Single Valued Neutrosophic Hesitant Fuzzy Computational Algorithm for Multiobjective Nonlinear Optimization Problem

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Abstract: In many real-life situations, it is often observed that the degree of indeterminacy (neutrality) plays an important role along with the satisfaction and dissatisfaction levels of the decision maker(s) (DM(s)) in any decision making process. Due to some doubt or hesitation, it may necessary for DM(s) to take opinions from experts which leads towards a set of conflicting values regarding satisfaction, indeterminacy and dis-satisfaction level of DM(s). In order to highlight the above-mentioned insight, we have developed an effective framework which reflects the reality involved in any decision-making process. In this study, a multiobjective nonlinear programming problem (MO-NLPP) has been formulated in the manufacturing system. A new algorithm, neutrosophic hesitant fuzzy programming approach (NHFPA), based on single-valued neutrosophic hesitant fuzzy decision set has been proposed which contains the concept of indeterminacy hesitant degree along with truth and falsity hesitant degrees of different objectives. In order to show the validity and applicability of the proposed approach, a numerical example has been presented. The superiority of the proposed approach has been shown by comparing with other existing approaches. Based on the present work, conclusions and future scope have been presented.

Keywords: Indeterminacy hesitant membership function, Neutrosophic hesitant fuzzy programming, Multiobjective nonlinear programming problem.

1 Introduction

Many decision-making processes inherently involved different conflicting objectives which are to be optimized (maximize/minimize) under given circumstances. In the present competitive era, it is indispensable for decision maker(s) (DM(s)) to obtain better possible outcomes/results when dealing with multiple objectives at a time. Although, it is quite difficult to have an optimal solution which satisfies all the objectives efficiently a compromise solution is possible which is accepted by DM(s) up to some extent. Literature reveals various approaches for multiobjective optimization problem and continuous effort have been made to obtain the best compromise solution. It is often observed that the modeling and formulation of the problem arising in agriculture production planning, manufacturing system etc. takes the form of nonlinear programming problem with multiple objective which is realistic in nature. Thus, multiobjective nonlinear programming problem (MO-NLPP) is also a challenging problem due to its local and global optimal concept, unlike multiobjective linear programming problem.

Bellman and Zadeh [5] introduced fuzzy set (FS) and based on that set Zimmermann [27] proposed fuzzy programming approach (FPA) for multiobjective optimization problems. The FPA deals only degree of belongingness but sometimes it may necessary to deal with non-membership function (non-belongingness) in order to obtain the results in the more realistic way. To overcome the above fact, Atanassov [4] introduced the intuitionistic fuzzy set (IFS) which is the extension of the FS. The IFS is based on more intuition as compared to FS because it also deals with the non-membership function (non-belongingness) of the element in the set. Based on IFS, intuitionistic fuzzy programming approach (IFPA) gained its own popularity among the existing multiobjective optimization techniques. Angelov [3] first used the optimization technique under intuitionistic fuzzy environment. Mahmoodirad et al. [15] proposed a new approach for the balanced transportation problem by considering all parameters and variables are of triangular intuitionistic fuzzy values and pointed out some shortcomings of existing approaches. Singh and Yadav [19] discussed multiobjective nonlinear programming problem in the manufacturing system and solved by using three approaches namely; *Zimmerman's technique, γ- operator* and *Min. bounded sum operator* with intuitionistic fuzzy parameters. Bharati and Singh [6] also proposed a new computational algorithm for multiobjective linear programming problem in the interval-valued intuitionistic fuzzy environment.

In recent years, the extensions or generalizations of FS and IFS have been presented with the fact that indeterminacy degree exists in real life and as a result, a set named neutrosophic set came in existence. Smarandache [20] introduced the concept of the neutrosophic set (NS). The term neutrosophic is the combination of two words, *neutre* from French meaning, neutral, and *sophia* from Greek meaning, skill/wisdom. Thus neutrosophic literally means *knowledge of neutral thoughts* which well enough differentiate it from FS and IFS. The neutrosophic set involves three membership functions, namely; maximization of truth (belongingness), indeterminacy (belongingness to some extent) and minimization of falsity (non-belongingness) in an efficient manner. Based on NS, neutrosophic programming approach (NPA) came into existence and extensively used in real life applications. Abdel-Basset et al. [1] proposed a novel approach to solving fully neutrosophic linear programming problem and applied to production planning problem. Rizk-Allah et al. [16] solved the MO-TPs under neutrosophic environment and compared the obtained results with the existing approach by measuring the ranking degree using TOPSIS approach. Ye et al. [23] formulated neutrosophic number nonlinear programming problem (NN-NPP) and proposed an effective method to solve the problem under neutrosophic number environments. Liu and You [12] extended Muirhead mean to interval neutrosophic set and developed some new operator named as interval neutrosophic Muirhead mean operators which have been further applied to multi-attribute decision making (MADM) problem. Liu et al. [14] have combined the power average operator with Herorian mean operator which results in linguistic neutrosophic power Herorian aggregation operator and extended them for neutrosophic information process. Ahmad and Adhami [2] have also solved the nonlinear transportation problem with fuzzy parameters using neutrosophic programming approach and compared the solution results with other existing approaches. Liu and Shi [10] have introduced the valued neutrosophic uncertain linguistic set and developed some operators which have been further used to multi-attribute group decision making (MAGDM) problem. Liu and Teng [11] have proposed some normal neutrosophic operator based on normal neutrosophic numbers and developed an MADM method based on neutrosophic number generalized weighted power averaging operator. Zhang et al. [25] have proposed some new MAGDM methods in which the attributes are interactive in the form of the interval-valued hesitant uncertain linguistic number. Liu and Zhang [13] have extended the Maclaurian symmetric mean operator to single-valued trapezoidal neutrosophic numbers and developed a method to deal with MAGDM problem based on single-valued trapezoidal neutrosophic weighted Maclaurian symmetric mean operator.

Sometimes, the DM(s) is(are) not sure about the single specific value of the parameters in the set due to doubt or incomplete information but a set of different conflicting values may possible to represent the membership degree for any element to the set. In order to deal with the above fact, Torra and Narukawa [21] introduced the concept of the hesitant fuzzy set (HFS). The HFS is the generalization of fuzzy set and is very useful tools by ensuring the active involvement of different experts' opinions in the decision-making process. Based on HFS, hesitant fuzzy programming approach (HFPA) has been developed which incontinently allows the DM(s) to collaborate with experts in order to collect their incompatible opinions. Bharati [7] developed the hesitant computational algorithm for multiobjective linear programming problem and applied to production planning problem. Zhang et al. [24] developed a hesitant fuzzy programming technique to deal with multi-criteria decision-making problems within the hesitant fuzzy elements environment. Zhou and Xu [26] proposed new portfolio selection and risk investment approaches under hesitant fuzzy environment. All the above-discussed sets have its own limitations regarding the existence of each element in the set. In brief, FS deals only the membership degree of the element in the set whereas IFS considers both membership and non-membership degree of the element in the set simultaneously. NS is the generalization of FS and IFS because it allows the DM(s) to implement the thoughts of neutrality which gives the indeterminacy membership degree for an element to the set. Furthermore, HFS is also an extension of FS as its membership is represented by a set of different conflicting values in the set. Based on the above-mentioned sets, various optimization techniques such as fuzzy optimization techniques, intuitionistic fuzzy optimization techniques, neutrosophic optimization techniques, and hesitant fuzzy optimization techniques have been developed and widely used to solve multiobjective optimization problem which usually exists in real life.

