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New Operators on Interval Valued Neutrosophic Sets

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Abstract. As a generalization of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets have been developed by F. Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth-membership function, an indeterminacy-membership function, and a falsity-membership function. An interval neutrosophic set is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In this paper we have defined some new operators on interval valued neutrosophic sets and studied their properties. In addition, we give numerical examples to illustrate the defined operations.

Keywords: Neutrosophic set, new operators on interval valued neutrosophic sets.

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1 Introduction

In 1999, a Russian scientist Molodstov [1] initiated the concept of soft set theory as a fundamental mathematical tool for modelling uncertainty, vague concepts and not clearly defined objects. Although various traditional tools, including but not limited to rough set theory [2], fuzzy set theory [3], intuitionistic fuzzy set theory [4] etc. have been used by many researchers to extract useful information hidden in the uncertain data, but there are inherent complications connected with each of these theories. Additionally, all these approachess lack in parameterizations of the tools and hence they couldn't be applied effectively in real life problems, especially in areas like environmental, economic and social problems. Soft set theory is standing uniquely in the sense that it is free from the above mentioned impediments and obliges approximate illustration of an object from the beginning, which makes this theory a natural mathematical formalism for approximate reasoning.

The notion of intuitionistic fuzzy set (IFS) was initiated by Atanassov as a significant generalization of fuzzy set. Intuitionistic fuzzy sets are very useful in situations when description of a problem by a linguistic variable, given in terms of a membership function only, seems too complicated. Recently intuitionistic fuzzy sets have been applied to many fields such as logic programming, medical diagnosis, decision making problems etc. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership (or simply membership) and falsity membership (or non-membership) values. But it doesn't handle the indeterminate and inconsistent information which exists in belief system. In 1995, F. Smarandache [05, 06] introduced the concept of neutrosphic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. This concept has been successfully applied to many fields such as databases [7, 8], medical diagnosis problem [9], decision making problem [10], topology [11], control theory [12] etc.

Presently works on the neutrosophic set theory is progressing rapidly. Bhowmik and Pal [13, 14] defined intuitionistic neutrosophic set. Later on Salam and Alblowi [15] introduced another concept called Generalized neutrosophic set. Wang et al. [16] proposed another extension of neutrosophic set which is single valued neutrosophic. Also Wang et al. [17] introduced the notion of interval valued neutrosophic set which is an instance of neutrosophic set. It is characterized by an interval membership degree, interval indeterminacy degree and interval non-membership degree. Ye [18, 19] defined similarity measures between interval neutrosophic sets and their multicriteria decision-making method. Majumdar and Samanta [20] proposed some types of similarity and entropy of neutrosophic sets. Broumi and Smarandache [21, 22, 23] proposed several similarity measures of neutrosophic sets. S. Broumi and F. Smarandache defined four new operations on interval-valued intuitionistic hesitant fuzzy sets and studied their important properties. F.G. Lupianez [24] defined the notion of neutrosophic topology on the non-standard interval. Majumder [25] discussed the distance and similarity between two neutrosophic sets . He also introduced the notion of entropy to measure the amount of uncertainty expressed by a neutrosophic set. H. Zhang et al. [26] defined operations for interval neutrosophic sets and a comparison approach was put forward based on the related research of interval valued intuitionistic fuzzy sets. He also developed two interval neutrosophic number aggregation operators and using these, a multi-criteria decision making problem was explored. H.Wang et al. [27] presented various properties of interval neutrosophic sets based on set theoretic operators. In 2017, Bera and Mahapatra [28] initiated the concept of neutrosophic soft matrix and they successfully applied it to solve decision making problems. Song et al. [29] applied neutrosophic sets to ideals in BCK/BCI algebras. Shahzadi et al [30] applied single valued neutrosophic sets in medical diagnosis. Recently, Thao and Smaran [31] proposed the concept of divergence measure on neutrosophic sets with an application to medical problem. Some recent applications of neutrosophic sets can be found in [32-39].

