NSS

University of New Mexico

Truss Design Optimization using Neutrosophic Optimization Technique

Mridula Sarkar[1],Samir Dey[2], and Tapan Kumar Roy[3]

[1] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur,P.O.-Botanic Garden, Howrah-711103, West Bengal, India.E-mail:mridula.sarkar86@yahoo.com

[2] Department of Mathematics, Asansol Engineering College,Vivekananda Sarani, Asansol-713305, West Bengal, India. E-mail:samir_besus@rediffmail.com

[3] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur,P.O.-Botanic Garden, Howrah-711103, West Bengal, India.E-mail:roy_t_k@yahoo.co.in

Abstract: In this paper, we develop a neutrosophic optimization (NSO) approach for optimizing the design of plane truss structure with single objective subject to a specified set of constraints. In this optimum design formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. A classical truss optimization example is presented to demonstrate the efficiency of the neutrosophic optimization approach. The test problem includes a two-bar planar truss subjected to a single load condition. This single-objective structural optimization model is solved by fuzzy and intuitionistic fuzzy optimization approach as well as neutrosophic optimization approach. A numerical example is given to illustrate our NSO approach. The result shows that the NSO approach is very efficient in finding the best optimal solutions.

Keywords: Neutrosophic Set, Single Valued Neutrosophic Set, Neutrosophic Optimization, Non-linear Membership Function, Structural Optimization.

1 Introduction

In the field of civil engineering nonlinear structural design optimizations are of great importance. So the description of structural geometry and mechanical properties like stiffness are required for a structural system. However the system description and system inputs may not be exact due to human errors or some unexpected situations. At this juncture fuzzy set theory provides a method which deals with ambiguous situations like vague parameters, non-exact objective and constraint. In structural engineering design problems, the input data and parameters are often fuzzy/imprecise with nonlinear characteristics that necessitate the development of fuzzy optimum structural design method. Fuzzy set (FS) theory has long been introduced to handle inexact and imprecise data by Zadeh [2], Later on Bellman and Zadeh [4] used the fuzzy set theory to the decision making problem. The fuzzy set theory also found application in structural design. Several researchers like Wang et al. [8] first applied α-cut method to structural designs where the non-linear problems were solved with various design levels α, and then a sequence of solutions were obtained by setting different level-cut value of α. Rao [3] applied the same α-cut method to design a four–bar mechanism for function generating problem. Structural optimization with fuzzy parameters was developed by Yeh et al. [9]. Xu [10] used two-phase method for fuzzy optimization of structures. Shih et al. [5] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources. Shih et al. [6] developed an alternative α-level-cuts methods for optimum structural design with fuzzy resources. Dey et al. [11] used generalized fuzzy number in context of a structural design. Dey et al used basic t-norm based fuzzy optimization technique for optimization of structure. Dey et al. [13] developed parameterized t-norm based fuzzy optimization method for optimum structural design. Also, Dey et.al [14] Optimized shape design of structural model with imprecise coefficient by parametric geometric programming.

In such extension, Atanassov [1] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory and is characterized by a membership function, a non- membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. A transportation model was solved by Jana et al.[15]using multi-objective intuitionistic fuzzy linear programming. Dey et al. [12] solved two bar truss non-linear problem by using intuitionistic fuzzy optimization problem. Dey et al. [16] used intuitionistic fuzzy optimization technique for multi objective optimum structural design. Intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information.

In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophic theory was introduced by Smarandache [7]. The motivation of the present study is to give computational algorithm for solving multi-objective structural problem by single valued neutrosophic optimization approach. Neutrosophic optimization technique is very rare in application to structural optimization. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership function in such optimization process. The results are compared numerically both in fuzzy optimization technique, intuitionistic fuzzy optimization technique and neutrosophic optimization technique. From our numerical result, it is clear that neutrosophic optimization technique provides better results than fuzzy optimization and intuitionistic fuzzy optimization.

.

2 Single-objective structural model

In sizing optimization problems,the aim is to minimize single objective function,usually the weight of the structure under certain behavioural constraints on constraint and displacement. The design variables are most frequently chosen to be dimensions of the cross sectional areas of the members of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness of cross-sections to a certain set. A discrete structural optimization problem can be formulated in the following form

$$Minimize\ WT(A)$$

$$subject\ to\ \sigma_i(A) \le 0, i = 1,2,........,m$$

$$A_j \in R^d,\quad j = 1,2,..........,n$$

where $WT(A)$ represents objective function, $\sigma_i(A)$ is the behavioural constraints, m and n are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective function is taken as $WT(A)=\sum_{i=1}^{m}\rho_i l_i A_i$ and constraint are chosen to be stress of structures as follows $\sigma_i(A) \lesssim \sigma_i$ with allowable tolerance σ_i^0 for $i = 1,2,........,m$ where ρ_i and l_i are weight of unit volume and length of i^{th} element respectively, m is the number of structural element, σ_i and σ_i^0 are the i^{th} stress ,allowable stress respectively.