In real life, hesitancy is the most trivial issue in the decision-making process. To deal with it, HFS may be used as an appropriate tool by assigning a set of different membership degree for an element in the set. The limitation of HFS is that it only represents the truth hesitant membership degree and does not deals with indeterminacy hesitant membership degree and falsity hesitant membership degree for an element in the set which arises due to inconsistent, imprecise, inappropriate and incomplete information. On the other hand, a single-valued neutrosophic set (SVNS) is a special case of NS which provides an additional opportunity to the DM(s) by incorporating the thoughts of neutrality. It is only confined to the truth, indeterminacy and a falsity membership degree for an element to the set. It can not ensure the interference of a set of membership values due to doubt and consequently the involvement of different experts' opinions in the decision-making process. The crucial situation arises when the two aspects namely; hesitations and neutral thoughts exist simultaneously in the decision-making process. In this case, HFS and SVNS may not be an appropriate tool to represent the situation in an efficient and effective manner. Thus, this kind of situations are beyond the scope of FS, IFS, SVNS, and HFS and consequently beyond the scope of FPA, IFPA, NPA, and HFPA to decision making process respectively. Therefore, truth, indeterminacy and the falsity situations under hesitant uncertainty is more practical terminology in real life optimization problems.

To get rid of the above limitations, Ye [22] investigated a new set named single-valued neutrosophic hesitant fuzzy set (SVNHFS) which is the combination of HFS and SVNS respectively. The SVNHFS contemplate over truth hesitant fuzzy membership, indeterminacy hesitant fuzzy membership and the falsity hesitant fuzzy membership degrees for an element to the set. Biswas et al. [8] discussed multi-attribute decision-making problems in which the rating values are expressed with single-valued neutrosophic hesitant fuzzy set information and proposed grey relational analysis method for multi-attribute decision making. Şahin and Liu [17] investigated correlation and correlation coefficient of SVNHFSs and discussed its applications in the decision-making process. Biswas et al. [9] proposed a variety of distance measures for single-valued neutrosophic sets and applied these measures to multi-attribute decision-making problems. In this present study, a new computational method, neutrosophic hesitant fuzzy programming approach (NHFPA) has been proposed to obtain the best possible solution of MO-NLPP which is based on SVNHFS. The proposed NHFPA involves the three membership function, namely; maximization of truth hesitant fuzzy (belongingness), indeterminacy hesitant fuzzy (belongingness to some extent) and minimization of falsity hesitant fuzzy (non-belongingness) in an emphatic manner.

To best of our knowledge, no such method has been proposed in the literature to solve the MO-NLPP. The proposed method covers different aspects of impreciseness, vagueness, inaccuracy, the incompleteness that are often encountered in real life optimization problems and provides flexibility in the decision-making process. The remarkable point is that the proposed approach actively seeks opinions from different experts under the neutrosophic environment which is more practical in real life situations and strongly concerned with the involvement of distinguished experts in order to make the fruitful decision. The neutral/indeterminacy hesitant fuzzy concept involved in single-valued neutrosophic hesitant fuzzy set leads towards the future research scope in this domain.

The rest of the paper has been summarized as follows:
In section 2, the preliminaries regarding neutrosophic set, hesitant fuzzy set, and single-valued neutrosophic hesitant fuzzy set have been discussed while section 3 represents the problem formulation and development of the proposed neutrosophic hesitant fuzzy programming approach (NHFPA). In section 4, a numerical study has been presented in order to show the applicability and validity of the proposed approach. A comparative study has also done with other existing approaches. Finally, conclusions and future scope have been discussed based on the present work in section 5.

2 Preliminaries

2.1 Neutrosophic Set (NS)

Definition 2.1.1: [20] Let X be a universe discourse such that x ∈ X, then a neutrosophic set A in X is defined by three membership functions namely, truth $T_A(x)$, indeterminacy $I_A(x)$ and a falsity $F_A(x)$ and is denoted by the following form:

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\} \tag{1}$$

where $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets belong to $]0^-, 1^+[$, also given as, $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so we have,

$$0^- \leq \; sup\, T_A(x) + I_A(x) + \; sup\, F_A(x) \leq 3^+ \tag{2}$$

Definition 2.1.2: [20] A single valued neutrosophic set A over universe of discourse X is defined as

$$A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\} \tag{3}$$

where $T_A(x), I_A(x)$ and $F_A(x) \in [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for each $x \in X$.

2.2 Hesitant Fuzzy Set (HFS)

Definition 2.2.1: [21] Let there be a fixed set X; a hesitant fuzzy set A on X is defined in terms of a function $h_A(x)$ that when applied to X returns a finite subset of [0,1] and mathematically can be represented as follows:

$$A = \{< x, h_A(x) > | x \in X\} \tag{4}$$

where $h_A(x)$ is a set of some different values in [0,1], denoting the possible membership degrees of the element $x \in X$ to A. Also, we call $h_A(x)$ a hesitant fuzzy element.

Definition 2.2.2: [21] For a given hesitant fuzzy element h, its lower and upper bounds are defined as $h^-(x) = min\ h(x)$ and $h^+(x) = max\ h(x)$, respectively.

2.3 Single Valued Neutrosophic Hesitant Fuzzy Set (SVNHFS)

Definition 2.3.1: [22] Let there be a fixed set X; an SVNHFS on X is defined as follows:

$$N_h = \{< x, T_h(x), I_h(x), F_h(x) > | x \in X\} \tag{5}$$

where $T_h(x)$, $I_h(x)$ and $F_h(x)$ are three sets of some values in [0,1], denoting the possible truth hesitant membership degree, indeterminacy hesitant membership degree and the falsity hesitant membership degree of the element $x \in X$ to the set N_h, respectively, with the conditions $0 \leq \alpha,\ \beta,\ \gamma \leq 1$ and $0 \leq \alpha^+,\ \beta^+,\ \gamma^+ \leq 3$, where $\alpha \in T_h(x)$, $\beta \in I_h(x)$, $\gamma \in F_h(x)$ with $\alpha^+ \in T_h^+(x) = \cup_{\alpha \in T_h(x)} max\{\alpha\}$, $\beta^+ \in I_h^+(x) = \cup_{\beta \in I_h(x)} max\{\beta\}$ and $\gamma^+ \in F_h^+(x) = \cup_{\gamma \in F_h(x)} max\{\gamma\}$ for all $x \in X$.
For simplicity, the three-tuple $N_h(x) = \{T_h(x), I_h(x), F_h(x)\}$ is called a single-valued neutrosophic hesitant fuzzy element (SVNHFE) or triple hesitant fuzzy element.

From Definition 2.3.1, it is clear that the SVNHFS comprises three different kinds of membership functions, namely; truth hesitant membership function, indeterminacy hesitant membership function and the falsity hesitant membership function, which consequently results in a more reliable framework and provides pliable access to assign values for each element in the domain, and can deal with three kind of hesitancy in this situation at a time. Thus, classical sets, including fuzzy sets, intuitionistic fuzzy sets, single-valued neutrosophic sets, hesitant fuzzy sets, can be considered as special cases of SVNHFSs (see [22]). Fig. 1 shows the graphical representation of classical sets to SVNHFSs.

