On single-valued co-neutrosophic graphs

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Abstract: In this paper, we introduce the notion of a single-valued co-neutrosophic graphs and study some methods of construction of new single-valued co-neutrosophic graphs. We compute degree of a vertex, strong single-valued co-neutrosophic graphs and complete single-valued co-neutrosophic graphs. We also introduce and give properties of regular and totally regular single-valued co-neutrosophic graphs.

Keywords: Single-valued neutrosophic graphs; degree of a vertex; strong single-valued co-neutrosophic graphs; complete single-valued co-neutrosophic graphs; regular and totally regular single-valued co-neutrosophic graphs.

1 Introduction and preliminaries

Zadeh [21] introduced the concepts of fuzzy set theory as a generalized concept of crisp set theory. The concept of fuzzy graph theory as a generalization of Eulers graph theory was first introduced by Rosenfeld [17] in 1975. Later, Bhattacharya [5] gave some remarks on fuzzy graphs. The concept of cofuzzy graphs by M. Akram [1]. The concepts of intuitionistic cofuzzy graph by Dhavaseelan [9]. Smarandache [20] introduced the concept of neutrosophic sets. Certain types of neutrosophic graphs were introduced by R. Dhavaseelan et al. [10].Some more work in single valued neutrosophic set,interval valued neutrosophic set and their application may be found in Karaaslan,et .al., [13], Hamidi,et .al., [11, 14], Broumi, et.al., [6–8, 15] and Shimaa Fathi,et.al [18]. Kandasamy, et.al [12], introduced the new dimension of neutrosophic graph.

In this paper, we introduce the notion of a single-valued co-neutrosophic graphs and study some methods of construction of new single-valued co-neutrosophic graphs. We compute degree of a vertex, strong single-valued co-neutrosophic graphs and complete single-valued co-neutrosophic graphs. We also introduce and give properties of regular and totally regular single-valued co-neutrosophic graphs.

Definition 1.1. [19] Let X be a space of points. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. The functions $T_A(x), I_A(x)$and $F_A(x)$ are real standard or non standard subsets of $]0^-, 1^+[$. That is,

$T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$,$F_A(x) : X \to]0^-, 1^+[$ and $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point view, the neutrosophic set takes the value from real standard or non standard subsets of $]0^-, 1^+[$. In real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non standard subset of $]0^-, 1^+[$.

Definition 1.2. $[2, 4]$ A single-valued neutrosophic graph is a pair $G = (A, B)$, where $A : V \to [0, 1]$ is single-valued neutrosophic set in V and $B : V \times V \to [0, 1]$ is single-valued neutrosophic relation on V such that

$$T_B(xy) \leq \min\{T_A(x), T_A(y)\}\ I_B(xy) \leq \min\{I_A(x), I_A(y)\}\ F_B(xy) \geq \max\{F_A(x), F_A(y)\}$$

for all $x, y \in V$. A is called single-valued neutrosophic vertex set of G and B is called single-valued neutrosophic edge set of G, respectively. We note that B is symmetric single-valued neutrosophic relation on A. If B is not symmetric single-valued neutrosophic relation on A, then $G = (A, B)$ is called a single-valued neutrosophic directed graph.

2 Single-valued co-neutrosophic graphs

Definition 2.1. A single-valued co-neutrosophic graph is a pair $G = (A, B)$, where $A : V \to [0, 1]$ is a single-valued co-neutrosophic set in V and $B : V \times V \to [0, 1]$ is a single-valued co-neutrosophic relation on V such that

$$T_B(xy) \geq \max\{T_A(x), T_A(y)\}$$
$$I_B(xy) \geq \max\{I_A(x), I_A(y)\}$$
$$F_B(xy) \leq \min\{F_A(x), F_A(y)\}$$

for all $x, y \in V$. A and B are called the single-valued co-neutrosophic vertex set of G and the single-valued co-neutrosophic edge set of G, respectively. We note that B is a symmetric single-valued co-neutrosophic relation on A. If B is not a symmetric single-valued co-neutrosophic relation on A, then $G = (A, B)$ is called a single-valued co-neutrosophic directed graph.

Notation 2.1. The triples $\langle T_A(x), I_A(x), F_A(x)\rangle$ denotes the degree of membership, an indeterminacy membership and nonmembership of vertex x, The triples $\langle T_B(xy), I_B(xy), F_B(xy)\rangle$ denote the degree of membership, an indeterminacy membership and nonmembership of edge relation $xy = (x, y)$ on V.

