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On Symbolic n-Plithogenic Random Variables Using a Generalized Isomorphism

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Abstract: In this work, we present a generalized isomorphism between field of symbolic n-plithogenic set and R^{n+1}, use it to study the most general form of symbolic plithogenic random variables and study its probabilistic properties including expectation, variance and moments generating function. We also use this isomorphism to study symbolic n-plithogenic probability density function and present many theorems related to it. As an application to this new theory, we study exponential distribution in its symbolic n-plithogenic form and derive its properties, like expected value and variance. Many examples were presented and solved successfully. This paper closes the grand gap in-plithogenic probability theory and paves the way to study many related theories like stochastic modeling and its applications.

Keywords: Plithogenic; Exponential Distribution; Expected Value; Variance; Isomorphism.

1. Introduction

Professor Florentin Smarandache presented a new set of numbers called neutrosophic numbers similar to hypercomplex numbers presented by Kantor, I.L. and Solodovnikov, A.S. [1] where this new set is defined by $R(I) = \{a + bI\ ; I^2 = I, a, b \in R\}$ [2]–[6]. This theory built new algebraic structures and new geometry. Hence, new theories in algebra, real analysis, probability, etc.

In neutrosophic probability theory, or as it is called by researchers "literal neutrosophic probability theory", many continuous probability distributions have been studied well, estimation theory was rebuilt under indeterminacy and many methods of estimation were well-defined including: maximum likelihood, moments and bayes. Researchers developed strong theories and many applications in real-life. From our point of view, the most important applications of this theory are in stochastic processes and stochastic modelling. [7]–[17].

Another extension to this set was then developed by professor Smarandache to what is known by plithogenic sets and it is said to be the most general form of a set until this moment. Plithogenic set is defined by $R(P_1, P_2, \dots, P_n) = \{a_0 + a_1P_1 + a_2P_2 + \dots + a_nP_n; a_0, a_2, \dots, a_n \in R\}$; $P_i^2 = P_i, P_iP_j = P_jP_i = P_{max(i,j)}$ and $i = 1,2, \dots, n$, $j = 1,2, \dots, n$. This last set was studied in many fields of mathematics but with $n = 2$. [18]–[34].

This paper can be considered a generalization of our work in [22] where we first presented the symbolic 2 plithogenic probability theory and studied its properties. This paper will close the gap in

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symbolic n-plithogenic probability theory and pave the way for many researches related to it including statistical inference, stochastic modelling, sampling theory, queueing theory, distributions theory, stable distributions, reliability theory, etc.

2. Preliminaries

Definition 2.1

Let $\boldsymbol{R(I)} = \{\boldsymbol{a} + \boldsymbol{bI}; \boldsymbol{I^2} = \boldsymbol{I}\}$, we call $\boldsymbol{R(I)}$ the neutrosophic field of reals.

Definition 2.2

Set of symbolic n-plithogenic real numbers is defined as follows:

$$R(\mathbb{P}) = R(P_1, P_2, \dots, P_n) = \{a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n; a_0, a_2, \dots, a_n \in R\}$$

Where:

$$P_i^2 = P_i, P_iP_j = P_jP_i = P_{\max(i,j)}; i = 1,2, \dots, n, j = 1,2, \dots, n$$

Definition 2.3

Symbolic 2 plithogenic random variable is defined as follows:

$$X_{2P}: \Omega_{2P} \to R(P_1, P_2); \Omega_{2P} = \Omega_0 \times \Omega_1(P_1) \times \Omega_2(P_2);$$

$$X_{2P} = X_0 + X_1P_1 + X_2P_2; P_1^2 = P_1, P_2^2 = P_2, P_1P_2 = P_2P_1 = P_2$$

Where random variables X_0, X_1, X_2 are classical random variables defined on $\Omega_0, \Omega_1, \Omega_2$ respectively.

3. Symbolic n-plithogenic random variables

Definition 3.1

Let $R(\mathbb{P})$ be the symbolic n-plithogenic set of reals, we define B isomorphism and its inverse B^{-1} between $R(\mathbb{P})$ and R^{n+1} as follows:

$$B: R(\mathbb{P}) \to R^{n+1};$$

$$B(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n) = (a_0, a_0 + a_1, \dots, a_0 + a_1 + \cdots + a_n)$$

$$B^{-1}: R^{n+1} \to R(\mathbb{P});$$

$$B^{-1}(a_0, a_1, \dots, a_n) = a_0 + (a_1 - a_0)P_1 + (a_2 - a_1)P_2 + \cdots + (a_n - a_{n-1})P_n$$

Theorem 3.1

Isomorphism presented in definition 3.1 is an algebraic isomorphism.