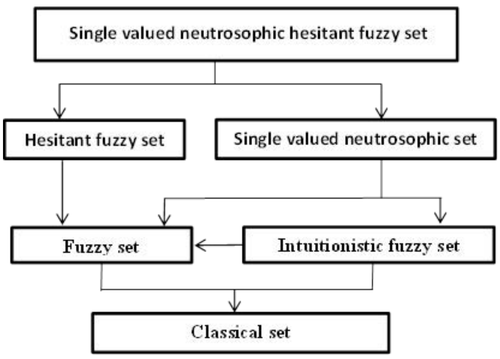

Figure 1: Diagrammatic coverage of classical sets to SVNHFSs.

Definition 2.3.2: [22] Let N_{h_1} and N_{h_2} be two SVNHFSs in a fixed set X; then their union can be defined as follows:

$$\begin{aligned} N_{h_1} \cup N_{h_2} =& \{T_h \in (T_{h_1} \cup T_{h_2}) | T_h \geq max\ (min\ \{T_{h_1} \cup T_{h_2}\}), \\ & I_h \in (I_{h_1} \cup I_{h_2}) | I_h \leq min\ (max\ \{I_{h_1} \cup I_{h_2}\}), \\ & F_h \in (F_{h_1} \cup F_{h_2}) | F_h \leq min\ (max\ \{F_{h_1} \cup F_{h_2}\})\} \end{aligned}$$

Definition 2.3.3: [22] Let N_{h_1} and N_{h_2} be two SVNHFSs in a fixed set X; then their intersection can be defined as follows:

$$\begin{aligned} N_{h_1} \cap N_{h_2} =& \{T_h \in (T_{h_1} \cap T_{h_2}) | T_h \leq min\ (max\ \{T_{h_1} \cap T_{h_2}\}), \\ & I_h \in (I_{h_1} \cap I_{h_2}) | I_h \geq max\ (min\ \{I_{h_1} \cap I_{h_2}\}), \\ & F_h \in (F_{h_1} \cap F_{h_2}) | F_h \geq max\ (min\ \{F_{h_1} \cap F_{h_2}\})\} \end{aligned}$$

3 Problem formulation and solution algorithm

3.1 General mathematical model of multiobjective nonlinear programming problem (MO-NLPP)

Generally, a mathematical programming problem is said to be nonlinear programming problem (NLPP) if either objective function, constraints or both are real-valued nonlinear functions. The objective function(s) is (are) to be optimized (minimize or maximize) under the given constraints. The classical multiobjective nonlinear programming problem (MO-NLPP) is represented in $\mathbf{M}_1$.

$$\begin{aligned}\mathbf{M}_1 :\ & Optimize\ Z_k(x),\ \ k = 1, 2, ..., K,\\ & s.t\ \ g_j(x) \leq d_j,\ \ j = 1, 2, ..., m_1,\\ & g_j(x) \geq d_j, \qquad j = m_1 + 1, m_1 + 2, ..., m_2,\\ & g_j(x) = d_j, \qquad j = m_2 + 1, m_2 + 2, ..., m,\\ & x \geq 0.\end{aligned}$$

where, either $Z_k,\ (k = 1, 2, ..., K), g_j,\ (j = 1, 2, ..., m)$ or both may be real valued nonlinear functions. $x = (x_1, x_2, ..., x_q)$ is a set of decision variables.

3.2 Development of proposed neutrosophic hesitant fuzzy programming approach (NHFPA)

In this study, a new approach based on single-valued neutrosophic hesitant fuzzy set to solve MO-NLPP has been investigated. The proposed approach is based on the hybrid combination of the two sets, namely; neutrosophic set (Smarandache [20]) and hesitant fuzzy set (Torra and Narukawa [21]) respectively. The proposed neutrosophic hesitant fuzzy programming approach (NHFPA) introduces more realistic aspects in dealing with the indeterminacy hesitation present in the decision-making problem. The interesting point is that the proposed NHFPA also considers the conflicting opinions of different experts regarding some parameters in real life problem which enables the DM(s) to obtain the adequate results under neutrosophic environment.

According to Bellman and Zadeh [5], the fuzzy set includes three concepts, namely; fuzzy decision (D), fuzzy goal (G) and fuzzy constraints (C) and incorporated these concepts in many real-life applications of decision-making under fuzzy environment. So, the fuzzy decision set is defined as follows:

$$D = G \cap C \tag{6}$$

Consequently, the neutrosophic hesitant fuzzy decision set D_h^N, with neutrosophic hesitant objectives and constraints, is defined as follows:

$$\begin{aligned} D_h^N = G \cap C &= (\cap_{k=1}^{K} D_k)(\cap_{i=1}^{m} C_i)\\ &= \{x,\ T_D(x),\ I_D(x),\ F_D(x)\}\\ &= \{T_D \in (T_{G_h} \cap T_{C_h}) \mid T_D \leq min\ (max\ \{T_{G_h} \cap T_{C_h}\}),\\ &\quad I_D \in (I_{G_h} \cap I_{C_h}) \mid I_D \geq max\ (min\ \{I_{G_h} \cap I_{C_h}\}),\\ &\quad F_D \in (F_{G_h} \cap F_{C_h}) \mid F_D \geq max\ (min\ \{F_{G_h} \cap F_{C_h}\})\}\end{aligned}$$

Where, $T_D(x),\ I_D(x)$ and $F_D(x)$ are a set of degree of acceptance of neutrosophic hesitant fuzzy decision solution under single-valued neutrosophic hesitant fuzzy decision set. Fig.2 shows the neutrosophic hesitant fuzzy membership degree for the objective function.

On solving each objective function individually, we have k solutions set, $X^1, X^2, ..., X^k$, after that the obtained solutions are substituted in each objective function to determine the lower and upper bound for each objective as given below:

$$U_k = max[Z_k(X^k)]\ \ and\ \ L_k = min[Z_k(X^k)]\ \ \forall\, k = 1, 2, 3, ..., K. \tag{7}$$

Now, we can define the different hesitant membership function more elaborately under neutrosophic hesitant fuzzy environment as follows:

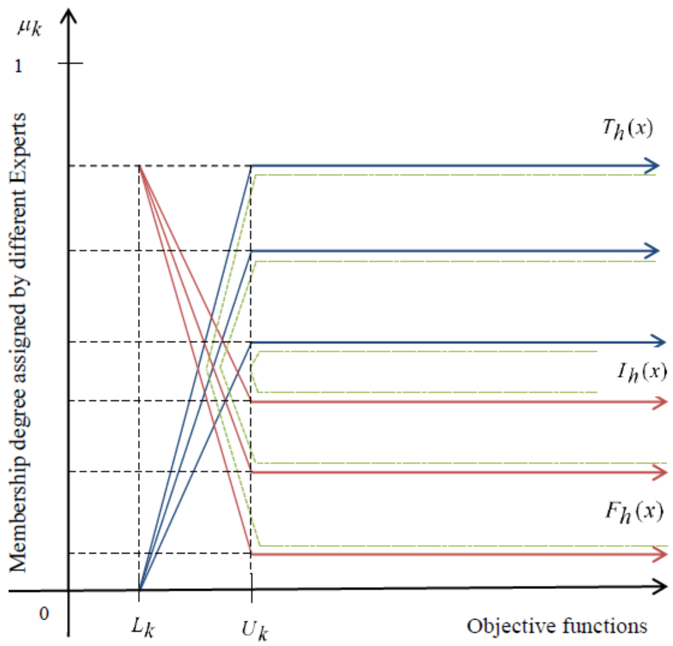

Figure 2: Graphical representation of neutrosophic hesitant fuzzy membership of objective function.

