Neutrosophic Weibull distribution and Neutrosophic Family Weibull Distribution

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Abstract: Many problems in life are filled with ambiguity, uncertainty, impreciseness …etc., therefore we need to interpret these phenomena. In this paper, we will focus on studying neutrosophic Weibull distribution and its family, through explaining its special cases , and the functions' relationship with neutrosophic Weibull such as Neutrosophic Inverse Weibull, Neutrosophic Rayleigh, Neutrosophic three parameter Weibull, Neutrosophic Beta Weibull, Neutrosophic five Weibull, Neutrosophic six Weibull distributions (various parameters).This study will enable us to deal with indeterminate or inaccurate problems in a flexible manner. These problems will follow this family of distributions. In addition, these distributions are applied in various domains, such as reliability, electrical engineering, Quality Control ….. etc. Some properties and examples for these distributions are discussed.

Keywords: Weibull distribution, Neutrosophic logic, Neutrosophic number, Neutrosophic Weibull, Neutrosophic inverse Weibull, Neutrosophic Rayleigh, Neutrosophic Weibull with (three, four, five, six) parameters.

1. Introduction

The real world is overstuffed with vague, unclear, fuzzy (problems, situations, ideas). The classical probability ignores extreme, aberrant, unclear values, and therefore a new adequate tool had to emerge. Neutrosophic logic was introduced by Smarandache in 1995, as a generalization for the fuzzy logic and intuitionistic fuzzy logic [5, 6]. Smarandache [3, 7, 8] and Salamaa.et.al [3, 4] were presented the fundamental concepts of neutrosophic set. Smarandache extended the fuzzy set to the neutrosophic set [1, 3], introducing the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where]-0, 1+[is the non-standard unit interval. Smarandache presented the neutrosophic statistics, which the data can be enigmatic, vague, imprecise, incomplete, even unknown.

The extension of classical distributions according to the neutrosophic logic means that the parameters of classical distribution take undetermined values[1,2,3,10], which allows dealing with all the situations that one may encounter while working with statistical data and especially when working with vague and inaccurate statistical data, such as the sample size may not be exactly known. The sample size could be between 50 and 70; the statistician is not sure about 20 sample persons if they belong or not to the population of interest; or because the 20 sample persons only partially belong to the population of interest, while partially they don't belong. This mean, in classical statistics all data are determined, while in neutrosophic statistic the data or a part of it are indeterminate in some degree. The neutrosophic researchers presented studies in objects different in neutrosophic statistic, such as Salama, Rafief [29], Abdel-Basset and others, see [20-28]. For more than a decade, Weibull distribution has been applied extensively in many areas and particularly used in the analysis of lifetime data for reliability engineering or biology (Rinne, 2008). However, the Weibull distribution has a weakness for modeling phenomenon with non-monotone failure rate. In this paper, we will define and study the Neutrosophic Weibull distribution, Neutrosophic family Weibull distribution for varies cases as Neutrosophic Weibull, Neutrosophic beta Weibull, Neutrosophic inverse Weibull, Neutrosophic Rayleigh, Neutrosophic with (three, four, five, six) parameters, and discuss some properties of these distributions, illustrated through examples and graphs.

2. Terminologies

In this section, we present some basic axioms of neutrosophic logic, and in particular, the work of Smarandache in [3, 7, 8] and Salama et al. [3, 4]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where] 0-,1+[is nonstandard unit interval.

2.1 Some definitions

Definition 1 [1, 2, 3] "Neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their".

Definition 2 [1, 2, 3] Let T, I,F be real standard or nonstandard subsets of] 0-,1+[, with
Sup_T=t_sup, inf_T=t_inf
Sup_I=i_sup, inf_I=i_inf
Sup_F=f_sup, inf_F=f_inf
n-sup=t_sup+i_sup+f_sup
n-inf=t_inf+i_inf+f_inf,
T, I, F are called neutrosophic components.

Definition 3 [4, 5] Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form {x, $(\mu_A(x), \delta_A(x), \gamma_A(x)): x \in X\}$, where $\mu_A(x)$, $\delta_A(x)$ *and* $\gamma_A(x)$ which represent the degree of member ship function , the degree of indeterminacy , and the degree of non-member ship , respectively of each element x ∈ X to the set A .