Definition 2.2. A partial single-valued co-neutrosophic subgraph of single-valued co-neutrosophic graph $G = (A, B)$ is a single-valued co-neutrosophic graph $H = (V', E')$ such that

(i) $V' \subseteq V$, where $T'_A(v_i) \leq T_A(v_i), I'_A(v_i) \leq I_A(v_i), F'_A(v_i) \geq F_A(v_i)$ for all $v_i \in V$.

(ii) $T_B(xy)' \leq T_B(xy)$;$I_B(xy)' \leq I_B(xy)$; $F_B(xy)' \geq F_B(xy)$ for every x and y

Definition 2.3. A single-valued co-neutrosophic graph $H = \langle A', B'\rangle$ is said to be a single-valued co-neutrosophic subgraph of the single-valued co-neutrosophic graph $G = \langle A, B\rangle$ if $A' \subseteq A$ and $B' \subseteq B$. In other words if $T_A{}'(x) = T_A(x)$;$I_A{}'(x) = I_A(x)$; $F'_A(x) = F_A(x)$ and $T_B(xy)' = T_B(xy)$;$I_B(xy)' = I_B(xy)$; $F_B(xy)' = F_B(xy)$ for every x and y

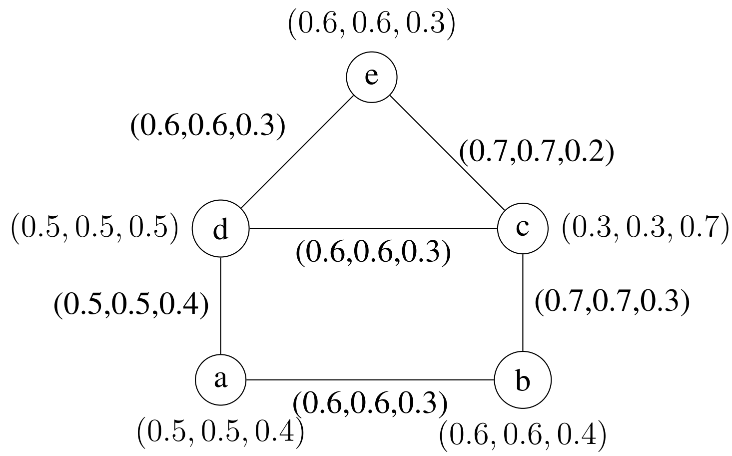


Figure 1: G : Single-valued co-neutrosophic graph

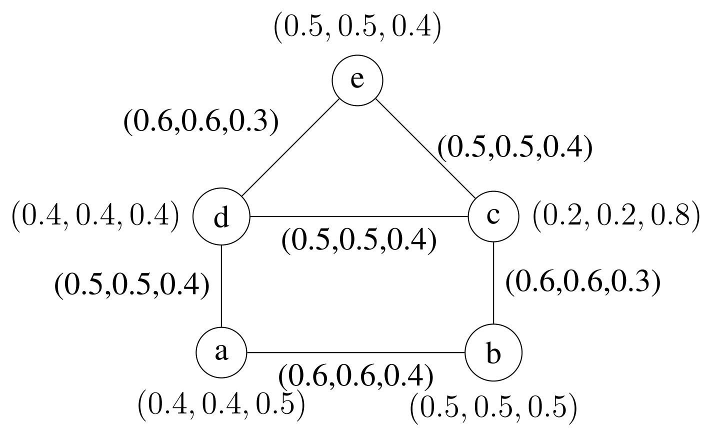


Figure 2: H : Single-valued co-neutrosophic partial subgraph ($H \subseteq G$)

Definition 2.4. A single-valued co-neutrosophic graph $G = \langle A, B\rangle$ is said to be strong single-valued co-neutrosophic graph if $T_B(xy) = max(T_A(x), T_A(y))$,$I_B(xy) = max(I_A(x), I_A(y))$ and $F_B(xy) = min(F_A(x), F_A(y))$, for all $(xy) \in E$.