C$ase - I$: For maximization type objective function.

The truth hesitant-membership functions:

$$T^{E_1}_{h^+}(Z_k(x)) = \begin{cases} 0 & if\ \ Z_k(x) < L_k \\ \alpha_1\ \frac{(Z_k(x))^t-(L_k)^t}{(U_k)^t-(L_k)^t} & if\ \ L_k \leq Z_k(x) \leq U_k \\ 1 & if\ \ Z_k(x) > U_k \end{cases} \tag{8}$$

$$T^{E_2}_{h^+}(Z_k(x)) = \begin{cases} 0 & if\ \ Z_k(x) < L_k \\ \alpha_2\ \frac{(Z_k(x))^t-(L_k)^t}{(U_k)^t-(L_k)^t} & if\ \ L_k \leq Z_k(x) \leq U_k \\ 1 & if\ \ Z_k(x) > U_k \end{cases} \tag{9}$$

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$$T^{E_n}_{h^+}(Z_k(x)) = \begin{cases} 0 & if\ \ Z_k(x) < L_k \\ \alpha_n\ \frac{(Z_k(x))^t-(L_k)^t}{(U_k)^t-(L_k)^t} & if\ \ L_k \leq Z_k(x) \leq U_k \\ 1 & if\ \ Z_k(x) > U_k \end{cases} \tag{10}$$

The indeterminacy hesitant-membership functions:

$$I^{E_1}_{h^+}(Z_k(x)) = \begin{cases} 0 & if\ \ Z_k(x) < L_k \\ \beta_1\ \frac{(Z_k(x))^t-(L_k)^t}{(s_k)^t} & if\ \ L_k \leq Z_1(x) \leq L_k + s_k \\ 1 & if\ \ Z_k(x) > L_k + s_k \end{cases} \tag{11}$$

$$I^{E_2}_{h^+}(Z_k(x)) = \begin{cases} 0 & if\ \ Z_k(x) < L_k \\ \beta_2\ \frac{(Z_k(x))^t-(L_k)^t}{(s_k)^t} & if\ \ L_k \leq Z_k(x) \leq L_k + s_k \\ 1 & if\ \ Z_k(x) > L_k + s_k \end{cases} \tag{12}$$

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$$I^{E_n}_{h^+}(Z_k(x)) = \begin{cases} 0 & if\ \ Z_k(x) < L_k \\ \beta_n\ \frac{(Z_k(x))^t-(L_k)^t}{(s_k)^t} & if\ \ L_k \leq Z_k(x) \leq L_k + s_k \\ 1 & if\ \ Z_k(x) > L_k + s_k \end{cases} \tag{13}$$

The falsity hesitant-membership functions:

$$F^{E_1}_{h^+}(Z_k(x)) = \begin{cases} 1 & if\ \ Z_k(x) < L_k + t_k \\ \gamma_1\ \frac{(U_k)^t-(Z_k(x))^t-(t_k)^t}{(U_k)^t-(L_k)^t-(t_k)^t} & if\ \ L_k + t_k \leq Z_k(x) \leq U_k \\ 0 & if\ \ Z_k(x) > U_k \end{cases} \tag{14}$$

$$F^{E_2}_{h^+}(Z_k(x)) = \begin{cases} 1 & if\ \ Z_k(x) < L_k + t_k \\ \gamma_2\ \frac{(U_k)^t-(Z_k(x))^t-(t_k)^t}{(U_k)^t-(L_k)^t-(t_k)^t} & if\ \ L_k + t_k \leq Z_k(x) \leq U_k \\ 0 & if\ \ Z_k(x) > U_k \end{cases} \tag{15}$$

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$$F^{E_n}_{h^+}(Z_k(x)) = \begin{cases} 1 & if\ \ Z_k(x) < L_k + t_k \\ \gamma_n\ \frac{(U_k)^t-(Z_k(x))^t-(t_k)^t}{(U_k)^t-(L_k)^t-(t_k)^t} & if\ \ L_k + t_k \leq Z_k(x) \leq U_k \\ 0 & if\ \ Z_k(x) > U_k \end{cases} \tag{16}$$

where parameter $t > 0$ and $s_k, t_k \in (0, 1)\ \forall k$, are indeterminacy and falsity tolerance values, which is assigned by DM(s) and h^+ represents the maximization type hesitant objective function.
$T^{E_1}_{h^+}(Z_k(x)),\ I^{E_1}_{h^+}(Z_k(x)),\ F^{E_1}_{h^+}(Z_k(x))$ are truth, indeterminacy and the falsity-hesitant-membership degrees assigned by 1^{st} expert.
$T^{E_2}_{h^+}(Z_k(x)),\ I^{E_2}_{h^+}(Z_k(x)),\ F^{E_2}_{h^+}(Z_k(x))$ are truth, indeterminacy and the falsity-hesitant-membership degrees assigned by 2^{nd} expert.
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$T^{E_n}_{h^+}(Z_k(x)),\ I^{E_n}_{h^+}(Z_k(x)),\ F^{E_n}_{h^+}(Z_k(x))$ are truth, indeterminacy and the falsity-hesitant-membership degrees assigned by n^{th} expert.

$\mathbf{C}ase - II$: For minimization type objective function.
The truth hesitant-membership functions:

$$T^{E_1}_{h^-}(Z_k(x)) = \begin{cases} 1 & if\ \ Z_k(x) < L_k \\ \alpha_1\ \frac{(U_k)^t-(Z_k(x))^t}{(U_k)^t-(L_k)^t} & if\ \ L_k \leq Z_k(x) \leq U_k \\ 0 & if\ \ Z_k(x) > U_k \end{cases} \tag{17}$$

$$T^{E_2}_{h^-}(Z_k(x)) = \begin{cases} 1 & if\ \ Z_k(x) < L_k \\ \alpha_2\ \frac{(U_k)^t-(Z_k(x))^t}{(U_k)^t-(L_k)^t} & if\ \ L_k \leq Z_k(x) \leq U_k \\ 0 & if\ \ Z_k(x) > U_k \end{cases} \tag{18}$$

$$\begin{matrix} . & .. & . \\ . & .. & . \\ . & .. & . \end{matrix}$$

$$T_{h^-}^{E_n}(Z_k(x)) = \begin{cases} 1 & if \ \ Z_k(x) < L_k \\ \alpha_n \ \frac{(U_k)^t - (Z_k(x))^t}{(U_k)^t - (L_k)^t} & if \ \ L_k \leq Z_k(x) \leq U_k \\ 0 & if \ \ Z_k(x) > U_k \end{cases} \tag{19}$$