Definition 4 [4, 5] The NSS 0_N and 1_N in X as follows:

0_N may be defined as:

$$0_1 = \{x \;\; 0,0,1: x \in X\}$$
$$0_2 = \{x \;\; 0,1,1: x \in X\}$$
$$0_3 = \{x \;\; 0,1,0: x \in X\}$$
$$0_4 = \{x \;\; 0,0,0: x \in X\}$$

1_N may be defined as:

$$1_1 = \{x \;\; 1,0,0: x \in X\}$$
$$1_2 = \{x \;\; 1,0,1: x \in X\}$$
$$1_3 = \{x \;\; 1,0,0: x \in X\}$$
$$1_4 = \{x \;\; 1,1,1: x \in X\}$$

2.2 *Neutrosophic probability*

Neutrosophic probability is a generalization of the classical probability in which the chance that event $A = \{ X, A_1, A_2, A_3\}$ occurs is P(A1) true, P(A2) indeterminate , P(A3) false on a space X, then $NP(A) = \{ X, P(A_1), P(A_2), P(A_3)\}$.

Definition 5 [3,4]

Let A and B be a neutrosophic events on a space X, then $NP(A) = \{ X, P(A_1), P(A_2), P(A_3)\}$

And $NP(B) = \{ X, P(B_1), P(B_2), P(B_3)\}$ their neutrosophic probabilities.

Definition 6 [3,4]

Let A and B be a neutrosophic events on a space X, and$NP(A) = \{ X, P(A_1), P(A_2), P(A_3)\}$, and $NP(B) = \{ X, P(B_1), P(B_2), P(B_3)\}$ are neutrosophic probabilities. Then we define

$NP(A \cap B) = \{ X, P(A_1 \cap B_1), P(A_2 \cap B_2), P(A_3 \cap B_3)\}$

$NP(A \cup B) = \{ X, P(A_1 \cup B_1), P(A_2 \cup B_2), P(A_3 \cup B_3)\}$

$NP(A^c) = \{ X, P({A_1}^c), P({A_2}^c) , P({A_3}^c)\}$

3 Weibull Distribution

Weibull distribution is one of most important distributions because it is widely used in reliability analysis, industrial and electrical engineering, in distribution of life time, in extreme value theory, … etc.; this distribution has various cases dependent on number of parameters such as two or three or five parameters α is the scale parameter, β is the shape parameter and γ is the location parameter. Also, it can be used to model a state where the failure function increases, decreases or remains constant with time.

4 Neutrosophic Weibull Distribution

A neutrosophic Weibull distribution (Neut-Weibull) of a continuous variable X is a classical Weibull distribution of x, but such that its mean α or β or γ are unclear or imprecise.

For example, α or β or γ can be an interval (open or closed or half open or half close) or can be set(s) with two or more elements. Then, the probability density function (p.d.f.) is:

$f_N(X) = \frac{\beta_N}{\alpha_N^{\beta_N}} X^{\beta_N - 1} e^{-(X/\alpha_N)^{\beta_N}}$, $X > 0$, Where β_N: is the shape parameter of distribution Net-Weibull, α_N: is the scale parameter of distribution Net- Weibull, such that N is a neutrosophic number.

4.1 Properties of Neutrosophic Weibull Distribution

- The distribution function (c.d.f.) is:

  $F_N(X) = 1 - e^{-(X/\alpha_N)^{\beta_N}}$,

  $E_N(X) = \alpha_N \Gamma\left(\frac{\beta_N + 1}{\beta_N}\right)$,

  $V_N(X) = {\alpha^2}_N \left[\Gamma\left(\frac{\beta_N + 2}{\beta_N}\right)\right] - \left[\Gamma\left(\frac{\beta_N + 1}{\beta_N}\right)\right]^2$.

- The hazard function is:

  $h_N = \beta_N X^{\beta_N - 1} X^{(\beta_N - 1/\alpha_N)^{\beta_N}}$.

- The moment rth about mean is:

  ${\alpha_N}^r \Gamma\left(\beta_N + \frac{r}{\beta_N}\right)$

- So, the reliability or survival function is:

$\overline{F_N}(X) = e^{-(X/\alpha_N)^{\beta_N}}$.

Now, we put β_N=1 in the formula (1), and we get the neutrosophic exponential distribution [13].

4.2 Example of Neutrosophic Weibull distribution

Let the product be an electric generator produced with high capacity of trademark that has a Weibull distribution with parameter α=1, β=[1.5,2]. Compute the probability of electric generator fails before the expiration of a five years warranty.

Solution :

In this example, we note that the shape parameter is indeterminate.