Definition 2.5. A single-valued co-neutrosophic graph $G = \langle A, B\rangle$ is said to be complete single-valued co-neutrosophic graph if $T_B(xy) = max(T_A(x), T_A(y))$, $I_B(xy) = max(I_A(x), I_A(y))$ and $F_B(xy) = min(F_A(x), F_A(y))$, for every $x, y \in V$.

Definition 2.6. Let $G = \langle A, B\rangle$ be a single-valued co-neutrosophic graph. Then the degree of a vertex v is defined by $d(v) = (d_T(v), d_I(v), d_F(v))$, where $d_T(v) = \sum_{u\neq v} T_B(u, v)$, $d_I(v) = \sum_{u\neq v} I_B(u, v)$ and $d_F(v) = \sum_{u\neq v} F_B(u, v)$

Definition 2.7. The minimum degree of G is $\delta(G) = (\delta_T(G), \delta_I(G), \delta_F(G))$, where $\delta_T(G) = min\{d_T(v)|v \in V\}$, $\delta_I(G) = min\{d_I(v)|v \in V\}$ and $\delta_F(G) = max\{d_F(v)|v \in V\}$

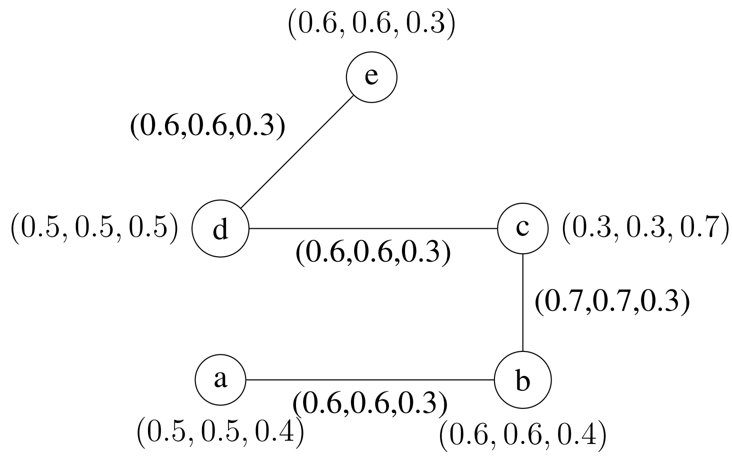


Figure 3: H : Single-valued co-neutrosophic subgraph

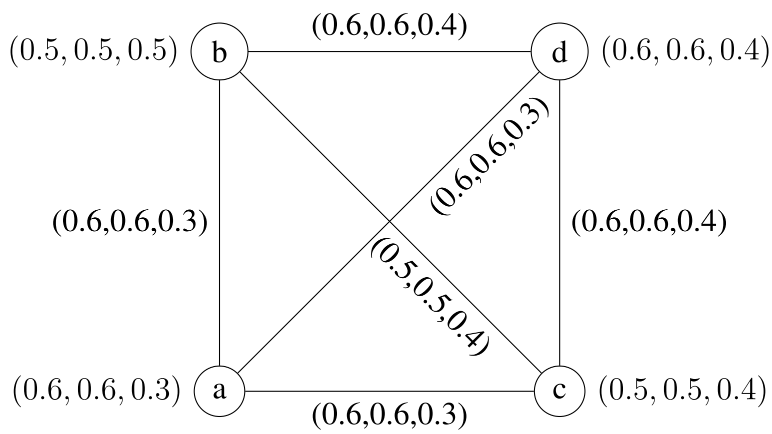


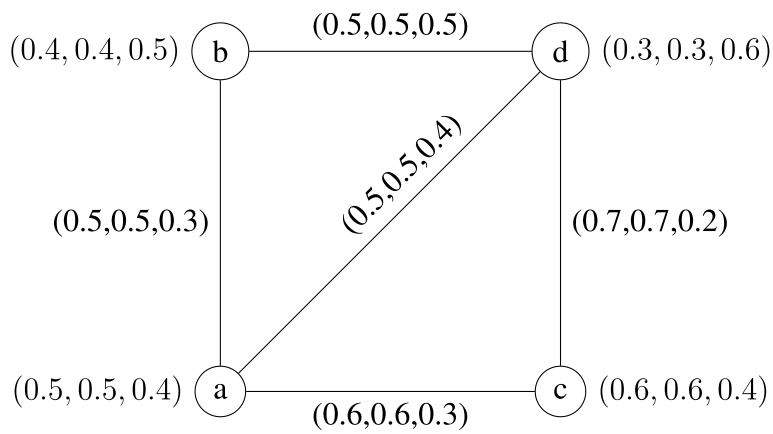
Figure 4: Complete single-valued co-neutrosophic graph