The indeterminacy hesitant-membership functions:

$$I_{h^-}^{E_1}(Z_k(x)) = \begin{cases} 1 & if \ \ Z_k(x) < U_k - s_k \\ \beta_1 \ \frac{(U_k)^t - (Z_k(x))^t}{(s_k)^t} & if \ \ U_k - s_k \leq Z_k(x) \leq U_k \\ 0 & if \ \ Z_k(x) > U_k \end{cases} \tag{20}$$

$$I_{h^-}^{E_2}(Z_k(x)) = \begin{cases} 1 & if \ \ Z_k(x) < U_k - s_k \\ \beta_2 \ \frac{(U_k)^t - (Z_k(x))^t}{(s_k)^t} & if \ \ U_k - s_k \leq Z_k(x) \leq U_k \\ 0 & if \ \ Z_k(x) > U_k \end{cases} \tag{21}$$

$$\begin{matrix} . & ... \\ . & ... \\ . & ... \end{matrix}$$

$$I_{h^-}^{E_n}(Z_k(x)) = \begin{cases} 1 & if \ \ Z_k(x) < U_k - s_k \\ \beta_n \ \frac{(U_k)^t - (Z_k(x))^t}{(s_k)^t} & if \ \ U_k - s_k \leq Z_k(x) \leq U_k \\ 0 & if \ \ Z_k(x) > U_k \end{cases} \tag{22}$$

The falsity hesitant-membership functions:

$$F_{h^-}^{E_1}(Z_k(x)) = \begin{cases} 0 & if \ \ Z_k(x) < L_k + t_k \\ \gamma_1 \ \frac{(Z_k(x))^t - (L_k)^t - (t_k)^t}{(U_k)^t - (L_k)^t - (t_k)^t} & if \ \ L_k + t_k \leq Z_k(x) \leq U_k \\ 1 & if \ \ Z_k(x) > U_k \end{cases} \tag{23}$$

$$F_{h^-}^{E_2}(Z_k(x)) = \begin{cases} 0 & if \ \ Z_k(x) < L_k + t_k \\ \gamma_2 \ \frac{(Z_k(x))^t - (L_k)^t - (t_k)^t}{(U_k)^t - (L_k)^t - (t_k)^t} & if \ \ L_k + t_k \leq Z_k(x) \leq U_k \\ 1 & if \ \ Z_k(x) > U_k \end{cases} \tag{24}$$

$$\begin{matrix} . & ... \\ . & ... \\ . & ... \end{matrix}$$

$$F_{h^-}^{E_n}(Z_k(x)) = \begin{cases} 0 & if \ \ Z_k(x) < L_k + t_k \\ \gamma_n \ \frac{(Z_k(x))^t - (L_k)^t - (t_k)^t}{(U_k)^t - (L_k)^t - (t_k)^t} & if \ \ L_k + t_k \leq Z_k(x) \leq U_k \\ 1 & if \ \ Z_k(x) > U_k \end{cases} \tag{25}$$

where parameter $t > 0$ and $s_k, t_k \in (0, 1) \ \forall k$, are indeterminacy and falsity tolerance values, which is assigned by DM(s) and h^- represents the minimization type hesitant objective function.
$T_{h^-}^{E_1}(Z_k(x)), \ I_{h^-}^{E_1}(Z_k(x)), \ F_{h^-}^{E_1}(Z_k(x))$ are truth, indeterminacy and the falsity-hesitant-membership degrees assigned by 1^{st} expert.
$T_{h^-}^{E_2}(Z_k(x)), \ I_{h^-}^{E_2}(Z_k(x)), \ F_{h^-}^{E_2}(Z_k(x))$ are truth, indeterminacy and the falsity-hesitant-membership degrees assigned by 2^{nd} expert.
......
......
$T_{h^-}^{E_n}(Z_k(x)), \ I_{h^-}^{E_n}(Z_k(x)), \ F_{h^-}^{E_n}(Z_k(x))$ are truth, indeterminacy and the falsity-hesitant-membership degrees assigned by n^{th} expert.

Let $T_h^{E_n} = min \ (T_{h^+}^{E_n}, \ T_{h^-}^{E_n})$, $I_h^{E_n} = min \ (I_{h^+}^{E_n}, \ I_{h^-}^{E_n})$ and $F_h^{E_n} = max \ (F_{h^+}^{E_n}, \ F_{h^-}^{E_n}) \ \forall \ k = 1, 2, ..., K$. Now, the motive is to determine the highest degree of satisfaction for DM(s) by establishing a balance between objectives and constraints.
The neutrosophic hesitant fuzzy model for MO-NLPP ($\mathbf{M}_1$) can be represented as follows:

$$\begin{aligned} \mathbf{M}_2 : & Max \ \ min_{k=1,2,3,...,K} \ \ T_h^{E_n}(Z_k(x)) \\ & Max \ \ min_{k=1,2,3,...,K} \ \ I_h^{E_n}(Z_k(x)) \\ & Min \ \ max_{k=1,2,3,...,K} \ \ F_h^{E_n}(Z_k(x)) \\ & s.t \ \ g_j(x) \leq d_j, \ \ j = 1, 2, ..., m_1, \\ & g_j(x) \geq d_j, \qquad j = m_1 + 1, m_1 + 2, ..., m_2, \\ & g_j(x) = d_j, \qquad j = m_2 + 1, m_2 + 2, ..., m, \\ & x \geq 0. \end{aligned}$$

With the help of auxiliary parameters, model $\mathbf{M}_2$ can be transformed into the following form $\mathbf{M}_3$.

$$\begin{aligned}
\mathbf{M}_3 :& Max \ \frac{\sum \alpha_n}{n} \\
& Max \ \frac{\sum \beta_n}{n} \\
& Min \ \frac{\sum \gamma_n}{n} \\
& s.t. \ T_{h^+}^{E_n}(Z_k(x)) \geq \alpha_n, \quad I_{h^+}^{E_n}(Z_k(x)) \geq \beta_n, \quad F_{h^+}^{E_n}(Z_k(x)) \leq \gamma_n \\
& T_{h^-}^{E_n}(Z_k(x)) \geq \alpha_n, \quad I_{h^-}^{E_n}(Z_k(x)) \geq \beta_n, \quad F_{h^-}^{E_n}(Z_k(x)) \leq \gamma_n \\
& g_j(x) \leq d_j, \ \ j = 1, 2, ..., m_1, \\
& g_j(x) \geq d_j, \ \ j = m_1+1, m_1+2, ..., m_2, \\
& g_j(x) = d_j, \ \ j = m_2+1, m_2+2, ..., m, \\
& x \geq 0, \quad \alpha_n, \beta_n, \gamma_n \in (0,1) \\
& \alpha_n + \beta_n + \gamma_n \leq 3, \quad \alpha_n \geq \beta_n, \quad \alpha_n \geq \gamma_n, \quad \forall \ n.
\end{aligned}$$

Using linear membership function, model $\mathbf{M}_3$ can be written as in $\mathbf{M}_4$.