3 Mathematical preliminaries

3.1 Fuzzy set

Let X be a fixed set. A fuzzy set A set of X is an object having the form $\tilde{A} = \{(x, T_A(x)) : x \in X\}$ where the function $T_A : X \to [0,1]$ defined the truth membership of the element $x \in X$ to the set A .

3.2 Intuitionistic fuzzy set

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form

$$\tilde{A}^i = \{< X, T_A(x), F_A(x) > | x \in X\} \text{ where}$$

$$T_A : X \to [0,1] \text{ and } F_A : X \to [0,1]$$

define the truth membership and falsity membership respectively, for every element of $x \in X$ $0 \le T_A + F_A \le 1$.

3.3 Neutrosophic set

Let a set X be a space of points (objects) and $x \in X$ .A neutrosophic set $\tilde{A}^n$ in X is defined by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$,and denoted by $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$.

$T_A(x)$ $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ .That is $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$, .There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

3.4 Single valued neutrosophic set

Let a set X be the universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ where $T_A : X \to [0,1]$, $I_A : X \to [0,1]$, and $F_A : X \to [0,1]$ with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

3.5 Complement of neutrosophic Set

Complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - F_A(x)$, $F_{c(A)}(x) = T_A(x)$.

3.6 Union of neutrosophic sets

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cup B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \max(T_A(x), T_B(x))$,

$I_{c(A)}(x) = \max(I_A(x), I_B(x))$,

$F_{c(A)}(x) = \min(F_A(x), F_B(x))$ for all $x \in X$.

3.7 Intersection of neutrosophic sets

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cap B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \min(T_A(x), T_B(x))$,

$I_{c(A)}(x) = \min(I_A(x), I_B(x))$,

$F_{c(A)}(x) = \max(F_A(x), F_B(x))$ for all $x \in X$.

4 Mathematical analyses

4.1 Neutrosophic optimization technique to solve minimization type Single-Objective

Let a nonlinear single-objective optimization problem be

$$Minimize \quad f(x) \tag{2}$$

Such that

$$g_j(x) \le b_j \qquad j = 1, 2, ..., m$$

$$x \ge 0$$

Usually constraints goals are considered as fixed quantity .But in real life problem ,the constraint goal cannot be always exact. So we can consider the constraint goal for less than type constraints at least b_j and it may possible to extend to $b_j + b_j^0$.This fact seems to take the constraint goal as a neutrosophic fuzzy set and which will be more realistic descriptions than others. Then the NLP becomes NSO problem with neutrosophic resources, which can be described as follows

$$Minimize \quad f(x) \tag{3}$$

Such that

$$g_j(x) \le \tilde{b}_j^n \qquad j = 1, 2, ..., m$$

$$x \ge 0$$

To solve the NSO (3), we are presenting a solution procedure for single-objective NSO problem (3) as follows

Step-1: Following warner's approach solve the single objective non-linear programming problem without tolerance in constraints $(i.e\ g_j(x) \le b_j)$, with tolerance of acceptance in constraints (i.e $g_j(x) \le b_j + b_j^0$) by appropriate non-linear programming technique

Here they are
Sub-problem-1

$$Minimize \quad f(x) \tag{4}$$

Such that

$$g_j(x) \le b_j \qquad j = 1, 2, ..., m$$

$$x \ge 0$$

Sub-problem-2

$$Minimize \quad f(x) \tag{5}$$

Such that

$$g_j(x) \le b_j + b_j^0, \quad j = 1, 2, ..., m$$

$$x \ge 0$$

We may get optimal solution $x^* = x^1, f(x^*) = f(x^1)$ and $x^* = x^1, f(x^*) = f(x^1)$

Step-2: From the result of step 1 we now find the lower bound and upper bound of objective functions. If $U_{f(x)}^T, U_{f(x)}^I, U_{f(x)}^F$ be the upper bounds of truth, indeterminacy , falsity function for the objective respectively and $L_{f(x)}^T, L_{f(x)}^I, L_{f(x)}^F$ be the lower bound of truth, indeterminacy, falsity membership functions of objective respectively. then

$$U_{f(x)}^F = U_{f(x)}^T, L_{f(x)}^F = L_{f(x)}^T + \varepsilon_{f(x)} \ \ where\ 0 < \varepsilon_{f(x)} < \left(U_{f(x)}^T - L_{f(x)}^T\right)$$

$$U_{f(x)}^F = U_{f(x)}^T, L_{f(x)}^F = L_{f(x)}^T + \varepsilon_{f(x)} \ \ where\ 0 < \varepsilon_{f(x)} < \left(U_{f(x)}^T - L_{f(x)}^T\right)$$

$$L_{f(x)}^I = L_{f(x)}^T, U_{f(x)}^I = L_{f(x)}^T + \xi_{f(x)} \ \ where\ 0 < \xi_{f(x)} < \left(U_{f(x)}^T - L_{f(x)}^T\right)$$

