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Single Valued Neutrosophic Hyperbolic Sine Similarity Measure Based MADM Strategy

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Abstract: In this paper, we introduce new type of similarity measures for single valued neutrosophic sets based on hyperbolic sine function. The new similarity measures are namely, single valued neutrosophic hyperbolic sine similarity measure and weighted single valued neutrosophic hyperbolic sine similarity measure. We prove the basic properties of the proposed similarity measures. We also develop a multi-attribute decision-making strategy for single valued neutrosophic set based on the proposed weighted similarity measure. We present a numerical example to verify the practicability of the proposed strategy. Finally, we present a comparison of the proposed strategy with the existing strategies to exhibit the effectiveness and practicality of the proposed strategy.

Keywords: Single valued neutrosophic set, Hyperbolic sine function, Similarity measure, MADM, Compromise function

1 Introduction

Smarandache [1] introduced the concept of neutrosophic set (NS) to deal with imprecise and indeterminate data. In the concept of NS, truth-membership, indeterminacy-membership, and falsity-membership are independent. Indeterminacy plays an important role in many real world decision-making problems. NS generalizes the Cantor set discovered by Smith [2] in 1874 and introduced by German mathematician Cantor [3] in 1883, fuzzy set introduced by Zadeh [4], intuitionistic fuzzy set proposed by Atanassov [5]. Wang et al. [6] introduced the concept of single valued neutrosophic set (SVNS) that is the subclass of a neutrosophic set. SVNS is capable to represent imprecise, incomplete, and inconsistent information that manifest the real world.

Neutrosophic sets and its various extensions have been studied and applied in different fields such as medical diagnosis [7, 8, 9], decision making problems [10, 11, 12, 13, 14], social problems [15, 16], educational problem [17, 18], conflict resolution [19], image processing [20, 21, 22], etc.

The concept of similarity is very important in studying almost every scientific field. Many strategies have been proposed for measuring the degree of similarity between fuzzy sets studied by Chen [23], Chen et al. [24], Hyung et al. [25], Pappis and Karacapilidis [26], Pramanik and Roy [27], etc. Several strategies have been proposed for measuring the degree of similarity between intuitionistic fuzzy sets studied by Xu [28], Papakostas et al. [29], Biswas and Pramanik [30], Mondal and Pramanik [31], etc. However, these strategies are not capable of dealing with the similarity measures involving indeterminacy. SVNS can handle this situation. In the literature, few studies have addressed similarity measures for neutrosophic sets and single valued neutrosophic sets [32, 33, 34, 35].

Ye [36] proposed an MADM method with completely unknown weights based on similarity measures under SVNS environment. Ye [37] proposed vector similarity measures of simplified neutrosophic sets and applied it in multi-criteria decision making problems. Ye [38] developed improved cosine similarity measures of simplified neutrosophic sets for medical diagnosis. Ye [39] also proposed exponential similarity measure of neutrosophic numbers for fault diagnoses of steam turbine. Ye [40] developed clustering algorithms based on similarity measures for SVNSs. Ye and Ye [41] proposed Dice similarity measure between single valued neutrosophic multisets. Ye et al. [42] proposed distance-based similarity measures of single valued neutrosophic multisets for medical diagnosis. Ye and Fu [43] developed a single valued neutrosophic similarity measure based on tangent function for multi-period medical diagnosis.

In hybrid environment Pramanik and Mondal [44] proposed cosine similarity measure of rough neutrosophic sets and provided its application in medical diagnosis. Pramanik and Mondal [45] also proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.

Research gap: MADM strategy using similarity measure based on hyperbolic sine function under single valued neutrosophic environment is yet to appear.

Research questions:

- Is it possible to define a new similarity measure between single valued neutrosophic sets using hyperbolic sine function?
- Is it possible to develop a new MADM strategy based on the proposed similarity measures in single valued neutrosophic environment?

Having motivated from the above researches on neutrosophic similarity measures, we have introduced the concept of hyperbolic sine similarity measure for SVNS environment. The new similarity measures called single valued neutrosophic hyperbolic sine similarity measure (SVNHSSM) and single valued neutrosophic weighted hyperbolic sine similarity measure (SVNWHSSM). The properties of hyperbolic sine similarity are established. We have developed a MADM model using the proposed SVNWHSSM. The proposed hyperbolic sine similarity measure is applied to multi-attribute decision making.

