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Neutrosophic Crisp Bi-Topological Spaces

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Abstract. In this paper, neutrosophic crisp bi-topological spaces, new types of open and closed sets in neutrosophic crisp bi-topological spaces, the closure and interior neutrosophic crisp set and a new concept of open and closed sets are introduced. The basic properties of these types of open and closed sets and their properties are studied.

Keywords: Neutrosophic crisp bi-topological spaces, neutrosophic crisp bi-open set, neutrosophic crisp bi-closed set, neutrosophic crisp S-open sets and neutrosophic crisp S-closed.

1. Introduction

Smarandache [1, 2] proposed a new branch of philosophy, called "Neutrosophy". From neutrosophy, Smarandache [1, 2, 3, 4] defined neutrosophic set. Neutrosophic set consists of three independent components *T, I*, and *F* which represent the membership, indeterminacy, and non membership values respectively. *T, I*, and *F* assumes the values from the non-standard unit interval$]^-0,1^+[$. Smarandache [1, 2] made the foundation of neutrosophic logic which generalizes fuzzy logic [5] and intuitionistic fuzzy logic [6]. Salama, Smarandache proposed the Neutrosophic Crisp Set Theory [16].

Alblowi, Salama and M. Eisa [7, 8] defined studied on neutrosophic sets and defined normal neutrosophic set, convex set, the concept of α-cut and neutrosophic ideals. Hanafy, Salama and Mahfouz [9] considered some possible definitions for basic concepts of the Neutrosophic Crisp Data And Its Operations.

Salama and Alblowi [10] defined neutrosophic topological spaces and established some of its properties. Salama and Alblowi 11] defined generalized neutrosophic set and defined generalized neutrosophic topological spaces. In the same study, Salama and Alblowi [11] established some properties of generalized neutrosophic topological spaces. Salama and Elagamy [12] introduced the notion of filters on neutrosophic sets and studied several relations between different neutrosophic filters and neutrosophic topologies. Salama and Smarandache [13] studied several relations between different neutrosophic crisp filters and neutrosophic topologies.

Salama, Smarandache and Kroumov [14] generalized the crisp topological spaces to the notion of neutrosophic crisp topological space. In the same study, Salama, Smarandache and Kroumov [14] introduced the definitions of neutrosophic crisp continuous function and neutrosophic crisp compact spaces.

In this paper we introduce the concept of neutrosophic crisp bi-topological spaces as generalization of neutrosophic crisp topological spaces. We introduce few new types of open and closed sets as neutrosophic crisp bi-open sets, neutrosophic crisp bi-closed sets, neutrosophic crisp S-open sets and neutrosophic crisp S-closed sets. We investigate the properties of these new four types of neutrosophic crisp sets.

Rest of the paper is organized as follows: Section 2 presents preliminaries of neutrosophic crisp set, neutrosophic crisp topology. Section 3 presents Neutrosophic crisp bi-topological spaces. Section 4 devotes the closure and the interior via neutrosophic crisp bi-open sets (Bi-NCOS) and neutrosophic crisp bi-closed (Bi-NCCS). Section 5 devotes the neutrosophic crisp S-open sets (S-NCOS) and neutrosophic crisp S-closed sets (S-NCOS). Section r presents conclusion of the paper.

2. Preliminaries Of Neutrosophic Crisp Sets:

Definition 2.1. [14] Let X be a non-empty fixed set. A neutrosophic crisp set (NCS) A is an object having the form $A = \{A_1, A_2, A_3\}$, where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_1 = \phi$.

Definition 2.2. [14, 15] Types of $NCSs$ ϕ_N and X_N in X

1. ϕ_N may be defined in many ways as a NCS as follows:
   1. $\phi_N = (\phi, \phi, X)$ or
   2. $\phi_N = (\phi, X, X)$ or
   3. $\phi_N = (\phi, X, \phi)$ or
   4. $\phi_N = (\phi, \phi, \phi)$.
2. X_N may be defined in many ways as a NCS, as follows:
   1. $X_N = (X, \phi, \phi)$ or
   2. $X_N = (X, X, \phi)$ or
   3. $X_N = (X, X, X)$.

Definition 2.3. [14] A neutrosophic set A is a subset of a neutrosophic set B denoted by $A \subseteq B$, may be defined as:

1. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1,\ A_2 \subseteq B_2\ and\ B_3 \subseteq A_3$.
2. $A \subseteq B \Leftrightarrow A_1 \subseteq B_1,\ B_2 \subseteq A_2\ and\ B_3 \subseteq A_3$.

Definition 2.4. [14] Let X be a non-empty set, and the NCSs A and B in the form $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$. Then:

1. $A \cap B$ may be defined in two ways:

i) $A \cap B = (A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3)$

ii) $A \cap B = (A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3)$.