Step-3: In this step we calculate membership for truth, indeterminacy and falsity membership function of objective as follows

$$T_{f(x)}(f(x)) = \begin{cases} 1 & if\ \ f(x) \le L_{f(x)}^T \\ 1 - \exp\left\{-\psi\left(\dfrac{U_{f(x)}^T - f(x)}{U_{f(x)}^T - L_{f(x)}^T}\right)\right\} & if\ \ L_{f(x)}^T \le f(x) \le U_{f(x)}^T \\ 0 & if\ \ f(x) \ge U_{f(x)}^T \end{cases}$$

$$I_{f(x)}\left(f(x)\right)=$$
$$\begin{cases} 1 & if \quad f(x)\le L^{I}_{f(x)} \\ \exp\left\{\dfrac{U^{I}_{f(x)}-f(x)}{U^{I}_{f(x)}-L^{I}_{f(x)}}\right\} & if \quad L^{I}_{f(x)}\le f(x)\le U^{I}_{f(x)} \\ 0 & if \quad f(x)\ge U^{I}_{f(x)} \end{cases}$$

$$F_{f(x)}\left(f(x)\right)=$$
$$\begin{cases} 0 & if \quad f(x)\le L^{F}_{f(x)} \\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(f(x)-\dfrac{U^{F}_{f(x)}+L^{F}_{f(x)}}{2}\right)\tau_{f(x)}\right\} & if \quad L^{F}_{f(x)}\le f(x)\le U^{F}_{f(x)} \\ 1 & if \quad f(x)\ge U^{F}_{f(x)} \end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker.

Step-4: In this step using exponential and hyperbolic membership function we calculate truth , indeterminacy and falsity membership function for constraints as follows

$$T_{g_j(x)}\left(g_j(x)\right)=$$
$$\begin{cases} 1 & if \quad g_j(x)\le b_j \\ 1-\exp\left\{-\psi\left(\dfrac{U^{T}_{g_j(x)}-g_j(x)}{U^{T}_{g_j(x)}-L^{T}_{g_j(x)}}\right)\right\} & if \quad b_j\le g_j(x)\le b_j+b_j^0 \\ 0 & if \quad g_j(x)\ge b_j+b_j^0 \end{cases}$$

$$I_{g_j(x)}\left(g_j(x)\right)=$$
$$\begin{cases} 1 & if \quad g_j(x)\le b_j \\ \exp\left\{\dfrac{\left(b_j+\xi_{g_j(x)}\right)-g_j(x)}{\xi_{g_j(x)}}\right\} & if \quad b_j\le g_j(x)\le b_j+\xi_{g_j(x)} \\ 0 & if \quad g_j(x)\ge b_j+\xi_{g_j(x)} \end{cases}$$

$$F_{g_j(x)}\left(g_j(x)\right)=$$
$$\begin{cases} 0 & if \quad g_j(x)\le b_j+\varepsilon_{g_j(x)} \\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(g_j(x)-\dfrac{2b_j+b_j^0+\varepsilon_{g_j(x)}}{2}\right)\tau_{g_j(x)}\right\} & if \quad b_j+\varepsilon_{g_j(x)}\le g_j(x)\le b_j+b_j^0 \\ 1 & if \quad g_j(x)\ge b_j+b_j^0 \end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for $j=1,2,.....,m$ $0<\varepsilon_{g_j(x)},\xi_{g_j(x)}<b_j^0$.

Step-5: Now using NSO for single objective optimization technique the optimization problem (2) can be formulated as

$$Maximize\ (\alpha+\gamma-\beta) \tag{6}$$

Such that

$$T_{f(x)}(x)\ge\alpha;\ \ T_{g_j}(x)\ge\alpha;$$
$$I_{f(x)}(x)\ge\gamma;\ \ I_{g_j}(x)\ge\gamma;$$
$$F_{f(x)}(x)\le\beta;\ \ F_{g_j}(x)\le\beta;$$
$$\alpha+\beta+\gamma\le 3;\ \ \alpha\ge\beta;\alpha\ge\gamma;$$
$$\alpha,\beta,\gamma\in[0,1]$$