$$\begin{aligned}
\mathbf{M}_4 :& Max \ \chi = \frac{\alpha_1 + \alpha_2 + ... + \alpha_n}{n} + \frac{\beta_1 + \beta_2 + ... + \beta_n}{n} - \frac{\gamma_1 + \gamma_2 + ... + \gamma_n}{n} \\
& s.t. \ T_{h^+}^{E_1}(Z_k(x)) \geq \alpha_1, \ T_{h^+}^{E_2}(Z_k(x)) \geq \alpha_2, \ ..., \ T_{h^+}^{E_n}(Z_k(x)) \geq \alpha_n \\
& I_{h^+}^{E_1}(Z_k(x)) \geq \beta_1, \ I_{h^+}^{E_2}(Z_k(x)) \geq \beta_2, \ ..., \ I_{h^+}^{E_n}(Z_k(x)) \geq \beta_n \\
& F_{h^+}^{E_1}(Z_k(x)) \leq \gamma_1, \ F_{h^+}^{E_2}(Z_k(x)) \leq \gamma_2, \ ..., \ F_{h^+}^{E_n}(Z_k(x)) \leq \gamma_n \\
& T_{h^-}^{E_1}(Z_k(x)) \geq \alpha_1, \ T_{h^-}^{E_2}(Z_k(x)) \geq \alpha_2, \ ..., \ T_{h^-}^{E_n}(Z_k(x)) \geq \alpha_n \\
& I_{h^-}^{E_1}(Z_k(x)) \geq \beta_1, \ I_{h^-}^{E_2}(Z_k(x)) \geq \beta_2, \ ..., \ I_{h^-}^{E_n}(Z_k(x)) \geq \beta_n \\
& F_{h^-}^{E_1}(Z_k(x)) \leq \gamma_1, \ F_{h^-}^{E_2}(Z_k(x)) \leq \gamma_2, \ ..., \ F_{h^-}^{E_n}(Z_k(x)) \leq \gamma_n \\
& g_j(x) \leq d_j, \qquad j = 1, 2, ..., m_1, \\
& g_j(x) \geq d_j, \qquad j = m_1+1, m_1+2, ..., m_2, \\
& g_j(x) = d_j, \qquad j = m_2+1, m_2+2, ..., m, \\
& x \geq 0, \ \ 0 \leq \alpha_1, \alpha_2, ..., \alpha_n \leq 1, \ \ 0 \leq \beta_1, \beta_2, ..., \beta_n \leq 1 \\
& 0 \leq \gamma_1, \gamma_2, ..., \gamma_n \leq 1, \ \ \alpha_n \geq \beta_n, \quad \alpha_n \geq \gamma_n, \\
& \alpha_n + \beta_n + \gamma_n \leq 3, \quad \forall \ n.
\end{aligned}$$

Finally, model $\mathbf{M}_4$ gives the compromise solution to MO-NLPP.

3.3 Proposed NHFPA algorithm for MO-NLPP

The whole procedure from problem formulation to final solvable model $\mathbf{M}_4$ discussed in section 3 is summarized as step-wise algorithm.

Step-1. Formulate the multiobjective nonlinear programing problems as in $\mathbf{M}_1$.

Step-2. Determine the bounds U_k and L_k, for each objective by using equation (7).

Step-3. By using U_k and L_k, define the upper and lower bound for truth hesitant, indeterminacy hesitant and falsity hesitant membership functions as given in equation (8)-(25).

Step-4. Ask for the truth hesitant, indeterminacy hesitant and the falsity hesitant membership degrees from different experts or DM(s).

Step-5. Formulate MO-NLPP under neutrosophic hesitant fuzzy environment defined in $\mathbf{M}_4$.

Step-6. Solve the multiobjective nonlinear programing problem in order to obtain the compromise solution using suitable techniques or some optimizing software packages.

4 Experimental study

In order to show the efficiency and validity of the proposed method, we adopted the numerical example of the manufacturing system discussed by Singh and Yadav [19]. The DM(s) of the company intends to maximize the total profit incurred over products and minimize the total time required for each product. Also, assumed that the DM(s) seeks three experts' opinion in the decision-making process. Therefore, the crisp multiobjective non-linear programming problem formulation [19] is given as follows:

$$\begin{aligned}
\mathbf{M}_1 : \ & Max \ Z_1(x) = 99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3 \\
& Min \ Z_2(x) = 3.875x_1 + 5.125x_2 + 5.9375x_3 \\
& s.t \ \ 2.0625x_1 + 3.875x_2 + 2.9375x_3 \leq 333.125 \\
& 3.875x_1 + 2.0625x_2 + 2.0625x_3 \leq 365.625 \\
& 2.9375x_1 + 2.0625x_2 + 2.9375x_3 \geq 360 \\
& x_1, \ x_2, \ x_3 \geq 0.
\end{aligned}$$

On solving each objective function individually given in $(\mathbf{M}_1)$, we get the following individual best solution, lower and upper bound for each objective.
$X^1 = (57.82, \ 13.09, \ 55.53)$, $X^2 = (62.26, \ 0, \ 60.28)$ along with $L_1 = 180.72$, $U_1 = 516.70$, $L_2 = 599.23$ and $U_2 = 620.84$.

Since, the first objective $Z_1(x)$ is of maximization type and the satisfaction level of Experts or DMs increases if the values of objective function tends towards its upper bound. Therefore the truth hesitant membership, indeterminacy hesitant membership and falsity hesitant membership functions of upper bound can be represented as follows:

For Z_1: The upper and lower bound for first objective and its membership functions.

$$T_{h^+}^{E_1}(Z_1(x)) = \begin{cases} 0 & if \;\; Z_1(x) < 180.72 \\ 0.98 \; \frac{(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t-(180.72)^t}{(516.70)^t-(180.72)^t} & if \;\; 180.72 \leq Z_1(x) \leq 516.70 \\ 1 & if \;\; Z_1(x) > 516.70 \end{cases} \tag{26}$$

$$T_{h^+}^{E_2}(Z_1(x)) = \begin{cases} 0 & if \;\; Z_1(x) < 180.72 \\ 0.99 \; \frac{(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t-(180.72)^t}{(516.70)^t-(180.72)^t} & if \;\; 180.72 \leq Z_1(x) \leq 516.70 \\ 1 & if \;\; Z_1(x) > 516.70 \end{cases} \tag{27}$$

$$T_{h^+}^{E_3}(Z_1(x)) = \begin{cases} 0 & if \;\; Z_1(x) < 180.72 \\ \frac{(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t-(180.72)^t}{(516.70)^t-(180.72)^t} & if \;\; 180.72 \leq Z_1(x) \leq 516.70 \\ 1 & if \;\; Z_1(x) > 516.70 \end{cases} \tag{28}$$

$$I_{h^+}^{E_1}(Z_1(x)) = \begin{cases} 0 & if \;\; Z_1(x) < 180.72 \\ 0.98 \; \frac{(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t-(180.72)^t}{(s_1)^t} & if \;\; 180.72 \leq Z_1(x) \leq 180.72 + s_1 \\ 1 & if \;\; Z_1(x) > 180.72 + s_1 \end{cases} \tag{29}$$

$$I_{h^+}^{E_2}(Z_1(x)) = \begin{cases} 0 & if \;\; Z_1(x) < 180.72 \\ 0.99 \; \frac{(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t-(180.72)^t}{(s_1)^t} & if \;\; 180.72 \leq Z_1(x) \leq 180.72 + s_1 \\ 1 & if \;\; Z_1(x) > 180.72 + s_1 \end{cases} \tag{30}$$

$$I_{h^+}^{E_3}(Z_1(x)) = \begin{cases} 0 & if \;\; Z_1(x) < 180.72 \\ \frac{(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t-(180.72)^t}{(s_1)^t} & if \;\; 180.72 \leq Z_1(x) \leq 180.72 + s_1 \\ 1 & if \;\; Z_1(x) > 180.72 + s_1 \end{cases} \tag{31}$$