2. $A \cup B$ may be defined in two ways as a NCSs.

i) $A \cup B = (A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3)$

ii) $A \cup B = (A1 \cup B1, A2 \cup B2, A3 \cap B3)$.

Definition 2.5. [14] A neutrosophic crisp topology (NCT) on a non-empty set X is a family Γ of neutrosophic crisp subsets in X satisfying the following axioms:

1. $\phi_N, X_N \in \Gamma$.
2. $A_1 \cap A_2 \in \Gamma$, for any A_1 and $A_2 \in \Gamma$.
3. $\bigcup A_j \in \Gamma, \forall \{A_j : j \in J\} \subseteq \Gamma$.

The pair (X, Γ) is said to be a neutrosophic crisp topological space (NCTS) in X, a set of elements in Γ is said to be a neutrosophic crisp open set (NCOS), neutrosophic crisp set F is closed (NCCS) if and only if its complement F^c is an open neutrosophic crisp set.

Definition 2.6. [14] Let X be a non-empty set, and the NCS A in the form $A = \{A_1, A_2, A_3\}$. *Then* A^c may be defined in three ways as an NCS, as follows:

$i)\ A^c = < A_1^c, A_2^c, A_3^c >$ or

$ii)\ A^c = < A_3, A_2, A_1 >$ or

$iii)\ A^c = < A_3, A_2^c, A_1 >$.

3. Neutrosophic Crisp Bi-Topological Space

In this section, we introduce neutrosophic bi-topological crisp spaces. Moreover we introduce new types of open and closed sets in neutrosophic bi-topological crisp spaces.

Definition 3.1. Let Γ_1, Γ_2 be any two neutrosophic crisp topology (NCT) on a nonempty set X. Then (X, Γ_1, Γ_2) is a neutrosophic crisp bi-topological space (Bi-NCTS for short).

Example 3.1. Let $X = \{1, 2, 3, 4\}$,

$\Gamma_1 = \{\phi_N, X_N, \text{D, C}\}$, $\Gamma_2 = \{\phi_N, X_N, \text{A, B}\}$,
$\text{A} = < \{1\}, \{2, 4\}, \{3\} > = C, B = < \{1\}, \{2\{, \{2, 3\} >$, $D = < \{1\}, \{2\{, \{3\} >$.

Then $(X, \Gamma_1), (X, \Gamma_2)$ are two neutrosophic crisp spaces. Therefore (X, Γ_1, Γ_2) is a neutrosophic crisp bi-topological space (Bi-NCTS).

Definition 3.2. Let (X, Γ_1, Γ_2) *be a neutrosophic* crisp Bi-topological space (Bi-NCTS).

The elements in $\Gamma_1 \cup \Gamma_2$ are said to be neutrosophic crisp bi-open sets (Bi-NCOS for short). A neutrosophic crisp set F is closed (Bi-NCCS for short) if and only if its complement F^c is an neutrosophic crisp bi-open set.

- the family of all neutrosophic crisp bi-open sets is denoted by (Bi-NCOS(X)).

- the family of all neutrosophic crisp bi-closed sets is denoted by (Bi-NCCS(X)).

Example 3.2. In Example 3.1, the neutrosophic crisp bi-open sets (Bi-NCOS) are :

Bi-NCOS(X) = $\Gamma_1 \cup \Gamma_2 = \{\phi_N, X_N, A, B, C, D\}$

the neutrosophic crisp bi-closed sets (Bi-NCCS) are :

Bi-NCCS(X) = $\Gamma_1 \cup \Gamma_2 = \{\phi_N, X_N, A_1, B_1, C_1, D_1\}$ where: $A_1 = < \{2, 3, 4\}, \{1, 3\}, \{1, 2, 4\} > = C_1$,

$B_1 = < \{2, 3, 4\}, \{1, 3, 4\}, \{1, 2\} >$,

$D_1 =< \{2,3,4\},\{1,3,4\},\{1,2,4\} >$

Remark 3.1.

1) Every neutrosophic crisp open set in (X,Γ_1) *or* (X,Γ_2) is a neutrosophic crisp bi-open set.
2) Every neutrosophic crisp closed set in (X,Γ_1) *or* (X,Γ_2) is a neutrosophic crisp bi-closed set.

Remark 3.2.

Every *neutrosophic crisp bi-topological space* (X,Γ_1,Γ_2) *induces* two *neutrosophic crisp topological spaces* as $(X,\Gamma_1),(X,\Gamma_2)$.

Remark 3.3.

If (X,Γ) be a neutrosophic crisp topological space then (X,Γ,Γ) is a neutrosophic crisp Bi-topological space.

Theorem 3.1. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS). Then, the union of two neutrosophic crisp bi-open (bi-closed) sets is not a neutrosophic crisp bi-open (bi-closed) set.

The proof of the theorem 3.1 follows from the example 3.3.