Proof

Let $a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n, b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n \in R(\mathbb{P})$.

$$\begin{aligned}
&B(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n + b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n) \\
&= B([a_0 + b_0] + [a_1 + b_1]P_1 + .. + [a_n + b_n]P_n) \\
&= (a_0 + b_0, a_0 + b_0 + a_1 + b_1, \dots, a_0 + b_0 + a_1 + b_1 + \cdots + a_n + b_n) \\
&= (a_0, a_0 + a_1, \dots, a_0 + a_1 + \cdots + a_n) + (b_0, b_0 + b_1, \dots, b_0 + b_1 + \cdots + b_n) \\
&= B(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n) + B(b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n).
\end{aligned}$$

We also have:

$$\begin{aligned}
&B([a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n] \cdot [b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n]) \\
&= B(a_0b_0 + [a_0b_1 + a_1b_1 + a_1b_0]P_1 + [a_0b_2 + a_1b_2 + a_2b_2 + a_2b_0 + a_2b_1]P_2 + \cdots \\
&+ [a_0b_n + a_1b_n + \cdots + a_nb_n + a_nb_{n-1} + a_nb_{n-2} + \cdots + a_nb_0]P_n) \\
&= (a_0b_0, a_0b_0 + a_0b_1 + a_1b_1 + a_1b_0, a_0b_0 + a_0b_1 + a_1b_1 + a_1b_0 + a_0b_2 + a_1b_2 \\
&+ a_2b_2 + a_2b_0 + a_2b_1, \dots, a_0b_0 + a_0b_1 + a_1b_1 + a_1b_0 + a_0b_n + a_1b_n + \cdots + a_nb_n \\
&+ a_nb_{n-1} + a_nb_{n-2} + \cdots + a_nb_0) \\
&= B(a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n)B(b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n).
\end{aligned}$$

Also, B is correspondence one-to-one because $Ker(B) = \{0\}$ and for every $(a_0, a_1, \dots, a_n) \in R^{n+1}$ exists $a_0 + (a_1 - a_0)P_1 + (a_2 - a_1)P_2 + \cdots + (a_n - a_{n-1})P_n \in R(\mathbb{P})$ that satisfies $B(a_0 + (a_1 - a_0)P_1 + (a_2 - a_1)P_2 + \cdots + (a_n - a_{n-1})P_n) = (a_0, a_1, \dots, a_n) \in R^{n+1}$ so B is an algebraic isomorphism.

Definition 3.2

We say that $\boldsymbol{a_0 + a_1P_1 + a_2P_2 + \cdots + a_nP_n \geq_P b_0 + b_1P_1 + b_2P_2 + \cdots + b_nP_n}$ if $\boldsymbol{a_0 \geq b_0, a_0 + a_1 \geq b_0 + b_1, \dots, a_0 + a_1 + \cdots + a_n \geq b_0 + b_1 + \cdots + b_n}$.

Theorem 3.2

Relation defined in definition 3.2 is a partial order relation.

Proof

Straightforward.

Definition 3.3

Symbolic n-plithogenic random variable is defined by:

$$X_P: \Omega_P \to R(\mathbb{P}); \Omega_P = \Omega_0 \times \Omega_1(P_1) \times \Omega_2(P_2) \dots \times \Omega_n(P_n);$$

$$X_P = X_0 + X_1P_1 + X_2P_2 + \cdots + X_nP_n; P_i^2 = P_i, P_iP_j = P_jP_i = P_{\max(i,j)}\ ; i = 1,2, \dots, n\ , j = 1,2, \dots, n$$

Where $X_0, X_1, X_2, \dots, X_n$ are classical random variables defined on $\Omega_0, \Omega_1, \Omega_2, \dots, \Omega_n$ respectively.