This paper is an attempt to define some new operators on interval valued neutrosophic sets and to study their properties. In addition to that, we have given numerical examples to illustrate the defined operations. The organization of this paper is as follow: In section 2, we briefly present some basic definitions which will be used in the rest of the paper. In section 3, we define some new operations on interval valued neutrosophic sets and discuss their properties. In section 5, conclusion is given. Lastly all the related references are given.

2 Preliminaries

2.1 Definition [3]:

Let U be a non empty set. Then a fuzzy set τ on U is a set having the form $\tau=\{(x,\ \mu_\tau(x)) : x \in U\}$

where the function $\mu_\tau : U \to [0, 1]$ is called the membership function and $\mu_\tau(x)$ represents the degree of membership of each element $x \in U$.

2.2 Definition [4]:

Let U be a non empty set. Then an intuitionistic fuzzy set (IFS for short) τ is an object having the form $\tau=\{\langle x,\ \mu_\tau(x),\ \gamma_\tau(x)\rangle :\ x \in U\}$ where the functions $\mu_\tau : U \to [0, 1]$ and $\gamma_\tau : U \to [0, 1]$ are called membership function and non-membership function respectively. $\mu_\tau(x)$ and $\gamma_\tau(x)$ represent the degree of membership and the degree of non-membership respectively of each element $x \in U$ and $0 \le \mu_\tau(x) + \gamma_\tau(x) \le 1$ for each $x \in U$.

We denote the class of all intuitionistic fuzzy sets on U by IFS^U.

2.3 Definition [5, 6]:

Let U be a non empty set. Then a neutrosophic set (NS for short) Γ is an object having the form $\Gamma=\{\langle \mathrm{x}, \mu_\Gamma(\mathrm{x}), \gamma_\Gamma(\mathrm{x}), \delta_\Gamma(\mathrm{x})\rangle : \mathrm{x}\in \mathrm{U}\}$ where the functions $\mu_\Gamma, \gamma_\Gamma, \delta_\Gamma : \mathrm{U}\to]^-0, 1^+[$ and $^-0 \le \mu_\Gamma(\mathrm{x})+\gamma_\Gamma(\mathrm{x})+\delta_\Gamma(\mathrm{x}) \le 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life applications in scientific and engineering problems it is difficult to use neutrosophic sets with value from real standard or non-standard subsets of $]^-0, 1^+[$. Hence we consider the neutrophic set which takes the value from the subset of [0, 1] i.e; $0 \le \mu_\Gamma(\mathrm{x})+\gamma_\Gamma(\mathrm{x})+\delta_\Gamma(\mathrm{x}) \le 3$ where $\mu_\Gamma, \gamma_\Gamma$ and δ_Γ are called truth membership function, indeterminacy membership function and falsity function respectively.

We denote the class of all neutrosophic sets on U by NS^U.

2.4 Definition [17]:

Let U be a non empty set. Then an interval valued neutrosophic set (IVNS for short) Γ is an object having the form

$$\Gamma=\{\langle \mathrm{x}, [\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})], [\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})], [\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})]\rangle : \mathrm{x}\in\mathrm{U}\}$$

where the functions $\mu_\Gamma, \gamma_\Gamma, \delta_\Gamma : \mathrm{U}\to Int([0, 1])$ and $0 \le \sup\mu_\Gamma(\mathrm{x})+\sup\gamma_\Gamma(\mathrm{x})+\sup\delta_\Gamma(\mathrm{x}) \le 3$.

We denote the class of all interval valued neutrosophic sets on U by $IVNS^U$.

2.5 Definition [17]:

Let Γ, Ω be two interval neutrosophic sets on U. Then

(a) Γ is called a subset of Ω, denoted by $\Gamma \subseteq \Omega$ if

$\inf\mu_\Gamma(\mathrm{x}) \le \inf\mu_\Omega(\mathrm{x})$, $\sup\mu_\Gamma(\mathrm{x}) \le \sup\mu_\Omega(\mathrm{x})$, $\inf\gamma_\Gamma(\mathrm{x}) \ge \inf\gamma_\Omega(\mathrm{x})$, $\sup\gamma_\Gamma(\mathrm{x}) \ge \sup\gamma_\Omega(\mathrm{x})$,
$\inf\delta_\Gamma(\mathrm{x}) \le \inf\delta_\Omega(\mathrm{x})$, $\sup\delta_\Gamma(\mathrm{x}) \le \sup\delta_\Omega(\mathrm{x})\ \forall\, \mathrm{x}\in\mathrm{U}$.

