NSS

University of New Mexico

On Single-Valued Neutrosophic Entropy of order α

Harish Garg, Nancy

School of Mathematics, Thapar University Patiala - 147004, Punjab, India. E-mail: harishg58iitr@gmail.com

Abstract: Entropy is one of the measures which is used for measuring the fuzziness of the set. In this article, we have presented an entropy measure of order α under the single-valued neutrosophic set environment by considering the pair of their membership functions as well as the hesitation degree between them. Based on this measure, some of its desirable properties have been proposed and validated by taking an example of structure linguistic variable. Furthermore, an approach based on the proposed measure has been presented to deal with the multi criteria decision-making problems. Finally, a practical example is provided to illustrate the decision-making process.

Keywords: Entropy measure, neutrosophic set, multi criteria decision-making, linguistic variable.

1 Introduction

In a real world, due to complexity of decision making or various constraints in today's life, it is difficult for the decision makers to give their opinions in a precise form. To handle these situations, fuzzy set (FS) theory [1], intuitionistic fuzzy set (IFS) theory [2] are successful theories for dealing the uncertainties in the data. After their pioneer works, various researchers have worked on these theories under the different domains such as on entropy measures, on correlation coefficients, on aggregation operators, and many others [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. However, both FS and IFS theories are not able to deal with the indeterminate and inconsistent information. For example, if an expert take an opinion from a certain person about the certain object, then a person may say that 0.5 is the possibility that statement is true, 0.7 say that the statement is false and 0.2 says that he or she is not sure of it. To resolve this, Smarandache [13] introduced a new component called as "indeterminacy-membership function" and added into the "truth membership function" and "falsity membership function", all are independent components lies in $]0^+, 1^+[$, and hence the corresponding set is known as Neutrosophic sets (NSs), which is the generalization of IFS and FS. However, without specification, NSs are difficult to apply in real-life problems. Thus, an extension of the NS, called a single-valued NSs (SVNSs) has been proposed by Wang et al. [14]. After their pioneer work, researchers are engaged in their extensions and their applications in the different disciplines. However, the most important task for the decision maker is to rank the objects so as to get the desired one(s). For it, researchers have incorporating the idea of SVNS theory into the measure theory and applied in many practically uncertain situations such as decision making, pattern recognition, medical diagnosis by using similarity measures [15, 16], distance measures [17, 18], cosine similarity measure [19, 20, 21, 22]. Thus, it has been concluded that the information measures such as entropy, divergence, distance, similarity etc., are of key importance in a number of theoretical and applied statistical inference and data processing problems.

But it has been observed from the above studies that all their measures do not incorporate the idea of the decision-maker preferences into the measure. Furthermore, the existing measure is in linear order, and hence it does not give the exact nature of the alternative. Therefore, keeping the criteria of flexibility and efficiency of neutrosophic sets, this paper presents a new parametric entropy measure of order α for measuring the fuzziness degree of a set. For this, a entropy measure of order α has been presented which makes the decision makers more reliable and flexible for the different values of these parameters. Based on it, some desirable properties of these measures have been studied.

The rest of the manuscript is summarized as follows. Section 2 presents some basic definition about the NS. In Section 3, a new entropy of order α is proposed and its axiomatic justification is established. Further, various desirable properties of it in terms of joint, and conditional entropies have been studied. An illustrative example to show their superiority has been described for structural linguistic variable. Section 4 presents the MCDM method based on the proposed generalized entropy measure along with an illustrative example for selecting the best alternative. Finally a conclusion has been drawn in Section 5.

2 Preliminaries

In this section, some needed basic concepts and definitions related to neutrosophic sets (NS) are introduced.

Definition 2.1. *[13] A NS 'A' in X is defined by its "truth membership function" $(T_A(x))$, a "indeterminacy-membership function" $(I_A(x))$ and a "falsity membership function" $(F_A(x))$ where all are the subset of $]0^-, 1^+[$ such that $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ for all $x \in X$.*

Definition 2.2. *[14] A NS 'A' is defined by*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

and is called as SVNS where $T_A(x), I_A(x), F_A(x) \in [0, 1]$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. The pairs of these is called as single-valued neutrosophic numbers (SVNNs) denoted by

$$\alpha = \langle \mu_A(x), \rho_A(x), \nu_A(x) \mid x \in X\rangle$$

and class of SVNSs is denoted by $\Phi(X)$.

Definition 2.3. *Let* $A=\langle \mu_A(x),\rho_A(x),\nu_A(x)\mid x\in X\rangle$ *and* $B=\langle \mu_B(x),\rho_B(x),\nu_B(x)\mid x\in X\rangle$ *be two SVNSs. Then the following expressions are defined by [14]*

(i) $A\subseteq B$ *if and only if* $\mu_A(x)\le\mu_B(x),\rho_A(x)\ge\rho_B(x)$ *and* $\nu_A(x)\ge\nu_B(x)$ *for all x in X;*

(ii) $A=B$ *if and only if* $A\subseteq B$ *and* $B\subseteq A$.

(iii) $A^c=\{\langle\nu_A(x),\rho_A(x),\mu_A(x)\mid x\in X\rangle\}$

(iv) $A\cap B=\langle\min(\mu_A(x),\mu_B(x)),\max(\rho_A(x),\rho_B(x)),\max(\nu_A(x),\nu_B(x))\rangle$

(v) $A\cup B=\langle\max(\mu_A(x),\mu_B(x)),\min(\rho_A(x),\rho_B(x)),\min(\nu_A(x),\nu_B(x))\rangle$

Majumdar and Samant [16] define the concept of entropy for neutrosophic sets which has been defined as below.