Theorem 3.3

Let X_p be a symbolic n-plithogenic random variable then the following equations hold:

1. $E(X_P) = E(X_0) + \sum_{i=1}^{n} E(X_i)P_i$.
2. $Var(X_0) + \sum_{i=1}^{n}\left[Var\left(\sum_{j=0}^{i} X_j\right) - Var\left(\sum_{j=0}^{i-1} X_j\right)\right]P_i$.
3. $\sigma(X_0) + \sum_{i=1}^{n}\left[\sigma\left(\sum_{j=0}^{i} X_j\right) - \sigma\left(\sum_{j=0}^{i-1} X_j\right)\right]P_i$.

Proof

Without loss of generality, we can prove the theorem assuming that X_p is a discrete random variable.

1. $E(X_P) = \sum_{x_P} x_P f(x_P) = \sum_{x_P}(x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n) f(x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n)$

The isomorphic expectation of last equation is:

$$B[E(X_P)] = B\left[\sum_{x_P}(x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n) f(x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n)\right]$$

$$= \sum_{x_P} B[(x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n) f(x_0 + x_1P_1 + x_2P_2 + \cdots + x_nP_n)]$$

$$= \left(\sum_{x_0} x_0 f(x_0), \sum_{x_0+x_1}(x_0 + x_1) f(x_0 + x_1), \dots, \sum_{x_0+x_1+\cdots+x_n}(x_0 + x_1 + \cdots + x_n) f(x_0 + x_1 + \cdots + x_n)\right) = \big(E(X_0), E(X_0 + X_1), \dots, E(X_0 + X_1 + \cdots + X_n)\big)$$

$$= \big(E(X_0), E(X_0) + E(X_1), \dots, E(X_0) + E(X_1) + \cdots + E(X_n)\big)$$

Taking B^{-1}:

$$E(X_P) = B^{-1}\big(E(X_0), E(X_0) + E(X_1), \dots, E(X_0) + E(X_1) + \cdots + E(X_n)\big)$$

$$= E(X_0) + [E(X_0) + E(X_1) - E(X_0)]P_1 + \cdots + [E(X_0) + E(X_1) + \cdots + E(X_n) - E(X_0) - E(X_1) - \cdots - E(X_{n-1})]P_n$$

$$= E(X_0) + E(X_1)P_1 + \cdots + E(X_n)P_2$$

2. $E(X_P^2) = E(X_0 + X_1P_1 + \cdots + X_nP_n)^2 = \sum_{x_P}(x_0 + x_1P_1 + \cdots + x_nP_n)^2 f(x_0 + x_1P_1 + \cdots + x_nP_n)$

Taking B:

$$B[E(X_P^2)] = B\left[\sum_{x_P}(x_0 + x_1P_1 + \cdots + x_nP_n)^2 f(x_0 + x_1P_1 + \cdots + x_nP_n)\right]$$

$$= \sum_{x_P} B[(x_0 + x_1P_1 + \cdots + x_nP_n)^2 f(x_0 + x_1P_1 + \cdots + x_nP_n)] =$$

$$= \left(\sum_{x_0} x_0^2 f(x_0), \sum_{x_0+x_1}(x_0 + x_1)^2 f(x_0 + x_1), \dots, \sum_{x_0+x_1+\cdots+x_n}(x_0 + x_1 + \cdots + x_n)^2 f(x_0 + x_1 + \cdots + x_n)\right) = (E(X_0^2), E(X_0 + X_1)^2, \dots, E(X_0 + X_1 + \cdots + X_n)^2)$$

Now by taking the inverse isometry we get:

$$E(X_P^2) = B^{-1}(E(X_0^2), E(X_0 + X_1)^2, \dots, E(X_0 + X_1 + \cdots + X_n)^2)$$
$$= E(X_0^2) + [E(X_0 + X_1)^2 - E(X_0^2)]P_1 + \cdots$$
$$+ [E(X_0 + X_1 + \cdots + X_n)^2 - E(X_0 + X_1 + \cdots + X_{n-1})^2]P_n$$

Also, we can prove in similar way that:

$$[E(X_P)]^2 = [E(X_0)]^2 + [[E(X_0 + X_1)]^2 - [E(X_0)]^2]P_1 + \cdots$$
$$+ [[E(X_0 + X_1 + \cdots + X_n)]^2 - [E(X_0 + X_1 + \cdots + X_{n-1})]^2]P_n$$