Example 3.3.

$X=\{1,2,3,4\}$, $\Gamma_1=\{\phi_N, X_N, \text{D}, \text{C}\}$, $\Gamma_2=\{\phi_N, X_N, \text{A}, \text{B}\}$,
$\text{A} =< \{3\},\{2,4\},\{1\} >, D =< \{1\},\{2\},\{3\} >$, $C =< \{1\},\{2,4\},\{3\} >$

It is clear that $(X,\Gamma_1),(X,\Gamma_2)$ are neutrosophic crisp topological spaces. Therefore (X,Γ_1,Γ_2) is a neutrosophic crisp bi-topological space

A, D are two neutrosophic crisp bi-open sets but $A \cup D =< \{1,3\},\{2,4\},\phi >$ is not neutrosophic crisp bi-open set. $A^c =< \{1,2,4\},\{1,3\},\{2,3,4\} >$, $D^c =< \{2,3,4\},\{1,3,4\},\{1,2,4\} >$

are two neutrosophic crisp bi-closed sets but $A^c \cup D^c =< X,\{1,3\},\{2,4\} >$ is not a neutrosophic crisp bi-closed set.

Theorem 3.2. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS). Then, the intersection of two neutrosophic crisp bi-open (bi-closed) sets is a neutrosophic crisp bi-open (bi-closed) set.

The proof of the theorem 3.2 follows from the example 3.4

Example 3.4. In example 3.3, A, D are two neutrosophic crisp bi-open sets but $A \cap D =< \varnothing,\{2\},\{1,3\} >$ is not a neutrosophic crisp bi-open set.

$A^c =< \{1,2,4\},\{1,3\},\{2,3,4\} >$,

$D^c =< \{2,3,4\},\{1,3,4\},\{1,2,4\} >$, are two neutrosophic crisp bi-closed sets but $A^c \cap D^c =< \{2,4\},\{1,3\},X >$ is not a neutrosophic crisp bi-closed set.

4. The closure and the interior via neutrosophic crisp bi-open sets (Bi-NCOS) and neutrosophic crisp bi-closed (Bi-NCCS)

In this section, we use this new concept of open and closed sets in the definition of closure and interior neutrosophic crisp set, where we define the closure and interior neutrosophic crisp set based on these new varieties of open and closed neutrosophic crisp sets. Also we introduce the basic properties of closure and the interior.

Definition 4. 1. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS) and A is a neutrosophic crisp set. Then, the union of neutrosophic crisp bi-open sets containing A is called neutrosophic crisp bi-interior of A ($NC^{Bi}Int(A)$ for short).

$NC^{Bi}Int(A) = \cup\{B : B \subseteq A$; B is neutrosophic crisp bi-open set$\}$.

Theorem 4.1. Let (X,Γ_1,Γ_2) be neutrosophic crisp bi-topological space (Bi-NCTS), A is neutrosophic crisp set then:

1. $NC^{Bi}Int(A) \subseteq A$.
2. $NC^{Bi}Int(A)$ is not neutrosophic crisp bi-open set .

Proof :

1. It follows from the definition of $NC^{Bi}Int(A)$ as a union of *neutrosophic crisp bi-open set*s contains A.
2. Follow from Theorem 3.2.

Theorem 4.2. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS), A and B are neutrosophic crisp sets. Then,

$A \subset B \Rightarrow NC^{Bi}Int(A) \subset NC^{Bi}Int(B)$ **.**

Proof: The Proof is obvious.

Definition 4.2. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS), A is neutrosophic crisp set. Then, the intersection of neutrosophic crisp bi-open sets, contained A is called neutrosophic crisp Bi-closure of A (NC^{Bi}-Cl(A)for short).

$NC^{Bi}\text{-}Cl(A) = \cap\{B : B \supseteq A$; B is a neutrosophic bi-closed set$\}$.

Theorem 4. 3. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS), and A is neutrosophic crisp set. Then

1. $A \subseteq NC^{Bi}cl(A)$.
2. $NC^{Bi}cl(A)$ is not a neutrosophic crisp bi-closed set.

Proof :

1. It follow from the definition of $NC^{Bi}cl(A)$ as an intersection of neutrosophic crisp bi-closed sets, contained in A**.**
2. It follows from the Theorem 3.2.

5. The neutrosophic crisp S-open sets (S-NCOS) and neutrosophic crisp S-closed sets (S-NCOS):

We introduce new concept of open and closed sets in neutrosophic crisp bi-topological space in this section, as neutrosophic crisp S-open sets (S-NCOS) and neutrosophic crisp S-closed sets (S-NCCS). Also we introduce the basic properties of this new concept of open and closed sets in bi-NCTS, and their relationship with neutrosophic crisp bi-open sets and neutrosophic crisp bi-closed sets.

