $U= [u_{ij}]_{m*n}$ of R. Where, $u_{ij}= \omega_i r_{ij}$ ($i \in I_m$ for ISQ criteria , $j \in I_n$ information systems),

$\omega = (\omega_1, \omega_2 \dots \omega_m)$ be the weight vector of ISQ criteria, where $\omega_i \in [0, 1]$, $i \in I_m$ and $\sum_{i=1}^{m} \omega_i = 1$

Then, calculate the comprehensive values S_j as:

$$S_j = \sum_{i=1}^{m} u_{ij} = ((\sum_{i=1}^{m} \omega_i r_{ij}, \sum_{i=1}^{m} \omega_i r_{ij}, \sum_{i=1}^{m} \omega_i r_{ij}); \text{Min } w_{\bar{a}ij}, \text{Max } u_{\bar{a}ij}, \text{Max } y_{\bar{a}ij}) \quad (j \in I_n) \qquad (3.1)$$

Phase 4: Evaluate and Rank ISQ

This phase aims to introduce two evaluating and ranking methods: (1) - weighted value and ambiguity based method, (2) the value index and ambiguity index method to give more than one option for evaluating and ranking ISQ.

(1)- Weighted value and ambiguity method

Firstly, calculate the value of truth-membership-membership degree, and indeterminacy-membership, and falsity-membership degree for each comprehensive value based on "Eq. (2.7)" "Eq. (2.9)"and "Eq. (2.11)", respectively, as the follows:

$$V_\mu (S_j) = ((a + 4b + c) (w_{sj})^2)/6 \qquad (3.2)$$

$$V_\upsilon (S_j) = ((a + 4b + c) (1-u_{sj})^2)/6 \qquad (3.3)$$

$$V_\lambda (S_j) = ((a + 4b + c) (1-y_{sj})^2)/6 \qquad (3.4)$$

And, calculate the ambiguity of truth-membership-membership degree, and indeterminacy-membership, and falsity-membership degree for each comprehensive value based on "Eq. (2.8)" "Eq. (2.10)"and "Eq. (2.12)", respectively, as the follows:

$$A_\mu (S_j) = ((c-a) (w_{sj})^2)/6 \qquad (3.5)$$

$$A_\upsilon (S_j) = ((c-a) (1-u_{sj})^2)/6 \qquad (4.6)$$

$$A_\lambda (S_j) = ((c-a) (1-y_{sj})^2)/6 \qquad (3.7)$$

Secondly, calculate the weighted values (θ - weighted value) for each alternative as the follows:

the θ -weighted value of each comprehensive value S_j is defined as:

$$V_\theta (S_j) = V_\mu (S_j)\, \theta + V_\upsilon (S_j)(1- \theta) + V_\lambda (S_j) (1- \theta) \qquad (3.8)$$

The θ - weighted ambiguity of a comprehensive value S_j can be defined as:

$$A_\theta (S_j) = (c-a) /6\ [\theta w_j^2 + (1-\theta) (1-u_{sj})^2 + (1-\theta) (1-y_{sj})^2] \qquad (3.9)$$

$$= A_\mu (S_j)\, \theta + A_\upsilon (S_j) +(1- \theta)\ A_\lambda (S_j) (1- \theta) \qquad (3.10)$$

4. Case study

An IS evaluation committee wants to evaluate quality of three IS centers at three universities according eight quality characteristics based ISO/IEC 25010: C= (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8) be quality characteristics: functionality c_1, reliability c_2, usability c_3, efficiency c_4, maintainability c_5, portability c_6, security c_7, compatibility c_8. The weight vector of the eight quality characteristics is ω = (.25, .25, .30, .20, .25, .20, .20, and .15).

Phase I: Using Single Valued Triangular Neutrosophic Numbers with scale based approach