The electric generator can work through to one year:

$$f_N(X) = \frac{[1.5,2]}{\alpha_N^{[1.5,2]}} X^{[1.5,2]-1} e^{-(X/\alpha_N)^{[1.5,2]}}$$

If we take $\beta = 1.5$, and $\alpha = 1$

$$f_N(X = 1) = 0.5518$$

the probability of electric generator fails before the expiration of a five years warranty:

$$P(X \leq 5) = 1 - e^{-(5/1)^{1.5}}, = 0.999986$$

If we take $\beta = 2$, and $\alpha = 1$

$$f_N(X = 1) = 0.7357$$

$$P(X \leq 5) = 1 - e^{-(5/1)^{5}}, = 0.999999$$

Thus, the probability that the electric machine fails has the range between [0.5518, 0.7357].

Now, suppose $\beta = 2$ and $= [1,2]$, i.e the scale parameter α is indeterminate.

We take $\alpha = 1$ and $\beta = 2$

$$f_N(X = 1) = \frac{2}{e^1} = 0.7357$$

We take $\alpha = 2$ and $\beta = 2$

$$f_N(X = 1) = \frac{1}{2e^{1/4}} = 0.3894$$

In this case, the probability that the electric machine fails has the range between [0.7357, 0.3894]. Also, we can take more values of X, showed in Figure (1).

Now, we can compute

$F_N(X) = 1 - 1/e = 0.6321\,, \quad if\ \alpha = 1$

$F_N(X) = 1 - e^{1}/_{1.2840} = 0.2212\,, \quad if\ \alpha = 2.$

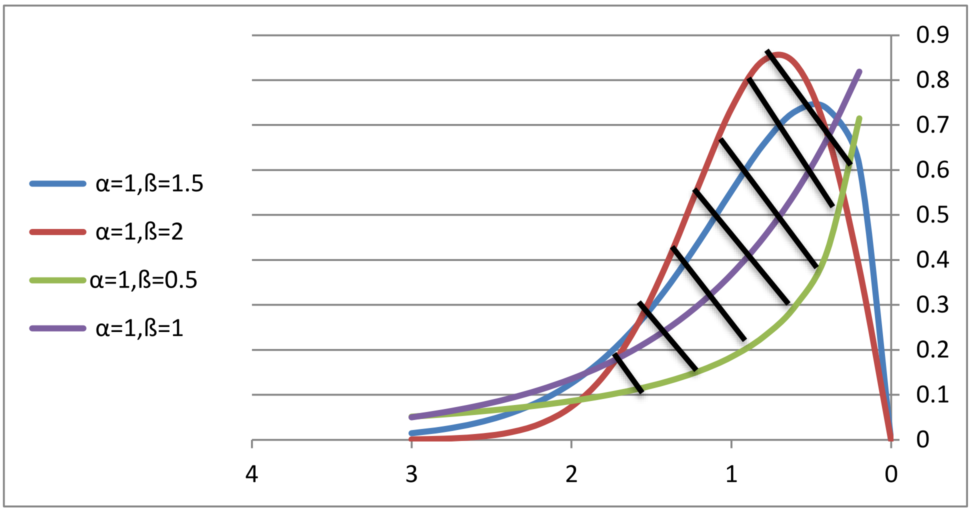

Figure 1: Neutrosophic Weibull distribution.

4.3 Comparison between Neutrosophic Weibull distribution and Weibull distribution

1- In classical Weibull, we noted that if the $\beta = 3.6$ *or more*, the probability distribution function (p.d.f) takes value error because it is greater than one, and this contradicts with law of probability,consedered Extreme values, while in neutrosophic Weibull this is applicable. See Figure (2).
2- In classical Weibull distribution, when X is increasing, the p.d.f. is decreasing, while in Neutrosophic Weibull distribution the p.d.f is unpredictable because of the aberrant values.
3- Many values that are larger than one are neglected in Weibull distribution, meanwhile in Neutrosophic Weibull these values are considered.
4- When $\alpha = \beta = 1$, the p.d.f. will equal zero when X=701,while in neutrosophic Weibull X can be of other values such as X={701,100} or [701,100] in this case p.d.f can be of different values.