The objectives of the paper:

- To define hyperbolic sine similarity measures for SVNS environment and prove some of it's basic properties.
- To define conpromise function for determining unknown weight of attributes.
- To develop a multi-attribute decision making model based on proposed similarity measures.
- To present a numerical example for the efficiency and effectiveness of the proposed strategy.

Rest of the paper is structured as follows. Section 2 presents preliminaries of neutrosophic sets and single valued neutrosophic sets. Section 3 is devoted to introduce hyperbolic sine similarity measure for SVNSs and some of its properties. Section 4 presents a method to determine unknown attribute weights. Section 5 presents a novel decision making strategy based on proposed neutrosophic hyperbolic sine similarity measure. Section 6 presents an illustrative example for the application of the proposed method. Section 7 presents a comparison analysis for the applicability of the proposed strategy. Section 8 presents the main contributions of the proposed strategy. Finally, section 9 presents concluding remarks and scope of future research.

2 Neutrosophic preliminaries

2.1 Neutrosophic set (NS)

Definition 2.1 [1] Let U be a universe of discourse. Then the neutrosophic set P can be presented of the form:

$P = \{< x{:}T_P(x), I_P(x), F_P(x)> \mid x \in U\}$, where the functions T, I, F: $U\to\ ]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set P satisfying the following the condition.

$$^-0 \le \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \le 3^+$$

2.2 Single valued neutrosophic set (SVNS)

Definition 2.2 [6] Let X be a space of points with generic elements in X denoted by x. A SVNS P in X is characterized by a truth-membership function $T_P(x)$, an indeterminacy-membership function $I_P(x)$, and a falsity membership function $F_P(x)$, for each point x in X.

$T_P(x)$, $I_P(x)$, $F_P(x) \in [0, 1]$. When X is continuous, a SVNS P can be written as follows:

$$P = \int_X \frac{<T_P(x), I_P(x), F_P(x)>}{x} : x \in X$$

When X is discrete, a SVNS P can be written as follows:

$$P = \sum_{i=1}^{n} \frac{<T_P(x_i), I_P(x_i), F_P(x_i)>}{x_i} : x_i \in X$$

For two SVNSs,

$P_{SVNS} = \{<x{:}\ T_P(x), I_P(x), F_P(x)> \mid x \in X\}$ and $Q_{SVNS} = \{<x, T_Q(x), I_Q(x), F_Q(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $P_{SVNS} \subseteq Q_{SVNS}$ if and only if $T_P(x) \le T_Q(x)$, $I_P(x) \ge I_Q(x)$, $F_P(x) \ge F_Q(x)$

(2) $P_{SVNS} = Q_{SVNS}$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$ for any $x \in X$.

3. Hyperbolic sine similarity measures for SVNSs

Let $A = <x(T_A(x), I_A(x), F_A(x))>$ and $B = <x(T_B(x), I_B(x), F_B(x))>$ be two SVNSs. Now hyperbolic sine similarity function which measures the similarity between two SVNSs can be presented as follows (see Eqn. 1):

$$\text{SVNHSSM}(A,B) = 1-\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sinh\begin{pmatrix}|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|\\ +|F_A(x_i)-F_B(x_i)|\end{pmatrix}}{11}\right) \quad (1)$$

Theorem 1. The defined hyperbolic sine similarity measure SVNHSSM(A, B) between SVNSs A and B satisfies the following properties:

1. $0 \le$ SVNHSSM(A, B) ≤ 1
2. SVNHSSM(A, B) = 1 if and only if $A = B$
3. SVNHSSM (A, B) = SVNHSSM(B, A)
4. If R is a SVNS in X and $A \subset B \subset R$ then SVNHSSM(A, R) $\le$ SVNHSSM(A, B) and SVNHSSM(A, R) $\le$ SVNHSSM(B, R).