Apply the scale based approach to enable the IS evaluator to give every quality attribute one of the following categories: Low, Not low, Very low, Completely low, More or less low, Fairly low, Essentially low, Neither low nor high, High, Not high, Very high, Completely high, More or less high, Fairly high, Essentially high, having corresponding values ((4.6; 5.5; 8.6); 0.4; 0.7; 0.2), ((4.7; 6.9; 8.5); 0.7; 0.2; 0.6), ((6.2; 7.6; 8.2); 0.4; 0.1; 0.3), ((7.1; 7.7; 8.3); 0.5; 0.2; 0.4), ((5.8; 6.9; 8.5); 0.6; 0.2; 0.3), ((5.5; 6.2; 7.3); 0.8; 0.1; 0.2), ((5.3; 6.7; 9.9); 0.3; 0.5; 0.2), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((6.2; 8.9; 9.1); 0.6; 0.3; 0.5), ((4.4; 5.9; 7.2); 0.7; 0.2; 0.3), ((6.6; 8.8; 10); 0.6; 0.2; 0.2), ((6.3; 7.5; 8.9); 0.7; 0.4; 0.6), ((5.3; 7.3; 8.7); 0.7; 0.2; 0.8), ((6.5; 6.9; 8.5); 0.6; 0.8; 0.1), ((7.5; 7.9; 8.5); 0.8; 0.5; 0.4). The quality attributes of the three information systems can be presented based on the scale based approach as the follows:

- **The first information system**

The following table represents the quality attributes of the first information system based on the scale based approach.

Table (1): The quality attributes of the first information system

ISQ characteristics	Low	Not low	Very low	Completely low	More or less low	Fairly low	Essentially low	Neither low nor high	High	Not high	Very high	Completely high	More or less high	Fairly high	Essentially high	Linguistic values
C1	√															((4.6; 5.5; 8.6); 0.4; 0.7; 0.2)
C2			√													((6.2; 7.6; 8.2); 0.4; 0.1; 0.3)
C3								√								((6.2; 8.9; 9.1); 0.6; 0.3; 0.5)
C4				√												((7.1; 7.7; 8.3); 0.5; 0.2; 0.4)
C5										√						((4.4; 5.9; 7.2); 0.7; 0.2; 0.3)
C6					√											((5.8; 6.9; 8.5); 0.6; 0.2; 0.3)
C7			√													((6.2; 7.6; 8.2); 0.4; 0.1; 0.3)
C8			√													((6.2; 7.6; 8.2); 0.4; 0.1; 0.3)

- **The second information system**

The following table represents the quality attributes of the second information system based on the scale based approach.

Table (2): The quality attributes of the second information system

ISQ characteristics	Low	Not low	Very low	Completely low	More or less low	Fairly low	Essentially low	Neither low nor high	High	Not high	Very high	Completely high	More or less high	Fairly high	Essentially high	Linguistic values
C1													√			((5.3; 7.3; 8.7); 0.7; 0.2; 0.8)
C2									√							((6.2; 8.9; 9.1); 0.6; 0.3; 0.5)
C3														√		((6.5; 6.9; 8.5); 0.6; 0.8; 0.1)
C4													√			((5.3; 7.3; 8.7); 0.7; 0.2; 0.8)
C5									√							((6.2; 8.9; 9.1); 0.6; 0.3; 0.5)
C6										√						((4.4; 5.9; 7.2); 0.7; 0.2; 0.3)
C7											√					((6.6; 8.8; 10); 0.6; 0.2; 0.2)
C8		√														((4.7; 6.9; 8.5); 0.7; 0.2; 0.6)

- **The third information system**

The following table represents the quality attributes of the third information system based on the scale based approach.

Table (3): The quality attributes of the third information system

ISQ characteristics	Low	Not low	Very low	Completely low	More or less low	Fairly low	Essentially low	Neither low nor high	High	Not high	Very high	Completely high	More or less high	Fairly high	Essentially high	Linguistic values
C1															√	((7.5; 7.9; 8.5); 0.8; 0.5; 0.4)
C2									√							((6.2; 8.9; 9.1); 0.6; 0.3; 0.5)
C3											√					((6.6; 8.8; 10); 0.6; 0.2; 0.2)
C4												√				((6.3; 7.5; 8.9); 0.7; 0.4; 0.6)
C5													√			((5.3; 7.3; 8.7); 0.7; 0.2; 0.8)
C6										√						((4.4; 5.9; 7.2); 0.7; 0.2; 0.3)
C7														√		((6.5; 6.9; 8.5); 0.6; 0.8; 0.1)
C8											√					((6.6; 8.8; 10); 0.6; 0.2; 0.2)