Hence, we have:

$$Var(X_P) = E(X_P^2) - [E(X_P)]^2$$
$$= E(X_0^2) + [E(X_0 + X_1)^2 - E(X_0^2)]P_1 + \cdots$$
$$+ [E(X_0 + X_1 + \cdots + X_n)^2 - E(X_0 + X_1 + \cdots + X_{n-1})^2]P_n$$
$$- \{[E(X_0)]^2 + [[E(X_0 + X_1)]^2 - [E(X_0)]^2]P_1 + \cdots$$
$$+ [[E(X_0 + X_1 + \cdots + X_n)]^2 - [E(X_0 + X_1 + \cdots + X_{n-1})]^2]P_n\}$$
$$= Var(X_0) + [Var(X_0 + X_1) - Var(X_0)]P_1 + \cdots$$
$$+ [Var(X_0 + X_1 + \cdots + X_n) - Var(X_0 + X_1 + \cdots + X_{n-1})]P_n$$
$$= Var(X_0) + \sum_{i=1}^{n}\left[Var\left(\sum_{j=0}^{i} X_j\right) - Var\left(\sum_{j=0}^{i-1} X_j\right)\right]P_i$$

3. Straightforward.

Theorem 3.4

A symbolic n-plithogenic function $f(x_P) = f(x_0 + x_1P_1 + \cdots + x_nP_n)$ is a probability density function in classical scene if and only if it satisfies the following conditions:

1. $f(x_0), f(x_0 + x_1), \dots, f(x_0 + x_1 + \cdots + x_n)$ are all continuous nonnegative functions.
2. $\int_{x_0} f(x_0)dx_0 = 1, \int_{x_0+x_1} f(x_0 + x_1)d(x_0 + x_1) = 1, \dots, \int_{x_0+x_1+\cdots+x_n} f(x_0 + x_1 + \cdots + x_n)d(x_0 + x_1 + \cdots + x_n) = 1.$

Proof

The isometric image of $f(x_P)$ is:

$$B\big(f(x_P)\big) = \big(f(x_0), f(x_0 + x_1), \dots, f(x_0 + x_1 + \dots + x_n)\big)$$

According to theorem 3.2 we can see that if $f(x_0), f(x_0 + x_1), \dots, f(x_0 + x_1 + \dots + x_n)$ are all nonnegative then $f(x_P)$ is a nonnegative function and vice-versa.

Also, according to the properties of the isomorphism B we can conclude that $f(x_P)$ will be a continuous function if and only if $f(x_0), f(x_0 + x_1), \dots, f(x_0 + x_1 + \dots + x_n)$ are all continuous functions.

Finally, let us assume that:

$$\int_{x_0} f(x_0)dx_0 = 1, \int_{x_0+x_1} f(x_0 + x_1)d(x_0 + x_1)$$

$$= 1, \dots, \int_{x_0+x_1+\dots+x_n} f(x_0 + x_1 + \dots + x_n)d(x_0 + x_1 + \dots + x_n) = 1.$$

Then taking B^{-1} yields to:

$$B^{-1}\left(\int_{x_0} f(x_0)dx_0, \int_{x_0+x_1} f(x_0 + x_1)d(x_0 + x_1), \dots, \int_{x_0+x_1+\dots+x_n} f(x_0 + x_1 + \dots + x_n)d(x_0 + x_1 + \dots + x_n)\right) = B^{-1}(1,1,\dots,1) = 1 + (1-1)P_1 + \dots + (1-1)P_n = 1$$

And this completes the proof.

Example

Let $f(x_P) = 2x_0 + (e^{-x_1} - 2x_0)P_1 + (1 - e^{-x_1})P_2; x_0 \in [0,1], x_0 + x_1 > 0, x_0 + x_1 + x_2 \in [0,1]$

1. prove that $f(x_P)$ is a probability density function.
2. Calculate the probability $P\left(X_P < \frac{1}{2} + P_1 - \frac{3}{4}P_2\right)$.

Solution

1. $B\big(f(x_P)\big) = B\big(2x_0 + \big(e^{-(x_0+x_1)} - 2x_0\big)P_1 + \big(1 - e^{-(x_0+x_1)}\big)P_2\big) = \Big(2x_0, 2x_0 + \big(e^{-(x_0+x_1)} - 2x_0\big), 2x_0 + \big(e^{-(x_0+x_1)} - 2x_0\big) + \big(1 - e^{-(x_0+x_1)}\big)\Big) = \big(2x_0, e^{-(x_0+x_1)}, 1\big)$

We conclude that:

$f(x_0) = 2x_0; x_0 \in [0,1]$

$f(x_0 + x_1) = e^{-(x_0+x_1)}; x_0 + x_1 > 0$

$f(x_0 + x_1 + x_2) = 1; x_0 + x_1 + x_2 \in [0,1]$

All previous functions are continuous nonnegative functions and integrate to one on their defined domain.