(b) The intersection of Γ and Ω is denoted by $\Gamma\cap\Omega$ and is defined by

$$\begin{aligned}\Gamma\cap\Omega=\{\langle&[\min(\inf\mu_\Gamma(\mathrm{x}),\inf\mu_\Omega(\mathrm{x})), \min(\sup\mu_\Gamma(\mathrm{x}),\sup\mu_\Omega(\mathrm{x}))],\\ &[\max(\inf\gamma_\Gamma(\mathrm{x}),\inf\gamma_\Omega(\mathrm{x})),\max(\sup\gamma_\Gamma(\mathrm{x}),\sup\gamma_\Omega(\mathrm{x}))],\\ &[\max(\inf\delta_\Gamma(\mathrm{x}),\inf\delta_\Omega(\mathrm{x})),\max(\sup\delta_\Gamma(\mathrm{x}),\sup\delta_\Omega(\mathrm{x}))]\rangle : \mathrm{x}\in\mathrm{U}\}.\end{aligned}$$

(c) The union of Γ and Ω is denoted by $\Gamma\cup\Omega$ and is defined by

$$\begin{aligned}\Gamma\cup\Omega=\{\langle&[\max(\inf\mu_\Gamma(\mathrm{x}),\inf\mu_\Omega(\mathrm{x})), \max(\sup\mu_\Gamma(\mathrm{x}),\sup\mu_\Omega(\mathrm{x}))],\\ &[\min(\inf\gamma_\Gamma(\mathrm{x}),\inf\gamma_\Omega(\mathrm{x})),\min(\sup\gamma_\Gamma(\mathrm{x}),\sup\gamma_\Omega(\mathrm{x}))],\\ &[\min(\inf\delta_\Gamma(\mathrm{x}),\inf\delta_\Omega(\mathrm{x})),\min(\sup\delta_\Gamma(\mathrm{x}),\sup\delta_\Omega(\mathrm{x}))]\rangle : \mathrm{x}\in\mathrm{U}\}.\end{aligned}$$

(d) The complement of Γ is denoted by Γ^c and is defined by

$$\Gamma^c=\{\langle \mathrm{x}, [\inf\delta_\Gamma(\mathrm{x}),\sup\delta_\Gamma(\mathrm{x})], [1\text{-}\sup\gamma_\Gamma(\mathrm{x}), 1-\inf\gamma_\Gamma(\mathrm{x})], [\inf\mu_\Gamma(\mathrm{x}),\sup\mu_\Gamma(\mathrm{x})]\rangle : \mathrm{x}\in\mathrm{U}\}$$

3. New Operators on Interval Valued Neutrosophic Sets

In this section we have proposed two new operators defined on interval valued neutrosophic sets. We also present their basic properties.

3.1 Definition:

The operator $\Box : IVNS^U \to IVNS^U$ is defined by

$$\Box\Gamma = \left\{ \left\langle x,\ \left[\inf\mu_\Gamma(x), \sup\mu_\Gamma(x)\right],\ \left[\inf\gamma_\Gamma(x), \sup\gamma_\Gamma(x)\right], \left[\inf\delta_\Gamma(x), 1-\sup\mu_\Gamma(x)\right] \right\rangle :\ x \in U \right\},$$

for $\Gamma \in IVNS^U$.

