Neutrosophic Soft Cubic Set in Topological Spaces

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Abstract: This research article lays the foundation to propose the new concept of neutrosophic soft cubic topology. Here we focus on the systematic study of neutrosophic soft cubic sets and deduce various properties which are induced by them. This enables us to introduce some equivalent characterizations and brings out the inter relations among them.

Keywords: Neutrosophic Soft Cubic Set, Neutrosophic Soft Cubic Topological Space.

1 Introduction

Molodtsov [11]proposed the concept of soft set theory in 1999 which is an entirely new approach for modeling various forms of vagueness and uncertainty of real life situations. Soft set theory has a rich potential to penetrate itself in several direction which is clearly figured out in Molodtsov's pioneer work[11]. Zadeh [19] in 1965 came out with a novel concept of fuzzy set which deals with the degree of membership function between [0,1]. In [9] Maji et al.intiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union , intersection, complement of fuzzy soft set. Moreover in [10] Maji et al. extended the idea of soft sets to Neutrosophic setting. Neutrosophic Logic has been proposed by Florentine Smarandache[15] which is based on nonstandard analysis that was given by Abraham Robinson[14] in 1960s.

Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency,redundancy, contradiction. The neutrosophic logic is formal frame to measure truth, indeterminacy and falsehood values . In Neutrosophic set, indeterminacy is quantified explicitly and the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.

Jun et al.[17]presented the concept of cubic set by combining the fuzzy sets and interval valued fuzzy set. Y.B.Jun et al.[18] extended this idea under neutrosophic environment and named it Neutrosophic cubic sets. Wang et al.[16] introduced the concept of interval neutrosophic set and studied some of its properties.Chinnadurai [7] investigated some characterizations of its properties of Neutrosophic cubic soft set. Following him Pramanik et al.[13] introduced further operations and brought some of the properties of Neutrosophic cubic soft set.

As a further development Anitha et al.[1] proposed the notion of neutrosophic soft cubic set and defined internal neutrosophic soft cubic set, external neutrosophic soft cubic set and studied some new type of internal neutrosophic cubic set (INSCS) and external neutrosophic cubic set (ENSCS) namely, $\frac{1}{3}$ INSCS or $\frac{2}{3}$, ENSCS , $\frac{2}{3}$ INSCS or $\frac{1}{3}$ ENSCS.Anitha et al.[2-3] has discussed various operations on Neutrosophic soft cubic sets and investigated several related properties. In [4] the author has presented an application of Neutrosophic soft Cubic set in pattern recognition. Neutrosophic soft Cubic set theory was applied in BCI/BCK algebra[5]. In this paper we define neutrosophic soft cubic topological space and we discuss some of its properties.

2 Preliminaries

Definition 2.1. [19]
Let E be a universe. Then a fuzzy set over E is defined by $X = \{\mu_x(x)/x : x\epsilon E\}$ where μ_x is called membership function of X and defined by $\mu_x : E \to [0, 1]$. For each $x\epsilon E$, the value $\mu_x(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [17]
Let X be a non-empty set. By a cubic set, we mean a structure $\Xi = \{\langle A(x), \mu(x)\rangle | x\epsilon X\}$ in which A is an interval valued fuzzy set (IVF) and $\mu_(x)$ is a fuzzy set. It is denoted by $\langle A, \mu\rangle$.

Definition 2.3. [8]
Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by$F : A \to P(U)$.

Definition 2.4. [15]
Let X be an universe. Then a neutrosophic set(NS) λ is an object having the form $\lambda = \{\langle x : T(x), I(x), F(x)\rangle : x\varepsilon X\}$ where the functions $T, I, F : X \to]0, 1^+[$ define respectively the degree of Truth, the degree of indeterminacy, and the degree of falsehood of the element $x\varepsilon X$ to the set λ with the condition
$-0 \leq T(x) + I(x) + F(x) \leq 3+$.

Definition 2.5. [16]
Let X be a non-empty set. An interval neutrosophic set (INS) A in X is characterized by the truth-membership function A_T, the indeterminacy-membership function A_I and the falsity-membership function A_F. For each point $x\epsilon X, A_T(x), A_I(x), A_F(x) \subseteq [0, 1]$.

Definition 2.6. [1]
Let U be an the initial universal set. Let $NC(U)$ denote the set of all neutrosophic cubic sets and E be a set of parameters. Let M ⊆ E then
$(P, M) = \{P(e) = \{\langle x, A_e(x), \lambda_e(x)\rangle : x\epsilon U\}e\epsilon M\}$
where $A_e(x) = \{\langle x, A_e^T(x), A_e^I(x), A_e^F(x)\rangle : x\epsilon U\}$ is an interval neutrosophic set and
$\lambda_e(x) = \{\langle x, \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x\epsilon U\}$ is an neutrosophic set.
Let P(U) denote the set of all neutrosophic cubic sets of U. The collection (P, M) is termed to be the neutrosophic soft cubic set over U, where F is a mapping given by F : A ⟶ P(U).
The neutrosophic soft cubic set is denoted by NSCset / NSCS. The collection of all neutrosophic soft cubic set over U is denoted by NSCS(U).

3 Some Results On Neutrosophic Soft Cubic Set

Definition 3.1. Let (P , E) be neutrosophic soft cubic set over U.

(i) (P , E) is called absolute or universal neutrosophic soft cubic set U if $P(e) = \hat{1} = \{(\widetilde{1}, \widetilde{1}, \widetilde{1}), (1, 1, 1)\}$ for all $e \in$ E. We denote it by U.

(ii) (P , E) is called null or empty neutrosophic soft cubic set U if $P(e) = \hat{0} = \{(\widetilde{0}, \widetilde{0}, \widetilde{0})(0, 0, 0)\}$ for all $e \in$ E. We denote it by Φ.
Obviously $\Phi^c = U$ and $U^c = \Phi$.

Definition 3.2. A neutrosophic soft cubic set (P , M) is said to be a subset of a neutrosophic soft cubic set (Q , N) if $M \subseteq N$ and $P(e) \subseteq Q(e)$ $\forall$ e $\in$ M , x $\in$ U if and only if $A_e(x) \subseteq B_e(x)$ and $\lambda_e(x) \subseteq \mu_e(x)$ $\forall$ e $\in$ M , u $\in$ U . We denote it by $(P, M)\widetilde{\subseteq}$ (Q , N). where $P(e) \subseteq Q(e)$ $\forall$ e $\in$ M , u $\in$ U if and only if $A_e(x) \subseteq B_e(x)$ and $\lambda_e(x) \subseteq \mu_e(x)$,

$$\begin{aligned}
A_e(x) \subseteq B_e(x) \quad &\Longrightarrow \quad A_e^{-T}(x) \leq B_e^{-T}(x),\\
&\quad\quad\; A_e^{+T}(x) \leq B_e^{+T}(x),\\
&\quad\quad\; A_e^{-I}(x) \geq B_e^{-I}(x),\\
&\quad\quad\; A_e^{+I}(x) \geq B_e^{+I}(x),\\
&\quad\quad\; A_e^{-F}(x) \geq B_e^{-F}(x),\\
&\quad\quad\; A_e^{+F}(x) \geq B_e^{+F}(x)\\
\text{and } \lambda_e(x) \subseteq \mu_e(x) \quad &\Longrightarrow \quad \lambda_e^{T}(x) \leq \mu_e^{T}(x),\\
&\quad\quad\; \lambda_e^{I}(x) \geq \mu_e^{I}(x),\\
&\quad\quad\; \lambda_e^{F}(x) \geq \mu_e^{F}(x),
\end{aligned}$$

Definition 3.3. The complement of neutrosophic soft cubic set (P , M) is denoted by $(P, M)^c$ and defined as $(P, M)^c = (P^c, \neg M)$ where $P^c : \neg A \longrightarrow P(U)$ is a mapping given by

$$\begin{aligned}
(P, M)^c &= \{\langle x, \tilde{A}_e^c(x), \lambda_e^c(x)\rangle : x\epsilon U\}e\epsilon M\}\\
&= \{\langle x, 1 - \tilde{A}_e^T(x), 1 - \tilde{A}_e^I(x), 1 - \tilde{A}_e^F(x),\\
&\quad 1 - \lambda_e^T(x), 1 - \lambda_e^I(x), 1 - \lambda_e^F(x)\rangle : x\epsilon U\}e\epsilon M\}\\
&= \{\langle x, 1 - A_e^{+T}, 1 - A_e^{-T}](x), [1 - A_e^{+I}, 1 - A_e^{-I}](x),\\
&\quad [1 - A_e^{-F}, 1 - A_e^{+F}](x), 1 - \lambda_e^T(x), 1 - \lambda_e^I(x),\\
&\quad 1 - \lambda_e^F(x)\rangle : x\epsilon U\}e\epsilon M\}
\end{aligned}$$

Remark 3.4. The complement of a neutrosophic soft cubic set (P , M) can also be defined as $(P, M)^c = U \backslash P(e)$ for all $e \in M$.

Definition 3.5. The union of two neutrosophic soft cubic sets (P, M)=$\{\langle x, A_e(x), \lambda_e(x)\rangle : x\epsilon U\}e\epsilon M\}$ and (Q, N)=$\{\langle x, A_e(x), \lambda_e(x)\rangle : x\epsilon U\}e\epsilon N\}$ over (U,E) is neutrosophic soft cubic set where C = M $\cup$ N ,$\forall e \in C$

$$H(e) = \begin{cases} P(e) & \text{if e} \in A - B \\ Q(e) & \text{if e} \in B - A \\ P(e) \cup Q(e) & \text{if e} \in A \cap B \end{cases}$$

and is written as (P , M)∪(Q , N)= (H,C).
where $H(e) = \langle x, max(\tilde{A}_e(x), \tilde{B}_e(x)), max(\lambda_e(x), \mu_e(x))\rangle$.

Definition 3.6. The intersection of two neutrosophic soft cubic sets (P , M)and (Q , N) over (U,E) is neutrosophic soft cubic set where C = M ∩ N ,$\forall e \in C$
$H(e) = P(e) \cap Q(e)$ and is written as (P , M)∩(Q , N)= (H,C).
where $H(e) = \langle x, min(\tilde{A}_e(x), \tilde{B}_e(x)), min(\lambda_e(x), \mu_e(x))\rangle$.