$$F_{h^+}^{E_1}(Z_1(x)) = \begin{cases} 1 & if \;\; Z_1(x) < 180.72 \\ 0.98 \; \frac{(516.70)^t-(t_1)^t-(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t}{(516.70)^t-(180.72)^t-(t_1)^t} & if \;\; 180.72 \leq Z_1(x) \leq 516.70 - t_1 \\ 0 & if \;\; Z_1(x) > 516.70 - t_1 \end{cases} \tag{32}$$

$$F_{h^+}^{E_2}(Z_1(x)) = \begin{cases} 1 & if \;\; Z_1(x) > 516.70 \\ 0.99 \; \frac{(516.70)^t-(t_1)^t-(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t}{(516.70)^t-(180.72)^t-(t_1)^t} & if \;\; 180.72 + t_1 \leq Z_1(x) \leq 516.70 \\ 0 & if \;\; Z_1(x) < 180.72 + t_1 \end{cases} \tag{33}$$

$$F_{h^+}^{E_3}(Z_1(x)) = \begin{cases} 1 & if \;\; Z_1(x) > 516.70 \\ \frac{(516.70)^t-(t_1)^t-(99.875x_1^{\frac{1}{2}}-8x_1+119.875x_2^{\frac{1}{2}}-10.125x_2+95.125x_3^{\frac{1}{3}}-8x_3)^t}{(516.70)^t-(180.72)^t-(t_1)^t} & if \;\; 180.72 + t_1 \leq Z_1(x) \leq 516.70 \\ 0 & if \;\; Z_1(x) < 180.72 + t_1 \end{cases} \tag{34}$$

Similarly, the second objective $Z_2(x)$ is of minimization type and the satisfaction level of Experts or DMs increases if the values of objective function tends towards its lower bound. Thus the truth hesitant membership, indeterminacy hesitant membership and falsity hesitant membership functions of lower bound can be represented as follows:

For Z_2: The upper and lower bound for second objective and its membership functions.

$$T_{h^-}^{E_1}(Z_2(x)) = \begin{cases} 1 & if \;\; Z_2(x) < 599.23 \\ 0.98 \; \frac{(620.84)^t-(3.875x_1+5.125x_2+5.9375x_3)^t}{(620.84)^t-(599.23)^t} & if \;\; 599.23 \leq Z_2(x) \leq 620.84 \\ 0 & if \;\; Z_2(x) > 620.84 \end{cases} \tag{35}$$

$$T_{h^-}^{E_2}(Z_2(x)) = \begin{cases} 1 & if \;\; Z_2(x) < 599.23 \\ 0.99 \; \frac{(620.84)^t-(3.875x_1+5.125x_2+5.9375x_3)^t}{(620.84)^t-(599.23)^t} & if \;\; 599.23 \leq Z_2(x) \leq 620.84 \\ 0 & if \;\; Z_2(x) > 620.84 \end{cases} \tag{36}$$

$$T_{h^-}^{E_3}(Z_2(x)) = \begin{cases} 1 & if \;\; Z_2(x) < 599.23 \\ \frac{(620.84)^t-(3.875x_1+5.125x_2+5.9375x_3)^t}{(620.84)^t-(599.23)^t} & if \;\; 599.23 \leq Z_2(x) \leq 620.84 \\ 0 & if \;\; Z_2(x) > 620.84 \end{cases} \tag{37}$$

$$I_{h^-}^{E_1}(Z_2(x)) = \begin{cases} 1 & if \;\; Z_2(x) < 620.84 - s_2 \\ 0.98 \; \frac{(620.84)^t-(3.875x_1+5.125x_2+5.9375x_3)^t}{(s_2)^t} & if \;\; 620.84 - s_2 \leq Z_2(x) \leq 620.84 \\ 0 & if \;\; Z_2(x) > 620.84 \end{cases} \tag{38}$$

$$I_{h^-}^{E_2}(Z_2(x)) = \begin{cases} 1 & if \;\; Z_2(x) < 620.84 - s_2 \\ 0.99 \; \frac{(620.84)^t-(3.875x_1+5.125x_2+5.9375x_3)^t}{(s_2)^t} & if \;\; 620.84 - s_2 \leq Z_2(x) \leq 620.84 \\ 0 & if \;\; Z_2(x) > 620.84 \end{cases} \tag{39}$$

$$I^{E_3}_{h^-}(Z_2(x)) = \begin{cases} 1 & if\ \ Z_2(x) < 620.84 - s_2 \\ \frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(s_2)^t} & if\ \ 620.84 - s_2 \leq Z_2(x) \leq 620.84 \\ 0 & if\ \ Z_2(x) > 620.84 \end{cases} \quad (40)$$

$$F^{E_1}_{h^-}(Z_2(x)) = \begin{cases} 0 & if\ \ Z_2(x) < 599.23 + t_2 \\ 0.98\ \frac{(3.875x_1 + 5.125x_2 + 5.9375x_3)^t - (599.23)^t - (t_2)^t}{(620.84)^t - (599.23)^t - (t_2)^t} & if\ \ 599.23 + t_2 \leq Z_2(x) \leq 620.84 \\ 1 & if\ \ Z_2(x) > 620.84 \end{cases} \quad (41)$$

$$F^{E_2}_{h^-}(Z_2(x)) = \begin{cases} 0 & if\ \ Z_2(x) < 599.23 + t_2 \\ 0.99\ \frac{(3.875x_1 + 5.125x_2 + 5.9375x_3)^t - (599.23)^t - (t_2)^t}{(620.84)^t - (599.23)^t - (t_2)^t} & if\ \ 599.23 + t_2 \leq Z_2(x) \leq 620.84 \\ 1 & if\ \ Z_2(x) > 620.84 \end{cases} \quad (42)$$

$$F^{E_3}_{h^-}(Z_2(x)) = \begin{cases} 0 & if\ \ Z_2(x) < 599.23 + t_2 \\ \frac{(3.875x_1 + 5.125x_2 + 5.9375x_3)^t - (599.23)^t - (t_2)^t}{(620.84)^t - (599.23)^t - (t_2)^t} & if\ \ 599.23 + t_2 \leq Z_2(x) \leq 620.84 \\ 1 & if\ \ Z_2(x) > 620.84 \end{cases} \quad (43)$$

The final solution model is given as follows:

$$\mathbf{M}_4 : Max\ \ \chi = \frac{\alpha_1 + \alpha_2 + \alpha_3}{3} + \frac{\beta_1 + \beta_2 + \beta_3}{3} - \frac{\gamma_1 + \gamma_2 + \gamma_3}{3}$$

$$s.t.\ 0.98\ \frac{(99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t - (180.72)^t}{(516.70)^t - (180.72)^t} \geq \alpha_1$$

$$0.99\ \frac{(99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t - (180.72)^t}{(516.70)^t - (180.72)^t} \geq \alpha_2$$

$$\frac{(99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t - (180.72)^t}{(516.70)^t - (180.72)^t} \geq \alpha_3$$

$$0.98\ \frac{(99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t - (180.72)^t}{(s_1)^t} \geq \beta_1$$

$$0.99\ \frac{(99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t - (180.72)^t}{(s_1)^t} \geq \beta_2$$

$$\frac{(99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t - (180.72)^t}{(s_1)^t} \geq \beta_3$$