Definition 2.4. *An entropy on* $SVNS(X)$ *is defined as real valued function* $E: SVNS(X)\to[0,1]$ *which satisfies following axioms [16]:*

(P1) $E(A)=0$ *if A is crisp set.*

(P2) $E(A)=1$ *if* $\mu_A(x)=\rho_A(x)=\nu_A(x)$

(P3) $E(A)=E_(A^c)$ *for all* $A\in SVNS(X)$

(P4) $E(A)\le E(B)$ *if* $A\subseteq B$ *that is ,* $\mu_A(x)\le\mu_B(x),\nu_A(x)\ge\nu_B(x)$ *and* $\rho_A(x)\ge\rho_B(x)$ *for* $\mu_B(x)\le\nu_B(x)$ *and* $\mu_B(x)\le\rho_B(x)$.

3 Entropy of order-α

In this section we proposed parametric entropy for $SVNS$

Definition 3.1. *The entropy of order-* α *for* $SVNS$ A *is defined as:*

$$\begin{aligned}E_\alpha(A) &= \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log_3\Big[\big(\mu_A^\alpha(x_i)+\rho_A^\alpha(x_i)+\nu_A^\alpha(x_i)\big)\\ &\times \big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\big)^{1-\alpha}\\ &+ 3^{1-\alpha}\big(1-\mu_A(x_i)-\rho_A(x_i)-\nu_A(x_i)\big)\Big], \qquad (1)\end{aligned}$$

where $\alpha>0,\alpha\ne1$.

Theorem 1. $E_\alpha(A)$ *as defined in Definition 3.1 is entropy for* $SVNS$.

Proof. In order to proof $E_\alpha(A)$ is a valid measure, we have to proof that it satisfies the axioms as given in Definition 2.4.

(P1) Let A be a crisp set i.e. $A=(1,0,0)$ or $A=(0,0,1)$. Then from Definition 3.1 we get $E_\alpha(A)=0$.

(P2) Let $\mu_A(x_i)=\rho_A(x_i)=\nu_A(x_i)$ for all $x_i\in X$ which implies that $E_\alpha(A)$ becomes

$$\begin{aligned}&E_\alpha(A)\\ &= \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log_3\Big[\big(\mu_A^\alpha(x_i)+\mu_A^\alpha(x_i)+\mu_A^\alpha(x_i)\big)\times\\ &\qquad +\big(\mu_A(x_i)+\mu_A(x_i)+\mu_A(x_i)\big)^{(1-\alpha)}\\ &\qquad +3^{1-\alpha}\big(1-\mu_A(x_i)-\mu_A(x_i)-\mu_A(x_i)\big)\Big]\\ &= \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log_3\Big[\big(3\mu_A^\alpha(x_i)\big)\big(3\mu_A(x_i)\big)^{1-\alpha}\\ &\qquad +3^{1-\alpha}\big(1-3\mu_A(x_i)\big)\Big]\\ &= \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log_3\Big[3^{2-\alpha}\mu_A(x_i)\\ &\qquad +3^{1-\alpha}-3^{2-\alpha}\mu_A(x_i)\Big]\\ &= 1\end{aligned}$$

Now, let $E_\alpha(A)=1$, that is,

$$\begin{aligned}&\sum_{i=1}^{n}\log_3\Big[\big(\mu_A^\alpha(x_i)+\rho_A^\alpha(x_i)+\nu_A^\alpha(x_i)\big)\big(\mu_A(x_i)+\rho_A(x_i)\\ &+\nu_A(x_i)\big)^{1-\alpha}+3^{1-\alpha}\big(1-\mu_A(x_i)-\rho_A(x_i)-\nu_A(x_i)\big)\Big]\\ &=n(1-\alpha)\\ \Rightarrow\ &\log_3\Big[\big(\mu_A^\alpha(x_i)+\rho_A^\alpha(x_i)+\nu_A^\alpha(x_i)\big)\big(\mu_A(x_i)+\rho_A(x_i)+\\ &\nu_A(x_i)\big)^{1-\alpha}+3^{1-\alpha}\big(1-\mu_A(x_i)-\rho_A(x_i)-\nu_A(x_i)\big)\Big]\\ &=(1-\alpha)\\ \Rightarrow\ &\big(\mu_A^\alpha(x_i)+\rho_A^\alpha(x_i)+\nu_A^\alpha(x_i)\big)\Big(\mu_A(x_i)+\rho_A(x_i)+\\ &\nu_A(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_A(x_i)-\rho_A(x_i)-\nu_A(x_i)\Big)\\ &=3^{1-\alpha}\\ \Rightarrow\ &\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)\Bigg[\frac{\mu_A^\alpha(x_i)+\rho^\alpha(x_i)+\nu_A^\alpha(x_i)}{3}\\ &-\Big(\frac{\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)}{3}\Big)^\alpha\Bigg]=0 \qquad (2)\end{aligned}$$

From Eq. (2) we get, either $\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)=0$ implies that

$$\mu_A(x_i)=\rho_A(x_i)=\nu_A(x_i)=0 \text{ for all } x_i\in X \qquad (3)$$

or

$$\frac{\mu_A^\alpha(x_i)+\rho^\alpha(x_i)+\nu_A^\alpha(x_i)}{3}-\Big(\frac{\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)}{3}\Big)^\alpha=0 \qquad (4)$$

Now, consider the following function

$$g(\zeta)=\zeta^\alpha \text{ where } \zeta\in[0,1]$$

Differentiate it with respect to ζ, we get

$$g'(\zeta) = \alpha\zeta^{\alpha-1}$$

$$g''(\zeta) = \alpha(\alpha-1)\zeta^{\alpha-2}$$

because $g''(\zeta) > 0$ for $\alpha > 1$ and $g''(\zeta) < 0$ for $\alpha < 1$ therefore $g(\zeta)$ is convex or concave according to $\alpha > 1$ or $\alpha < 1$. So, for any points ζ_1, ζ_2 and ζ_3 in $[0, 1]$, we have

$$\frac{g(\zeta_1)+g(\zeta_2)+g(\zeta_3)}{3} - g\Big(\frac{\zeta_1+\zeta_2+\zeta_3}{3}\Big) \geq 0 \text{ for } \alpha > 1 \quad (5)$$

$$\frac{g(\zeta_1)+g(\zeta_2)+g(\zeta_3)}{3} - g\Big(\frac{\zeta_1+\zeta_2+\zeta_3}{3}\Big) \leq 0 \text{ for } \alpha < 1 \quad (6)$$

In above, equality holds only if $\zeta_1 = \zeta_2 = \zeta_3$. Hence from Eqs. (3),(4), (5) and (6) we conclude Eqs. (2) and (4) holds only when $\mu_A(x_i) = \rho_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$.