Definition 3.7. If (P , M) and (Q , N) be two neutrosophic soft cubic sets then (P , M) AND (Q , N) is a NSCS denoted by
(P , M) ∧ (Q , N) and is defined by (P , M) ∧ (Q , N) = (H,M× N), where (H,A× B)= $P(\alpha_i) \cap F(\beta_i)$ the truth membership, indeterminacy membership and the falsity membership of (H,A× B) are as follows:
$H^T(a_i, b_i)(h) = min(\tilde{A}^T(a)(h), \tilde{A}^T(b)(h)), min(\lambda^T(a)(h), \lambda^T(b)(h))$
$H^I(a_i, b_i)(h) = min(\tilde{A}^I(a)(h), \tilde{A}^I(b)(h)), min(\lambda^I(a)(h), \lambda^I(b)(h))$ and
$H^F(a_i, b_i)(h) = min(\tilde{A}^F(a)(h), \tilde{A}^F(b)(h)), min(\lambda^F(a)(h), \lambda^F(b)(h))$
$forall(a_i, b_i)\epsilon M \times N.$

Definition 3.8. [2] If (P , M) and (Q , N) be two Neutrosophic soft cubic sets then (P , M)OR (Q , N) is a NSCS denoted by
(P , M) ∨ (Q , N) and is defined by (P , M) ∨ (Q , N) = (H,A× B), where the truth membership, indeterminacy membership and the falsity membership of (H,A× B) are as follows:
$H^T(a_i, b_i)(h) = max(\tilde{A}^T(a)(h), \tilde{A}^T(b)(h)), max(\lambda^T(a)(h), \lambda^T(b)(h))$
$H^I(a_i, b_i)(h) = max(\tilde{A}^I(a)(h), \tilde{A}^I(b)(h)), max(\lambda^I(a)(h), \lambda^I(b)(h))$ and
$H^F(a_i, b_i)(h) = max(\tilde{A}^F(a)(h), \tilde{A}^F(b)(h)), max(\lambda^F(a)(h), \lambda^F(b)(h))$
$forall(a_i, b_i)\epsilon M \times N.$

Proposition 3.9. Let U be an initial universal set and E be a set of parameters.Let (P , E) and (Q , E) be NSCS over U. Then the following are true.

(i) (P , E)⊆(Q , E) iff (P , E)∩(Q , E)= (P , E)

(ii) (P , E)⊆(Q , E) iff (P , E)∪(Q , E)= (Q , E)

Proof:

(i) Suppose that (P , E) ⊆(Q , E) then P(e)⊆Q(e) for all $e \in E$.
Let (P , E)∩(Q , E)=(H , E). Since H(e)=P(e)∩ Q(e) = P(e)for all $e \in E$ then (H , E)= (P , E).
Suppose that (P , E)∩ (Q , E) = (P , E) and let (P , E)∩(Q , E)= (H , E).
Since H(e)= P(e) ∩ Q(e) for all $e \in E$, we know that P(e)⊆ Q(e) for all $e \in E$.
Hence (P , E)⊆(Q , E).

(ii) Suppose that (P , E) ⊆(Q , E) then P(e)⊆Q(e) for all $e \in E$.
Let (P , E)∪(Q , E)=(H , E). Since H(e)=P(e)∪ Q(e) = Q(e)for all $e \in E$ then (H , E)= (Q , E).
Suppose that (P , E)∪ (Q , E) = (Q , E) and let (P , E)∪(Q , E)= (H , E).
Since H(e)= P(e) ∪ Q(e) for all $e \in E$, we know that P(e)⊆ Q(e) for all $e \in E$.
Hence (P , E)⊆(Q , E)..

Proposition 3.10. Let U be an initial universal set and E be a set of parameters.Let (P , E), (Q , E), (H , E) and (K , E) be NSCS over U. Then the following are true.

(i) If (P , E) $\cap$ (Q , E) = Φ, then (P , E)$\subseteq$(Q , E)^c

(ii) If (P , E) $\subseteq$ (Q , E) and (Q , E) $\subseteq$ (H , E) then (P , E) $\subseteq$ (H , E)

(iii) If (P , E) $\subseteq$ (Q , E) and (H , E) $\subseteq$ (K , E) then (P , E) $\cap$ (H , E)$\subseteq$ (Q , E) $\cap$ (K , E)

(iv) (P , E) $\subseteq$ (Q , E) iff (Q , E)^c $\subseteq$ (P , E)^c

Proof:

(i) Suppose that $(P, E) \cap (Q, E) = \Phi$, then P(e)$\cap$ Q(e) = Φ.
So $P(e) \subseteq U \setminus Q(e) = Q^c(e)$ for all $e \in E$.
Therefore we have $(P, E) \subseteq (Q, E)^c$.

(ii) (P , E) $\subseteq$ (Q , E) and (Q , E) $\subseteq$ (H , E)
$\Rightarrow$ P(e)$\subseteq$ Q(e) and Q(e)$\subseteq$ H(e)for all $e \in E$
$\Rightarrow$ P(e)$\subseteq$ Q(e)$\subseteq$ H(e)for all $e \in E$
$\Rightarrow$ P(e)$\subseteq$ H(e) for all $e \in E$.

(iii) (P , E) $\subseteq$ (Q , E) and (H , E) $\subseteq$ (K , E)for all $e \in E$.
$\Rightarrow$ P(e)$\subseteq$ Q(e) and H(e)$\subseteq$ K(e)for all $e \in E$
$\Rightarrow$ (P , E) $\cap$ (H , E) $\subseteq$ (Q , E) $\cap$ (K , E)for all $e \in E$..

(iv) (P , E) $\subseteq$ (Q , E) $\Leftrightarrow$ P(e)$\subseteq$ Q(e) for all $e \in E$
$\Leftrightarrow$ (Q(e))^c $\subseteq$ (P(e))^c for all $e \in E$
$\Leftrightarrow$ Q ^{c}(e) $\subseteq$ P ^{c}(e) for all $e \in E$
$\Leftrightarrow$ (Q , E)^c $\subseteq$ (P , E)^c.

Definition 3.11. Let I be an arbitrary indexed set and $\{(P_i, E)\}_{i \in I}$ be a subfamily of NSCS over U with parameter set E.

(i) The union of these NSCS is the NSCS (H , E) where $H(e) = \bigcup_{i \in I} P_i(e)$ for each $e \in E$. We write $\bigcup_{i \in I} (P_i, E) = (H, E)$.

(ii) The intersection of these NSCS is the NSCS (K , E) where $K(e) = \bigcap_{i \in I} P_i(e)$ for each $e \in E$. We write $\bigcap_{i \in I} (P_i, E) = (K, E)$.

Proposition 3.12. Let I be an arbitrary indexed set and $\{(P_i, E)\}_{i \in I}$ be a subfamily of NSCS over U with parameter set E. Then

(i) $\left(\bigcup_{i \in I} (P_i, E) \right)^c = \bigcap_{i \in I} (P_i, E)^c$

(ii) $\left(\bigcap_{i \in I} (P_i, E) \right)^c = \bigcup_{i \in I} (P_i, E)^c$

Proof:

(i) Let $\left(\bigcup_{i\in I}(P_i,E)\right)=(H,E)$ implies $\left(\bigcup_{i\in I}(P_i,E)\right)^c=(H,E)^c$.
Then $H^c(e)=U\setminus H(e)=U\setminus\bigcup_{i\in I}P_i(e)=\bigcap_{i\in I}(U\setminus P_i(e))$ for all $e\in E$ ——(1).
On the otherhand, $\bigcap_{i\in I}(P_i,E)^c=(K,E)$, by definition
$K(e)=\bigcap_{i\in I}P_i^c(e)=\bigcap_{i\in I}(U\setminus P_i(e))$ for all $e\in E$ ——(2).
From (1) and (2) we have the result.

(ii) Let $\left(\bigcap_{i\in I}(P_i,E)\right)=(H,E)$ implies $\left(\bigcap_{i\in I}(P_i,E)\right)^c=(H,E)^c$.
Then $H^c(e)=U\setminus H(e)=U\setminus\bigcap_{i\in I}P_i(e)=\bigcup_{i\in I}(U\setminus P_i(e))$ for all $e\in E$ ——(1).
On the otherhand, $\bigcup_{i\in I}(P_i,E)^c=(K,E)$, by definition
$K(e)=\bigcup_{i\in I}P_i^c(e)=\bigcup_{i\in I}(U\setminus P_i(e))$ for all $e\in E$ ——(2).
From (1) and (2) we have the result.

Proposition 3.13. Let U be an initial universal set and E be a set of parameters.

(i) $(\Phi,E)^c=(U,E)$.

(ii) $(U,E)^c=(\Phi,E)$.

Proof:

(i) Let $(\Phi,E)=(P,E)$, then for all $e\in E$,

$$\begin{aligned}
P(e) &= \{(x,\tilde{A}_e^T(x),\tilde{A}_e^I(x),\tilde{A}_e^F(x)),\\
&\quad \lambda_e^T(x),\lambda_e^I(x),\lambda_e^F(x):x\in U\}\\
&= \{x,(\tilde{0},\tilde{0},\tilde{0})(0,0,0);x\in U\}\\
(\Phi,E)^c &= (P,E)^c, \text{then for all} e\in E\\
&= \{(x,\tilde{A}_e^T(x),\tilde{A}_e^I(x),\tilde{A}_e^F(x)),\\
&\quad \lambda_e^T(x),\lambda_e^I(x),\lambda_e^F(x):x\in U\}^c\\
&= \{x,1-\tilde{A}_e^T(x),1-\tilde{A}_e^I(x),\\
&\quad 1-\tilde{A}_e^F(x),1-\lambda^T(e)(x),\\
&\quad 1-\lambda_e^I(x),1-\lambda_e^F(x):x\in U\}\\
&= \{x,[1-A_e^{+T},1-A_e^{-T}](x),\\
&\quad [1-A_e^{+I},1-A_e^{-I}](x),\\
&\quad [1-A_e^{-F},1-A_e^{+F}](x),\\
&\quad 1-\lambda^T(e)(x),1-\lambda_e^I(x),\\
&\quad 1-\lambda_e^F(x):x\in U\}\\
&= \{(x,(\hat{1},\hat{1},\hat{1}),(1,1,1));x\in U\}.
\end{aligned}$$

$$Thus(\Phi,E)^c=(U,E).$$

(ii) Let $(U, E) = (P, E)$, then for all $e \in E$,

$$\begin{aligned}
P(e) &= \{(x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \\
&\quad \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U\} \\
&= \{x, (\tilde{1}, \tilde{1}, \tilde{1})(1, 1, 1); x \in U\} \\
(U, E)^c &= (P, E)^c, then for all e \in E \\
&= \{(x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \\
&\quad \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x) : x \in U\}^c \\
&= \{x, 1 - \tilde{A}_e^T(x), 1 - \tilde{A}_e^I(x), \\
&\quad 1 - \tilde{A}_e^F(x), 1 - \lambda^T(e)(x), \\
&\quad 1 - \lambda_e^I(x), 1 - \lambda_e^F(x) : x \in U\} \\
&= \{x, [1 - A_e^{+T}, 1 - A_e^{-T}](x), \\
&\quad [1 - A_e^{+I}, 1 - A_e^{-I}](x), \\
&\quad [1 - A_e^{-F}, 1 - A_e^{+F}](x), \\
&\quad 1 - \lambda^T(e)(x), 1 - \lambda_e^I(x), \\
&\quad 1 - \lambda_e^F(x) : x \in U\} \\
&= \{(x, (\hat{0}, \hat{0}, \hat{0}), (0, 0, 0)); x \in U\}. \\
Thus (U, E)^c = (\Phi, E)
\end{aligned}$$

Proposition 3.14. Let U be an initial universal set and E be a set of parameters.