Proofs:

1. For two neutrosophic sets A and B,

$0 \le T_A(x_i), I_A(x_i), F_A(x_i), T_B(x_i), I_B(x_i), F_B(x_i) \le 1$

$$\Rightarrow 0 \le |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \le 3$$

$$\Rightarrow 0 \le \left(\frac{\sinh\left(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \right)}{11} \right) \le 1$$

Hence $0 \le$ SVNHSSM(A, B) ≤ 1

2. For any two SVNSs A and B, if $A = B$,

$\Rightarrow T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

$\Rightarrow |T_A(x) - T_B(x)| = 0$, $|I_A(x) - I_B(x)| = 0$,

$|F_A(x) - F_B(x)| = 0$

Hence SVNHSSM(A, B) = 1.

Conversely,

SVNHSSM(A, B) = 1

$\Rightarrow |T_A(x) - T_B(x)| = 0$, $|I_A(x) - I_B(x)| = 0$,

$|F_A(x) - F_B(x)| = 0$.

This implies, $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$.

Hence $A = B$.

3. Since,

$|T_A(x) - T_B(x)| = |T_B(x) - T_A(x)|$,

$|I_A(x) - I_B(x)| = |I_B(x) - I_A(x)|$,

$|F_A(x) - F_B(x)| = |F_B(x) - F_A(x)|$.

We can write, SVNHSSM(A, B) = SVNHSSM(B, A).

4. $A \subset B \subset R$

$\Rightarrow T_A(x) \le T_B(x) \le T_R(x)$, $I_A(x) \ge I_B(x) \ge I_R(x)$, $F_A(x) \ge F_B(x) \ge F_R(x)$ for $x \in X$.

Now we have the following inequalities:

$|T_A(x) - T_B(x)| \le |T_A(x) - T_R(x)|$,

$|T_B(x) - T_R(x)| \le |T_A(x) - T_R(x)|$;

$|I_A(x) - I_B(x)| \le |I_A(x) - I_R(x)|$,

$|I_B(x) - I_R(x)| \le |I_A(x) - I_R(x)|$;

$|F_A(x) - F_B(x)| \le |F_A(x) - F_R(x)|$,

$|F_B(x) - F_R(x)| \le |F_A(x) - F_R(x)|$.

Thus, SVNHSSM(A, R) $\le$ SVNHSSM(A, B) and SVNHSSM(A, R) $\le$ SVNHSSM(B, R).

3.1 Weighted hyperbolic sine similarity measures for SVNSs

Let $A = \langle x(T_A(x), I_A(x), F_A(x))\rangle$ and $B = \langle x(T_B(x), I_B(x), F_B(x))\rangle$ be two SVNSs. Now weighted hyperbolic sine similarity function which measures the similarity between two SVNSs can be presented as follows (see Eqn. 2):

SVN WHSSM (A, B) =

$$1 - \sum_{i=1}^{n} w_i \left(\frac{\sinh\left(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \right)}{11} \right) \quad (2)$$

Here, $0 \le w_i \le 1$, $\sum_{i=1}^{n} w_i = 1$.

Theorem 2. The defined weighted hyperbolic sine similarity measure SVNWHSSM(A, B) between SVNSs A and B satisfies the following properties:

1. $0 \le$ SVNWHSSM(A, B) ≤ 1
2. SVNWHSSM (A, B) = 1 if and only if $A = B$
3. SVNWHSSM (A, B) = SVNWHSSM (B, A)
4. If R is a SVNS in X and $A \subset B \subset R$ then SVNWHSSM (A, R) $\le$ SVNWHSSM(A, B) and SVNWHSSM (A, R) $\le$ SVNWHSSM (B, R).

Proofs:

1. For two neutrosophic sets A and B,

$0 \le T_A(x_i), I_A(x_i), F_A(x_i), T_B(x_i), I_B(x_i), F_B(x_i) \le 1$

$$\Rightarrow 0 \le |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \le 3$$

$$\Rightarrow 0 \le \left(\frac{\sinh\left(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \right)}{11} \right) \le 1$$

Again, $0 \le w_i \le 1$, $\sum_{i=1}^{n} w_i = 1$.

Hence $0 \le$ SVNWHSSM(A, B) ≤ 1

2. For any two SVNSs A and B, if $A = B$,

$\Rightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x)$

$\Rightarrow |T_A(x) - T_B(x)| = 0$, $|I_A(x) - I_B(x)| = 0$,

$|F_A(x) - F_B(x)| = 0$

Hence SVNWHSSM(A, B) = 1.