where

$$\alpha=T_{\tilde{D}^n}(x)=\min\left\{T_{f(x)}\left(f(x)\right),T_{g_j(x)}\left(g_j(x)\right)\right\}$$

for $j=1,2,...,m$

$$\gamma=I_{\tilde{D}^n}(x)=\min\left\{I_{f(x)}\left(f(x)\right),I_{g_j(x)}\left(g_j(x)\right)\right\}$$

for $j=1,2,...,m$ and

$$\beta=F_{\tilde{D}^n}(x)=\min\left\{F_{f(x)}\left(f(x)\right),F_{g_j(x)}\left(g_j(x)\right)\right\}$$ for $j=1,2,...,m$

are the truth ,indeterminacy and falsity membership function of decision set $\tilde{D}^n=f^n(x)\bigcap_{j=1}^{m}g^n{}_j(x)$. Now if non-linear membership be considered the above problem (6) can be reduced to following crisp linear programming problem

$$Maximize\ (\theta+\kappa-\eta) \tag{7}$$

Such that

$$f(x)+\theta\frac{\left(U^{T}_{f(x)}-L^{T}_{f(x)}\right)}{\psi}\le U^{T}_{f(x)};$$
$$f(x)+\kappa\xi_{f(x)}\le U^{T}_{f(x)};\text{ b}$$
$$f(x)+\frac{\eta}{\tau_{f(x)}}\le\frac{U^{T}_{f(x)}+L^{T}_{f(x)}+\varepsilon_{f(x)}}{2};$$
$$g_j(x)+\theta\frac{b_j^0}{\psi}\le b_j+b_j^0;$$
$$g_j(x)+\kappa\xi_{g_j(x)}\le b_j^0+\xi_{g_j(x)};$$
$$g_j(x)+\frac{\eta}{\tau_{g(x)}}\le\frac{2b_j+b_j^0+\varepsilon_{g_j(x)}}{2};$$
$$\theta+\kappa+\eta\le 3;$$
$$\theta\ge\kappa;\theta\ge\eta;$$

$$\theta,\kappa,\eta\in[0,1]$$

where $\theta=-\ln(1-\alpha)$; $\psi=4$; $\tau_{f(x)}=\frac{6}{\left(U^F_{f(x)}-L^F_{f(x)}\right)}$;

$\tau_{g_j(x)}=\frac{6}{\left(b_j^0-\varepsilon_j\right)}$, *for* $j=1,2,........,m$ $\kappa=\ln\gamma$;

$\eta=-\tanh^{-1}(2\beta-1)$.

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

5. Solution of Single-objective Structural Optimization Problem (SOSOP) by Neutrosophic Optimization Technique

To solve the SOSOP (1), step 1 of 4 is used and we will get optimum solutions of two sub problem as A^1 and A^2 .After that according to step 2 we find upper and lower bound of membership function of objective function as $U^T_{WT(A)},U^I_{WT(A)},U^F_{WT(A)}$ and $U^T_{WT(A)},U^I_{WT(A)},U^F_{WT(A)}$ where

$$U^T_{WT(A)}=\max\left\{WT\left(A^1\right),WT\left(A^2\right)\right\},L^T_{WT(A)}=\min\left\{WT\left(A^1\right),WT\left(A^2\right)\right\},$$

$$U^F_{WT(A)}=U^T_{WT(A)},L^F_{WT(A)}=L^T_{WT(A)}+\varepsilon_{WT(A)}\ \ where\ 0<\varepsilon_{WT(A)}<\left(U^T_{WT(A)}-L^T_{WT(A)}\right)$$

$$L^I_{WT(A)}=L^T_{WT(A)},U^I_{WT(A)}=L^T_{WT(A)}+\xi_{WT(A)}\ \ where\ 0<\xi_{WT(A)}<\left(U^T_{WT(A)}-L^T_{WT(A)}\right)$$

Let the non-linear membership function for objective function $WT(A)$ be

$$T_{WT(A)}\left(WT(A)\right)=\begin{cases}1 & if\ \ WT(A)\le L^T_{WT(A)}\\ 1-\exp\left\{-\psi\left(\frac{U^T_{WT(A)}-WT(A)}{U^T_{WT(A)}-L^T_{WT(A)}}\right)\right\} & if\ \ L^T_{WT(A)}\le WT(A)\le U^T_{WT(A)}\\ 0 & if\ \ WT(A)\ge U^T_{WT(A)}\end{cases}$$

$$I_{WT(A)}\left(WT(A)\right)=\begin{cases}1 & if\ \ WT(A)\le L^I_{WT(A)}\\ \exp\left\{\frac{U^I_{WT(A)}-WT(A)}{U^I_{WT(A)}-L^I_{WT(A)}}\right\} & if\ \ L^I_{WT(A)}\le WT(A)\le U^I_{WT(A)}\\ 0 & if\ \ WT(A)\ge U^I_{WT(A)}\end{cases}$$