(P3) Since $A^c = \{\langle x, \nu_A(x), \rho_A(x), \mu_A(x) \mid x \in X\rangle\}$ which implies that $E_\alpha(A^c) = E_\alpha(A)$.

(P4) Rewrite the entropy function as

$$\begin{aligned} f(x,y,z) &= \\ \frac{1}{1-\alpha}\sum_{i=1}^{n}\log_3 &\Big[\big(x^\alpha+y^\alpha+z^\alpha\big)\big(x+y+z\big)^{1-\alpha} \\ &+3^{1-\alpha}(1-x-y-z)\Big] \end{aligned} \quad (7)$$

where $x, y, z \in [0, 1]$. In order to proof the proposed entropy satisfies (P4), it is sufficient to prove that the function f defined in Eq. (7) is an increasing function with respect to x and decreasing with respect to y and z. For it, take a partial derivative of the function with respect to x, y and z and hence we get.

$$\frac{\partial f}{\partial x} = \frac{(1-\alpha)(x^\alpha+y^\alpha+z^\alpha)(x+y+z)^{-\alpha} + \alpha(x+y+z)^{1-\alpha}x^{\alpha-1} - 3^{1-\alpha}}{(x^\alpha+y^\alpha+z^\alpha)(x+y+z)^{1-\alpha} + 3^{1-\alpha}(1-x-y-z)]} \quad (8)$$

$$\frac{\partial f}{\partial y} = \frac{(1-\alpha)(x^\alpha+y^\alpha+z^\alpha)(x+y+z)^{-\alpha} + \alpha(x+y+z)^{1-\alpha}y^{\alpha-1} - 3^{1-\alpha}}{(1-\alpha)[(x^\alpha+y^\alpha+z^\alpha)(x+y+z)^{1-\alpha} + 3^{1-\alpha}(1-x-y-z)]} \quad (9)$$

$$\frac{\partial f}{\partial z} = \frac{(1-\alpha)(x^\alpha+y^\alpha+z^\alpha)(x+y+z)^{-\alpha} + \alpha(x+y+z)^{1-\alpha}z^{\alpha-1} - 3^{1-\alpha}}{(1-\alpha)[(x^\alpha+y^\alpha+z^\alpha)(x+y+z)^{1-\alpha} + 3^{1-\alpha}(1-x-y-z)]} \quad (10)$$

After setting $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$ and $\frac{\partial f}{\partial z} = 0$, we get $x = y = z$. Also,

$$\frac{\partial f}{\partial x} \geq 0, \text{ whenever } x \leq y, x \leq z, \alpha > 0, \alpha \neq 0 \quad (11)$$

$$\frac{\partial f}{\partial x} \leq 0, \text{ whenever } x \geq y, x \geq z, \alpha > 0, \alpha \neq 0. \quad (12)$$

Thus, $f(x, y, z)$ is increasing function with respect to x for $x \leq y, x \leq z$ and decreasing when $x \geq y, x \geq z$. Similarly, we have

$$\frac{\partial f}{\partial y} \leq 0 \text{ and } \frac{\partial f}{\partial z} \leq 0, \text{whenever } x \leq y, x \leq z. \quad (13)$$

$$\frac{\partial f}{\partial y} \geq 0 \text{ and } \frac{\partial f}{\partial z} \geq 0, \text{whenever } x \geq y, x \geq z. \quad (14)$$

Thus, $f(x, y, z)$ is decreasing function with respect to y and z for $x \leq y, x \leq z$ and increasing when $x \geq y, x \geq z$.

Therefore from monotonicity of function f, and by taking two $SVNSs$ $A \subseteq B$, i.e., $\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x)$ and $\rho_A(x) \geq \rho_B(x)$ for $\mu_B(x) \leq \nu_B(x)$ and $\mu_B(x) \leq \rho_B(x)$, we get $E_\alpha(A) \leq E_\alpha(B)$.

□

Example 3.1. *Let A be $SVNS$ in universe of discourse $X = \{x_1, x_2, x_3, x_4\}$ defined as $A = \{\langle x_1, 0.4, 0.3, 0.9\rangle, \langle x_2, 0.7, 0.5, 0.3\rangle, \langle x_3, 0.2, 0.9, 0.8\rangle, \langle x_4, 0.5, 0.4, 0.6\rangle\}$. Then entropies values for different values of α is $E_{0.2}(A) = 0.9710; E_{0.5}(A) = 0.9303; E_2(A) = 0.7978; E_5(A) = 0.7246; E_{10}(A) = 0.7039$. It is clearly seen from this result that with the increase of α, the values of $E_\alpha(A)$ is decreases.*

The above proposed entropy measure of order α satisfies the following additional properties.