$$0.98\ \frac{(516.70)^t - (t_1)^t - (99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t}{(516.70)^t - (180.72)^t - (t_1)^t} \leq \gamma_1$$

$$0.99\ \frac{(516.70)^t - (t_1)^t - (99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t}{(516.70)^t - (180.72)^t - (t_1)^t} \leq \gamma_2$$

$$\frac{(516.70)^t - (t_1)^t - (99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3)^t}{(516.70)^t - (180.72)^t - (t_1)^t} \leq \gamma_3$$

$$0.98\ \frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(620.84)^t - (599.23)^t} \geq \alpha_1$$

$$0.99\ \frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(620.84)^t - (599.23)^t} \geq \alpha_2$$

$$\frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(620.84)^t - (599.23)^t} \geq \alpha_3$$

$$0.98\ \frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(s_2)^t} \geq \beta_1$$

$$0.99\ \frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(s_2)^t} \geq \beta_2$$

$$\frac{(620.84)^t - (3.875x_1 + 5.125x_2 + 5.9375x_3)^t}{(s_2)^t} \geq \beta_3$$

$$0.98\ \frac{(3.875x_1 + 5.125x_2 + 5.9375x_3)^t - (599.23)^t - (t_2)^t}{(620.84)^t - (599.23)^t - (t_2)^t} \leq \gamma_1$$

$$0.99\ \frac{(3.875x_1 + 5.125x_2 + 5.9375x_3)^t - (599.23)^t - (t_2)^t}{(620.84)^t - (599.23)^t - (t_2)^t} \leq \gamma_2$$

$$\frac{(3.875x_1 + 5.125x_2 + 5.9375x_3)^t - (599.23)^t - (t_2)^t}{(620.84)^t - (599.23)^t - (t_2)^t} \leq \gamma_3$$

$$2.0625x_1 + 3.875x_2 + 2.9375x_3 \leq 333.125$$

$$3.875x_1 + 2.0625x_2 + 2.0625x_3 \leq 365.625$$

$$2.9375x_1 + 2.0625x_2 + 2.9375x_3 \geq 360$$

$$x_1,\ x_2,\ x_3 \geq 0,\quad 0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1,\ \ 0 \leq \beta_1, \beta_2, \beta_3 \leq 1,$$
$$0 \leq \gamma_1, \gamma_2, \gamma_3 \leq 1,\ \ 0 \leq s_1, t_1 \leq 1,\ \ 0 \leq s_2, t_2 \leq 1,$$
$$\alpha_n \geq \beta_n,\ \ \alpha_n \geq \gamma_n,\ \ \alpha_n + \beta_n + \gamma_n \leq 3,\quad \forall\ n = 1, 2, 3.$$

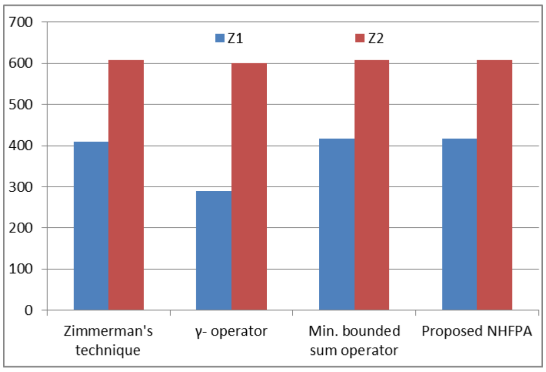
The multiobjective nonlinear programming problem $\mathbf{M}_4$ has been written in AMPL language and solved using solvers available on NEOS server online facility provided by Wisconsin Institutes for Discovery at the University of Wisconsin in Madison for solving Optimization problems, see (Server [18]).
At $t = 2$, the optimal solution of the multiobjective nonlinear programming problem by using the proposed neutrosophic hesitant fuzzy programming approach (NHFPA) is $x = (60.48,\ 5.26,\ 58.37)$, $Z_1 = 416.58,\ Z_2 = 607.88$ with the degree of satisfaction $\chi = 1.20$ respectively.

4.1 Comparative study

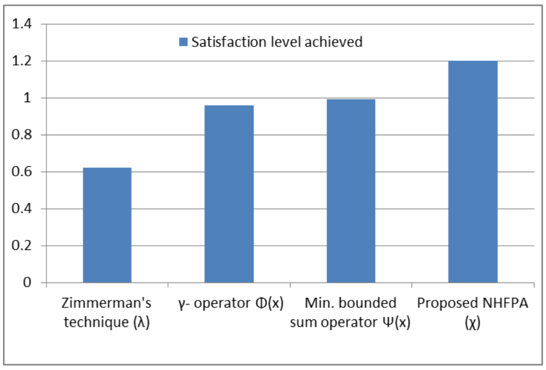
The multiobjective nonlinear programming problem of manufacturing system with conflicting objectives have been solved by using proposed neutrosophic hesitant fuzzy programming approach (NHFPA). The solution results obtained by proposed method and with other existing approaches discussed in [19] have been summarized in Table-1. From the table, it is clear that the minimum deviation from ideal solution of each objective function is 100.12 and 0.41 by using proposed NHFPA and γ- operator respectively. Furthermore, the highest satisfaction level has been attained by proposed approach i.e; χ=1.20, which reveals the superiority of proposed NHFPA over other existing approaches in terms of satisfactory degree of DM(s). Fig-3 shows the graphical representation of the objective functions and satisfaction level obtained by different approaches.

Table 1: Comparison of results with existing methods.

Solution method	Objective values		Deviations from ideal solutions		Satisfaction level
	Max. Z_1	Min. Z_2	$(U_1 - Z_1)$	$(Z_2 - L_2)$	
Zimmerman's technique [19]	409.70	607.28	107	8.05	λ= 0.62
γ- operator [19]	288.86	599.64	227.84	0.41(min.)	$\phi(x)$= 0.96
Min. bounded sum operator [19]	416.58	607.88	100.12	8.65	$\psi(x)$= 0.99
Proposed NHFPA	416.58	607.88	100.12(min.)	8.65	χ= 1.20 (max.)



(a) Objective functions obtained by different approaches.



(b) Satisfaction level achieved by different approaches.

Figure 3: Comparison of results with proposed NHFPA and different existing approaches.

5 Conclusions

In this study, a new approach has been suggested to solve the multiobjective nonlinear programming problem in the neutrosophic hesitant fuzzy environment. The proposed neutrosophic hesitant fuzzy programming approach (NHFPA) comprises three different membership functions, namely; truth hesitant, indeterminacy hesitant and a falsity hesitant membership function which contains a set of different values between 0 and 1. The proposed approach provides the more realistic framework and considers various aspects of the DM's neutral thoughts with hesitations in the decision-making process. The main contribution by introducing the proposed approach is that it allows the DM(s) to express his/her(their) degree of hesitation and neutral thoughts according to the need of adverse situations in a convenient manner. In order to show the superiority of proposed NHFPA, it is applied to solve multiobjective nonlinear programming problem in the manufacturing system. To best of our knowledge, no such approach is suggested in the literature to solve MO-NLPP in such an efficient and effective manner.

Therefore, the proposed NHFPA will be very helpful in such a typical situation when the DM(s) have some neutral thoughts and also with a set some hesitation values in the decision-making process. In future, the proposed approach may be applied to the multiobjective fractional programming problem, bi-level nonlinear programming problem, multilevel fractional programming problem etc.

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