(i) $(P, E) \cup (\Phi, E) = (P, E)$.

(ii) $(P, E) \cup (U, E) = (U, E)$.

Proof:

(i) $(P, E) = \{(x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)) : x \in U\}\ \forall e \in E$
$(\Phi, E) = \{(x, (\tilde{0}, \tilde{0}, \tilde{0})(0, 0, 0)) : x \in U\}\ \forall e \in E$
$(P, E) \cup (\Phi, E)$
$= \{(x, max(\tilde{A}_e^T(x), \tilde{0}), max(\tilde{A}_e^I(x), \tilde{0}), max(\tilde{A}_e^F(x), \tilde{0}), max(\lambda_e^T(x), 0), max(\lambda_e^I(x), 0), max(\lambda_e^F(x), 0) :$
$x \in U\}\ \forall e \in E$
= $\{(x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)); x \in U\}\ \forall e \in E$
= (P, E).
Thus $(P, E) \cup (\Phi, E) = (P, E)$

(ii) $(P, E) = \{(x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)) : x \in U\}\ \forall e \in E$
$(\Phi, E) = \{(x, (\tilde{1}, \tilde{1}, \tilde{1})(1, 1, 1)) : x \in U\}\ \forall e \in E$
$(P, E) \cup (U, E)$
$= \{(x, max(\tilde{A}_e^T(x), \tilde{1}), max(\tilde{A}_e^I(x), \tilde{1}), max(\tilde{A}_e^F(x), \tilde{1}), max(\lambda_e^T(x), 1), max(\lambda_e^I(x), 1), max(\lambda_e^F(x), 1)) :$
$x \in U\}\ \forall e \in E$
= $\{(x, (\tilde{1}, \tilde{1}, \tilde{1})(1, 1, 1)) : x \in U\}\ \forall e \in E$
= (U, E).
Thus $(P, E) \cup (U, E) = (U, E)$

Proposition 3.15. Let U be an initial universal set and E be a set of parameters.

(i) $(P, E) \cap (\Phi, E) = (\Phi, E)$.

(ii) $(P, E) \cap (U, E) = (P, E)$.

Proof:

(i) $(P, E) = \{e, (x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)) : x \in U\} \forall e \in E$
$(\Phi, E) = \{e, (x, (\tilde{0}, \tilde{0}, \tilde{0})(0, 0, 0)); x \in U\} \forall e \in E$
$(P, E) \cap (\Phi, E) = \{e, (x, min(\tilde{A}_e^T(x), \tilde{0}), min(\tilde{A}_e^I(x), \tilde{0}), min(\tilde{A}_e^F(x), \tilde{0}),$
$min(\lambda_e^T(x), 0), min(\lambda_e^I(x), 0), min(\lambda_e^F(x), 1) : x \in U\} \forall e \in E$
= $\{e, (x, (\tilde{0}, \tilde{0}, \tilde{0})(0, 0, 0)); x \in U\} \forall e \in E$ = (Φ, E).
Thus $(P, E) \cap (\Phi, E) = (\Phi, E)$

(ii) $(P, E) = \{e, (x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)) : x \in U\} \forall e \in E$
$(U, E) = \{e, (x, (\tilde{1}, \tilde{1}, \tilde{1})(1, 1, 1)); x \in U\} \forall e \in E$
$(P, E) \cap (U, E) = \{e, (x, min(\tilde{A}_e^T(x), \tilde{1}), min(\tilde{A}_e^I(x), \tilde{1}), min(\tilde{A}_e^F(x), \tilde{1}),$
$min(\lambda_e^T(x), 1), min(\lambda_e^I(x), 1), min(\lambda_e^F(x), 1) : x \in U\} \forall e \in E$
=$\{e, (x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)) : x \in U\} \forall e \in E$.
Thus $(P, E) \cap (U, E) = (P, E)$

Proposition 3.16. Let U be an initial universal set , E be a set of parameters and $M, N \subseteq E$.

(i) $(P, M) \cup (\Phi, N) = (P, M)$ iff $N \subseteq M$.

(ii) $(P, M) \cup (U, N) = (U, N)$ iff $M \subseteq N$.

Proof:

(i) For (P , M) we have
$P(e) = \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}\ \forall e \in M$
Also let $(\Phi, N) = (Q, N)$, then
$Q(e) = \{\langle x, (\tilde{0}, \tilde{0}, \tilde{0})(0, 0, 0)\rangle; x \in U\} \forall e \in N$
Let $(P, M) \cup (\Phi, N)$ = $(P, M) \cup (Q, N)$ = (H, C)
where $C = M \cup N$ and for all $e \in C$

$$H(e) = \begin{cases} P(e) & \text{if e} \in M - N \\ Q(e) & \text{if e} \in N - M \\ P(e) \cap Q(e) & \text{if e} \in M \cap N \end{cases}$$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), & \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}, & \text{if e} \in M - N \\ \{\langle x, \tilde{B}_e^T(x), \tilde{B}_e^I(x), \tilde{B}_e^F(x), & \\ \mu_e^T(x), \mu_e^I(x), \mu_e^F(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, max(\tilde{A}_e(x), \tilde{B}_e(x)), & \\ max(\lambda_e(x), \mu_e(x))\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\} & \text{if e} \in M - N \\ \{\langle x, (\tilde{0}, \tilde{0}, \tilde{0})(0,0,0)(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, max(\tilde{A}_e^T(x), \tilde{0}), \\ max(\tilde{A}_e^I(x), \tilde{0}), \\ max(\tilde{A}_e^F(x), \tilde{0}), \} \\ \{max(\lambda_e(x), 0), max(\lambda_e(x), 0), \\ max(\lambda_e(x), 0\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}, & \text{if e} \in M - N \\ \{\langle x, (\tilde{0}, \tilde{0}, \tilde{0})(0,0,0)(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

Let $N \subseteq M$, then

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}, & \text{if e} \in M - N \\ \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

=P(e),$\forall e \in M$.
Conversely, Let $(P, M)\cup(\Phi, N) = (P, M)$
Then $M = M \cup N \Rightarrow N \subseteq M$

(ii) For (P , M) we have
$P(e) = \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)), \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}\ \forall e \in M$
Also let $(U, N) = (Q, N)$, then
$Q(e) = \{\langle x, (\tilde{1}, \tilde{1}, \tilde{1})(1, 1, 1)\rangle; x \in U\}\forall e \in N$
Let $(P, M)\cup(U, N)$ = $(P, M)\cup(Q, N)$ = (H, C)
where $C = M \cup N$ and for all $e \in C$

$$H(e) = \begin{cases} P(e) & \text{if e} \in M - N \\ Q(e) & \text{if e} \in N - M \\ P(e) \cup Q(e) & \text{if e} \in M \cap N \end{cases}$$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}, & \text{if e} \in M - N \\ \{\langle x, \tilde{B}_e^T(x), \tilde{B}_e^I(x), \tilde{B}_e^F(x), \\ \mu_e^T(x), \mu_e^I(x), \mu_e^F(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, max(\tilde{A}_e(x), \tilde{B}_e(x)), \\ max(\lambda_e(x), \mu_e(x))\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\} & \text{if e} \in M - N \\ \{\langle x, (\tilde{1}, \tilde{1}, \tilde{1})(1,1,1)(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, max(\tilde{A}_e^T(x), \tilde{1}), \\ max(\tilde{A}_e^I(x), \tilde{1}), \\ max(\tilde{A}_e^F(x), \tilde{1}), \} \\ \{max(\lambda_e(x), 1), max(\lambda_e(x), 1), \\ max(\lambda_e(x), 1\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x), \\ \lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \in U\}, & \text{if e} \in M - N \\ \{\langle x, (\tilde{1}, \tilde{1}, \tilde{1})(1,1,1)(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, (\tilde{1}, \tilde{1}, \tilde{1})(1,1,1)(x)\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

Let $M \subseteq N$, then

$$H(e) = \begin{cases} \{\langle x, (\tilde{1}, \tilde{1}, \tilde{1})(1,1,1)(x)\rangle : x \in U\}, & \text{if e} \in N - M \\ \{\langle x, (\tilde{1}, \tilde{1}, \tilde{1})(1,1,1)(x)\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

=(U, N),$\forall e \in M$.

Conversely, Let $(P, M) \cup (U, N) = (U, N)$

Then $N = M \cup N \Rightarrow M \subseteq N$

Proposition 3.17. Let U be an initial universal set , E be a set of parameters and $M, N \subseteq E$.

(i) $(P, M) \cap (\Phi, N) = (\Phi, M \cap N)$.

(ii) $(P, M) \cap (U, N) = (P, M \cap N)$.

Proof:

(i) For (P , M) we have
$P(e) = \{\langle x, \tilde{A}_e^T(x), \tilde{A}_e^I(x), \tilde{A}_e^F(x)\lambda_e^T(x), \lambda_e^I(x), \lambda_e^F(x)\rangle : x \epsilon U\}\ e \epsilon M$
Also let $(\Phi, N) = (Q, N)$, then
$Q(e) = \{x, (\tilde{0}, \tilde{0}, \tilde{0}), (0, 0, 0); x \in U\}\ \forall e \in N$
Let $(P, M) \cap (\Phi, N)$ = $(P, M) \cap (Q, N)$ = (H, C)
where $C = M \cap N$ and for all $e \in C$

$H(e) = \Big\{\langle x, min(\tilde{A}_e(x), \tilde{B}_e(x)), min(\lambda_e(x), \mu_e(x))\rangle : x \in U\Big\}$
$= \{(x, min(\tilde{A}^T_e(x), \tilde{0}), min(\tilde{A}^I_e(x), \tilde{0}), min(\tilde{A}^F_e(x), \tilde{0}),$
$min(\lambda^T_e(x), 0), min(\lambda^I_e(x), 0), min(\lambda^F_e(x), 0)) : x \in U\}$
$= \{(x, (\tilde{0}, \tilde{0}, \tilde{0})(0, 0, 0)); x \in U\}$ for all $e \in C$.
= (Q , C) = (Φ, C).
Thus $(P, M)\cap(\Phi, N) = (\Phi, M \cap N)$.