Consider two $SVNSs$ A and B defined over $X = \{x_1, x_2, \ldots, x_n\}$. Take partition of X as $X_1 = \{x_i \in X : A \subseteq B\}$, $X_2 = \{x_i \in X : A \supseteq B\}$. Then we define the joint and conditional entropies between them as follows

(i) **Joint entropy**

$$\begin{aligned}
&E_\alpha(A\cup B) \\
=\ & \frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log_3\Big[\big(\mu^\alpha_{A\cup B}(x_i)+\rho^\alpha_{A\cup B}(x_i)+\nu^\alpha_{A\cup B}(x_i)\big)\times \\
&\big(\mu_{A\cup B}(x_i)+\rho_{A\cup B}(x_i)+\nu_{A\cup B}(x_i)\big)^{1-\alpha} \\
&+3^{1-\alpha}\big(1-\mu_{A\cup B}(x_i)-\rho_{A\cup B}(x_i)-\nu_{A\cup B}(x_i)\big)\Big] \\
=\ & \frac{1}{n(1-\alpha)}\Bigg\{\sum_{x_i\in X_1}\log_3\Bigg[\Big(\mu^\alpha_B(x_i)+\rho^\alpha_B(x_i)+\nu^\alpha_B(x_i)\Big)\times \\
&\Big(\mu_B(x_i)+\rho_B(x_i)+\nu_B(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_B(x_i) \\
&-\rho_B(x_i)-\nu_B(x_i)\Big)\Bigg]+\sum_{x_i\in X_2}\log_3\Bigg[\Big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i) \\
&+\nu^\alpha_A(x_i)\Big)\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)^{1-\alpha} \\
&+3^{1-\alpha}\Big(1-\mu_A(x_i)-\rho_A(x_i)-\nu_A(x_i)\Big)\Bigg]\Bigg\}
\end{aligned} \quad (15)$$

(ii) **Conditional entropy**

$$
\begin{aligned}
& E_\alpha(A|B) \\
= \; & \frac{1}{n(1-\alpha)} \sum_{x_i\in X_2} \Bigg\{ \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] - \log_3 \Bigg[\Big(\mu^\alpha_B(x_i) + \rho^\alpha_B(x_i) + \nu^\alpha_B(x_i) \Big) \times \\
& \Big(\mu_B(x_i) + \rho_B(x_i) + \nu_B(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_B(x_i) \\
& - \rho_B(x_i) - \nu_B(x_i) \Big) \Bigg] \Bigg\}
\end{aligned}
$$

and

$$
\begin{aligned}
& E_\alpha(B|A) \\
= \; & \frac{1}{n(1-\alpha)} \sum_{x_i\in X_1} \Bigg\{ \log_3 \Bigg[\Big(\mu^\alpha_B(x_i) + \rho^\alpha_B(x_i) + \nu^\alpha_B(x_i) \Big) \times \\
& \Big(\mu_B(x_i) + \rho_B(x_i) + \nu_B(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_B(x_i) \\
& - \rho_B(x_i) - \nu_B(x_i) \Big) \Bigg] - \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \Bigg\}
\end{aligned}
$$

Here $E_\alpha(A|B)$ is "entropy of A given B".

Theorem 2. *For $SVNSs$ A and B following statements hold*

(i) $E_\alpha(A\cup B) = E_\alpha(A) + E_\alpha(B|A)$

(ii) $E_\alpha(A\cup B) = E_\alpha(B) + E_\alpha(A|B)$

(iii) $E_\alpha(A\cup B) = E_\alpha(A) + E_\alpha(B|A) = E_\alpha(B) + E_\alpha(A|B)$

(iv) $E_\alpha(A\cup B) + E_\alpha(A\cap B) = E_\alpha(A) + E_\alpha(B)$.

Proof. (i) Here, we have to proof (i) only, (ii) and (iii) can be follows from it.

$$
\begin{aligned}
& E_\alpha(A) + E_\alpha(B|A) - E_\alpha(A\cup B) \\
= \; & \frac{1}{n(1-\alpha)} \sum_{i=1}^{n} \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \\
+ \; & \frac{1}{n(1-\alpha)} \sum_{x_i\in X_1} \Bigg\{ \log_3 \Bigg[\Big(\mu^\alpha_B(x_i) + \rho^\alpha_B(x_i) + \nu^\alpha_B(x_i) \Big) \times \\
& \Big(\mu_B(x_i) + \rho_A(x_i) + \nu_B(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_B(x_i) \\
& - \rho^{(}_B x_i) - \nu_B(x_i) \Big) \Bigg] - \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \Bigg\} \\
- \; & \frac{1}{n(1-\alpha)} \Bigg\{ \sum_{x_i\in X_1} \log_3 \Bigg[\Big(\mu^\alpha_B(x_i) + \rho^\alpha_B(x_i) + \nu^\alpha_B(x_i) \Big) \times \\
& \Big(\mu_B(x_i) + \rho_B(x_i) + \nu_B(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_B(x_i) \\
& - \rho_B(x_i) - \nu_B(x_i) \Big) \Bigg] - \sum_{x_i\in X_2} \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i)\nu^\alpha_A(x_i) \Big) \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \Bigg\} \\
= \; & \frac{1}{n(1-\alpha)} \Bigg\{ \sum_{x_i\in X_1} \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \nu_A(x_i) \Big) \Bigg] + \sum_{x_i\in X_2} \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \Bigg\} \\
+ \; & \frac{1}{n(1-\alpha)} \sum_{x_i\in X_1} \Bigg\{ \log_3 \Bigg[\Big(\mu^\alpha_B(x_i) + \rho^\alpha_B(x_i) + \nu^\alpha_B(x_i) \Big) \times \\
& \Big(\mu_B(x_i) + \rho_A(x_i) + \nu_B(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_B(x_i) \\
& - \rho_B(x_i) - \nu_B(x_i) \Big) \Bigg] - \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \rho^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \Bigg\} \\
- \; & \frac{1}{n(1-\alpha)} \Bigg\{ \sum_{x\in X_1} \log_3 \Bigg[\Big(\mu^\alpha_B(x_i) + \rho^\alpha_B(x_i) + \nu^\alpha_B(x_i) \Big) \times \\
& \Big(\mu_B(x_i) + \rho_B(x_i) + \nu_B(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_B(x_i) \\
& - \rho_B(x_i) - \nu_B(x_i) \Big) \Bigg] - \sum_{x_i\in X_2} \log_3 \Bigg[\Big(\mu^\alpha_A(x_i) + \nu^\alpha_A(x_i) \Big) \times \\
& \Big(\mu_A(x_i) + \rho_A(x_i) + \nu_A(x_i) \Big)^{1-\alpha} + 3^{1-\alpha} \Big(1 - \mu_A(x_i) \\
& - \rho_A(x_i) - \nu_A(x_i) \Big) \Bigg] \Bigg\} \\
= \; & 0
\end{aligned}
$$