$$I_{\sigma_i(A)}\left(\sigma_i(A)\right)=\begin{cases}1 & if\ \ \sigma_i(A)\le\sigma_i\\ \exp\left\{\frac{\left(\sigma_i+\xi_{g_j(x)}\right)-\sigma_i(A)}{\xi_{\sigma_i(x)}}\right\} & if\ \ \sigma_i\le\sigma_i(A)\le\sigma_i+\xi_{\sigma_i(x)}\\ 0 & if\ \ \sigma_i(A)\ge\sigma_i+\xi_{\sigma_i(x)}\end{cases}$$

$$F_{\sigma_i(A)}\left(\sigma_i(A)\right)=\begin{cases}0 & if\ \ \sigma_i(A)\le\sigma_i+\varepsilon_{\sigma_i(A)}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\sigma_i(A)-\frac{2b_j+b_j^0+\varepsilon_{\sigma_i}}{2}\right)\tau_{\sigma_i}\right\} & if\ \ \sigma_i+\varepsilon_{\sigma_i(A)}\le\sigma_i(A)\le\sigma_i+\sigma_i^0\\ 1 & if\ \ \sigma_i(A)\ge\sigma_i+\sigma_i^0\end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for $j=1,2,.....,m$ $0<\varepsilon_{\sigma_i(A)},\xi_{\sigma_i(A)}<\sigma_i^0$

then neutrosophic optimization problem can be formulated as

$$Max\ (\alpha+\beta-\gamma) \tag{8}$$

such that

$$T_{WT(A)}\left(WT(A)\right)\ge\alpha;\ T_{\sigma_i(A)}\left(\sigma_i(A)\right)\ge\alpha;$$

$$I_{WT(A)}\left(WT(A)\right)\ge\gamma;\ I_{\sigma_i(A)}\left(\sigma_i(A)\right)\ge\gamma;$$

$$F_{WT(A)}\left(WT(A)\right)\le\beta;\ F_{\sigma_i(A)}\left(\sigma_i(A)\right)\le\beta$$

$$\alpha+\beta+\gamma\le3;\alpha\ge\beta,\alpha\ge\gamma;$$

$$\alpha,\beta,\gamma\in[0,1]$$

The above problem can be reduced to following crisp linear programming problem, for non-linear membership as

$$Maximize\ (\theta+\kappa-\eta) \tag{9}$$

such that

$$WT(A)+\theta\frac{\left(U^T_{WT(A)}-L^T_{WT(A)}\right)}{\psi}\le U^T_{WT(A)};$$

$$WT(A)+\frac{\eta}{\tau_{WT(A)}}\le\frac{U^T_{WT(A)}+L^T_{WT(A)}+\varepsilon_{WT(A)}}{2};$$

$$WT(A)+\kappa\xi_{WT(A)}\le U^T_{WT(A)};$$

$$\sigma_i(A)+\theta\frac{\sigma_i^0}{\psi}\le\sigma_i+\sigma_i^0;$$

$$\sigma_i\left(A\right)+\kappa\xi_{\sigma_i(A)}\le\sigma_i^0+\xi_{\sigma_i(A)};$$

$$\sigma_i\left(A\right)+\frac{\eta}{\tau_{\sigma_i(A)}}\le\frac{2\sigma_i+\sigma_i^0+\varepsilon_{\sigma_i(A)}}{2};$$

$$\theta+\kappa-\eta\le 3;\ \theta\ge\kappa;\theta\ge\eta;$$

$$\theta,\kappa,\eta\in\left[0,1\right]$$

where

$$\theta=-\ln\left(1-\alpha\right);\ \psi=4;\ \tau_{WT(A)}=\frac{6}{\left(U_{WT(A)}^F-L_{WT(A)}^F\right)};$$

$$\kappa=\ln\gamma;\ \eta=-\tanh^{-1}\left(2\beta-1\right).\ and\ \tau_{\delta(A)}=\frac{6}{\left(U_{\delta(A)}^F-L_{\delta(A)}^F\right)};$$

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

6 Numerical illustration

A well-known two-bar [17] planar truss structure is considered. The design objective is to minimize weight of the structural $WT\left(A_1,A_2,y_B\right)$ of a statistically loaded two-bar planar truss subjected to stress $\sigma_i\left(A_1,A_2,y_B\right)$ constraints on each of the truss members $i=1,2$.