(ii) For (P , M) we have
$P(e) = \{\langle x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)\lambda^T_e(x), \lambda^I_e(x), \lambda^P_e(x)\rangle : x\epsilon U\}\ e\epsilon M$
Also let $(U, N) = (Q, N)$, then
$Q(e) = \{x, (\tilde{1}, \tilde{1}, \tilde{1}), (1, 1, 1); x \in U\}\ \forall e \in N$
Let $(P, M)\cap(U, N)$ = $(P, M)\cap(Q, N)$ = (H, C)
where $C = M \cap N$ and for all $e \in C$
$H(e) = \Big\{\langle x, min(\tilde{A}_e(x), \tilde{B}_e(x)), min(\lambda_e(x), \mu_e(x))\rangle : x \in U\Big\}$
$= \{(x, min(\tilde{A}^T_e(x), \tilde{1}), min(\tilde{A}^I_e(x), \tilde{1}), min(\tilde{A}^F_e(x), \tilde{1}),$
$min(\lambda^T_e(x), 1), min(\lambda^I_e(x), 1), min(\lambda^F_e(x), 1)) : x \in U\}$
= $\{\langle x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x)\lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x)\rangle : x\epsilon U\}$ for all $e \in C$
= $P(e) = (P, C)$.
Thus $(P, M)\cap(U, N) = (P, M \cap N)$.

Proposition 3.18. Let U be an initial universal set, E be a set of parameters and $M, N \subseteq E$.

(i) $((P, M)\cup(Q, N))^c \subseteq (P, M)^c \cup (Q, N)^c$.

(ii) $(P, M)^c \cap (Q, N)^c \subseteq ((P, M)\cap(Q, N))^c$.

Proof:

(i) Let $(P, M)\cup(Q, N) = (H, C)$ where $C = M \cup N$ and $\forall e \in C$

$$H(e) = \begin{cases} \{\langle x, \tilde{A}^T_e(x), \tilde{A}^I_e(x), \tilde{A}^F_e(x), \lambda^T_e(x), \lambda^I_e(x), \lambda^F_e(x)\rangle : x \in U\}, & \text{if e} \in M - N \\ \{\langle x, \tilde{B}^T_e(x), \tilde{B}^I_e(x), \tilde{B}^F_e(x), \mu^T_e(x), \mu^I_e(x), \mu^F_e(x)\rangle : x \in U\} & \text{if e} \in N - M \\ \{\langle x, max(\tilde{A}_e(x), \tilde{B}_e(x)), max(\lambda_e(x), \mu_e(x))\rangle : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

Thus $((P, M)\cup(Q, N))^c = (H, C)^c$, where $C = M \cup N$ and $\forall e \in C$

$$(H(e))^c = \begin{cases} \{(P(e))^c, & \text{if e} \in M - N \\ \{(Q(e))^c & \text{if e} \in N - M \\ \{(P(e) \cup Q(e))^c & \text{if e} \in N \cap M \end{cases}$$

$$= \begin{cases} \{(x, 1 - \tilde{A}^T_e(x), 1 - \tilde{A}^I_e(x), 1 - \tilde{A}^F_e(x), \\ 1 - \lambda^T(e)(x), 1 - \lambda^I_e(x), 1 - \lambda^F_e(x) : x \in U\} & \text{if e} \in M - N \\ \{(x, 1 - \tilde{B}^T_e(x), 1 - \tilde{B}^I_e(x), 1 - \tilde{B}^F_e(x), \\ 1 - \mu^T(e)(x), 1 - \mu^I_e(x), 1 - \mu^F_e(x) : x \in U\} & \text{if e} \in N - M \\ \{(x, 1 - max(\tilde{A}^T_e(x), \tilde{B}^T_e(x)), 1 - max(\tilde{A}^I_e(x), \tilde{B}^I_e(x)), 1 - max(\tilde{A}^F_e(x), \tilde{B}^F_e(x)), \\ 1 - max(\lambda^T_e(x), \mu^T_e(x)), 1 - max(\lambda^I_e(x), \mu^I_e(x)), 1 - max(\lambda^F_e(x), \mu^F_e(x)) : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

Again $(P,M)^c \cup (Q,N)^c = (I,J)$ say J = M $\cup$ N and $\forall e \in J$

$$I(e) = \begin{cases} \{(P(e))^c, & \text{if e} \in M-N \\ \{(Q(e))^c & \text{if e} \in N-M \\ \{(P(e) \cup Q(e))^c & \text{if e} \in M \cap N \end{cases}$$

$$= \begin{cases} \{(x, 1-P_e^T(x), 1-P_e^I(x), 1-P_e^F(x)) : x \in U\}, & \text{if e} \in M-N \\ \{(x, 1-Q_e^T(x), 1-Q_e^I(x), 1-Q_e^F(x)) : x \in U\} & \text{if e} \in N-M \\ \{(x, 1-max(\tilde{A}_e^T(x), \tilde{B}_e^T(x)), 1-max(\tilde{A}_e^I(x), \tilde{B}_e^I(x)), 1-max(\tilde{A}_e^F(x), \tilde{B}_e^F(x)), & \\ 1-max(\lambda_e^T(x), \mu_e^T(x)), 1-max(\lambda_e^I(x), \mu_e^I(x)), 1-max(\lambda_e^F(x), \mu_e^F(x)) : x \in U\} & \text{if e} \in M \cap N \end{cases}$$

C $\subseteq$ J $\forall e \in J$. $(H(e))^c \subseteq I(e)$. Thus $((P,M) \cup (Q,N))^c \subseteq (P,M)^c \cup (Q,N)^c$

(ii) Let $(P,M) \cap (Q,N) = (H,C)$ where $C = M \cap N$ and $\forall e \in C$ and
H(e)= P(e)$\cap$ Q(e) =
$\{(x, min(P_e^T(x), Q_e^T(x)), min(P_e^I(x), Q_e^I(x)), \min(P_e^F(x), Q_e^F(x)))\}$
where
$min(P_e^T(x), Q_e^T(x))= min(\tilde{A}_e^T(x), \tilde{B}_e^T(x)), min(\lambda_e^T(x), \mu_e^T(x)),$
$min(P_e^I(x), Q_e^I(x)) = min(\tilde{A}_e^I(x), \tilde{B}_e^I(x)), min(\lambda_e^I(x), \mu_e^I(x))$
$min(P_e^F(x), Q_e^F(x)))=min(\tilde{A}_e^F(x), \tilde{B}_e^F(x)), min(\lambda_e^F(x), \mu_e^F(x))$
Thus $((P,M) \cap (Q,N))^c = (H,C)^c$, where $C = M \cap N$ and $\forall e \in C$
$(H(e))^c = \{(x, min(P_e^T(x), Q_e^T(x)), min(P_e^I(x), Q_e^I(x)), min(P_e^F(x), Q_e^F(x)))\}^c$
$= \{(x, 1-min(P_e^T(x), Q_e^T(x)), 1-min(P_e^I(x), Q_e^I(x)), 1-min(P_e^F(x), Q_e^F(x)))\}$
Again $(P,M)^c \cap (Q,N)^c = (I,J)$ say J = M $\cap$ N and $\forall e \in J$
$I(e) = (P(e))^c \cap (Q(e))^c$
$= \{(x, 1-min(P_e^T(x), Q_e^T(x)), 1-min(P_e^I(x), Q_e^I(x)), 1-min(P_e^F(x), Q_e^F(x)))\}$
We see that C = J and $\forall e \in J$, I(e)$\subseteq$ $(H(e))^c$.
Thus $(P,M)^c \cap (Q,N)^c \subseteq ((P,M) \cup (Q,N))^c$.

Proposition 3.19. (De Morgan's Law) For neutrosophic soft cubic sets (P , E)and (Q , E) over the same universe U with parameter set E, we have the following.

(i) $((P,E) \cup (Q,E))^c = (P,E)^c \cap (Q,E)^c$.

(ii) $((P,E) \cap (Q,E))^c = (P,E)^c \cup (Q,E)^c$.

Proof:

(i) Let $(P,E) \cup (Q,E) = (H,E)$ where $\forall e \in E$
$H(e) = P(e) \cup Q(e)$
$= \{(x, max(P_e^T(x), Q_e^T(x)), max(P_e^I(x), Q_e^I(x)), max(P_e^F(x), Q_e^F(x)))\}$
Thus $((P,E) \cup (Q,E))^c = (H,E)^c$ where $\forall e \in E$

$H(e)^c = (P(e) \cup Q(e))^c$
$=\{(x, max(P_e^T(x), Q_e^T(x)), max(P_e^I(x), Q_e^I(x)), max(P_e^F(x), Q_e^F(x)))\}^c$
$= \{(x, 1-max(P_e^T(x), Q_e^T(x)), 1-max(P_e^I(x), Q_e^I(x)), 1-max(P_e^F(x), Q_e^F(x)))\}$
Again $(P,E)^c \cap (Q,E)^c = (I,E)$ say $\forall e \in E$

$I(e) = (P(e))^c \cap (P(e))^c$
$= \{(x, min(1 - P_e^T(x), 1 - Q_e^T(x)), min(1 - P_e^I(x), 1 - Q_e^I(x)), min(1 - P_e^F(x), 1 - Q_e^F(x)))\}^c$
$= \{(x, 1 - max(P_e^T(x), Q_e^T(x)), 1 - max(P_e^I(x), Q_e^I(x)), 1 - max(P_e^F(x), Q_e^F(x))\}^c$
Thus $((P, E) \cup (Q, E))^c = (P, E)^c \cap (Q, E)^c$.