3.2 Example:

Let us consider an interval valued neutrosophic set Γ on U given by

$$\Gamma = \left\{ \left\langle a,\ [0.2, 0.4],\ [0.6, 0.3], [0.3, 0.5] \right\rangle, \left\langle b,\ [0.6, 0.8],\ [0.5, 0.6], [0.1, 0.4] \right\rangle \right\}.$$

Then we have $\Box\Gamma = \left\{ \left\langle a,\ [0.2, 0.4],\ [0.6, 0.3], [0.3, 0.5] \right\rangle, \left\langle b,\ [0.6, 0.8],\ [0.5, 0.6], [0.1, 0.6] \right\rangle \right\}.$

3.3 Definition:

The operator $\Diamond : IVNS^U \to IVNS^U$ is defined by

$$\Diamond\Gamma = \left\{ \left\langle x,\ \left[\inf\mu_\Gamma(x), 1-\sup\delta_\Gamma(x)\right],\ \left[\inf\gamma_\Gamma(x), \sup\gamma_\Gamma(x)\right], \left[\inf\delta_\Gamma(x), \sup\delta_\Gamma(x)\right] \right\rangle :\ x \in U \right\},$$

for $\Gamma \in IVNS^U$.

3.4 Example:

Let us consider an interval valued neutrosophic set Γ on U given by

$$\Gamma = \left\{ \left\langle a,\ [0.2, 0.4],\ [0.6, 0.3], [0.3, 0.5] \right\rangle, \left\langle b,\ [0.3, 0.8],\ [0.5, 0.6], [0.1, 0.4] \right\rangle \right\}.$$

Then we have $\Diamond\Gamma = \left\{ \left\langle a,\ [0.2, 0.5],\ [0.6, 0.3], [0.3, 0.5] \right\rangle, \left\langle b,\ [0.3, 0.6],\ [0.5, 0.6], [0.1, 0.4] \right\rangle \right\}.$

3.5 Theorem:

For $\Gamma \in IVNS^U$, we have the followings

(a) $\left(\Box\Gamma^c\right)^c = \Diamond\Gamma$

(b) $\left(\Diamond\Gamma^c\right)^c = \Box\Gamma$

(c) $\Box\Gamma \subseteq \Gamma \subseteq \Diamond\Gamma$

(d) $\Box\left(\Box\Gamma\right) = \Box\Gamma$

(e) $\Diamond\left(\Diamond\Gamma\right) = \Diamond\Gamma$

(f) $\Box\left(\Diamond\Gamma\right) = \Diamond\Gamma$

(g) $\Diamond\left(\Box\Gamma\right) = \Box\Gamma$

<u>Proof:</u>

(a) $\Gamma = \left\{ \left\langle x,\ \left[\inf\mu_\Gamma(x), \sup\mu_\Gamma(x)\right],\ \left[\inf\gamma_\Gamma(x), \sup\gamma_\Gamma(x)\right], \left[\inf\delta_\Gamma(x), \sup\delta_\Gamma(x)\right] \right\rangle :\ x \in U \right\}$

$\Rightarrow \Gamma^c = \left\{ \left\langle x,\ \left[\inf\delta_\Gamma(x), \sup\delta_\Gamma(x)\right],\ \left[1\text{-}\sup\gamma_\Gamma(x), 1-\inf\gamma_\Gamma(x)\right], \left[\inf\mu_\Gamma(x), \sup\mu_\Gamma(x)\right] \right\rangle :\ x \in U \right\}$

$\therefore \Box\Gamma^c = \left\{\left\langle \mathrm{x}, \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right], \left[1\text{-}\sup\gamma_\Gamma(\mathrm{x}), 1-\inf\gamma_\Gamma(\mathrm{x})\right], \left[\inf\mu_\Gamma(\mathrm{x}), 1-\sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}.$

Hence $\left(\Box\Gamma^c\right)^c$

$= \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), 1-\sup\delta_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\} = \Diamond\Gamma.$

(b) $\Gamma = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$\Rightarrow \Gamma^c = \left\{\left\langle \mathrm{x}, \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right], \left[1\text{-}\sup\gamma_\Gamma(\mathrm{x}), 1-\inf\gamma_\Gamma(\mathrm{x})\right], \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$\therefore \Diamond\Gamma^c = \left\{\left\langle \mathrm{x}, \left[\inf\delta_\Gamma(\mathrm{x}), 1-\sup\mu_\Gamma(\mathrm{x})\right], \left[1\text{-}\sup\gamma_\Gamma(\mathrm{x}), 1-\inf\gamma_\Gamma(\mathrm{x})\right], \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}.$

Hence $\left(\Diamond\Gamma^c\right)^c$

$= \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), 1-\sup\mu_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\} = \Box\Gamma.$

(c) Proof is straight forward.