(iv) For an $SVNSs$ A and B, we have

$$\begin{aligned}
&E_\alpha(A\cap B)\\
=\ &\frac{1}{n(1-\alpha)}\sum_{i=1}^{n}\log_3\Big[\Big(\mu^\alpha_{A\cap B}(x_i)+\rho^\alpha_{A\cap B}(x_i)+\nu^\alpha_{A\cap B}(x_i)\Big)\times\\
&\Big(\mu_{A\cap B}(x_i)+\rho_{A\cap B}(x_i)+\nu_{A\cap B}(x_i)\Big)^{1-\alpha}+\\
&3^{1-\alpha}\Big(1-\mu_{A\cap B}(x_i)-\nu_{A\cap B}(x_i)\Big)\Big]\\
=\ &\frac{1}{n(1-\alpha)}\Big\{\sum_{x\in X_1}\log_3\Big[\Big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i)+\nu^\alpha_A(x_i)\Big)\times\\
&\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)^{(1-\alpha)}+3^{1-\alpha}\Big(1-\mu_A(x_i)\\
&-\rho_A(x_i)-\nu_A(x_i)\Big)\Big]+\sum_{x\in X_2}\log_3\Big[\Big(\mu^\alpha_B(x_i)+\rho^\alpha_B(x_i)+\nu^\alpha_B(x_i)\Big)\\
&\Big(\mu_B(x_i)+\rho_B(x_i)+\nu_B(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_B(x_i)\\
&-\rho_B(x_i)-\nu_B(x_i)\Big)\Big]\Big\}
\end{aligned}$$

Hence, by the definition of joint entropy $E_\alpha(A\cup B)$ given in Eq. (15), we get

$$E_\alpha(A\cup B)+E_\alpha(A\cap B)=E_\alpha(A)+E_\alpha(B)$$

□

Theorem 3. *For $SVNSs$ A and B following statements holds*

(i) $E_\alpha(A)-E_\alpha(A\cap B)=E_\alpha(A|B)$

(ii) $E_\alpha(B)-E_\alpha(A\cap B)=E_\alpha(A|B)$

Proof. We prove (i) part only, other can be proven similarly.

Consider

$$\begin{aligned}
&E_\alpha(A)-E_\alpha(A\cap B)\\
=\ &\frac{1}{n(1-\alpha)}\Big\{\sum_{i=1}^{n}\log_3\Big[\big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i)+\nu^\alpha_A(x_i)\Big)\times\\
&\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_A(x_i)\\
&-\rho_A(x_i)-\nu_A(x_i)\Big)\Big]-\sum_{i=1}^{n}\log_3\Big[\Big(\mu^\alpha_{A\cap B}(x_i)+\rho^\alpha_{A\cap B}(x_i)\\
&+\nu^\alpha_{A\cap B}(x_i)\big)\Big(\mu_{A\cap B}(x_i)+\rho_{A\cap B}(x_i)+\nu_{A\cap B}(x_i)\Big)^{1-\alpha}\\
&+3^{1-\alpha}\Big(1-\mu_{A\cap B}(x_i)-\rho_{A\cap B}(x_i)-\nu_{A\cap B}(x_i)\Big)\Big]\Big\}\\
=\ &\frac{1}{n(1-\alpha)}\Big\{\sum_{x\in X_1}\log_3\Big[\Big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i)+\nu^\alpha_A(x_i)\Big)\times\\
&\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_A(x_i)\\
&-\nu_A(x_i)\Big)\Big]+\sum_{x\in X_2}\log_3\Big[\Big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i)+\nu^\alpha_A(x_i)\Big)\times\\
&\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i))^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_A(x_i)\\
&\rho_A(x_i)-\nu_A(x_i)\Big)\Big]-\sum_{x\in X_1}\log_3\Big[\Big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i)+\nu^\alpha_A(x_i)\Big)\times\\
&\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_A(x_i)\\
&-\rho_A(x_i)-\nu_A(x_i)\Big)\Big]-\sum_{x\in X_2}\log_3\Big[\Big(\mu^\alpha_B(x_i)+\rho^\alpha_B(x_i)+\nu^\alpha_B(x_i)\Big)\times\\
&\Big(\mu_B(x_i)+\rho_B(x_i)+\nu_B(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_B(x_i)\\
&-\rho_B(x_i)-\nu_B(x_i)\Big)\Big]\Big\}\\
+\ &\frac{1}{n(1-\alpha)}\sum_{x\in X_2}\Big\{\log_3\Big[\Big(\mu^\alpha_A(x_i)+\rho^\alpha_A(x_i)+\nu^\alpha_A(x_i)\Big)\times\\
&\Big(\mu_A(x_i)+\rho_A(x_i)+\nu_A(x_i)\Big)^{(1-\alpha)}+3^{1-\alpha}\Big(1-\mu_A(x_i)\\
&-\rho_A(x_i)-\nu_A(x_i)\Big)\Big]-\log_3\Big[\Big(\mu^\alpha_B(x_i)+\rho^\alpha_B(x_i)+\nu^\alpha_B(x_i)\Big)\times\\
&\Big(\mu_B(x_i)+\rho_B(x_i)+\nu_B(x_i)\Big)^{1-\alpha}+3^{1-\alpha}\Big(1-\mu_B(x_i)\\
&-\rho_B(x_i)-\nu_B(x_i)\Big)\Big]\Big\}\\
=\ &E_\alpha(A|B)
\end{aligned}$$