2. Calculating $P\left(X_P < \frac{1}{2} + P_1 - \frac{3}{4}P_2\right)$ is equivalent to calculating the following three probabilities:

$$\int_{0}^{\frac{1}{2}} 2x_0 dx_0 = x_0^2|_0^{\frac{1}{2}} = \frac{1}{4}, \int_{0}^{\frac{1}{2}+1} e^{-(x_0+x_1)} d(x_0+x_1) = \left[1-e^{-(x_0+x_1)}\right]_0^{\frac{3}{2}} = 1-e^{-\frac{3}{2}}, \int_{0}^{\frac{1}{2}+1-\frac{3}{4}} d(x_0+x_1+x_2) = \frac{3}{4}$$

So $P\left(X_P < \frac{1}{2} + P_1 - \frac{3}{4}P_2\right) = B^{-1}\left(\frac{1}{4}, 1-e^{-\frac{3}{2}}, \frac{3}{4}\right) = \frac{1}{4} + \left(1 - e^{-\frac{3}{2}} - \frac{1}{4}\right)P_1 + \left(\frac{3}{4} - 1 + e^{-\frac{3}{2}}\right)P_2 = \frac{1}{4} + \left(\frac{3}{4} - e^{-\frac{2}{3}}\right)P_1 + \left(e^{-\frac{3}{2}} - \frac{1}{4}\right)P_2.$

Theorem 3.5

Let X_P be a symbolic n-plithogenic random variable then its moments generating function is:

$$M_{X_P}(t) = M_{X_0}(t) + \sum_{i=1}^{n}\left[M_{\sum_{j=0}^{i} X_j}(t) - M_{\sum_{j=0}^{i-1} X_j}(t)\right]P_i$$

Proof

$$M_{X_P}(t) = E(e^{tX_P}) = \int_{-\infty}^{+\infty} e^{tx_P} f(x_P) dx_P = B^{-1}B\left[\int_{-\infty}^{+\infty} e^{tx_P} f(x_P) dx_P\right]$$

$$= B^{-1}\left(\int_{-\infty}^{+\infty} e^{tx_0} f(x_0) dx_0\,, \int_{-\infty}^{+\infty} e^{t(x_0+x_1)} f(x_0+x_1) d(x_0+x_1)\,, \dots, \int_{-\infty}^{+\infty} e^{t(x_0+x_1+\dots+x_n)} f(x_0+x_1+\cdots + x_n) d(x_0+x_1+\cdots+x_n)\right)$$

$$= B^{-1}\left(M_{X_0}(t), M_{X_0+X_1}(t), \dots, M_{X_0+X_1+\dots+X_n}(t)\right)$$

$$= M_{X_0}(t) + \left[M_{X_0+X_1}(t) - M_{X_0}(t)\right]P_1 + \cdots + \left[M_{X_0+X_1+\dots+X_n}(t) - M_{X_0+X_1+\dots+X_{n-1}}(t)\right]P_n = M_{X_P}(t)$$

$$= M_{X_0}(t) + \sum_{i=1}^{n}\left[M_{\sum_{j=0}^{i} X_j}(t) - M_{\sum_{j=0}^{i-1} X_j}(t)\right]P_i$$

Theorem 3.6

Let X_P be a symbolic n-plithogenic random variable and let its moments generating function be $M_{X_P}(t)$ then:

$$\frac{d^k}{dt^k} M_{X_P}(t)|_{t=0} = E(X_P^k)$$

Proof

We have

$$M_{X_P}(t) = M_{X_0}(t) + \sum_{i=1}^{n}\left[M_{\sum_{j=0}^{i} X_j}(t) - M_{\sum_{j=0}^{i-1} X_j}(t)\right]P_i$$