(ii) Let $(P, E) \cap (Q, E) = (H, E)$ where $\forall e \in E$
$H(e) = P(e) \cap Q(e)$
$= \{(x, min(P_e^T(x), Q_e^T(x)), min(P_e^I(x), Q_e^I(x)), min(P_e^F(x), Q_e^F(x)))\}$
Thus $((P, E) \cap (Q, E))^c = (H, E)^c$ where $\forall e \in E$
$H(e)^c = (P(e) \cap Q(e))^c$
$=\{(x, min(P_e^T(x), Q_e^T(x)), min(P_e^I(x), Q_e^I(x)), min(P_e^F(x), Q_e^F(x)))\}^c$
$= \{(x, 1 - min(P_e^T(x), Q_e^T(x)), 1 - min(P_e^I(x), Q_e^I(x)), 1 - min(P_e^F(x), Q_e^F(x)))\}$
Again $(P, E)^c \cup (Q, E)^c = (I, E)$ say $\forall e \in E$
$I(e) = (P(e))^c \cup (P(e))^c$
$= \{(x, max(1 - P_e^T(x), 1 - Q_e^T(x)), max(1 - P_e^I(x), 1 - Q_e^I(x)), max(1 - P_e^F(x), 1 - Q_e^F(x)))\}^c$
$= \{(x, 1 - min(P_e^T(x), Q_e^T(x)), 1 - min(P_e^I(x), Q_e^I(x)), 1 - min(P_e^F(x), Q_e^F(x))\}^c$
Thus $((P, E) \cap (Q, E))^c = (P, E)^c \cup (Q, E)^c$.

Proposition 3.20. Let U be an initial universal set , E be a set of parameters and $A, B \subseteq E$.

(i) $((P, M) \wedge (Q, N))^c = (P, M)^c \vee (Q, N)^c$.

(ii) $((P, M) \vee (Q, N))^c = (P, M)^c \wedge (Q, N)^c$.

Proof:

(i) Let $(P, M) \wedge (Q, N) = (H, M \times N)$ where
$H(m, n) =$
$\{(x, min(P_e^T(x), Q_e^T(x)), min(P_e^I(x), Q_e^I(x)), min(P_e^F(x), Q_e^F(x)))\}$
$\forall m \in M$ and $\forall n \in N$
$((P, M) \wedge (Q, N))^c = (H, M \times N)^c \; \forall (a, b) \in M \times N$
$(H(m, n))^c =$
$\{(x, 1{-}min(P_e^T(x), Q_e^T(x)), 1{-}min(P_e^I(x), Q_e^I(x)), 1{-}min(P_e^F(x), Q_e^F(x)))\}^c \; \forall m \in M$ and $\forall n \in N$
Let $(P, M)^c \vee (Q, N)^c = (R, M \times N)$ where
$R(m, n) =$
$\{(x, max(1 - P_e^T(x), 1 - Q_e^T(x)),$
$max(1 - P_e^I(x), 1 - Q_e^I(x)), max(1 - P_e^F(x), 1 - Q_e^F(x)))\} \; \forall m \in M$ and $\forall n \in N$
$= \{(x, 1 - min(P_e^T(x), Q_e^T(x)), 1 - min(P_e^I(x), Q_e^I(x)), 1 - min(P_e^F(x), Q_e^F(x)))\} \; \forall m \in M$ and $\forall n \in N$.
Thus $((P, M) \wedge (Q, N))^c = (P, M)^c \vee (Q, N)^c$

(ii) Let $(P, M) \vee (Q, N) = (H, M \times N)$ where
$H(m, n) =$
$\{(x, max(P_e^T(x), Q_e^T(x)), max(P_e^I(x), Q_e^I(x)), max(P_e^F(x), Q_e^F(x)))\}$
$\forall m \in M$ and $\forall n \in N$
$((P, M) \vee (Q, N))^c = (H, M \times N)^c \; \forall (a, b) \in M \times N$
$(H(m, n))^c =$

$\{(x, 1-max(P_e^T(x), Q_e^T(x)), 1-max(P_e^I(x), Q_e^I(x)), 1-max(P_e^F(x), Q_e^F(x)))\}^c$ $\forall m \in M$ and $\forall n \in N$
Let $(P, M)^c \wedge (Q, N)^c = (R, M \times N)$ where
$R(m, n) =$
$\{(x, min(1 - P_e^T(x), 1 - Q_e^T(x)),$
$min(1 - P_e^I(x), 1 - Q_e^I(x)), min(1 - P_e^F(x), 1 - Q_e^F(x)))\}$ $\forall m \in M$ and $\forall n \in N$
$= \{(x, 1 - max(P_e^T(x), Q_e^T(x)), 1 - max(P_e^I(x), Q_e^I(x)), 1 - max(P_e^F(x), Q_e^F(x)))\}$ $\forall m \in M$ and $\forall n \in N$.
Thus $((P, M) \vee (Q, N))^c = (P, M)^c \wedge (Q, N)^c$

4 Neutrosophic soft cubic topological spaces

In this section, we give the definition of neutrosophic soft cubic topological spaces with some examples and results

Let U be an universe set, E be the set of parameters,$\wp(U)$ be the set of all subsets of U, NSCS(U) be the set of all neutrosophic soft cubic sets in U and NSCS(U;E) be the family of all neutrosophic soft cubic sets over U via parameters in E.

Definition 4.1. Let(X, E)be an element of $NSCS(U; E)$, $\wp(X; E)$ be the collection of all neutrosophic soft cubic subsets of(X, E) .A sub family τ of $\wp(X; E)$is called neutrosophic soft cubic topology (in short NSCS-topology) on (X,E) if the following conditions hold.

(i) $(\Phi, E), (X, E) \in \tau$.

(ii) (P , E), (Q , E) $\in \tau$ implies $(P, E) \cap (Q, E) \in \tau$.

(iii) $\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau$ implies $\bigcup \{(P_\alpha, E); \alpha \in \Gamma\} \in \tau$

The triplet (X, τ, E) is called a neutrosophic soft cubic topological space (in short NSCTS) over (X,E).
Every member of τ is called a neutrosophic soft cubic open set in (X,E)(in short NSCOP(X)).
$\Phi : A \longrightarrow NSCS(U)$ is defined as
$\Phi(e) = \{x, ([0, 0], [0, 0], [0, 0]), (0, 0, 0) : x \in X\} \forall e \in A \subseteq E$.
A neutrosophic soft cubic subset (P , E) of (X,E) is called a neutrosophic soft cubic closed set in (X,E)(in short NSCCS(X)) if $(P, E) \in \tau^c$ where $\tau^c = \{(P, E)^c : (P, E) \in \tau\}$

Example 4.2. Let $X = \{x_1, x_2, x_3\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\}$. The tabular representations which are shown in Table1-6.

X	e_1	e_2	e_3
x_1	$([0.5, 0.8], [0.3, 0.5], [0.2, 0.7])(0.8, 0.4, 0.5)$	$([0.4, 0.7], [0.2, 0.3], [0.1, 0.3])(0.7, 0.7, 0.5)$	$([0.3, 0.9], [0, 0.1], [0, 0.2])(0.7, 0.5, 0.6)$
x_2	$([0.5, 1], [0, 0.1], [0.3, 0.6])(0.5, 0.4, 0.5)$	$([0.6, 0.8], [0.2, 0.4], [0.1, 0.3])(0.6, 0.7, 0.6)$	$([0.4, 0.9], [0.1, 0.3], [0.2, 0.4])(0.6, 0.5, 0.7)$
x_3	$([0.4, 0.7], [0.3, 0.4], [0.1, 0.2])(0.9, 0.5, 0.6)$	$([0.6, 0.9], [0.1, 0.2], [0.10.2])(0.8, 0.8, 0.5)$	$([0.4, 0.8], [0.1, 0.2], [0, 0.5])(0.8, 0.6, 0.6)$

Table 1: The tabular representation of (X, E).

Here the sub-family $\tau_1 = \{(\Phi, E), (X, E), (P, E), (Q, E), (H, E), (L, E)\}$ of $\wp(X, E)$ is a neutrosophic soft cubic topology on (X,E), as it satisfies the necessary three axioms of topology and (X, τ, E) is a NSCTS. But the sub-family $\tau_2 = \{(\Phi, E), (X, E), (P, E), (Q, E)\}$ of $\wp(X, E)$ is not a neutrosophic soft cubic topology on (X,E), as the union $(P, E) \cup (Q, E)$ does not belong to τ_2

X	e_1	e_2	e_3
x_1	$([0,0],[0,0],[0,0])(0,0,0)$	$([0,0],[0,0],[0,0])(0,0,0)$	$([0,0],[0,0],[0,0])(0,0,0)$
x_2	$([0,0],[0,0],[0,0])(0,0,0)$	$([0,0],[0,0],[0,0])(0,0,0)$	$([0,0],[0,0],[0,0])(0,0,0)$
x_3	$([0,0],[0,0],[0,0])(0,0,0)$	$([0,0],[0,0],[0,0])(0,0,0)$	$([0,0],[0,0],[0,0])(0,0,0)$

Table 2: The tabular representation of (Φ, E).

X	e_1	e_2	e_3
x_1	$([0.1,0.7],[0.4,0.8],[0.3,1])(0.8,0.4,0.5)$	$([0.1,0.3],[0.4,0.6],[0.2,0.6])(0.4,0.5,0.3)$	$([0.2,0.5],[0.8,0.9],[0.4,0.9])(0.5,0.5,0.4)$
x_2	$([0.4,0.8],[0.6,0.7],[0.6,0.9])(0.5,0.4,0.5)$	$([3,0.4],[0.4,0.7],[0.2,0.8])(0.5,0.3,0.3)$	$([0.1,0.3],[0.6,0.8],[0.3,0.7])(0.4,0.5,0.4)$
x_3	$([0.1,0.3],[0.6,0.7],[0.2,0.8])0.7,0.7,0.3$	$([0,0.5],[0.5,0.8],[0.4,1])(0.6,0.7,0.4)$	$([0,0.3],[0.6,0.9],[0.1,0.7])(0.6,0.5,0.5)$

Table 3: The tabular representation of (P, E).

X	e_1	e_2	e_3
x_1	$([0.4,0.7],[0.5,0.7],[0.4,0.9])(0.5,0.4,0.6)$	$([0.2,0.3],[0.4,0.5],[0.7,0.9])(0.5,0.4,0.4)$	$([0.3,0.7],[0.5,0.8],[0.1,0.2])(0.4,0.5,0.4)$
x_2	$([0.3,0.9],[0.1,0.2],[0.6,0.7])(0.6,0.7,0.3)$	$([5,0.6],[0.6,0.7],[0.3,0.4])(0.3,0.4,0.5)$	$([2,0.6],[0.3,0.5],[0.5,0.8])(0.4,0.7,0.5)$
x_3	$([0.3,0.5],[0.4,0.8],[0.1,0.4])(0.5,0.4,0.5)$	$([4,0.6],[0.3,0.5],[0.2,0.5])(0.4,0.6,0.3)$	$([0.1,0.3],[0.3,0.5],[0.6,0.8])(0.3,0.5,0.4)$

Table 4: The tabular representation of (Q, E).