Phase 2: Construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker

Let Q= (q_1, q_2, q_3) be a set of the three IS. C= (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8) be ISQ criteria: functionality c_1, reliability c_2, usability c_3, efficiency c_4, maintainability c_5, portability c_6, security c_7, compatibility c_8. Let A= $[A_{ij}]_{8*3}$ = ((a_{ij}, b_{ij}, c_{ij}); $w_{\bar{a}ij}$,$u_{\bar{a}ij}$,$y_{\bar{a}ij}$) (i∈ I_8 for ISQ criteria, j ∈ I_3 the three information systems) be a SVTrN-numbers. Then

$$\begin{bmatrix} ((4.6,5.5,8.6);0.4,0.7,0.2) & ((5.3,7.3,8.7);0.7,0.2,0.8) & ((7.5,7.9,8.5);0.8,0.5,0.4) \\ ((6.2,7.6,8.2);0.4,0.1,0.3) & ((6.2,8.9,9.1);0.6,0.3,0.5) & ((6.2,8.9,9.1);0.6,0.3,0.5) \\ ((6.2,8.9,9.1);0.6,0.3,0.5) & ((6.5,6.9,8.5);0.6,0.8,0.1) & ((6.6,8.8,10);0.6,0.2,0.2) \\ ((7.1,7.7,8.3);0.5,0.2,0.4) & ((5.3,7.3,8.7);0.7,0.2,0.8) & ((6.3,7.5,8.9);0.7,0.4,0.6) \\ ((4.4,5.9,7.2);0.7,0.2,0.3) & ((6.2,8.9,9.1);0.6,0.3,0.5) & ((5.3,7.3,8.7);0.7,0.2,0.8) \\ ((5.8,6.9,8.5);0.6,0.2,0.3) & ((4.4,5.9,7.2);0.7,0.2,0.3) & ((4.4,5.9,7.2);0.7,0.2,0.3) \\ ((6.2,7.6,8.2);0.4,0.1,0.3) & ((6.6,8.8,10);0.6,0.2,0.2) & ((6.5,6.9,8.5);0.6,0.8,0.1) \\ ((6.2,7.6,8.2);0.4,0.1,0.3) & ((4.7,6.9,8.5);0.7,0.2,0.6) & ((6.6,8.8,10);0.6,0.2,0.2) \end{bmatrix}$$

Phase 2: Construct the SVTrN-Multi-Criteria Decision Matrix of Decision Maker

Let Q= (q_1, q_2, q_3) be a set of the three IS. C= (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8) be ISQ criteria: functionality c_1, reliability c_2, usability c_3, efficiency c_4, maintainability c_5, portability c_6, security c_7, compatibility c_8. Let A= $[A_{ij}]_{8*3}$ = ((a_{ij}, b_{ij}, c_{ij}); $w_{\bar{a}ij}$,$u_{\bar{a}ij}$,$y_{\bar{a}ij}$) (i∈ I_8 for ISQ criteria, j ∈ I_3 the three information systems) be a SVTrN-numbers. Then

$$\begin{bmatrix} ((4.6,5.5,8.6);0.4,0.7,0.2) & ((5.3,7.3,8.7);0.7,0.2,0.8) & ((7.5,7.9,8.5);0.8,0.5,0.4) \\ ((6.2,7.6,8.2);0.4,0.1,0.3) & ((6.2,8.9,9.1);0.6,0.3,0.5) & ((6.2,8.9,9.1);0.6,0.3,0.5) \\ ((6.2,8.9,9.1);0.6,0.3,0.5) & ((6.5,6.9,8.5);0.6,0.8,0.1) & ((6.6,8.8,10);0.6,0.2,0.2) \\ ((7.1,7.7,8.3);0.5,0.2,0.4) & ((5.3,7.3,8.7);0.7,0.2,0.8) & ((6.3,7.5,8.9);0.7,0.4,0.6) \\ ((4.4,5.9,7.2);0.7,0.2,0.3) & ((6.2,8.9,9.1);0.6,0.3,0.5) & ((5.3,7.3,8.7);0.7,0.2,0.8) \\ ((5.8,6.9,8.5);0.6,0.2,0.3) & ((4.4,5.9,7.2);0.7,0.2,0.3) & ((4.4,5.9,7.2);0.7,0.2,0.3) \\ ((6.2,7.6,8.2);0.4,0.1,0.3) & ((6.6,8.8,10);0.6,0.2,0.2) & ((6.5,6.9,8.5);0.6,0.8,0.1) \\ ((6.2,7.6,8.2);0.4,0.1,0.3) & ((4.7,6.9,8.5);0.7,0.2,0.6) & ((6.6,8.8,10);0.6,0.2,0.2) \end{bmatrix}$$

Phase 3: Calculate the Comprehensive Values

Before calculating the comprehensive values, Compute the normalized decision-making matrix R= $[r_{ij}]_{8*3}$ and compute U= $[u_{ij}]_{8*3}$ as the follows:

Compute the normalized decision-making matrix R= $[r_{ij}]_{m*n}$ where

R= (($a_{ij}/\bar{a}^+$, $b_{ij}/\bar{a}^+$, $c_{ij}/\bar{a}^+$); $w_{\bar{a}ij}$,$u_{\bar{a}ij}$,$y_{\bar{a}ij}$), such that $\bar{a}$+= Max {c_{ij}. i∈ I_m, j ∈ I_n}

R=

$$\begin{bmatrix} ((.46,.55,.86);0.4,0.7,0.2) & ((.53,.73,.87);0.7,0.2,0.8) & ((.75,.79,.85);0.8,0.5,0.4) \\ ((.62,.76,.82);0.4,0.1,0.3) & ((.62,.89,.91);0.6,0.3,0.5) & ((.62,.89,.91);0.6,0.3,0.5) \\ ((.62,.89,.91);0.6,0.3,0.5) & ((.65,.69,.85);0.6,0.8,0.1) & ((.66,.88,1);0.6,0.2,0.2) \\ ((.71,.77,.83);0.5,0.2,0.4) & ((.53,.73,.87);0.7,0.2,0.8) & ((.63,.75,.89);0.7,0.4,0.6) \\ ((.44,.59,.72);0.7,0.2,0.3) & ((.62,.89,.91);0.6,0.3,0.5) & ((.53,.73,.87);0.7,0.2,0.8) \\ ((.58,.69,.85);0.6,0.2,0.3) & ((.44,.59,.72);0.7,0.2,0.3) & ((.44,.59,.72);0.7,0.2,0.3) \\ ((.62,.76,.82);0.4,0.1,0.3) & ((.66,.88,1);0.6,0.2,0.2) & ((.65,.69,.85);0.6,0.8,0.1) \\ ((.62,.76,.82);0.4,0.1,0.3) & ((.47,.69,.85);0.7,0.2,0.6) & (((.66,.88,1);0.6,0.2,0.2) \end{bmatrix}$$

Compute U= $[u_{ij}]_{m*n}$ of R. Where, u_{ij}= $\omega_i r_{ij}$ (i∈ I_m for ISQ criteria, j ∈ I_n information systems),

ω = (.35, .25, .30, .20, .25, .20, .30, .20) be the weight vector of ISQ criteria, where ω_i ∈[0, 1], i ∈ I_m , and

$$\sum_{i=1}^{m} \omega_i = 1$$

Calculate the comprehensive values S_j as:

$$S_j = \sum_{i=1}^{m} u_{ij} \quad (j \in I_n),$$

U=

$$\begin{bmatrix} ((.161,.192,.301);0.4,0.7,0.2) & ((.185,.255,.304);0.7,0.2,0.8) & ((.262,.276,.297);0.8,0.5,0.4) \\ ((.155,.190,.205);0.4,0.1,0.3) & ((.155,.222,.227);0.6,0.3,0.5) & ((.155,.222,.227);0.6,0.3,0.5) \\ ((.186,.267,.273);0.6,0.3,0.5) & ((.195,.207,.255);0.6,0.8,0.1) & ((.198,.264,.300);0.6,0.2,0.2) \\ ((.142,.154,.166);0.5,0.2,0.4) & ((.106,.146,.174);0.7,0.2,0.8) & ((.126,.150,.178);0.7,0.4,0.6) \\ ((.110,.147,.180);0.7,0.2,0.3) & ((.155,.222,.227);0.6,0.3,0.5) & ((.132,.182,.217);0.7,0.2,0.8) \\ ((.116,.138,.170);0.6,0.2,0.3) & ((.088,.118,.144);0.7,0.2,0.3) & ((.088,.118,.144);0.7,0.2,0.3) \\ ((.186,.228,.246);0.4,0.1,0.3) & ((.198,.264,.300);0.6,0.2,0.2) & ((.195,.207,.255);0.6,0.8,0.1) \\ ((.124,.152,.164);0.4,0.1,0.3) & ((.094,.138,.170);0.7,0.2,0.6) & (((.132,.176,.200);0.6,0.2,0.2) \end{bmatrix}$$

Then, calculate the comprehensive values S_j as:

$S_j =$

$$\sum_{i=1}^{m} u_{ij} = ((\sum_{i=1}^{m} \omega_i r_{ij}, \sum_{i=1}^{m} \omega_i r_{ij}, \sum_{i=1}^{m} \omega_i r_{ij}); \text{Min } w_{\tilde{a}ij}, \text{Max } u_{\tilde{a}ij}, \text{Max } y_{\tilde{a}ij})$$