By taking k[th] derivative of the last equation and substituting $t = 0$ we get:

$$\frac{d^k}{dt^k} M_{X_P}(t)|_{t=0} = \frac{d^k}{dt^k}\left(M_{X_0}(t) + \sum_{i=1}^{n}\left[M_{\sum_{j=0}^{i} x_j}(t) - M_{\sum_{j=0}^{i-1} x_j}(t)\right] P_i\right)_{t=0}$$

$$= \frac{d^k}{dt^k} M_{X_0}(0) + \sum_{i=1}^{n}\left[\frac{d^k}{dt^k} M_{\sum_{j=0}^{i} x_j}(0) - \frac{d^k}{dt^k} M_{\sum_{j=0}^{i-1} x_j}(0)\right] P_i$$

$$= E(X_0^k) + \sum_{i=1}^{n}\left[E\left(\sum_{j=0}^{i} X_j\right)^k - E\left(\sum_{j=0}^{i-1} X_j\right)^k\right] P_i = E(X_P^k)$$

4. Application to symbolic n-plithogenic exponential distribution

Definition 4.1

A symbolic n-plithogenic random variable is said to follow exponential distribution with parameter $\lambda_N = \lambda_0 + \lambda_1 P_1 + \cdots + \lambda_n P_n$ if its probability density function is given by:

$$f(x_P) = \lambda_0 e^{-\lambda_0 x_0} + \sum_{i=1}^{n}\left[\sum_{j=0}^{i} \lambda_j\, e^{-\sum_{j=0}^{i} \lambda_j \sum_{j=0}^{i} x_j} - \sum_{j=0}^{i-1} \lambda_j\, e^{-\sum_{j=0}^{i-1} \lambda_j \sum_{j=0}^{i-1} x_j}\right] P_i\, ; x_P, \lambda_P >_P 0$$

Theorem 4.1

If X_P is a symbolic n-plithogenic exponential random variable with parameter $\lambda_N = \lambda_0 + \lambda_1 P_1 + \cdots + \lambda_n P_n$ then:

1. $F(x_P) = 1 - \lambda_0 e^{-\lambda_0 x_0} + \sum_{i=1}^{n}\left[e^{-\sum_{j=0}^{i-1} \lambda_j \sum_{j=0}^{i-1} x_j} - e^{-\sum_{j=0}^{i} \lambda_j \sum_{j=0}^{i} x_j}\right] P_i\, ; x_P, \lambda_P >_P 0$
2. $E(X_P) = \frac{1}{\lambda_0} + \sum_{i=1}^{n}\left[\frac{1}{\sum_{j=0}^{i} \lambda_j} - \frac{1}{\sum_{j=0}^{i-1} \lambda_j}\right] P_i$
3. $Var(X_P) = \frac{1}{\lambda_0^2} + \sum_{i=1}^{n}\left[\frac{1}{\left(\sum_{j=0}^{i} \lambda_j\right)^2} - \frac{1}{\left(\sum_{j=0}^{i-1} \lambda_j\right)^2}\right] P_i$

Proof

1. $F(x_P) = \int_0^{x_P} f(x_P) dx_P = \int_0^{x_0 + x_1 P_1 + \cdots + x_n P_n} \left[\lambda_0 e^{-\lambda_0 x_0} + \sum_{i=1}^{n}\left[\sum_{j=0}^{i} \lambda_j\, e^{-\sum_{j=0}^{i} \lambda_j \sum_{j=0}^{i} x_j} - \sum_{j=0}^{i-1} \lambda_j\, e^{-\sum_{j=0}^{i-1} \lambda_j \sum_{j=0}^{i-1} x_j}\right] P_i\right] d(x_0 + x_1 P_1 + \cdots + x_n P_n) = B^{-1}\left[\int_0^{x_0} \lambda_0 e^{-\lambda_0 x_0} dx_0, \int_0^{x_0 + x_1} (\lambda_0 + \lambda_1) e^{-(\lambda_0 + \lambda_1)(x_0 + x_1)} d(x_0 + x_1), \ldots, \int_0^{x_0 + x_1 + \cdots + x_n} (\lambda_0 + \lambda_1 + \cdots + \lambda_n) e^{-(\lambda_0 + \lambda_1 + \cdots + \lambda_n)(x_0 + x_1 + \cdots + x_n)} d(x_0 + x_1 + \cdots + x_n)\right] = B^{-1}\left(1 - e^{-\lambda_0 x_0}, 1 - e^{-(\lambda_0 + \lambda_1)(x_0 + x_1)}, \ldots, 1 - e^{-(\lambda_0 + \lambda_1 + \cdots + \lambda_n)(x_0 + x_1 + \cdots + x_n)}\right) = 1 - \lambda_0 e^{-\lambda_0 x_0} + \sum_{i=1}^{n}\left[e^{-\sum_{j=0}^{i-1} \lambda_j \sum_{j=0}^{i-1} x_j} - e^{-\sum_{j=0}^{i} \lambda_j \sum_{j=0}^{i} x_j}\right] P_i$