Let $(H, E) = (P, E) \cap (Q, E)$ The tabular representation of (H, E) is given by

X	e_1	e_2	e_3
x_1	$([0.1,0.7],[0.4,0.7],[0.3,0.9])(0.5,0.4,0.5)$	$([0.1,0.3],[0.4,0.5],[0.2,0.6])(0.4,0.4,0.3)$	$([0.2,0.5],[0.5,0.8],[0.1,0.2])(0.4,0.5,0.4)$
x_2	$([0.3,0.8],[0.1,0.2],[0.6,0.7])(0.5,0.4,0.3)$	$([0.3,0.4],[0.4,0.7],[0.2,0.4])(0.3,0.3,0.3)$	$([0.1,0.3],[0.3,0.5],[0.3,0.7])(0.4,0.5,0.4)$
x_3	$([0.1,0.3],[0.4,0.7],[0.1,0.4])(0.5,0.4,0.3)$	$([0,0.5],[0.3,0.5],[0.2,0.5])(0.4,0.6,0.3)$	$([0.0,0.3],[0.3,0.5],[0.1,0.7])(0.3,0.5,0.4)$

Table 5: The tabular representation of (H, E)

Let $(L, E) = (P, E) \cup (Q, E)$ The tabular representation of (L, E) is given by

X	e_1	e_2	e_3
x_1	$([0.4,0.7],[0.5,0.8],[0.4,1])(0.8,0.4,0.6)$	$([0.2,0.3],[0.4,0.6],[0.7,0.9])(0.5,0.5,0.4)$	$([0.3,0.7],[0.8,0.9],[0.4,0.9])(0.5,0.5,0.4)$
x_2	$([0.4,0.9],[0.6,0.7],[0.6,0.9])0.6,0.7,0.5$	$([0.5,0.6],[0.6,0.7],[0.3,0.8])(0.5,0.4,0.5)$	$([0.2,0.6],[0.6,0.8],[0.5,0.8])(0.4,0.7,0.5)$
x_3	$([0.3,0.5],[0.6,0.8],[0.3,0.8])(0.7,0.7,0.5)$	$([0.4,0.6],[0.5,0.8],[0.4,1])(0.6,0.7,0.4)$	$([0.1,0.3],[0.6,0.9],[0.6,0.8])(0.3,0.5,0.4)$

Table 6: The tabular representation of (L, E)

Definition 4.3. As every NSC-topology on (X, E) must contain the sets $(\Phi, E), (X, E) \in \tau$ so the family $\tau = \{(\Phi, E), (X, E)\}$ forms a NSC-topology on (X, E). The topology is called indiscrete NSC-topology and the triplet(X, τ, E) is called an indiscrete neutrosophic soft cubic topological space (or simply indiscrete NSC-topological space).

Definition 4.4. Let ξ denote the family of all NSC-subsets of (X, E). Then we observe that ξ satisfies all the axioms of topology on(X, E). This topology is called discrete neutrosophic soft cubic topology and the triplet (X, ξ, E) is called discrete discrete neutrosophic soft cubic topological space (or simply discrete NSCTS).

Definition 4.5. Let (X, τ, E) be an NSC-topological space over (X, E). A neutrosophic soft cubic subset (P, E) of (X, E) is called neutrosophic soft cubic set (in short NSC-closed set) if its complement $(P, E)^c$ is a member of τ.

Example 4.6. Let us consider Example 4.2 then the NSC-closed set in (X, τ_1, E) are shown in Table7-12.

X	e_1	e_2	
x_1	$([0.5, 0.8], [0.3, 0.5], [0.2, 0.7])(0.8, 0.4, 0.5)$	$([0.4, 0.7], [0.2, 0.3], [0.1, 0.3])(0.7, 0.7, 0.5)$	$([0.3, 0.9], [0, 0.1], [0, 0.2])(0.7, 0.5, 0.6)$
x_2	$([0.5, 1], [0, 0.1], [0.3, 0.6])(0.5, 0.4, 0.5)$	$([0.6, 0.8], [0.2, 0.4], [0.1, 0.3])(0.6, 0.7, 0.6)$	$([0.4, 0.9], [0.1, 0.3], [0.2, 0.4])(0.6, 0.5, 0.7)$
x_3	$([0.4, 0.7], [0.3, 0.4], [0.1, 0.2])(0.9, 0.5, 0.6)$	$([0.6, 0.9], [0.1, 0.2], [0.1, 0.2])(0.8, 0.8, 0.5)$	$([0.4, 0.8], [0.1, 0.2], [0, 0.5])(0.8, 0.6, 0.6)$

Table 7: The tabular representation of $(X, E)^c$

X	e_1	e_2	e_3
x_1	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$
x_2	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$	$([1, 1], [, 1], [1, 1])(1, 1, 1)$	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$
x_3	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$	$([1, 1], [1, 1], [1, 1])(1, 1, 1))$	$([1, 1], [1, 1], [1, 1])(1, 1, 1)$

Table 8: The tabular representation of $(\Phi, E)^c$

X	e_1	e_2	
x_1	$([0.3, 0.9], [0.2, 0.6], [0, 0.7])(0.2, 0.6, 0.5)$	$([0.7, 0.9], [0.4, 0.6], [0.4, 0.8])(0.6, 0.5, 0.7)$	$([0.5, 0.8], [0.1, 0.2], [0.1, 0.6])(0.5, 0.5, 0.6)$
x_2	$([0.2, 0.6], [3, 0.4], [0.1, 0.4])(0.5, 0.6, 0.5)$	$([0.6, 0.7], [0.3, 0.6], [0.2, 0.8])(0.5, 0.7, 0.7)$	$([0.7, 0.9], [0.2, 0.4], [0.3, 0.7])(0.6, 0.5, 0.6)$
x_3	$([0.7, 0.9], [0.3, 0.4], [0.2, 0.8])(0.3, 0.3, 0.7)$	$([0.5, 1], [0.2, 0.5], [0, 0.6])(0.4, 0.3, 0.6)$	$([0.7, 1], [0.1, 0.4], [0.3, 0.9])(0.4, 0.6, 0.6)$

Table 9: The tabular representation of $(P, E)^c$

Proposition 4.7. Let (X, τ_1, E) and (X,τ_2,E) be two neutrosophic soft cubic topological spaces. Denote $\tau_1 \cap \tau_2 = \{(P, E) : (P, E) \in \tau_1 \ and \ (P, E) \in \tau_2\}$. Then $\tau_1 \cap \tau_2$ is a neutrosophic soft cubic topology.
Proof:

(i) Since (X, τ_1, E) is a neutrosophic soft cubic topological space then $(\Phi, E) \in \tau_1$,
(X, τ_2, E) is a neutrosophic soft cubic topological space then $(\Phi, E) \in \tau_2$.
Therefore $(\Phi, E) \in \tau_1 \cap \tau_2$
Since (X, τ_1, E) is a neutrosophic soft cubic topological space then $(X, E) \in \tau_1$,
(X, τ_2, E) is a neutrosophic soft cubic topological space then $(X, E) \in \tau_2$.
Therefore $(X, E) \in \tau_1 \cap \tau_2$

(ii Let (P , E), (Q , E) $\in \tau_1 \cap \tau_2$. Then (P , E), (Q , E) $\in \tau_1$ and (P , E), (Q , E) $\in \tau_2$, τ_1 and τ_2 are two neutrosophic soft cubic topologies on X. Then $(P, E) \cap (Q, E) \in \tau_1$ and $(P, E) \cap (Q, E) \in \tau_2$. Hence $(P, E) \cap (Q, E) \in \tau_1 \cap \tau_2$.

(iii Let $\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau_1 \cap \tau_2$. Then $\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau_1$ and $\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau_2$. Since τ_1 and τ_2 are two neutrosophic soft cubic topologies on X. Then $\bigcup\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau_1$ and $\bigcup\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau_2$.
Thus $\bigcup\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau_1 \cap \tau_2$.

Theorem 4.8. Let $\{\tau_i : i \in I\}$ be any collection of NSC-topology on (X, E). Then their intersection $\bigcap\limits_{i\in I} \tau_i$ is also a NSC-topology on (X, E).
Proof:

(i) Since$(\Phi, E), (X, E) \in \tau_i foreachi \in I$. Hence $(\Phi, E), (X, E) \in \bigcap\limits_{i\in I} \tau_i$.

(ii) Let $\{(P_\alpha, E); \alpha \in \Gamma\}$ be an arbitrary family of neutrosophic soft cubic sets where $\{(P_\alpha, E); \in \bigcap\limits_{i\in I} \tau\}$ for each $\alpha \in \Gamma$. Then for each $i \in I$ $\{(P_\alpha, E); \in \tau$ for $\alpha \in \Gamma\}$ and since for each $i \in I$ is a NSC-topology, therefore for each $\bigcup\limits_{i\in I}(P_\alpha, E); \in \tau_i$for each $i \in I$. Hence $\bigcup\limits_{i\in I}(P_\alpha, E); \in \bigcap\limits_{i\in I} \tau_i$.

(iii) Let $(P, E), (Q, E)X \in \bigcap\limits_{i\in I} \tau_i$, then $(P, E), (Q, E)X \in \tau_i$ for each$i \in I$. Since for each$i \in I$,τ_i is an NSC-topology, therefore $(P, E) \cap (Q, E) \in \tau_i$ for each $i \in I$. Hence $(P, E) \cap (Q, E)X \in \bigcap\limits_{i\in I} \tau_i$. Thus $\bigcap\limits_{i\in I} \tau_i$ satisfies all the axioms of topology. Hence $\bigcap\limits_{i\in I} \tau_i$ forms a NSC-topology. But union of NSC-topologies need not be a NSC-topology. Let us show this with the following example

Remark 4.9. If τ_1 and τ_2 be two neutrosophic soft cubic topologies on (X,E).

(i) $\tau_1 \vee \tau_2 = \{(P, E)\Cup(Q, E) : (P, E) \in \tau_1 \; and \; (Q, E) \in \tau_2\}$.