(d) $\Gamma = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$\Rightarrow \Box\Gamma = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), 1-\sup\mu_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$\Rightarrow \Box(\Box\Gamma) = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), 1-\sup\mu_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$= \Box\Gamma.$

(e) Proof is similar to (d).

(f) $\Gamma = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$\Rightarrow \Diamond\Gamma = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), 1-\sup\delta_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$\Rightarrow \Box(\Diamond\Gamma) = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), 1-\sup\delta_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$

$= \Diamond\Gamma.$

(g) Proof is similar to (f).

3.6 Theorem:

For $\Gamma, \Omega \in IVNS^U$, we have the followings

(a) $\Box(\Gamma\cup\Omega) = \Box\Gamma\cup\Box\Omega$

(b) $\Box(\Gamma\cap\Omega) = \Box\Gamma\cap\Box\Omega$

(c) $\Diamond(\Gamma\cup\Omega) = \Diamond\Gamma\cup\Diamond\Omega$

(d) $\Diamond(\Gamma\cap\Omega) = \Diamond\Gamma\cap\Diamond\Omega$

Proof:

We have,

$\Gamma = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Gamma(\mathrm{x}), \sup\mu_\Gamma(\mathrm{x})\right], \left[\inf\gamma_\Gamma(\mathrm{x}), \sup\gamma_\Gamma(\mathrm{x})\right], \left[\inf\delta_\Gamma(\mathrm{x}), \sup\delta_\Gamma(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$ and

$\Omega = \left\{\left\langle \mathrm{x}, \left[\inf\mu_\Omega(\mathrm{x}), \sup\mu_\Omega(\mathrm{x})\right], \left[\inf\gamma_\Omega(\mathrm{x}), \sup\gamma_\Omega(\mathrm{x})\right], \left[\inf\delta_\Omega(\mathrm{x}), \sup\delta_\Omega(\mathrm{x})\right]\right\rangle: \mathrm{x}\in\mathrm{U}\right\}$.

(a) $\Gamma\cup\Omega$

$$=\Big\{\Big\langle\big[\max\big(\inf\mu_\Gamma(x),\inf\mu_\Omega(x)\big),\ \max\big(\sup\mu_\Gamma(x),\sup\mu_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\gamma_\Gamma(x),\inf\gamma_\Omega(x)\big),\min\big(\sup\gamma_\Gamma(x),\sup\gamma_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\delta_\Gamma(x),\inf\delta_\Omega(x)\big),\min\big(\sup\delta_\Gamma(x),\sup\delta_\Omega(x)\big)\ \big]\Big\rangle: x\in U\Big\}.$$

Hence $\Box(\Gamma\cup\Omega)$

$$=\Big\{\Big\langle\big[\max\big(\inf\mu_\Gamma(x),\inf\mu_\Omega(x)\big),\ \max\big(\sup\mu_\Gamma(x),\sup\mu_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\gamma_\Gamma(x),\inf\gamma_\Omega(x)\big),\min\big(\sup\gamma_\Gamma(x),\sup\gamma_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\delta_\Gamma(x),\inf\delta_\Omega(x)\big),1-\sup\big(\max\big(\sup\mu_\Gamma(x),\sup\mu_\Omega(x)\big)\big)\big]\Big\rangle: x\in U\Big\}$$
$$=\Big\{\Big\langle\big[\max\big(\inf\mu_\Gamma(x),\inf\mu_\Omega(x)\big),\ \max\big(\sup\mu_\Gamma(x),\sup\mu_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\gamma_\Gamma(x),\inf\gamma_\Omega(x)\big),\min\big(\sup\gamma_\Gamma(x),\sup\gamma_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\delta_\Gamma(x),\inf\delta_\Omega(x)\big),\min\big(1-\sup\mu_\Gamma(x),1-\sup\mu_\Omega(x)\big)\big]\Big\rangle: x\in U\Big\}.$$