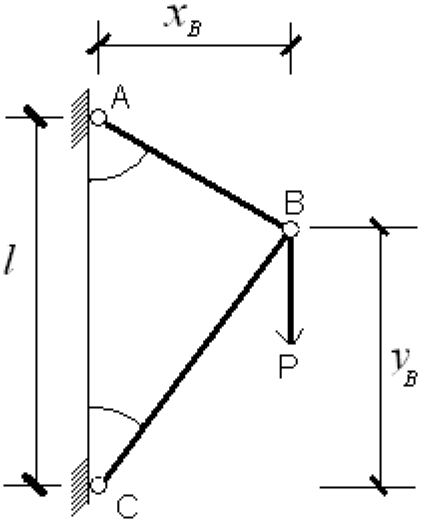

Figure 1. Design of the two-bar planar truss

The multi-objective optimization problem can be stated as follows

$$Minimize\ WT\left(A_1,A_2,y_B\right)=\rho\left(A_1\sqrt{x_B^2+\left(l-y_B\right)^2}+A_2\sqrt{x_B^2+{y_B}^2}\right)\quad(10)$$

Such that

$$\sigma_{\text{AB}}\left(A_1,A_2,y_B\right)\equiv\frac{P\sqrt{x_B^2+\left(l-y_B\right)^2}}{lA_1}\le\left[\sigma_{AB}^T\right];$$

$$\sigma_{\text{BC}}\left(A_1,A_2,y_B\right)\equiv\frac{P\sqrt{x_B^2+{y_B}^2}}{lA_2}\le\left[\sigma_{BC}^C\right];$$

$$0.5\le y_B\le 1.5$$

$$A_1>0,A_2>0;$$

where $P=$ nodal load ; $\rho=$ volume density ; $l=$ length of AC ; $x_B=$ perpendicular distance from AC to point B . $A_1=$ Cross section of bar- AB ; $A_2=$ Cross section of bar- BC . $\left[\sigma_T\right]=$ maximum allowable tensile stress , $\left[\sigma_C\right]=$ maximum allowable compressive stress and $y_B=y$ -co-ordinate of node B .

Input data for crisp model (10) is in table 1.

Solution : According to step 2 of 4,we find upper and lower bound of membership function of objective function as

$$U_{WT(A)}^T,U_{WT(A)}^I,U_{WT(A)}^F$$

and $L_{WT(A)}^T,L_{WT(A)}^I,L_{WT(A)}^F$ where

$$U_{WT(A)}^T=14.23932=U_{WT(A)}^F$$

$$L_{WT(A)}^T=12.57667=L_{WT(A)}^I$$

$$L_{WT(A)}^F=12.57667+\varepsilon_{WT(A)},\ 0<\varepsilon_{WT(A)}<1.66265$$

$$U_{WT(A)}^I=L_{WT(A)}^T+\xi_{WT(A)},\ 0<\xi_{WT(A)}<1.66265$$

Now using the bounds we calculate the membership functions for objective as follows

$$T_{WT(A_1,A_2,y_B)}\left(WT\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ WT\left(A_1,A_2,y_B\right)\le 12.57667\\ 1-\exp\left\{-4\left(\dfrac{14.23932-WT\left(A_1,A_2,y_B\right)}{1.66265}\right)\right\} & if\ 12.57667\le WT\left(A_1,A_2,y_B\right)\le 14.23932\\ 0 & if\ WT\left(A_1,A_2,y_B\right)\ge 14.23932\end{cases}$$

$$I_{WT(A_1,A_2,y_B)}\left(WT\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ WT\left(A_1,A_2,y_B\right)\le 12.57667\\ \exp\left\{\dfrac{\left(12.57667+\xi_{WT}\right)-WT\left(A_1,A_2,y_B\right)}{\xi_{WT}}\right\} & if\ 12.57667\le WT\left(A_1,A_2,y_B\right)\le 12.57667+\xi_{WT}\\ 0 & if\ WT\left(A_1,A_2,y_B\right)\ge 12.57667+\xi_{WT}\end{cases}$$

$$F_{WT(A_1,A_2,y_B)}\left(WT\left(A_1,A_2,y_B\right)\right)=\begin{cases}0 & if\ WT\left(A_1,A_2,y_B\right)\le 12.57667+\varepsilon_{WT}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(WT\left(A_1,A_2,y_B\right)-\frac{\left(26.81599+\varepsilon_{WT}\right)}{2}\right)\frac{6}{1.66265-\varepsilon_{WT}}\right\} & if\ 12.57667+\varepsilon_{WT}\le WT\left(A_1,A_2,y_B\right)\le 14.23932\\ 1 & if\ WT\left(A_1,A_2,y_B\right)\ge 14.23932\end{cases}$$