2. $E(X_P) = \int_0^{\infty} x_P f(x_P) dx_P = \int_0^{\infty} (x_0 + x_1 P_1 + \cdots + x_n P_n) \left[\lambda_0 e^{-\lambda_0 x_0} + \sum_{i=1}^{n}\left[\sum_{j=0}^{i} \lambda_j\, e^{-\sum_{j=0}^{i} \lambda_j \sum_{j=0}^{i} x_j} - \sum_{j=0}^{i-1} \lambda_j\, e^{-\sum_{j=0}^{i-1} \lambda_j \sum_{j=0}^{i-1} x_j}\right] P_i\right] d(x_0 + x_1 P_1 + \cdots + x_n P_n) = B^{-1}\left[\int_0^{\infty} x_0 \lambda_0 e^{-\lambda_0 x_0} dx_0, \int_0^{\infty} (x_0 + x_1)(\lambda_0 + \lambda_1) e^{-(\lambda_0 + \lambda_1)(x_0 + x_1)} d(x_0 + x_1), \ldots, \int_0^{\infty} (x_0 + x_1 + \cdots + x_n)(\lambda_0 + \lambda_1 + \cdots +\right.$

$\lambda_n)e^{-(\lambda_0+\lambda_1+\cdots+\lambda_n)(x_0+x_1+\cdots+x_n)}d(x_0+x_1+\cdots+x_n)\Big] = B^{-1}\left(\frac{1}{\lambda_0},\frac{1}{\lambda_0+\lambda_1},\dots,\frac{1}{\lambda_0+\lambda_1+\cdots+\lambda_n}\right) = \frac{1}{\lambda_0} + \sum_{i=1}^{n}\left[\frac{1}{\sum_{j=0}^{i}\lambda_j} - \frac{1}{\sum_{j=0}^{i-1}\lambda_j}\right]P_i$

3. $$Var(X_P) = B^{-1}B\left(\int_0^\infty [x_P - E(X_P)]^2\lambda_P e^{-\lambda_P x_P}dx_P\right) = B^{-1}\Bigg(\int_0^\infty \left(x_0 - \frac{1}{\lambda_0}\right)^2 \lambda_0 e^{-\lambda_0 x_0}dx_0, \int_0^\infty \left(x_0+x_1-\frac{1}{\lambda_0+\lambda_1}\right)^2(\lambda_0+\lambda_1)e^{-(\lambda_0+\lambda_1)(x_0+x_1)}d(x_0+x_1), \dots, \int_0^\infty\left(x_0+x_1+\cdots+x_n-\frac{1}{\lambda_0+\lambda_1+\cdots+\lambda_n}\right)^2(\lambda_0+\lambda_1+\cdots+\lambda_n)e^{-(\lambda_0+\lambda_1+\cdots+\lambda_n)(x_0+x_1+\cdots+x_n)}d(x_0+x_1+\cdots+x_n)\Bigg) =$$

$$B^{-1}\left(\frac{1}{\lambda_0^2},\frac{1}{(\lambda_0+\lambda_1)^2},\dots,\frac{1}{(\lambda_0+\lambda_1+\cdots+\lambda_n)^2}\right) = \frac{1}{\lambda_0^2} + \sum_{i=1}^{n}\left[\frac{1}{\left(\sum_{j=0}^{i}\lambda_j\right)^2} - \frac{1}{\left(\sum_{j=0}^{i-1}\lambda_j\right)^2}\right]P_i$$

5. Conclusion

We have presented an important introduction to symbolic n-plithogenic probability theory and studied random variables related to it. Many theorems were demonstrated and proved successfully. As an application to this new theory, exponential distribution was defined and its properties were studied. Many examples have been solved successfully. In future researches, we are going to study symbolic n-plithogenic stochastic processes and its real-life applications in communication using queueing theory.

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