Definition 2.8. The maximum degree of G is $\Delta(G) = (\Delta_T(G), \Delta_I(G), \Delta_F(G))$, where $\Delta_T(G) = max\{d_T(v)|v \in V\}$, $\Delta_I(G) = max\{d_I(v)|v \in V\}$ and $\Delta_F(G) = min\{d_F(v)|v \in V\}$

Example 2.1. Let $G = \langle A, B\rangle$ be a single-valued co-neutrosophic graph. Draw as below

The degrees are $d_T(a) = 1.6, d_I(a) = 1.6, d_F(a) = 1.0, d_T(c) = 1.3, d_I(c) = 1.3, d_F(c) = 0.5$, $d_T(d) = 1.7, d_I(d) = 1.7, d_F(d) = 1.1, d_T(b) = 1.0, d_I(b) = 1.0, d_F(b) = 0.8$.
Minimum degree of a graph is $\delta_T(G) = 1.0, \delta_I(G) = 1.0, \delta_F(G) = 1.1$
Maximum degree of a graph is $\Delta_T(G) = 1.7, \Delta_I(G) = 1.7, \Delta_F(G) = 0.5$

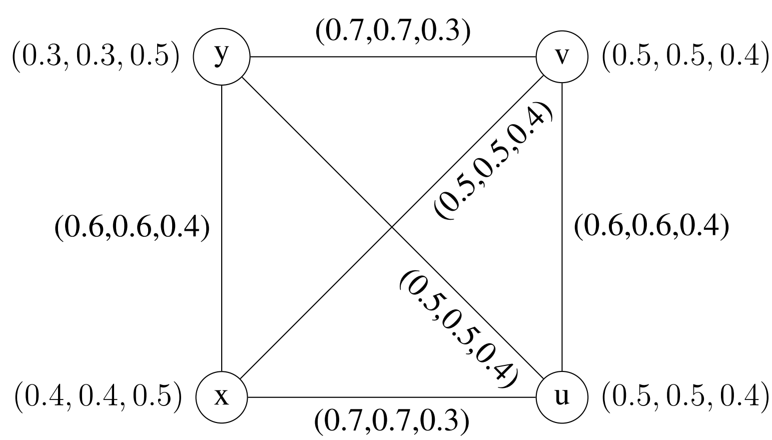
Definition 2.9. Let $G = \langle A, B\rangle$ be a single-valued co-neutrosophic graph. The total degree of a vertex $v \in V$ is defined as :
$Td(v) = Td_T(v)+Td_I(v)+Td_F(v)$, where $Td_T(v) = \sum_{(u,v)\in E} T_B(u, v)+T_A(v)$, $Td_I(v) = \sum_{(u,v)\in E} I_B(u, v)+I_A(v)$ and $Td_F(v) = \sum_{(u,v)\in E} F_B(u, v) + F_A(v)$.



If each vertex of G has the same total degree (r_1, r_2, r_3), then G is said to be an (r_1, r_2, r_3) totally regular single-valued co-neutrosophic graph.

Definition 2.10. Let $G = \langle A, B \rangle$ be a single-valued co-neutrosophic graph. If each vertex has same degree (r, s, t), then G is called (r, s, t) regular single-valued co-neutrosophic graph. Thus $r = d_T(v), s = d_I(v), t = d_F(v)$;for $v \in V$.

Example 2.2. Let $G = \langle A, B \rangle$ be a single-valued co-neutrosophic graph. Draw as below



$d(y) = (1.8, 1.8, 1.1), d(v) = (1.8, 1.8, 1.1), d(u) = (1.8, 1.8, 1.1), d(x) = (1.8, 1.8, 1.1)$. So, G is a regular single-valued co-neutrosophic graph. But G is not totally regular single-valued co-neutrosophic graph. Since $Td(y) = 5.8 \neq 6.1 = Td(v)$.

Remark 2.1. (a) For a single-valued co-neutrosophic graph, H = (A,B) to be both regular & totally regular, the number of vertices in each edge must be same.

(b) And also each vertex lies in exactly same number of edges.