Again

$$\Box\Gamma=\Big\{\Big\langle x,\ \big[\inf\mu_\Gamma(x),\sup\mu_\Gamma(x)\big],\ \big[\inf\gamma_\Gamma(x),\sup\gamma_\Gamma(x)\big],\big[\inf\delta_\Gamma(x),1-\sup\mu_\Gamma(x)\big]\Big\rangle:\ x\in U\Big\}$$

and

$$\Box\Omega=\Big\{\Big\langle x,\ \big[\inf\mu_\Omega(x),\sup\mu_\Omega(x)\big],\ \big[\inf\gamma_\Omega(x),\sup\gamma_\Omega(x)\big],\big[\inf\delta_\Omega(x),1-\sup\mu_\Omega(x)\big]\Big\rangle:\ x\in U\Big\}$$

.

Hence $\Box\Gamma\cup\Box\Omega$

$$=\Big\{\Big\langle\big[\max\big(\inf\mu_\Gamma(x),\inf\mu_\Omega(x)\big),\ \max\big(\sup\mu_\Gamma(x),\sup\mu_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\gamma_\Gamma(x),\inf\gamma_\Omega(x)\big),\min\big(\sup\gamma_\Gamma(x),\sup\gamma_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\delta_\Gamma(x),\inf\delta_\Omega(x)\big),\min\big(1-\sup\mu_\Gamma(x),1-\sup\mu_\Omega(x)\big)\big]\Big\rangle: x\in U\Big\}.$$

Consequently, $\Box(\Gamma\cup\Omega)=\Box\Gamma\cup\Box\Omega$.

(b) Proof is similar to (a).

(c) $\Gamma\cup\Omega$

$$=\Big\{\Big\langle\big[\max\big(\inf\mu_\Gamma(x),\inf\mu_\Omega(x)\big),\ \max\big(\sup\mu_\Gamma(x),\sup\mu_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\gamma_\Gamma(x),\inf\gamma_\Omega(x)\big),\min\big(\sup\gamma_\Gamma(x),\sup\gamma_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\delta_\Gamma(x),\inf\delta_\Omega(x)\big),\min\big(\sup\delta_\Gamma(x),\sup\delta_\Omega(x)\big)\ \big]\Big\rangle: x\in U\Big\}.$$

Hence $\Diamond(\Gamma\cup\Omega)$

$$=\Big\{\Big\langle\big[\max\big(\inf\mu_\Gamma(x),\inf\mu_\Omega(x)\big),\ 1-\sup\big(\min\big(\sup\delta_\Gamma(x),\sup\delta_\Omega(x)\big)\big)\big],$$
$$\big[\min\big(\inf\gamma_\Gamma(x),\inf\gamma_\Omega(x)\big),\min\big(\sup\gamma_\Gamma(x),\sup\gamma_\Omega(x)\big)\big],$$
$$\big[\min\big(\inf\delta_\Gamma(x),\inf\delta_\Omega(x)\big),\min\big(\sup\delta_\Gamma(x),\sup\delta_\Omega(x)\big)\big]\Big\rangle: x\in U\Big\}$$

$$=\Big\{\Big\langle\Big[\max\big(\inf\mu_{\Gamma}(x),\inf\mu_{\Omega}(x)\big),\ \max\big(1-\sup\mu_{\Gamma}(x),1-\sup\mu_{\Omega}(x)\big)\Big],$$
$$\Big[\min\big(\inf\gamma_{\Gamma}(x),\inf\gamma_{\Omega}(x)\big),\min\big(\sup\gamma_{\Gamma}(x),\sup\gamma_{\Omega}(x)\big)\Big],$$
$$\Big[\min\big(\inf\delta_{\Gamma}(x),\inf\delta_{\Omega}(x)\big),\min\big(\sup\delta_{\Gamma}(x),\sup\delta_{\Omega}(x)\big)\Big]\Big\rangle:x\in U\Big\}.$$