(ii) $\tau_1 \wedge \tau_2 = \{(P, E)\Cap(Q, E) : (P, E) \in \tau_1 \; and \; (Q, E) \in \tau_2\}$.

Example 4.10. Let (P , E) and (Q , E) be neutrosophic soft cubic set as in Example 28.
Define $\tau_1 = \{(\Phi, E), (X, E), (P, E)\}$, $\tau_2 = \{(\Phi, E), (X, E), (Q, E)\}$.
Then $\tau_1 \cap \tau_2 = \{(\Phi, E), (X, E)\}$ is neutrosophic soft cubic topology on X.
But $\tau_1 \cup \tau_2 = \{(\Phi, E), (X, E), (P, E), (Q, E)\}$,
$\tau_1 \vee \tau_2 = \{(\Phi, E), (X, E), (P, E), (Q, E), (P, E)\Cup(Q, E)\}$,
$\tau_1 \wedge \tau_2 = \{(\Phi, E), (X, E), (P, E), (Q, E), (P, E)\Cap(Q, E)\}$ are not neutrosophic soft cubic topology on(X,E).

Definition 4.11. Let (X, τ, E) be a neutrosophic soft cubic topological space on $(X, E)$and $(Y, E) \in \wp(X, E)$. Then the collection $\tau_Y = \{(Y, E)\Cap(Q, E) : (Q, E) \in \tau\}$ is called a neutrosophic soft cubic subspace topology on (X,E). Hence (Y, τ_Y, E) is called a neutrosophic soft cubic topological subspace of (X, τ, E)

Theorem 4.12. Let (X, τ, E) be a neutrosophic soft cubic topological space and $e \in E$, $\tau(e) = \{P(e) : (P, E) \in \tau\}$ is a neutrosophic soft cubic topology on (X,E).
Proof: Let $e \in E$

X	e_1	e_2	e_3
x_1	$([0.3, 0.6], [0.3, 0.5], [0.1, 0.6])(0.5, 0.6, 0.4)$	$([0.7, 0.8], [0.5, 0.6], [0.1, 0.4])(0.5, 0.6, 0.6)$	$([0.3, 0.7], [0.2, 0.5], [0.8, 0.9])(0.6, 0.5, 0.6)$
x_2	$([0.1, 0.7], [0.8, 0.9], [0.3, 0.4])(0.4, 0.3, 0.7)$	$([0.4, 0.5], [0.3, 0.4], [0.6, 0.7])(0.7, 0.6, 0.5)$	$([0.4, 0.8], [0.5, 0.7], [0.2, 0.5])(0.6, 0.3, 0.5)$
x_3	$([0.5, 0.7], [0.2, 0.6], [0.6, 0.9])(0.5, 0.6, 0.5)$	$([0.4, 0.6], [0.5, 0.7], [0.5, 0.8])(0.6, 0.4, 0.7)$	$([0.7, 0.9], [0.5, 0.7], [0.2, 0.4])(0.7, 0.5, 0.6)$

Table 10: The tabular representation of $(Q, E)^c$

(i) Let (Φ, E),$(X, E) \in \tau$, (Φ, E), $(X, E) \in \tau(e)$.

(ii) Let V, W $\in \tau$, then there exist (P , E), (Q , E) $\in \tau$ such that V = P(e)and W = Q(e).
Since τ is a neutrosophic soft cubic topology on X, (P , E)⋂(Q , E)$\in \tau$.
Put (H , E)= (P , E)⋂(Q , E). Then (H , E) $\in \tau$.
We have V $\cap$ W = P(e)⋂ Q(e)=H(e) and $\tau(e) = \{P(e) : (P, E) \in \tau\}$.
Then V $\cap$ W $\in \tau$.

(iii) Let $\{(V_\alpha, E); \alpha \in \Gamma\} \in \tau(e)$ .Then for every $\alpha \in \Gamma$, there exist $(P_\alpha, E) \in \tau$ such that $V_\alpha = P_\alpha(e)$.
Since τ is a neutrosophic soft cubic topological space on X, $\bigcup\{(P_\alpha, E); \alpha \in \Gamma\} \in \tau$.
Put $(P, E) = \bigcup\{(P_\alpha, E); \alpha \in \Gamma\}$ then (P , E) $\in \tau$.
Note that $\bigcup\limits_{\alpha\in\Gamma} V_\alpha = \bigcup\{P_\alpha, (e); \alpha \in \Gamma\}$ = P(e) and $\tau(e) = \{P(e) : (P, E) \in \tau\}$. Then $\bigcup\limits_{\alpha\in\Gamma} V_\alpha \in \tau(e)$.
Therefore $\tau(e) = \{P(e) : (P, E) \in \tau\}$ is a neutrosophic soft cubic topology on X.

Definition 4.13. Let (X, τ, E) be a neutrosophic soft cubic topological space over (X,E) and $\mathcal{B} \subseteq \tau$. $\mathcal{B}$ is a basis on τ if for each $(Q, E) \in \tau$, there exist $\mathcal{B}' \subseteq \mathcal{B}$ such that $(Q, E) = \cup\mathcal{B}'$

Example 4.14. Let τ be a neutrosophic soft cubic topology as in Example 28. Then
$\mathcal{B} = \{(P, E),(Q, E),(L, E),(\Phi, E),(X, E)\}$ is a basis for τ

Theorem 4.15. Let $\mathcal{B}$ be a basis for neutrosophic soft cubic topology τ. Denote $\mathcal{B}_e = \{P(e) : (P, E) \in \mathcal{B}\}$ and $\tau(e) = \{P(e) : (P, E) \in \tau\}$ for any $e \in E$. Then $\mathcal{B}_e$ is a basis for neutrosophic soft cubic topology $\tau(e)$.
Proof: Let $e \in E$. For any V $\in \tau(e)$, V = Q(e) for $(Q, E) \in \tau$. Here $\mathcal{B}$ is a basis for τ. Then there exists $\mathcal{B}' \subseteq \mathcal{B}$ such that $(Q, E) = \cup\mathcal{B}'$. So $V = \cup\mathcal{B}'_e$ where $\mathcal{B}'_e = \{P(e) : (P, E) \in \mathcal{B}'\} \subseteq \mathcal{B}_e$. Thus $\mathcal{B}_e$ is a basis for neutrosophic soft cubic topology $\tau(e)$ for any $e \in E$

Definition 4.16. Let (X, τ, E) be a neutrosophic soft cubic topological space and let (P , E) be a neutrosophic soft cubic set over (X, E). Then the interior and closure of (P , E) denoted respectively by int(P , E) and cl(P , E) are defined as follows. $int(P, E) = \cup\{(Q, E) \in \tau : (Q, E)\subseteq(P, E)\}$
ie., $int(P, E) = \cup\{(Q, E) : (Q, E)\subseteq(P, E)$ and (Q, E) is $NSCOS\}$
$cl(P, E) = \cap\{(Q, E) \in \tau^c : (P, E)\subseteq(Q, E)\}$ ie., $cl(P, E) = \cap\{(Q, E) : (P, E)\subseteq(Q, E)$ and (Q, E) is $NSCCS\}$

Example 4.17. We consider the Example 4.2 and take NSCS (G,E) as shown in Table 13.
$int(G, E) = (P, E)$ and $cl(G, E) = (P, E)^c$.

Theorem 4.18. Let (X, τ, E) be a neutrosophic soft cubic topological space. Then the following properties hold.

(i) $(\Phi, E),(X, E) \in \tau$

(ii) The intersection of any number of neutrosophic soft cubic closed sets is a neutrosophic soft cubic closed set over X.

X	e_1	e_2	e_3
x_1	$([0.3, 0.9], [0.3, 0.6], [0.1, 0.7])(0.5, 0.6, 0.5)$	$([0.4, 0.7], [0.2, 0.3], [0.1, 0.3])(0.7, 0.7, 0.5)$	$([0.3, 0.9], [0, 0.1], [0, 0.2])(0.7, 0.5, 0.6)$
x_2	$([0.5, 1], [0, 0.1], [0.3, 0.6])(0.5, 0.6, 0.7)$	$([0.6, 0.8], [0.2, 0.4], [0.1, 0.3])(0.6, 0.7, 0.6)$	$([0.4, 0.9], [0.1, 0.3], [0.2, 0.4])(0.6, 0.5, 0.7)$
x_3	$([0.4, 0.7], [0.3, 0.4], [0.6, 0.9])(0.5, 0.6, 0.7)$	$([0.6, 0.9], [0.1, 0.2], [0.1, 0.2])(0.8, 0.8, 0.5)$	$([0.4, 0.8], [0.1, 0.2], [0, 0.5])(0.8, 0.6, 0.6)$

Table 11: The tabular representation of $(L, E)^c$

(iii) The union of any two neutrosophic soft cubic closed sets is a neutrosophic soft cubic closed set over X.

Proof:

(i) Since $(\Phi, E), (X, E) \in \tau$,therefore $(\Phi, E)^c, (X, E)^c$ are NSC-closed set.

(ii) Let $\{(P_\alpha, E); \alpha \in \Gamma\}$ be an arbitrary family of NSC-closed sets in (X, τ,E) and let $(P, E) = \bigcap\limits_{\alpha\in\Gamma} (P_\alpha, E)$. Now $(P, E)^c = (\bigcap\limits_{\alpha\in\Gamma} (P_\alpha, E))^c = \bigcup\limits_{\alpha\in\Gamma} (P_\alpha, E)^c$ and $(P_\alpha, E)^c \in \tau$ for each $\alpha \in \Gamma$, so $\bigcup\limits_{\alpha\in\Gamma} (P_\alpha, E)^c \in \tau$. Hence $(P_\alpha, E)^c \in \tau$. Thus $(P_\alpha, E)^c$ is NSC-closed set.

(iii) Let $\{(P_i, E) : i = 1, 2, 3, ...n\}$ be the family of NSCCS

Theorem 4.19. Let (X, τ,E) be a neutrosophic soft cubic topology on X and let (P , E) be neutrosophic soft cubic set over (X,E). Then the following properties hold.

(i) $int(P, E) \subseteq (P, E)$.

(ii) $(Q, E) \subseteq (P, E) \Rightarrow int(Q, E) \subseteq int(P, E)$.

(iii) (P, E) is a neutrosophic soft cubic open set $\Leftrightarrow int(P, E) = (P, E)$.

(iv) $int(int(P, E)) = int(P, E)$.

(v) $int((\Phi, E)) = (\Phi, E)$, $int((X, E)) = (X, E)$.