Proposition 2.1. Let $G = \langle A, B \rangle$ be a single-valued co-neutrosophic graph. Then $T_A : V \to [0, 1], I_A : V \to [0, 1], F_A : V \to [0, 1]$ is a constant function iff following are equivalent.

(1) G is a regular single-valued co-neutrosophic graph,

(2) G is a totally regular single-valued co-neutrosophic graph.

Proof. suppose that (T_A, I_A, F_A) is a constant function. Let $T_A(v_i) = k_1, I_A(v_i) = k_2, F_A(v_i) = k_3$ for all $v_i \in V$. Assume that G is a (r_1, r_2, r_3) regular single-valued co-neutrosophic graph. Then $d_T(v_i) = r_1, d_I(v_i) = r_2, d_F(v_i) = r_3$ for all $v_i \in V$. So $Td(v_i) = Td_T(v_i) + Td_I(v_i) + Td_F(v_i)$

$$\begin{aligned} Td_T(v_i) &= d_T(v_i) + T_A(v_i), \text{ for all } v_i \in V \\ &= r_1 + k_1 = c_1. \\ Td_I(v_i) &= d_I(v_i) + I_A(v_i), \text{ for all } v_i \in V \\ &= r_2 + k_2 = c_2. \\ Td_F(v_i) &= d_F(v_i) + F_A(v_i), \text{ for all } v_i \in V \\ &= r_3 + k_3 = c_3. \end{aligned}$$

Hence G is totally regular single-valued co-neutrosophic graph. Thus $(1) \Rightarrow (2)$ is proved.

Now, suppose that G is a (t_1, t_2, t_3) totally regular single-valued co-neutrosophic graph, then $Td_T(v_i) = t_1, Td_T(v_i) = t_2, Td_F(v_i) = t_3$ for all $v_i \in V$.

$$Td_T(v_i) = d_T(v_i) + T_A(v_i) = t_1,$$
$$\Rightarrow d_T(v_i) = t_1 - T_A(v_i) = t_1 - k_1, \text{ for all } v_i \in V.$$

Similarly, $Td_I(v_i) = d_I(v_i) + I_A(v_i) = t_2$,
$\Rightarrow d_I(v_i) = t_2 - I_A(v_i) = t_2 - k_2$, for all $v_i \in V$.

$Td_F(v_i) = d_F(v_i) + F_A(v_i) = t_3$,
$\Rightarrow d_F(v_i) = t_3 - F_A(v_i) = t_3 - k_3$, for all $v_i \in V$. So, G is a regular single-valued co-neutrosophic graph. Thus $(2) \Rightarrow (1)$ is proved. Hence (1) and (2) are equivalent. □

Proposition 2.2. If a single-valued co-neutrosophic graph is both regular and totally regular, then (T_A, I_A, F_A) is constant function.

Proof. Let G be a (r, s, t) regular and (k_1, k_2, k_3) totally regular single-valued co-neutrosophic graphs. So, $d_T(v_1) = r, d_I(v_1) = s, d_F(v_1) = t$ for $v_1 \in V$ and $Td_T(v_1) = k_1, Td_I(v_1) = k_2, Td_F(v_1) = k_3$ for all $v_1 \in V$. Now,

$$\begin{aligned} Td_T(v_1) &= k_1, \text{ for all } v_1 \in V, \\ d_T(v_1) + T_A &= k_1, \text{ for all } v_1 \in V, \\ r + T_A(v_1) &= k_1, \text{ for all } v_1 \in V, \\ T_A(v_1) &= k_1 - r, \text{ for all } v_1 \in V. \end{aligned}$$

Hence $T_A(v_1)$ is a constant function.

Similarly, $I_A(v_1) = k_2 - s$ for all $v_1 \in V$ and $F_A(v_1) = k_3 - t$ for all $v_1 \in V$. Hence (T_A, I_A, F_A) is a constant. □

3 Conclusion

In this paper, we introduced the notion of a single-valued co-neutrosophic graphs and study some methods of construction of new single-valued co-neutrosophic graphs. We computed degree of a vertex, strong single-valued co-neutrosophic graphs and complete single-valued co-neutrosophic graphs. Properties of regular and totally regular single-valued co-neutrosophic graphs are discussed. In future, we are introduce and discuss the energy of Single-valued co-neutrosophic graphs.

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