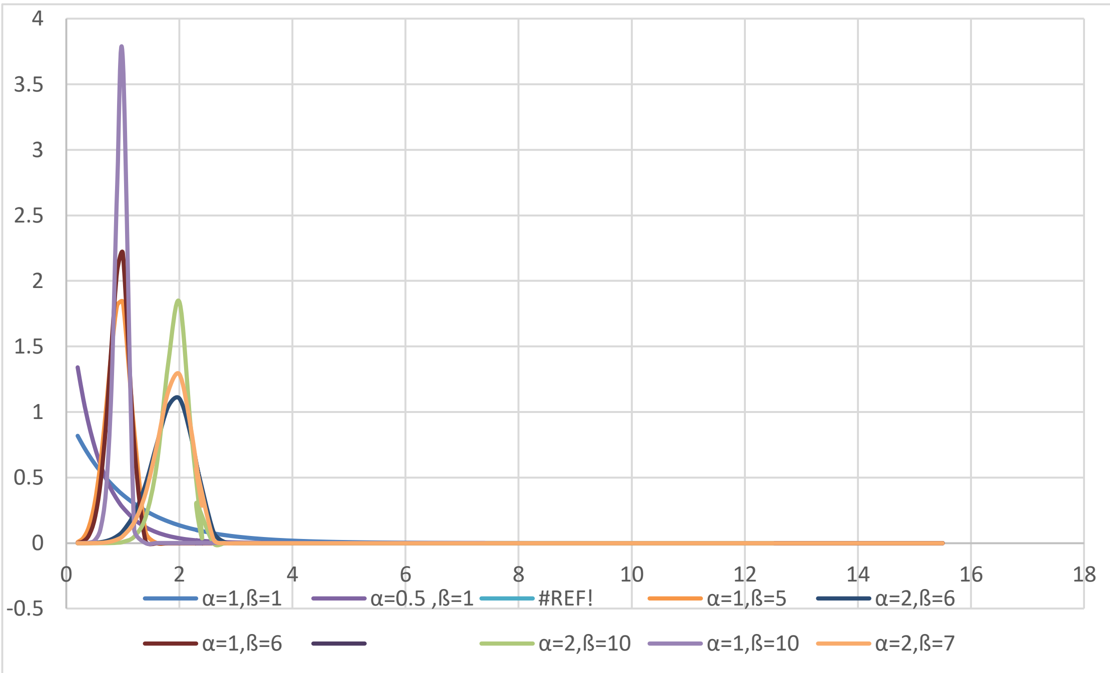

Figure 2: p.d.f of neutrosophic Weibull more than one.

5 The Family of Neutrosophic Weibull

In this section, we study the various types of Net-Weibull, such as neutrosophic Rayleigh distribution, neutrosophic inverse Weibull distribution, neutrosophic Beta-Weibull distribution and (three, four, five, six)-parameters Weibull distributions.

5.1 Neutrosophy Rayleigh Distribution

A Rayleigh distribution is often observed when the total size of a vector is linked to its directional components. Considering this distribution is important in the error analysis of various systems or individuals. It is also considered as a model for testing life failure/expiration. Rayleigh distribution is used in the study of the event of sea wave rise in the oceans and the study of wind speed, as well as in the information of the strength of signals and radiation at peak time of communications. The distribution is widely applied:

- In communications theory, to model multiple paths of dense scattered signals getting to a receiver;
- In the physics, to model wind speed, wave heights and sound/light radiation;
- In engineering, to measure the lifetime of an object, since the lifetime depends on the object's age (resistors, transformers, and capacitors in aircraft radar sets);
- In medical imaging examination, to study noise variance in magnetic resonance imaging.

Now, we define the probability density function of neutrosophic Rayleigh distribution as follows:

$R_N(X) = \frac{X}{{\delta_N}^2} e^{-X^2 \ /2{\delta_N}^2}$, $X > 0,\ \delta_N$ is the scale parameter.

this parameter δ_N can take the values of an interval or a set:
cumulative distribution is $F_N(X) = 1 - e^{-X^2 \ /2{\delta_N}^2}$,
the mean of Neutrosophic Rayleigh distribution is

$$E(X) = \delta_N \sqrt{\frac{\pi}{2}},$$

the variance: var(x) = 2-π/2 ${\delta_N}^2$.