This completes the proof. □

Let $A=\langle x,\mu_A(x),\rho_A(x),\nu_A(x)|x\in X\rangle$ be $SVNS$ in X. For n be any positive real number, Zhang et al. [23] defined A^n as follows

$$A^n=\langle x,\mu_A(x)^n,1-(1-\rho_A(x))^n,1-(1-\nu_A(x))^n\rangle \quad (16)$$

Definition 4. *Contraction of $SVNS$ A in universe of discourse X is defined by*

$$CON(A)=\langle x,\mu_{CON(A)}(x),\rho_{CON(A)}(x),\nu_{CON(A)}(x)\rangle$$

where $\mu_{CON(A)}(x)=[\mu_A(x)]^2$; $\rho_{CON(A)}(x)=1-[1-\rho_A(x)]^2$; $\nu_{CON(A)}(x)=1-[1-\nu_A(x)]^2$ *i.e.* $CON(A)=A^2$

Definition 5. *Dilation of $SVNS$ A in universe of discourse X is defined by*

$$DIL(A)=\langle x,\mu_{DIL(A)}(x),\rho_{DIL(A)}(x),\nu_{DIL(A)}(x)\rangle$$

where $\mu_{DIL(A)}(x)=[\mu_A(x)]^{1/2}$; $\rho_{DIL(A)}(x)=1-[1-\rho_A(x)]^{1/2}$; $\nu_{DIL(A)}(x)=1-[1-\nu_A(x)]^{1/2}$ *i.e.* $DIL(A)=$

$A^{1/2}$

An illustrative example has been tested on the concentration and dilation for comparing the performance of proposed entropy with the some existing entropies as given below.

(i) Entropy defined by [5];

$$E_{SK}(A) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\min(\mu_A(x_i), \nu_A(x_i)) + \pi_A(x_i)}{\max(\mu_A(x_i), \nu_A(x_i)) + \pi_A(x_i)}\right]$$

(ii) Entropy defined by [3];

$$E_{BB}(A) = \frac{1}{n}\sum_{i=1}^{n}\pi_A(x_i)$$

(iii) Entropy defined by [8];

$$E_{ZJ}(A) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\mu_A(x_i) \bigwedge \nu A(x_i)}{\mu_A(x_i) \bigvee \nu A(x_i)}\right)$$

(iv) Entropy defined by [4];

$$E_{ZL}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}|\mu_A(x_i) - \nu_A(x_i)|$$

Example 3.2.

Let $X = \{x_1, x_2, ..., x_5\}$ be universe of discourse and a $SVNS\ A$ "LARGE" on X may be defined as $A = \{\langle x_1, 0.1, 0.7, 0.8\rangle, \langle x_2, 0.3, 0.6, 0.5\rangle, \langle x_3, 0.5, 0.3, 0.4\rangle, \langle x_4, 0.9, 0.2, 0.0\rangle, \langle x_5, 1.0, 0.1, 0.0\rangle\}$. Using the operations defined in Eq. (16) on $SVNS$, we can generate following $SVNSs$

$$A, A^{1/2}, A^2, A^3$$

which can be defined as
$A^{1/2}$ may treated as "More or Less LARGE",
A^2 may treated as "Very LARGE",
A^3 may treated as "Quite Very LARGE"
and these corresponding sets are computed as

$A^{1/2} = \{\langle x_1, 0.3162, 0.4523, 0.5528\rangle, \langle x_2, 0.5477, 0.3675, 0.2929\rangle, \langle x_3, 0.7071, 0.1633, 0.2254\rangle, \langle x_4, 0.9487, 0.1056, 0\rangle, \langle x_5, 1.0000, 0.0513, 0\rangle\}$;
$A^1 = \{\langle x_1, 0.1, 0.7, 0.8\rangle, \langle x_2, 0.3, 0.6, 0.5\rangle, \langle x_3, 0.5, 0.3, 0.4\rangle, \langle x_4, 0.9, 0.2, 0.0\rangle, \langle x_5, 1.0, 0.1, 0\rangle\}$;
$A^2 = \{\langle x_1, 0.01, 0.91, 0.96\rangle, \langle x_2, 0.09, 0.84, 0.75\rangle, \langle x_3, 0.25, 0.51, 0.64\rangle, \langle x_4, 0.81, 0.36, 0\rangle, \langle x_5, 1.00, 0.19, 0\rangle\}$;
$A^3 = \{\langle x_1, 0.0010, 0.9730, 0.9920\rangle, \langle x_2, 0.0270, 0.9360, 0.8750\rangle, \langle x_3, 0.1250, 0.6570, 0.7840\rangle, \langle x_4, 0.7290, 0.4880, 0\rangle, \langle x_5, 1.000, 0.2710, 0\rangle\}$

The entropy measures values corresponding to existing measures as well as the proposed measures for different values of α are summarized in Table 1 for these different linguistic variable $SVNSs$. From this table, it has been concluded that with the increase of the parameter α, the entropy measure for the linguistic variable "More or Less LARGE", "LARGE', "VERY LARGE" are decreases. Also it has been observed that whenever the values of α are increases from 0 to 15 then the pattern for the variable "LARGE" is $E_\alpha(A) > E_\alpha(A^{1/2}) > E_\alpha(A^2) > E_\alpha(A^3)$ and the results coincides with the existing measures results. On the other hand, whenever the value of α are increases beyond the 15 then the order the patterns are slightly different. Hence the proposed entropy measure is used as an alternative measure for computing the order value of the linguistic variable as compared to existing. Moreover, the proposed measure is more generalized as the different different values of α will give the different choices of the decision-maker for assessing the results, and hence more reliable from linguistic variable point-of-view.