Conversely,

SVNWHSSM(A, B) = 1

$\Rightarrow |T_A(x) - T_B(x)| = 0$, $|I_A(x) - I_B(x)| = 0$,

$|F_A(x) - F_B(x)| = 0$.

This implies, $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$.

Hence $A = B$.

3. Since,

$|T_A(x) - T_B(x)| = |T_B(x) - T_A(x)|$,

$|I_A(x) - I_B(x)| = |I_B(x) - I_A(x)|$,

$|F_A(x) - F_B(x)| = |F_B(x) - F_A(x)|$.

We can write, SVNWHSSM(A, B) = SVNWHSSM(B, A).

4. $A \subset B \subset R$

$\Rightarrow T_A(x) \le T_B(x) \le T_R(x),\ I_A(x) \ge I_B(x) \ge I_R(x),$
$F_A(x) \ge F_B(x) \ge F_R(x)$ for $x \in X$.

Now we have the following inequalities:

$|T_A(x) - T_B(x)| \le |T_A(x) - T_R(x)|$,

$|T_B(x) - T_R(x)| \le |T_A(x) - T_R(x)|$;

$|I_A(x) - I_B(x)| \le |I_A(x) - I_R(x)|$,

$|I_B(x) - I_R(x)| \le |I_A(x) - I_R(x)|$;

$|F_A(x) - F_B(x)| \le |F_A(x) - F_R(x)|$,

$|F_B(x) - F_R(x)| \le |F_A(x) - F_R(x)|$.

Thus SVNWHSSM(A, R) $\le$ SVNWHSSM(A, B) and SVNWHSSM(A, R) $\le$ SVNWHSSM(B, R).

4. Determination of unknown attribute weights

When attribute weights are completely unknown to decision makers, the entropy measure [46] can be used to calculate attribute weights. Biswas et al. [47] employed entropy measure for MADM problems to determine completely unknown attribute weights of SVNSs.

4.1 Compromise function

The compromise function of a SVNS $A = \langle T_{ij}^A, I_{ij}^A, F_{ij}^A \rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) is defined as follows (see Eqn. 3):

$$C_j(A) = \sum_{i=1}^{m} \left(2 + T_{ij}^A - I_{ij}^A - F_{ij}^A\right) \Big/ 3 \qquad (3)$$

The weight of j-th attribute is defined as follows (see Eqn. 4).

$$w_j = \frac{C_j(A)}{\sum_{j=1}^{n} C_j(A)} \qquad (4)$$

Here, $\sum_{j=1}^{n} w_j = 1.$

Theorem 3. The compromise function $C_j(A)$ satisfies the following properties:

P1. $C_j(A) = 1$, if $T_{ij} = 1, F_{ij} = I_{ij} = 0$.

P2. $C_j(A) = 0$, if $\langle T_{ij}, I_{ij}, F_{ij} \rangle = \langle 0, 1, 1 \rangle$.

P3. $C_j(A) \ge E_j(B)$, if $T_{ij}^A \ge T_{ij}^B$ and $I_{ij}^A + F_{ij}^A \le I_{ij}^B + F_{ij}^B$.

Proofs.

P1. $T_{ij} = 1, F_{ij} = I_{ij} = 0$

$$\Rightarrow C_j(A) = \frac{1}{m} \sum_{i=1}^{m} 3/3 = \frac{1}{m}.m = 1$$

P2. $\langle T_{ij}, I_{ij}, F_{ij} \rangle = \langle 0, 1, 1 \rangle$.

$$\Rightarrow C_j(A) = \frac{1}{m} \sum_{i=1}^{m} 0/3 = 0$$

P3. $C_j(A) - C_j(B)$

$$\Rightarrow \left\{ \frac{1}{m} \sum_{i=1}^{m} \left(2 + T_{ij}^A - I_{ij}^A - F_{ij}^A\right)\Big/3 - \frac{1}{m} \sum_{i=1}^{m} \left(2 + T_{ij}^B - I_{ij}^B - F_{ij}^B\right)\Big/3 \right\} > 0$$

$\Rightarrow C_j(A) - C_j(B) > 0$, Since, $T_{ij}^A > T_{ij}^B$ and $I_{ij}^A + F_{ij}^A < I_{ij}^B + F_{ij}^B$.