S_1= ((1.18, 1.468, 1.705); .4, .7, .5)
S_2= ((1.176, 1.572, 1.801); .6, .8, .8)
S_3= ((1.288, 1.592, 1.818); .6, .8, .8)

Phase 4: Rank ISQ

Apply the two evaluating and ranking methods: (1) - weighted value and ambiguity based method, (2) the value index and ambiguity index method

1. Weighted value and ambiguity method

Calculate the weighted value and ambiguity of truth-membership and indeterminacy membership, and falsity-membership degree for each comprehensive value

$V_\mu (S_1) = 1.459\,(.4)^2 = .233$
$V_\upsilon(S_1) = 1.459\,(1-.7)^2 = .131$
$V_\lambda (S_1) = 1.459\,(1-.5)^2 = .364$

$V_\mu (S_2) = 1.544\,(.6)^2 = .555;$
$V_\upsilon (S_2) = 1.544\,(1-.8)^2 = .061;$
$V_\lambda (S_2) = 1.544\,(1-.8)^2 = .061$

$V_\mu (S_3) = 1.581\,(.6)^2 = .569;$
$V_\upsilon (S_3) = 1.581\,(1-.8)^2 = .063;$
$V_\lambda (S_3) = 1.581\,(1-.8)^2 = .063$

$V_\theta = .233\,\theta + .131(1-\theta) + .364(1-\theta)$
$V_\theta = .555\,\theta + .061\,(1-\theta) + .061(1-\theta)$
$V_\theta = .569\,\theta + .063(1-\theta) + .063(1-\theta)$

Thirdly, graphically represents weighted values for evaluating and ranking quality of IS. The following figure represents the weighted values of the S_1, S_2 and S_3

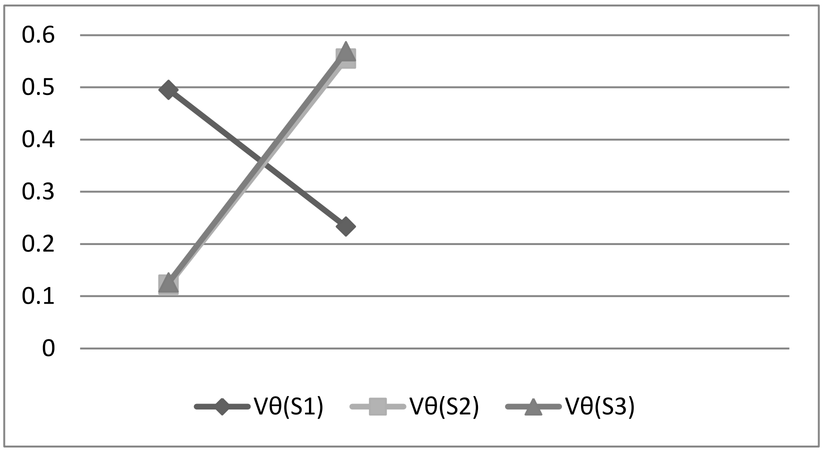

Fig. 1. The weighted values of the S_1, S_2 and S_3

- From figure (1) for any $\theta \in [0, .523]$ the weighted values of the S_1, S_2 and S_3 can ranked as the follows: $V_\theta (S_1) > V_\theta (S_3) > V_\theta (S_2)$. Consequently, the quality of the first information system > the quality of the third information system > the quality of the second information system
- From figure (1), the weighted values of S_1 and S_3 have equal values at $\theta = .523$. The weighted ambiguities of S_1 and S_3 can be calculated based on Eq. (3.9) as follows:

$A_{.523} (S_1) = .0212$
$A_{.523} (S_3) = .0198$

Therefore, $S_1 > S_3$, Consequently, the quality of the first information system is greater than the quality of the third information system

- From figure (1) for any $\theta \in [.523, .536]$ the weighted values of the S_1, S_2 and S_3 can ranked as the follows: $V_\theta (S_1) > V_\theta (S_3) > V_\theta (S_2)$.