Table 1: Values of different entropy measure for IFS

Entropy measure	$A^{1/2}$	A	A^2	A^3	Ranking
E_{BB}[3]	0.0818	0.100	0.0980	0.0934	(2341)
E_{ZL}[4]	0.4156	0.4200	0.2380	0.1546	(2134)
E_{SK}[5]	0.3446	0.3740	0.1970	0.1309	(2134)
${E_{hc}}^2$[7]	0.3416	0.3440	0.2610	0.1993	(2134)
${E_r}^{1/2}$[7]	0.6672	0.6777	0.5813	0.4805	(2134)
E_{ZJ}[8]	0.2851	0.3050	0.1042	0.0383	(2134)
$E_\alpha(A)$ (Proposed measure)					
$\alpha = 0.3$	0.7548	0.7566	0.6704	0.5774	(2134)
$\alpha = 0.5$	0.7070	0.7139	0.6101	0.5137	(2134)
$\alpha = 0.8$	0.6517	0.6637	0.5579	0.4731	(2134)
$\alpha \to 1$	0.6238	0.6385	0.5372	0.4611	(2134)
$\alpha = 2$	0.5442	0.5727	0.4956	0.4513	(2134)
$\alpha = 5$	0.4725	0.5317	0.4858	0.4793	(2341)
$\alpha = 10$	0.4418	0.5173	0.4916	0.4999	(2431)
$\alpha = 15$	0.4312	0.5112	0.4937	0.5064	(2431)
$\alpha = 50$	0.4166	0.4994	0.4937	0.5064	(4231)
$\alpha = 100$	0.4137	0.4965	0.4612	0.5112	(4231)

4 MCDM problem on proposed entropy measure

In this section, we discuss the method for solving the MCDM problem based on the proposed entropy measure.

4.1 MCDM method based on proposed Entropy measure

Consider the set of different alternatives $A = \{A_1, A_2, ..., A_m\}$ having the different criteria $C = \{C_1, C_2, ..., C_n\}$ in neutrosophic environment and the steps for computing the best alternative is summarized as follows

Step 1: Construction of decision making matrix :
Arrange the each alternatives A_i under the criteria C_j according to preferences of the decision maker in the form of neutrosophic matrix $D_{m\times n} = \langle \mu_{ij}, \nu_{ij}, \rho_{ij}\rangle$ where μ_{ij} represents the degree that alternative A_i satisfies the criteria C_j, ρ_{ij} represents the degree that alternative A_i indeterminant about the criteria C_j and ν_{ij} represents the degree that alternative A_i doesn't satisfies the criteria C_j, where $0 \le \mu_{ij}, \rho_{ij}, \nu_{ij} \le 1$ and $\mu_{ij} + \rho_{ij} + \nu_{ij} \le 3$;

$i = 1, 2, ..., m$; $j = 1, 2, ...n$. The decision matrix given below

$$D_{m\times n}(x_{ij}) = \begin{bmatrix} \langle\mu_{11},\rho_{11},\nu_{11}\rangle & \langle\mu_{12},\rho_{12},\nu_{12}\rangle & \cdots & \langle\mu_{1n},\rho_{1n},\nu_{1n}\rangle \\ \langle\mu_{21},\rho_{21},\nu_{21}\rangle & \langle\mu_{22},\rho_{22},\nu_{22}\rangle & \cdots & \langle\mu_{2n},\rho_{2n},\nu_{2n}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle\mu_{m1},\rho_{m1},\nu_{m1}\rangle & \langle\mu_{m2},\rho_{m2},\nu_{m2}\rangle & \cdots & \langle\mu_{mn},\rho_{mn},\nu_{mn}\rangle \end{bmatrix}$$

Step 2: Normalized the decision making : Criterion of alternatives may be of same type or of different types . If the all criterion are of same kind then there is no need of normalization. On the other hand , we should convert the benefit type criterion values to the cost types in C by using the following method-

$$r_{ij} = \begin{cases} \beta_{ij}^c; & j \in B \\ \beta_{ij}; & j \in C \end{cases} \tag{17}$$

where $\beta_{ij}^c = \langle\nu_{ij}, \rho_{ij}, \mu_{ij}\rangle$ is complement of $\beta_{ij} = \langle\mu_{ij}, \rho_{ij}, \nu_{ij}\rangle$. Hence, we obtain the normalized NS decision making $R = [r_{ij}]_{m\times n}$.

Step 3: Compute the aggregated value of the alternatives: By using the proposed entropy measure aggregated the rating values corresponding to each alternatives $A_i(i = 1, 2, ..., m)$ and get the overall value r_i.

Step 4: Rank the Alternatives: Rank all the alternatives $A_i(i = 1, 2, ..., m)$ according to the values of proposed entropy obtained from Step 3 and get the most desirable alternative.

4.2 Illustrative Example

Let us consider multi-criteria decision making problem. There is investment company,which wants to invest a sum of money in best option. There is a panel with four possible alternatives to invest the money, namely

(i) A_1 is food company;

(ii) A_2 is transport company;

(iii) A_3 is an electronic company;

(iv) A_4 is an tyre company.

Decision maker take decision according to three criteria given below:

a) C_1 is growth analysis;

b) C_2 is risk analysis;

c) C_3 is enviroment impact analysis.

Then the following procedure has been followed for computing the best alternative as an investment.

Step 1: The value of an alternative $A_i(i = 1, 2, 3, 4)$ with respect to criteria $C_j(j = 1, 2, 3)$ obtained from questionnaire of domain expert. Thus, when the four possible alternatives with respect to the above three criteria are evaluated by the expert, we obtain the following single valued neutrosophic decision matrix:

$$D = \begin{bmatrix} \langle 0.5, 0.2, 0.3\rangle & \langle 0.5, 0.1, 0.4\rangle & \langle 0.7, 0.1, 0.2\rangle \\ \langle 0.4, 0.2, 0.3\rangle & \langle 0.3, 0.2, 0.4\rangle & \langle 0.8, 0.3, 0.2\rangle \\ \langle 0.4, 0.3, 0.1\rangle & \langle 0.5, 0.1, 0.3\rangle & \langle 0.5, 0.1, 0.4\rangle \\ \langle 0.6, 0.1, 0.2\rangle & \langle 0.2, 0.2, 0.5\rangle & \langle 0.4, 0.3, 0.2\rangle \end{bmatrix}$$