Hence, $C_j(A) \ge C_j(B)$.

5. Decision making procedure

Let A_1, A_2 , ..., A_m be a discrete set of alternatives, C_1, C_2, ..., C_n be the set of attributes of each alternative. The values associated with the alternatives A_i (i = 1, 2,..., m) against the attribute C_j (j = 1, 2, ..., n) for MADM problem is presented in a SVNS based decision matrix.

The steps of decision-making (see Figure 2) based on single valued neutrosophic weighted hyperbolic sine similarity measure (SVNWHSSM) are presented using the following steps.

Step 1: Determination of the relation between alternatives and attributes

The relation between alternatives A_i (i = 1, 2, ..., m) and the attribute C_j (j = 1, 2, ..., n) is presented in the Eqn. (5).

$$D[A\,|\,C]=\begin{pmatrix} & C_1 & C_2 & \cdots & C_n \\ A_1 & \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ A_2 & \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \langle T_{m1}, I_{m1}, F_{1m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{pmatrix} \quad (5)$$

Here $\langle T_{ij}, I_{ij}, F_{ij}\rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) be SVNS assessment value.

Step 2: Determine the weights of attributes

Using the Eqn. (3) and (4), decision-maker calculates the weight of the attribute C_j (j = 1, 2, …, n).

Step 3: Determine ideal solution

Generally, the evaluation attribute can be categorized into two types: benefit type attribute and cost type attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit type attributes and a minimum operator for the cost type attributes to determine the best value of each attribute among all the alternatives. Therefore, we define an ideal alternative as follows:

$A^* = \{C_1^*, C_2^*, \ldots, C_m^*\}$.

Here, benefit attribute C_j^* can be presented as follows:

$$C_j^*=\left\lfloor \max_i T_{C_j}{}^{(A_i)}, \min_i I_{C_j}{}^{(A_i)}, \min_i F_{C_j}{}^{(A_i)} \right\rfloor \quad (6)$$

for j = 1, 2, ..., n.

Similarly, the cost attribute C_j^* can be presented as follows:

$$C_j^*=\left\lfloor \min_i T_{C_j}{}^{(A_i)}, \max_i I_{C_j}{}^{(A_i)}, \max_i F_{C_j}{}^{(A_i)} \right\rfloor \quad (7)$$

for j = 1, 2, ..., n

Step 4: Determine the similarity values

Using Eqns. (2) and (5), calculate SVNWHSSM values for each alternative between positive (or negative) ideal solutions and corresponding single valued neutrosophic from decision matrix $D[A|C]$.

Step 5: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures. Highest value indicates the best alternative.

Step 6: End

6. Numerical example

In this section, we illustrate a numerical example as an application of the proposed approach. We consider a decision-making problem stated as follows. Suppose a person who wants to purchase a SIM card for his/her mobile connection. Therefore, it is necessary to select suitable SIM card for his/her mobile connection. After initial screening, there are four possible alternatives (SIM cards) for mobile connection. The alternatives (SIM cards) are presented as follows:

- A_1: Airtel
- A_2: Vodafone
- A_3: BSNL
- A_4: Reliance Jio

The person must take a decision based on the following five attributes of SIM cards:

- C_1: Service quality
- C_2: Cost
- C_3: Initial talk time
- C_4: Call rate per second
- C_5: Internet and other facilities

The decision-making strategy is presented using the following steps.

Step 1: Determine the relation between alternatives and attributes

The relation between alternatives A_1, A_2, A_3, and A_4 and the attributes C_1, C_2, C_3, C_4, C_5 is presented in the Eqn. (8).

$$D[A\,|\,C_1, C_2, C_3, C_4, C_5]=\begin{pmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ A_1 & \langle .7, .3, .3\rangle & \langle .6, .4, .3\rangle & \langle .8, .1, .1\rangle & \langle .5, .4, .4\rangle & \langle .5, .3, .2\rangle \\ A_2 & \langle .5, .3, .1\rangle & \langle .7, .1, .3\rangle & \langle .7, .3, .1\rangle & \langle .6, .1, .1\rangle & \langle .5, .2, .3\rangle \\ A_3 & \langle .8, .2, .2\rangle & \langle .6, .4, .3\rangle & \langle .6, 0, .1\rangle & \langle .7, .3, 0\rangle & \langle .5, .3, .4\rangle \\ A_4 & \langle .6, .1, .3\rangle & \langle .5, .1, .2\rangle & \langle .6, .3, .1\rangle & \langle .5, .1, .2\rangle & \langle .9, .1, .1\rangle \end{pmatrix} \quad (8)$$