Again

$$\Diamond\Gamma=\Big\{\Big\langle x,\ \big[\inf\mu_{\Gamma}(x),1-\sup\delta_{\Gamma}(x)\big],\ \big[\inf\gamma_{\Gamma}(x),\sup\gamma_{\Gamma}(x)\big],\big[\inf\delta_{\Gamma}(x),\sup\delta_{\Gamma}(x)\big]\Big\rangle:\ x\in U\Big\}$$

and

$$\Diamond\Omega=\Big\{\Big\langle x,\ \big[\inf\mu_{\Omega}(x),1-\sup\delta_{\Omega}(x)\big],\ \big[\inf\gamma_{\Omega}(x),\sup\gamma_{\Omega}(x)\big],\big[\inf\delta_{\Omega}(x),\sup\delta_{\Omega}(x)\big]\Big\rangle:\ x\in U\Big\}$$

Hence $\Box\Gamma\cup\Box\Omega$

$$=\Big\{\Big\langle\Big[\max\big(\inf\mu_{\Gamma}(x),\inf\mu_{\Omega}(x)\big),\ \max\big(1-\sup\delta_{\Gamma}(x),1-\sup\delta_{\Omega}(x)\big)\Big],$$
$$\Big[\min\big(\inf\gamma_{\Gamma}(x),\inf\gamma_{\Omega}(x)\big),\min\big(\sup\gamma_{\Gamma}(x),\sup\gamma_{\Omega}(x)\big)\Big],$$
$$\Big[\min\big(\inf\delta_{\Gamma}(x),\inf\delta_{\Omega}(x)\big),\min\big(\sup\delta_{\Gamma}(x),\sup\delta_{\Omega}(x)\big)\Big]\Big\rangle:x\in U\Big\}.$$

Consequently, $\Box\big(\Gamma\cup\Omega\big)=\Box\Gamma\cup\Box\Omega$.

(d) Proof is similar to (c).

3.7 Definition:

The operator $\circ: IVNS^{U}\rightarrow IFS^{U}$ is defined by

$$\circ\Gamma=\Big\{\Big\langle x,\ \inf\mu_{\Gamma}(x),\ \inf\gamma_{\Gamma}(x),\inf\delta_{\Gamma}(x)\Big\rangle:\ x\in U\Big\},\ \Gamma\in IVNS^{U}.$$

3.8 Example:

Let us consider an interval valued neutrosophic set Γ on U given by

$$\Gamma=\Big\{\Big\langle a,\ [0.2,0.4],\ [0.6,0.3],[0.3,0.5]\Big\rangle,\Big\langle b,\ [0.6,0.8],\ [0.5,0.6],[0.1,0.4]\Big\rangle\Big\}.$$

Then we have $\circ\Gamma=\Big\{\Big\langle a,\ 0.2,\ 0.6,0.3\Big\rangle,\Big\langle b,\ 0.6,\ 0.5,0.1\Big\rangle\Big\}.$

3.9 Theorem:

For $\Gamma\in IVNS^{U}$, we have

(a) $\circ\big(\Box\Gamma\big)=\circ\Gamma$

(b) $\circ\big(\Diamond\Gamma\big)=\circ\Gamma$

Proof:

We have,

$$\Gamma=\Big\{\Big\langle x,\ \big[\inf\mu_{\Gamma}(x),\sup\mu_{\Gamma}(x)\big],\ \big[\inf\gamma_{\Gamma}(x),\sup\gamma_{\Gamma}(x)\big],\big[\inf\delta_{\Gamma}(x),\sup\delta_{\Gamma}(x)\big]\Big\rangle:\ x\in U\Big\}.$$