Similarly the membership functions for tensile stress are

$$T_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ \sigma_T\left(A_1,A_2,y_B\right)\le 130\\ 1-\exp\left\{-4\left(\frac{150-\sigma_T\left(A_1,A_2,y_B\right)}{20}\right)\right\} & if\ 130\le\sigma_T\left(A_1,A_2,y_B\right)\le 150\\ 0 & if\ \sigma_T\left(A_1,A_2,y_B\right)\ge 150\end{cases}$$

$$I_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ \sigma_T\left(A_1,A_2,y_B\right)\le 130\\ \exp\left\{\frac{\left(130+\xi_{\sigma_T}\right)-\sigma_T\left(A_1,A_2y_B\right)}{\xi_{\sigma_T}}\right\} & if\ 130\le\sigma_T\left(A_1,A_2,y_B\right)\le 130+\xi_{\sigma_T}\\ 0 & if\ \sigma_T\left(A_1,A_2,y_B\right)\ge 130+\xi_{\sigma_T}\end{cases}$$

$$F_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T\left(A_1,A_2,y_B\right)\right)=\begin{cases}0 & if\ \sigma_T\left(A_1,A_2,y_B\right)\le 130+\varepsilon_{\sigma_T}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\sigma_T\left(A_1,A_2,y_B\right)-\left(\frac{280+\varepsilon_{\sigma_T}}{2}\right)\right)\frac{6}{20-\varepsilon_{\sigma_T}}\right\} & if\ 130+\varepsilon_{\sigma_T}\le\sigma_T\left(A_1,A_2,y_B\right)\le 150\\ 1 & if\ \sigma_T\left(A_1,A_2,y_B\right)\ge 150\end{cases}$$

where $0<\varepsilon_{\sigma_T},\xi_{\sigma_T}<20$

and the membership functions for compressive stress constraint are

$$T_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ \sigma_C\left(A_1,A_2,y_B\right)\le 90\\ 1-\exp\left\{-4\left(\frac{100-\sigma_C\left(A_1,A_2,y_B\right)}{10}\right)\right\} & if\ 90\le\sigma_C\left(A_1,A_2,y_B\right)\le 100\\ 0 & if\ \sigma_C\left(A_1,A_2,y_B\right)\ge 100\end{cases}$$

$$I_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ \sigma_C\left(A_1,A_2,y_B\right)\le 90\\ \exp\left\{\frac{\left(90+\xi_{\sigma_C}\right)-\sigma_C\left(A_1,A_2,y_B\right)}{\xi_{\sigma_C}}\right\} & if\ 90\le\sigma_C\left(A_1,A_2,y_B\right)\le 90+\xi_{\sigma_C}\\ 0 & if\ \sigma_C\left(A_1,A_2,y_B\right)\ge 90+\xi_{\sigma_C}\end{cases}$$

$$F_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C\left(A_1,A_2,y_B\right)\right)=\begin{cases}0 & if\ \sigma_C\left(A_1,A_2,y_B\right)\le 90+\varepsilon_{\sigma_C}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\sigma_C\left(A_1,A_2,y_B\right)-\left(\frac{190+\varepsilon_{\sigma_C}}{2}\right)\right)\frac{6}{10-\varepsilon_{\sigma_C}}\right\} & if\ 90+\varepsilon_{\sigma_C}\le\sigma_C\left(A_1,A_2,y_B\right)\le 100\\ 1 & if\ \sigma_C\left(A_1,A_2,y_B\right)\ge 100\end{cases}$$

where $0<\varepsilon_{\sigma_C},\xi_{\sigma_C}<10$.

Now , using above mentioned truth, indeterminacy and falsity membership function NLP (7) can be solved by NSO technique for different values of $\varepsilon_{WT},\varepsilon_{\sigma_T},\varepsilon_{\sigma_C}$ and $\xi_{WT},\xi_{\sigma_T},\xi_{\sigma_C}$. The optimum solution of SOSOP(10) is given in table (2) and the solution is compared with fuzzy and intuitionistic fuzzy optimization technique.

Table 1: Input data for crisp model (10)

Applied load $P\left(KN\right)$	Volume density $\rho\left(KN/m^3\right)$	Length $l\left(m\right)$	Maximum allowable tensile stress $\left[\sigma_T\right]\left(Mpa\right)$	Maximum allowable compressive stress $\left[\sigma_C\right]\left(Mpa\right)$	Distance of x_B from AC $\left(m\right)$
100	7.7	2	130 with tolerance 20	90 with tolerance 10	1