Step 2: Determine the weights of attributes

Using the Eq. (3) and (4), we calculate the weight of the attributes C_1, C_2, C_3, C_4, C_5 as follows:

$[w_1, w_2, w_3, w_4, w_5]$ =

[0.2023, 0.1917, 0.2078, 0.2009, 0.1973]

Step 3: Determine ideal solution

In this problem, attributes C_1, C_3, C_4, C_5 are benefit type attributes and , C_2 is the cost type attribute.

A^* = {(0.8, 0.1, 0.1), (0.5, 0.4, 0.3), (0.8, 0.0, 0.1), (0.7, 0.1, 0.0), (0.9, 0.1, 0.1)}.

Step 4: Determine the weighted similarity values

Using Eq. (2) and Eq. (8), we calculate similarity measure values for each alternative as follows.

SVNWHSSM(A^*, A_1) = 0.92422

SVNWHSSM(A^*, A_2) = 0.95629

SVNWHSSM(A^*, A_3) = 0.97866

$\text{SVNWHSSM}(A^*, A_4) = 0.96795$

Step 5: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures (see Figure 1). Now the final ranking order will be as follows.

$A_3 \succ A_4 \succ A_2 \succ A_1$

Highest value indicates the best alternative.

Step 6: End

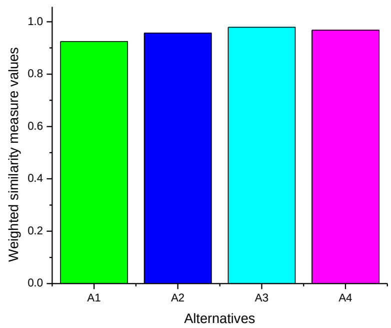

FIGURE 1: Graphical representation of alternatives versus weighted similarity measures.

7. Comparison analysis

The ranking results calculated from proposed strategy and different existing strategies [38, 48, 49, 50] are furnished in Table 1. We observe that the ranking results obtained from proposed and existing strategies in the literature differ.

The proposed strategy reflects that the optimal alternative is A_3. The ranking result obtained from Ye [38] is similar to the proposed strategy. The ranking results obtained from Ye and Zhang [48] and Mondal and Pramanik [49] differ from the optimal result of the proposed strategy. In Ye [50], the ranking order differs but the best alternative is the same to the proposed strategy.

Table 1 The ranking results of existing strategies

Strategies	Ranking results
Ye and Zhang[48]	$A_4 \succ A_2 \succ A_3 \succ A_1$
Mondal and Pramanik [49]	$A_4 \succ A_3 \succ A_2 \succ A_1$
Ye [38]	$A_3 \succ A_4 \succ A_2 \succ A_1$
Ye [50]	$A_3 \succ A_2 \succ A_4 \succ A_1$
Proposed strategy	$A_3 \succ A_4 \succ A_2 \succ A_1$

8. Contributions of the proposed strategy

1) SVNHSSM and SVNWHSSM in SVNS environment are firstly defined in the literature. We have also proved their basic properties.
2) We have proposed 'compromise function' for calculating unknown weights structure of attributes in SVNS environment.
3) We develop a decision making strategy based on the proposed weighted similarity measure (SVNWHSSM).
4) Steps and calculations of the proposed strategy are easy to use.
5) We have solved a numerical example to show the feasibility, applicability, and effectiveness of the proposed strategy.

9. Conclusion

In the paper, we have proposed hyperbolic sine similarity measure and weighted hyperbolic sine similarity measures for SVNSs and proved their basic properties. We have proposed compromise function to determine unknown weights of the attributes in SVNS environment. We have developed a novel MADM strategy based on the proposed weighted similarity measure to solve decision problems. We have solved a numerical problem and compared the obtained result with other existing strategies to demonstrate the effectiveness of the proposed MADM strategy. The proposed MADM strategy can be applied in other decision-making problem such as supplier selection, pattern recognition, cluster analysis, medical diagnosis, weaver selection [51-53], fault diagnosis [54], brick selection [55-56], data mining [57], logistic centre location selection [58-60], teacher selection [61, 62], etc.