Proof: (i) and (v) follows from definition [4.16].

(ii) $int(Q, E) = \cup\{(K, E) : (K, E) \subseteq (Q, E)$ and (K, E) is NCSOS in X$\}$
$int(P, E) = \cup\{(S, E) : (S, E) \subseteq (P, E)$ and (S, E) is NCSOS in X$\}$
Now $int(Q, E) \subseteq (Q, E) \subseteq (P, E) \Rightarrow int(Q, E) \subseteq (P, E)$.
Since $int(P, E)$ is the largest NCSOS contained in (P , E). Therefore $int(Q, E) \subseteq int(P, E)$

(iii) Let (P , E) be a neutrosophic soft cubic open set. Then it is the largest neutrosophic soft cubic open set contained in (P , E) and hence (P , E)= int(P , E). Conversely let (P , E)= int(P , E), since int(P , E)is the union of neutrosophic soft cubic open sets which is neutrosophic soft cubic open set.Hence (P , E) is neutrosophic soft cubic open set.

(iv) $int(P, E) = \cup\{(S, E) : (S, E) \subseteq int(P, E)$ and (S, E) is NCSOS in X$\}$
Since $int(P, E)$is the largest neutrosophic soft cubic open set contained in int (P , E).
Therefore$int(int(P, E)) = int(P, E)$.

Theorem 4.20. Let (X,τ,E) be a neutrosophic soft cubic topological space and (P , E) be a neutrosophic soft cubic set over (X,E). Then the following properties hold.

X	e_1	e_2	e_3
x_1	$([0.5, 0.8], [0.3, 0.5], [0.2, 0.7])(0.8, 0.4, 0.5)$	$([0.4, 0.7], [0.2, 0.3], [0.1, 0.3])(0.7, 0.7, 0.5)$	$([0.3, 0.9], [0, 0.1], [0, 0.2])(0.7, 0.5, 0.6)$
x_2	$([0.5, 1], [0, 0.1], [0.3, 0.6])(0.5, 0.4, 0.5)$	$([0.6, 0.8], [0.2, 0.4], [0.1, 0.3])(0.6, 0.7, 0.6)$	$([0.4, 0.9], [0.1, 0.3], [0.2, 0.4])(0.6, 0.5, 0.7)$
x_3	$([0.4, 0.7], [0.3, 0.4], [0.1, 0.2])(0.9, 0.5, 0.6)$	$([0.6, 0.9], [0.1, 0.2], [0.1, 0.2])(0.8, 0.8, 0.5)$	$([0.4, 0.8], [0.1, 0.2], [0, 0.5])(0.8, 0.6, 0.6)$

Table 12: The tabular representation of $(H, E)^c$

(i) $(P,E)\subseteq cl(P,E)$.

(ii) $(Q,E)\subseteq(P,E) \Rightarrow cl(Q,E)\subseteq cl(P,E)$.

(iii) (P,E) is a neutrosophic soft cubic closed set $\Leftrightarrow cl(P,E) = (P,E)$.

(iv) $cl(cl(P,E)) = cl(P,E)$.

(v) $cl((\Phi,E)) = (\Phi,E)$, $cl((X,E)) = (X,E)$.

Proof:

(i) From the definition [4.16] $(P,E)\subseteq cl(P,E)$.

(ii) $cl(Q,E) = \cap\{(K,E) : (Q,E)\subseteq(K,E)$ and (K,E) is NCSCS in X$\}$
$cl(P,E) = \cap\{(S,E) : (P,E)\subseteq(S,E)$ and (S,E) is NCSCS in X$\}$
Since $(Q,E)\subseteq cl(Q,E)$ and $(P,E)\subseteq cl(P,E) \Rightarrow (Q,E)\subseteq(P,E)\subseteq cl(P,E) \Rightarrow (Q,E)\subseteq cl(P,E)$.
Since $cl(P,E)$ is the smallest neutrosophic soft cubic closed set containing (P,E). Hence $cl(Q,E)\subseteq cl(P,E)$.

(iii) Let (P,E) be neutrosophic soft cubic closed set. Then it is the smallest neutrosophic soft cubic closed set containing itself and hence $(P,E) = cl(P,E)$.
Conversely let $(P,E) = cl(P,E)$, since $cl(P,E)$ being the intersection of neutrosophic soft cubic closed sets is neutrosophic soft cubic closed set. Hence $(P,E)$is neutrosophic soft cubic closed set.

(iv) $cl(cl(P,E)) = \cap\{(S,E) : (P,E)\subseteq(S,E)$ and (S,E) is NCSCS in X$\}$
Since $cl(P,E)$ is the smallest closed neutrosophic soft closed set containing cl (P , E).
Therefore $cl(cl(P,E)) = cl(P,E)$.

(v) $cl((\Phi,E)) = (\Phi,E)$, $cl((X,E)) = (X,E)$ are follows from definition[4.16].

Theorem 4.21. Let (X,τ,E) be neutrosophic soft cubic topological space and let (P , E) and (Q , E) are neutrosophic soft cubic sets over (X,E). Then the following properties hold.

(i) $int(Q,E)\cap int(P,E) = int((Q,E)\cap(P,E))$

(ii) $int(Q,E)\cup int(P,E)\subseteq int((Q,E)\cup(P,E))$

(iii) $cl(Q,E)\cup cl(P,E) = cl((Q,E)\cup(P,E))$

(iv) $cl((Q,E)\cap(P,E))\subseteq cl(Q,E)\cap cl(P,E)$

(v) $[int(Q,E)]^c = cl[(Q,E)^c]$

(vi) $[cl(Q,E)]^c = int[(Q,E)^c]$

X	e_1	e_2	e_3
x_1	$([0.2,0.8],[0.3,0.6],[0.2,0.8])(0.8,0.3,0.4)$	$([0.2,0.4],[0.4,0.6],[0.2,0.4])(0.6,0.4,0.3)$	$([0.2,0.6],[0.7,0.8],[0.3,0.4])(0.6,0.5,0.4)$
x_2	$([0.5,0.8],[0.5,0.6],[0.5,0.8])(0.6,0.3,0.5)$	$([0.1,0.4],[0.4,0.6],[0.1,0.5])(0.7,0.3,0.3)$	$([0.0.2,0.5],[0.5,0.8],[0.2,0.4])(0.5,0.5,0.4)$
x_3	$([0.1,0.6],[0.4,5],[0.2,0.7])(0.7,0.4,0.3)$	$([0.2,0.6],[0.5,0.7],[0.1,0.7])(0.7,0.6,0.4)$	$([0.1,0.4],[0.2,0.5],[0.1,0.5])(0.7,0.4,0.4)$

Table 13: The tabular representation of (G,E)

Proof:

(i) Since $(Q,E)\cap(P,E)\subseteq(Q,E)$ and $(Q,E)\cap(P,E)\subseteq(Q,E)$,
by Theorem 4.19(ii), $(Q,E)\subseteq(P,E) \Rightarrow int(Q,E)\subseteq int(P,E)$,
then $int((Q,E)\cap(P,E))\subseteq int(Q,E)$ and $int((Q,E)\cap(P,E))\subseteq int(P,E)$
$\Rightarrow int((Q,E)\cap(P,E))\subseteq int(Q,E)\cap int(P,E)$.
Now int(Q , E) and int(P , E) are NCSOSs
$\Rightarrow int(Q,E)\cap int(P,E)$ is NCSOS,
then $int(Q,E)\subseteq(Q,E)$ and $int(P,E)\subseteq(P,E) \Rightarrow int(Q,E)\cap int(P,E)\subseteq int((Q,E)\cap(P,E))$.
Therefore $int(Q,E)\cap int(P,E) = int((Q,E)\cap(P,E))$.

(ii) $(Q,E)\subseteq(P,E)\cup(Q,E)$ and $(P,E)\subseteq(P,E)\cup(Q,E)$
by Theorem 4.19(ii), $(Q,E)\subseteq(P,E) \Rightarrow int(Q,E)\subseteq int(P,E)$,
then $int(Q,E)\subseteq int((P,E)\cup(Q,E))$ and $int(P,E)\subseteq int((P,E)\cup(Q,E))$.
Hence $int(Q,E)\cup int(P,E)\subseteq int((Q,E)\cup(P,E))$

(iii) Since $(Q,E)\subseteq cl(Q,E)$ and $(P,E)\subseteq cl(P,E)$.
We have $(Q,E)\cup(P,E) = cl(Q,E)\cup cl(P,E)$
$\Rightarrow cl((Q,E)\cup(P,E)) = cl(Q,E)\cup cl(P,E)$ .....(1)
$cl(Q,E)\cup cl(P,E) = cl((Q,E)\cup(P,E))$ And since $(Q,E)\subseteq cl(Q,E)$ and $(P,E)\subseteq cl(P,E)$
so $cl(Q,E)\subseteq cl((Q,E)\cup(P,E))$ and $cl(P,E)\subseteq cl((Q,E)\cup(P,E))$ ......(2)
Therefore$cl(Q,E)\cup cl(P,E)\subseteq cl((Q,E)\cup(P,E))$
From (1) and (2) $cl(Q,E)\cup cl(P,E) = cl((Q,E)\cup(P,E))$.

(iv) Since $(Q,E)\cap(P,E)\subseteq(Q,E)$ and $(Q,E)\cap(P,E)\subseteq(P,E)$
and so $cl((Q,E)\cap(P,E))\subseteq cl(Q,E)$ and $cl((Q,E)\cap(P,E))\subseteq\cap cl(P,E)$
Hence $cl((Q,E)\cap(P,E))\subseteq cl(Q,E)\cap cl(P,E)$

(v) $[int(Q,E)]^c = [\cup\{(K,E) : (K,E)\subseteq(Q,E)$ and (K,E) is NCSOS in X$\}]^c$
$= \cap\{(K,E)^c : (Q,E)^c\subseteq(K,E)^c$ and $(K,E)^c$ is NCSCS in X$\}]\}$
$= cl[(Q,E)^c]$.

(vi) $[cl(Q,E)]^c = [\cap\{(K,E) : (K,E)\subseteq(Q,E)$ and (K,E) is NCSCS in X$\}]^c$
$= \cup\{(K,E)^c : (K,E)^c\subseteq(Q,E)^c$ and $(K,E)^c$ is NCSOS in X$\}]^c\}$
$= int[(Q,E)^c]$..

5 Conclusions

This paper lays the foundation for the further study on different separation axioms via this sets. Further various types of relation between neutrosophic soft cubic topological sets can be analysed under various mappings.

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