5.2 Neutrosophic Weibull with 3 Parameters

We can obtain the neutrosophic Weibull with 3-parameters by relaying on Weibull with 2-parameters and adding the third parameter, namely the location parameter (γ), this is in classical probability . Now, we define the neutrosophic Weibull with three parameters (an indeterminacy may exist in one parameter or in all parameters). Neutrosophic Weibull with 3-parameters is defined as follows:

$$f_N(X) = [\beta_N \frac{(X-\gamma_N)^{\beta_N - 1}}{\alpha_N^{\beta_N}}] e^{-((X-\gamma_N)/\alpha_N)^{\beta_N}} \quad , \ \gamma_N \le X$$

- The distribution function is:

  $F_N(X) = 1 - e^{-((X-\gamma_N)/\alpha_N)^{\beta_N}}$, $\gamma_N \le X$
- The hazard function is:

  $h_N(X) = \beta_N \ \ (X - \gamma_N)^{\beta_N} (1 \ /\alpha_N)^{\beta_N}$, $\gamma_N \le X$
- The survival function is

  $\overline{F_N}(X) = e^{-((X-\gamma_N)/\alpha_N)^{\beta_N}}$
- The variance

$$V_N(X) = \alpha^2{}_N\left[\Gamma\left(\frac{\beta_N+2}{\beta_N}\right)\right] - \left[\Gamma\left(\frac{\beta_N+1}{\beta_N}\right)\right]^2,$$

- The expected value $E_N(X) = \gamma_N + \alpha_N\Gamma\,(\frac{\beta_N+1}{\beta_N})$.

5.3 Four-Parameter Neutrosophic-Beta-Weibull

The Beta-Weibull was first proposed by Famoye et al. (2005) [11,12, 15]. We now define the new density function of neutrosophic-Beta-Weibull distribution (NBW) in neutrosophic logic with indeterminacy points for random variable or parameters as follows:

$$f(X) = \frac{\Gamma\,(c_N+\gamma_N)}{\Gamma\,(c_N)\Gamma(\gamma_N)}\frac{\alpha_N}{\beta_N}\left(\frac{X}{\beta_N}\right)^{\alpha_N-1}[1-e^{-(X/\beta_N)^{\alpha_N}}]^{c_N-1}e^{-\gamma_N(X/\beta_N)^{\alpha_N}}$$

$$X > 0,\ \gamma_N, \beta_N, \alpha_N > 0$$

where these parameters $\gamma_N, \beta_N, \alpha_N$ can be set(s) or interval (closed or open or half):

$$\text{Because } \lim_{x\to 0} f(X) = \lim_{x\to 0}\frac{\Gamma\,(c_N+\gamma_N)}{\Gamma\,(c_N)\Gamma(\gamma_N)}\frac{\alpha_N}{\beta_N}\left(\frac{X}{\beta_N}\right)^{\alpha_N-1}\left[1-e^{-(X/\beta_N)^{\alpha_N}}\right]^{c_N-1}e^{-\gamma_N(X/\beta_N)^{\alpha_N}}$$

$$= \frac{\Gamma\,(c_N+\gamma_N)}{\Gamma\,(c_N)\Gamma(\gamma_N)}\frac{\alpha_N}{\beta_N}\left(\frac{X}{\beta_N}\right)^{\alpha_N-1}e^{-\gamma_N(X/\beta_N)^{\alpha_N}}[1-e^{-(X/\beta_N)^{\alpha_N}}]^{c_N-1}$$

$$= \frac{\Gamma\,(c_N+\gamma_N)}{\Gamma\,(c_N)\Gamma(\gamma_N)}\frac{\alpha_N}{\beta_N}\left(\frac{X}{\beta_N}\right)^{\alpha_N-1}e^{-\gamma_N(X/\beta_N)^{\alpha_N}}\left[1-\frac{1}{2!}\left(\frac{X}{\beta_N}\right)^{\alpha_N}+\frac{1}{3!}\left(\frac{X}{\beta_N}\right)^{2\alpha_N}-\frac{1}{4!}\left(\frac{X}{\beta_N}\right)^{3\alpha_N}+\cdots\right]^{c_N-1}$$

Then the probability of density function is equal to

$$= \lim_{x\to 0}\frac{\alpha_N}{\beta_N}\frac{\Gamma\,(c_N+\gamma_N)}{\Gamma\,(c_N)\Gamma(\gamma_N)}\left(\frac{X}{\beta_N}\right)^{\alpha_N-1} = \begin{cases}\infty & \alpha_N\, c_N < 1\\ \frac{\alpha_N}{\beta_N}\frac{\Gamma\,(c_N+\gamma_N)}{\Gamma\,(c_N)\Gamma(\gamma_N)} & \alpha_N\, c_N = 1\\ 0 & \alpha_N\, c_N > 1\end{cases}$$

where $\beta_N, c_N, \gamma_N, \alpha_{N,}$ are Neutrosophy numbers.