Consequently, the quality of the first information system > the quality of the third information system > the quality of the second information system

- From figure (1), the weighted values of S_1 and S_2 have equal values at $\theta = .536$. The weighted ambiguities of S_1 and S_2 can be calculated based on Eq. (4.9) as follows:

$A_\theta(S_j) = (c\text{-}a)/6\,[\theta w_j^2 + (1-\theta)(1-u_{sj})^2 + (1-\theta)(1-y_{sj})^2]$

$A_{.536}(S_1) = .0210$

$A_{.536}(S_2) = .0237$

Therefore, $S_2 > S_1$, Consequently, the quality of the second information system is greater than the quality of the first information system

- From figure (1) for any $\theta \in [.536, 1]$ the weighted values of the S_1, S_2 and S_3 can ranked as the follows: $V_\theta(S_3) > V_\theta(S_2) > V_\theta(S_1)$. Consequently, the quality of the third information system > the quality of the second information system > the quality of the first information system

This method gives more attention to uncertainty in decision making as well as it takes into account cut sets of SVTrN numbers that can reflect the information on membership degrees and non-membership degrees. However, the calculations and graphically representation of this method become complex when alternatives increase.

1. The value index and ambiguity index method

Apply the value index and ambiguity index method to rank Information Systems Quality (ISQ) as the follows:

$V_\mu(S_1) = 1.459\,(.4)^2 = .233$

$V_\upsilon(S_1) = 1.459\,(1-.7)^2 = .131$

$V_\lambda(S_1) = 1.459\,(1-.5)^2 = .364$

$V_\mu(S_2) = 1.544\,(.6)^2 = .555$;

$V_\upsilon(S_2) = 1.544\,(1-.8)^2 = .061$;

$V_\lambda(S_2) = 1.544\,(1-.8)^2 = .061$

$V_\mu(S_3) = 1.581\,(.6)^2 = .569$;

$V_\upsilon(S_3) = 1.581\,(1-.8)^2 = .063$;

$V_\lambda(S_3) = 1.581\,(1-.8)^2 = .063$

$V(S_1, \lambda) = .233\,\lambda + .131(1-\lambda) + .364(1-\lambda)$

$V(S_2, \lambda) = .555\,\lambda + .061\,(1-\lambda) + .061(1-\lambda)$

$V(S_3, \lambda) = .569\lambda + .063(1-\lambda) + .063(1-\lambda)$

Table (4): Ranking results based on the Weighted Values and Ambiguities index method of SVTrN-numbers

λ	*V (S₁, λ)*	*V (S₂, λ)*	*V (S₃, λ)*	*Ranking results*
$.1 \in [0,1/2]$	.468	.165	.170	$S_1 > S_3 > S_2$
$.3 \in [0,1/2]$	.416	.251	.258	$S_1 > S_3 > S_2$
.5	.364	.338	.347	$S_1 > S_3 > S_2$
$.7 \in [1/2,1]$	.311	.425	.436	$S_3 > S_2 > S_1$
$.8 \in [1/2,1]$	.285	.468	.480	$S_3 > S_2 > S_1$

(1) From table (4) values: .1 and .3 where $\lambda \in [0, 1/2]$, the results show when the decision maker prefers negative feeling, the ranking of quality of the three information systems is $S_1 > S_3 > S_2$, Consequently, the quality of the first IS > the quality of the third IS > the quality of the second IS.

(2) From table (4) where $\lambda = ½$ shows that the decision maker is indifferent between positive feeling and negative feeling, the ranking of quality of the three information systems is $S_1 > S_3 > S_2$, Consequently, the quality of the first IS > the quality of the third IS > the quality of the second IS.

(3) From table (4) values: .7 and .8 where $\lambda \in [1/2,1]$, the results show when the decision maker prefers positive feeling, evaluation and ranking of quality of the three information systems is $S_3 > S_2 > S_1$, Consequently, the quality of the third IS > the quality of the second IS > the quality of the first IS.

This method focuses on value index and ambiguity index and it can reflect the decision maker's subjectivity attitude to the SVTrN- numbers.

5. Conclusion and Future Work

This work intended to introduce a framework with two ranking methods of SVTrN- numbers with the scale based approach for evaluating and ranking ISQ. The proposed framework consists of four phases. The results indicated that each ranking method has its own advantage that make. In this perspective, the weighted value and ambiguity based method gives more attention to uncertainty in ranking and evaluating ISQ as well as it takes into account cut sets of SVTrN numbers that can reflect the information on membership degrees and non-membership degrees. The value index and ambiguity index can handle indeterminacy and uncertainty and it can reflect the decision maker's subjectivity attitude to the SVTrN- numbers.

For future work, SVTrN-numbers can be applied widely for more real practical applications with adapting and generalizing existing methods of ranking fuzzy numbers and intuitionistic fuzzy number to give more efficient results.

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