Definition 5.1. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS). Then, a subset A of space X is said to be a neutrosophic crisp S-open set (S-NCOS for short) if $A\in\Gamma_1$ *and* $A\notin\Gamma_2$ or $A\in\Gamma_2$ *and* $A\notin\Gamma_1$ and its complement is said to be neutrosophic crisp S-closed set (S-NCCS for short).

* the family of all neutrosophic crisp S-open sets is denoted by (S-NCOS(X)).

* the family of all neutrosophic crisp S-closed sets is denoted by (S-NCCS(X)).

Example 5.1. In Example 3.1, B, D are two neutrosophic crisp S-open sets.

Theorem 5.1 Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS), then

1. Every S-NCOS is Bi-NCOS.
2. Every S-NCCS is Bi-NCCS.

Proof:

1. Let A be neutrosophic crisp S-open set ,then $A\in\Gamma_1$ *and* $A\notin\Gamma_2$ or

   $A\in\Gamma_2$ *and* $A\notin\Gamma_1$ therefore A is Bi-NCOS.

2. Let A be neutrosophic crisp S-closed set,then A^c is neutrosophic crisp S-open set therefore $A^c\in\Gamma_1$ *and* $A^c\notin\Gamma_2$ or $A^c\in\Gamma_2$ *and* $A^c\notin\Gamma_1$, so A^c is Bi-NCOS therefore A is a Bi-NCCS .

Remark 5.1. The converse of Theorem 5.1 is not true. It is shown in example 5.2.

Example 5.2. In any neutrosophic crisp bi-topo-logical space, ϕ_N ,X_N are two neutrosophic crisp bi-open sets, but ϕ_N ,X_N are not neutrosophic crisp bi-open sets.

Also ϕ_N ,X_N are two neutrosophic crisp bi-closed sets, but ϕ_N ,X_N are not neutrosophic crisp bi-closed sets.

Theorem 5.2. Let (X,Γ_1,Γ_2) be a neutrosophic crisp bi-topological space (Bi-NCTS). Then, the union of two neutrosophic crisp S-open (S-closed) sets is not a neutrosophic crisp S-open (S-closed) set.

Proof. The proof follows from the following example 5.3.

Example 5.3. In example 3.4

$X = \{1, 2, 3, 4\}$, $\Gamma_1 = \{\phi_N, X_N, \text{A}\}$,

$\Gamma_2 = \{\phi_N, X_N, \text{D,C}\}, A =< \{3\}, \{2,4\}, \{1\} >$.

$D =< \{1\}, \{2\}, \{3\} >, C =< \{1\}, \{2,4\}, \{3\} >$.

It is clear that $(X, \Gamma_1), (X, \Gamma_2)$ are neutrosophic crisp topological spaces, therefore (X, Γ_1, Γ_2) is a neutrosophic crisp bi-topological space.

A, D are two neutrosophic crisp S-open sets but $A \cup D =< \{1,3\}, \{2,4\}, \varnothing >$ is not a neutrosophic crisp S-open set.

$A^c =< \{1,2,4\}, \{1,3\}, \{2,3,4\} >$, $D^c =< \{2,3,4\}, \{1,3,4\}, \{1,2,4\} >$ are two *neutrosophic crisp S-closed* sets but $A^c \cup D^c =< X, \{1,3\}, \{2,4\} >$ is not a *neutrosophic crisp S-closed* set.

Theorem 5.3. Let (X, Γ_1, Γ_2) be a neutrosophic crisp Bi-topological space (Bi-NCTS), then the intersection of two *neutrosophic crisp* S-open (S-closed) *se*ts is *not a neutrosophic crisp* S-open (S-closed) *se*t.

Proof. The proof follows from the following example 5.4.

Example5.4 In example 3.4, A, D are two *neutrosophic crisp* S-*open* sets but $A \cap D =< \varnothing, \{2\}, \{1,3\} >$ is not a *neutrosophic crisp* S-*open* set.

$A^c =< \{1,2,4\}, \{1,3\}, \{2,3,4\} >$, $D^c =< \{2,3,4\}, \{1,3,4\}, \{1,2,4\} >$ are two *neutrosophic crisp* S-*closed* sets but $A^c \cap D^c =< \{2,4\}, \{1,3\}, X >$ is not a *neutrosophic crisp* S-*closed* set.

6 Conclusion

In this paper we have introduced neutrosophic crisp bi-topological space, neutrosophic crisp Bi-open, neutrosophic crisp bi-closed, neutrosophic crisp S-open, neutrosophic crisp S-open set's. Also we have studied some of their basic properties and their relationship with each other. Finally, these new concepts are going to pave the way for new types of open and closed sets as neutrosophic crisp bi-α-open sets, neutrosophic crisp bi-β-open sets, neutrosophic crisp bi-pre-open sets, neutrosophic crisp bi-semi-open sets.

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