- When $c_N = \gamma_N = 1$, then the (NBW) is reduced to neutrosophic Weibull distribution.
- When $\beta_N = \alpha_N = 1, c_N = 2, \gamma_N = \delta\sqrt{2}$, the NBW is reduced to neutrosophy Rayleigh.
- In (1958) Kies defined the survival function to Weibull with four parameters in classical distribution.

Here we define Neutrosophic survival function in Neutrosophic distribution as follows:

$$\overline{F_N}(X) = e^{\{-\gamma_N\left(\frac{x-\alpha_N}{\beta_N-x}\right)^{k_N}\}},\ \gamma_N > 0, k_N > 0,\ 0 < \alpha_N < X < \beta_N < \infty.$$

5.4 Neutrosophic Weibull Distribution with 5 Parameters

Phani in (1987) [14] suggested model with survival function has five parameters. We define the neutrosophic Weibull with 5-parameters:

$$\overline{F_N}(X) = e^{\frac{-\gamma_N\,[X-\alpha_N]^{b_1}}{[\beta_N - X]^{b_2}}},\quad \gamma_N, b_1, b_2 > 0,\quad 0 < \alpha_N < X < \beta_N < \infty.$$

5.5 Neutrosophic Weibull Distribution with 6 Parameters

T, W, and Uraiwan in (2014) [15] proposed a mixed distribution is Beta exponential Weibull Poisson distribution. We define the neutrosophic Beta exponential Weibull Poisson distribution as follows:

Let x be the neutrosophic random variable with parameters $\gamma_N, k_N,\ \alpha_N,\ \beta_N, b_1, b_2$;

$$f(x) = \frac{\beta_N \alpha_N \gamma_N \; {k_N}^{\beta_N} \; X^{\beta_N - 1} u (1-u)^{\alpha_N - 1} e^{\gamma_N (1-u)^{\alpha_N}}}{\mathrm{B}(b_1, b_2)(e^{\gamma_N} - 1)} . \left[\frac{e^{\gamma_N (1-u)^{\alpha_N}} - 1}{(e^{\gamma_N} - 1)}\right]^{\alpha_N - 1} \left[1 - \frac{e^{\gamma_N (1-u)^{\alpha_N}} - 1}{(e^{\gamma_N} - 1)}\right]^{b1_N - 1}$$

where $u = e^{-(Xk_N)^{\beta_N}}$.

5.6 Neutrosophic Inverse Weibull Distribution

Keller et al. (1985) used the inverse Weibull distribution for reliability analysis of commercial vehicle engines. Here, we define Neutrosophic inverse Weibull distribution as follows:

$f_N(t) = \beta_N \alpha_N^{\beta_N} t^{-\beta_N - 1} e^{-(\alpha_N/t)^{\beta_N}}$, $t > 0$, So the Hazard function is $h_N(t) = \frac{\beta_N \alpha_N^{\beta_N} t^{-\beta_N - 1} e^{-(\alpha_N/t)^{\beta_N}}}{1 - e^{-(\alpha_N/t)^{\beta_N}}}$.

6 Applications

Many applications of Weibull families distributions are suitable for modeling and analysis of floods, rainfall, sea, electronic, manufacturing products, navigation and transportation control. The theories and tools of reliability engineering are applied into widespread fields, such as electronic and manufacturing products, aerospace equipment, earthquake and volcano forecasting, communication, navigation and transportation control, medical processor to the survival analysis of human being or biological species, and so on [14]. So the neutrosophic has the multi-applied in Decision-making, introduced by Abdel-Basset and others.

7 Conclusions

The study of neutrosophic probability distributions gives us a more comprehensive space in the applied field, as it takes into account more than the value of the distribution parameters and not only one value as in the classical distributions, and thus we will be able to solve and explain many of the issues that have been hindering us or we tended to ignore in classical logic. In this paper, we defined th new neutrosophic clasical distribution, the neutrosophic Weibull distribution and neutrosophic family Weibull (neutrosophic inverse Weibull, Neutrosophic Rayleigh distribution, Neutrosophic Weibull distribution with (3, 4, 5, 6)-parameters, and give clear examples. Because the weibull distribution has many applications in different fields.such as control system, relability and others. We also study some properties of these distributions (mean, variance, failure function and reliability function). In the future, we will apply these distributions to many problems and we will examine other distributions in neutrosophic logic.

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