Table 2: Comparison of Optimal solution of SOSOP (10) based on different methods

Methods	$A_1\left(m^2\right)$	$A_2\left(m^2\right)$	$WT\left(A_1,A_2\right)\left(KN\right)$	$y_B\left(m\right)$
Fuzzy single-objective non-linear programming (FSONLP) with non-linear membership functions	.5883491	.7183381	**14.23932**	1.013955
Intuitionistic fuzzy single-objective nonlinear programming (IFSONLP) with non-linear membership functions $\varepsilon_1=0.8,\varepsilon_2=16,\varepsilon_3=8$	.6064095	.6053373	**13.59182**	.5211994
Neutosophic optimization(NSO) with non-linear membership functions $\varepsilon_1=0.8,\varepsilon_2=16,\varepsilon_3=8$ $\xi_1=0.66506,\xi_2=8,\xi_3=4$	.5451860	.677883	**13.24173**	.7900455

Here we get best solutions for the different tolerance ξ_1, ξ_2 and ξ_3 for indeterminacy exponential membership function of objective functions for this structural optimization problem. From the table 2, it shows that NSO technique gives better Pareto optimal result in the perspective of Structural Optimization.

7 Conclusions

The main objective of this work is to illustrate how neutrosophic optimization technique using non-linear membership function can be utilized to solve a nonlinear structural problem. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; but rather they are independent with degree of indeterminacy. The numerical illustration shows the superiority of neutrosophic optimization over fuzzy optimization as well as intuitionistic fuzzy optimization. The results of this study may lead to the development of effective neutrosophic technique for solving other model of nonlinear programming problem in other engineering field.

References

[1] Atanassov, K. T. Intuitionistic fuzzy sets. Fuzzy Sets and Systems,vol.20, Issue 1, (1986),87-96.

[2] Zadeh, L.A. Fuzzy set. Information and Control, vol.8, Issue 3.(1965), 338-353.

[3] Rao, S. S. Description and optimum design of fuzzy mechanical systems. Journal of Mechanisms, Transmissions, and Automation in Design, vol.109,Issue 1, (1987), 126-132.

[4] Bellman, R. E., &Zadeh, L. A. Decision-making in a fuzzy environment. Management science, vol.17, Issue 4, (1970), B-141.

[5] Shih, C. J., & Lee, H. W. Level-cut approaches of first and second kind for unique solution design in fuzzy engineering optimization problems. Tamkang Journal of Science and Engineering,vol.7,Issue 3, (2004),189-198.

[6] Shih, C. J., Chi, C. C., & Hsiao, J. H. Alternative α-level-cuts methods for optimum structural design with fuzzy resources. Computers & structures, vol.81,Issue 28, (2003), 2579-2587.

[7] Smarandache, F. Neutrosophy, neutrosophic probability, set and logic, Amer. Res. Press, Rehoboth, USA, (1998),105.

[8] Wang, G.Y. & Wang, W.Q. Fuzzy optimum design of structure. Engineering Optimization, vol.8, (1985),291-300.

[9] Yeh, Y.C. & Hsu, D.S. Structural optimization with fuzzy parameters. Computer and Structure,vol.37,Issue6, (1990),917–924.

[10] Changwen, X. Fuzzy optimization of structures by the two-phase method. Computers & Structures, vol.31,Issue 4, (1989),575-580.

[11] Dey, S., & Roy, T. K. A Fuzzy programming Technique for Solving Multi-objective Structural Problem. International Journal of Engineering and Manufacturing,vol. 4,Issue 5, (2014), 24.

[12] Dey, S., & Roy, T. K. Optimized solution of two bar truss design using intuitionistic fuzzy optimization technique. International Journal of Information Engineering and Electronic Business, vol.6,Issue 4, (2014), 45.

[13] Dey, S., & Roy, T. K. Multi-objective structural design problem optimization using parameterized t-norm based fuzzy optimization programming Technique. Journal of Intelligent and Fuzzy Systems, vol.30,Issue 2, (2016),971-982.

[14] Dey, S., & Roy, T. Optimum shape design of structural model with imprecise coefficient by parametric geometric programming. Decision Science Letters, vol.4,Issue 3, (2015), 407-418.

[15] Jana, B., & Roy, T. K. Multi-objective intuitionistic fuzzy linear programming and its application in transportation model. Notes on Intuitionistic Fuzzy Sets, vol.13,Issue 1, (2007). 34-51.

[16] Dey, S. & Roy, T. K. Multi-objective structural optimization using fuzzy and intuitionistic fuzzy optimization technique. International Journal of Intelligent systems and applications, vol.7, Issue 5, (2015). 57.

[17] Ali, N., Behdinan, K., & Fawaz, Z. Applicability and viability of a GA based finite element analysis architecture for structural design optimization. Computers & structures, vol.81,Issue 22, (2003), 2259-2271.

Received: November 22, 2016. Accepted: December 28, 2016