Then

(a) $\Box\Gamma=\Big\{\Big\langle x,\ \big[\inf\mu_{\Gamma}(x),\sup\mu_{\Gamma}(x)\big],\ \big[\inf\gamma_{\Gamma}(x),\sup\gamma_{\Gamma}(x)\big],\big[\inf\delta_{\Gamma}(x),1-\sup\mu_{\Gamma}(x)\big]\Big\rangle:\ x\in U\Big\}$

and so $\circ\big(\Box\Gamma\big)=\Big\{\Big\langle x,\ \inf\mu_{\Gamma}(x),\ \inf\gamma_{\Gamma}(x),\inf\delta_{\Gamma}(x)\Big\rangle:\ x\in U\Big\}=\circ\Gamma.$

(b) Proof is similar to (a).

3.10 Theorem:

For $\Gamma, \Omega \in IVNS^U$, we have the followings

(a) $\circ(\Gamma \cup \Omega) = \circ\Gamma \cup \circ\Omega$

(b) $\circ(\Gamma \cap \Omega) = \circ\Gamma \cap \circ\Omega$

Proof:

We have,

$\Gamma = \{\langle x, [\inf\mu_\Gamma(x), \sup\mu_\Gamma(x)], [\inf\gamma_\Gamma(x), \sup\gamma_\Gamma(x)], [\inf\delta_\Gamma(x), \sup\delta_\Gamma(x)]\rangle : x \in U\}$ and

$\Omega = \{\langle x, [\inf\mu_\Omega(x), \sup\mu_\Omega(x)], [\inf\gamma_\Omega(x), \sup\gamma_\Omega(x)], [\inf\delta_\Omega(x), \sup\delta_\Omega(x)]\rangle : x \in U\}$.

(a) $\Gamma \cup \Omega$

$$= \{\langle x, [\max(\inf\mu_\Gamma(x), \inf\mu_\Omega(x)), \max(\sup\mu_\Gamma(x), \sup\mu_\Omega(x))],$$
$$[\min(\inf\gamma_\Gamma(x), \inf\gamma_\Omega(x)), \min(\sup\gamma_\Gamma(x), \sup\gamma_\Omega(x))],$$
$$[\min(\inf\delta_\Gamma(x), \inf\delta_\Omega(x)), \min(\sup\delta_\Gamma(x), \sup\delta_\Omega(x))]\rangle : x \in U\}.$$

$\therefore \circ(\Gamma \cup \Omega)$

$$= \{\langle x, \max(\inf\mu_\Gamma(x), \inf\mu_\Omega(x)), \min(\inf\gamma_\Gamma(x), \inf\gamma_\Omega(x)), \min(\inf\delta_\Gamma(x), \inf\delta_\Omega(x))\rangle : x \in U\}.$$

Again we have, $\circ\Gamma \cup \circ\Omega$

$$= \{\langle x, \inf\mu_\Gamma(x), \inf\gamma_\Gamma(x), \inf\delta_\Gamma(x)\rangle : x \in U\} \cup \{\langle x, \inf\mu_\Omega(x), \inf\gamma_\Omega(x), \inf\delta_\Omega(x)\rangle : x \in U\}$$
$$= \{\langle x, \max(\inf\mu_\Gamma(x), \inf\mu_\Omega(x)), \min(\inf\gamma_\Gamma(x), \inf\gamma_\Omega(x)), \min(\inf\delta_\Gamma(x), \inf\delta_\Omega(x))\rangle : x \in U\}.$$

Consequently $\circ(\Gamma \cup \Omega) = \circ\Gamma \cup \circ\Omega$.

(b) Proof is similar to (a).

4. Conclusions

Neutrosophic set is a part of neutrosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. In this paper we have defined the set-theoretic operators on interval valued neutrosophic sets and studied some properties. We hope that this paper will promote the future study on interval valued neutrosophic sets to carry out a general framework for their application in practical life. Moreover, with the motivations of ideas presented in the paper, one can think of similar operations on interval valued neutrosophic sets of type-2, hesitant interval valued neutrosophic sets, interval valued neutrosophic soft sets and interval valued hesitant neutrosophic soft sets.

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