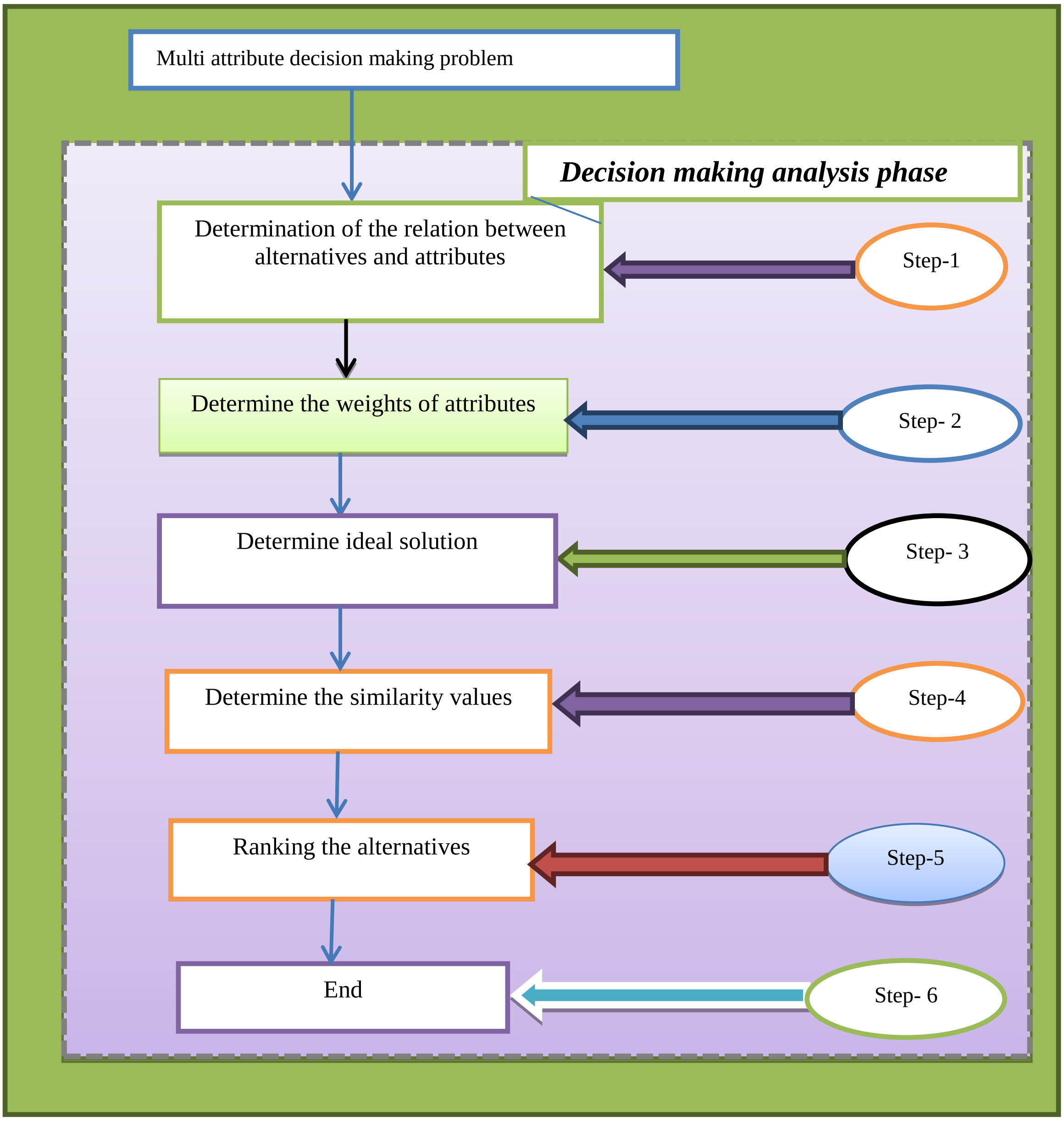

FIGURE 2: Phase diagram of the proposed decision making strategy

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