Step 2: Since the criteria C_1 is the benefit criteria and C_2,C_3 are cost criteria, so we above decision matrix transformed into following normalized matrix $R = \langle T_{ij}, I_{ij}, F_{ij}\rangle$ as follows

$$R = \begin{bmatrix} \langle 0.3, 0.2, 0.5\rangle & \langle 0.5, 0.1, 0.4\rangle & \langle 0.7, 0.1, 0.2\rangle \\ \langle 0.3, 0.2, 0.4\rangle & \langle 0.3, 0.2, 0.4\rangle & \langle 0.8, 0.3, 0.2\rangle \\ \langle 0.1, 0.3, 0.4\rangle & \langle 0.5, 0.1, 0.3\rangle & \langle 0.5, 0.1, 0.4\rangle \\ \langle 0.2, 0.1, 0.6\rangle & \langle 0.2, 0.2, 0.5\rangle & \langle 0.4, 0.3, 0.2\rangle \end{bmatrix}$$

Step 3: Utilizing the proposed entropy measure corresponding to $\alpha = 2$ to get the aggregated values r_{ij} of all the alternatives, which are as following $E_\alpha(A_1) = 0.7437$; $E_\alpha(A_2) = 0.8425$; $E_\alpha(A_3) = 0.8092$; $E_\alpha(A_4) = 0.8089$

Step 4: Based on above values, we conclude that ranking of given alternatives is

$$E_\alpha(A_2) > E_\alpha(A_3) > E_\alpha(A_4) > E_\alpha(A_1)$$

Hence, A_2 is best alternative i.e., Investment company should invest in transport company.

5 Conclusion

In this article, we have introduced the entropy measure of order α for single valued neutrosophic numbers for measuring the degree of the fuzziness of the set in which the uncertainties present in the data are characterized into the truth, the indeterminacy and the falsity membership degrees. Some desirable properties corresponding to these entropy have also been illustrated. A structure linguistic variable has been taken as an illustration. Finally, a decision-making method has been proposed based on entropy measures. To demonstrate the efficiency of the proposed coefficients, numerical example from the investment field has been taken. A comparative study as well as the effect of the parameters on the ranking of the alternative will support the theory and hence demonstrate that the proposed measures place an alternative way for solving the decision-making problems.

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

[2] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87 – 96.

[3] P. Burillo, H. Bustince, Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets, Fuzzy sets and systems 78 (3) (1996) 305 – 316.

[4] W. Zeng, H. Li, Relationship between similarity measure and entropy of interval-valued fuzzy sets, Fuzzy Sets and Systems 157 (11) (2006) 1477 – 1484.

[5] E. Szmidt, J. Kacprzyk, Entropy for intuitionistic fuzzy sets, Fuzzy Sets and Systems 118 (3) (2001) 467 – 477.

[6] H. Garg, N. Agarwal, A. Tripathi, Generalized intuitionistic fuzzy entropy measure of order α and degree β and its applications to multi-criteria decision making problem, International Journal of Fuzzy System Applications 6 (1) (2017) 86 – 107.

[7] W. L. Hung, M. S. Yang, Fuzzy entropy on intuitionistic fuzzy sets, Intenational Journal of Intelligent Systems 21 (2006) 443 – 451.

[8] Q. S. Zhang, S. Y. Jiang, A note on information entropy measure for vague sets, Information Science 178 (2008) 4184 – 4191.

[9] H. Garg, A new generalized improved score function of interval-valued intuitionistic fuzzy sets and applications in expert systems, Applied Soft Computing 38 (2016) 988 – 999.

[10] H. Garg, A new generalized pythagorean fuzzy information aggregation using einstein operations and its application to decision making, International Journal of Intelligent Systems 31 (9) (2016) 886 – 920.

[11] H. Garg, A novel correlation coefficients between pythagorean fuzzy sets and its applications to decision-making processes, International Journal of Intelligent Systems 31 (12) (2016) 1234 – 1253.

[12] H. Garg, Generalized intuitionistic fuzzy interactive geometric interaction operators using einstein t-norm and t-conorm and their application to decision making, Computer and Industrial Engineering 101 (2016) 53 – 69.

[13] F. Smarandache, A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.

[14] H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, Multispace Multistructure 4 (2010) 410 – 413.

[15] J. Ye, Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment, Journal of Intelligent and Fuzzy Systems 27 (6) (2014) 2927 – 2935.

[16] P. Majumdar, S. K. Samant, On similarity and entropy of neutrosophic sets, Journal of Intelligent and Fuzzy Systems 26 (3) (2014) 1245 – 1252.

[17] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, International Journal of Intelligent Fuzzy Systems 26 (1) (2014) 165 – 172.

[18] H. L. Huang, New distance measure of single-valued neutrosophic sets and its application, International Journal of Intelligent Systems 31 (10) (2016) 1021 – 1032.

[19] S. Broumi, F. Smarandache, Correlation coefficient of interval neutrosophic set, Applied Mechanics and Material 436 (2013) 511 – 517.

[20] I. Hanafy, A. Salama, K. Mahfouz, Correlation coefficients of neutrosophic sets by centroid method, International Journal of Probability and Statistics 2 (1) (2013) 9–12.

[21] S. Broumi, F. Smarandache, Cosine similarity measure of interval valued neutrosophic sets, Neutrosophic Sets and Systems 5 (2014) 15 – 20.

[22] J. Ye, Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses, Artificial Intelligence in Medicine 63 (2015) 171 – 179.

[23] H. Y. Zhang, J. Q. Wang, X. H. Chen. Interval neutrosophic sets and their application in multicriteria decision making problems, The Scientific World Journal Volume 2014 (2014) Article ID 645953, 15 pages.

Received: November 11, 2016. Accepted: November 17, 2016