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VIKOR Based MAGDM strategy under Bipolar Neutrosophic Set Environment

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Abstract. In this paper, we extend the VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) strategy to multiple attribute group decision-making (MAGDM) with bipolar neutrosophic set environment. In this paper, we first define VIKOR strategy in bipolar neutrosophic set environment to handle MAGDM problems, which means we combine the VIKOR with bipolar neutrosophic number to deal with MAGDM. We propose a new strategy for solving MAGDM. Finally, we solve MAGDM problem using our newly proposed VIKOR strategy under bipolar neutrosophic set environment. Further, we present sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives.

Keywords: Bipolar neutrosophic sets, VIKOR strategy, Multi attribute group decision making.

1 Introduction

In 1965, Zadeh [1] first introduced the fuzzy set to deal with the vague, imprecise data in real life specifying the membership degree of an element. Thereafter, in 1986 Atanassov [2] introduced intuitionistic fuzzy set to tackle the uncertainity in data in real life expressing membership degree and non-membership degree of an element as independent component. As a generalization of classical set, fuzzy set and intuitionistic fuzzy set, Smarandache [3] introduced the neutrosophic set by expressing the membership degree (truth membership degree), indeterminacy degree and non-membership degree (falsity membership degree) of an element independently. For real applications of neutrosophic set, Wang et al. [4] introduced the single valued neutrosophic set which is a sub class of neutrosophic set.

Decision making process involves seleting the best alternative from the set of feasible alternatives. There exist many decision making strategies in crisp set environment[5-7], fuzzy [8-12], intuitionistic fuzzy set environment [13-19]. vauge set environment [20, 21]. Theoretical as well as practical applications multi attribute decision making (MADM) of SVNS environment [22-42] and interval neutrosophic set (INS) environment [43-56] have been reported in the literaure. Recently, decision making in hybrid neutrosophic set environment have drawn much attention of the researches such as rough neutrosophic environment [57-73], neutrosophic soft set environment [74-80], neutrosophic soft expert set environment [81-82], neutrosophic hesitant fuzzy set environment [83-87], neutrosophic refined set environment [88-93], neutrosophic cubic set environment [94-104], etc. In 2015, Deli *et al.* [105] proposed bipolar neutrosophic set (BNS) using the concept of bipolar fuzzy sets [106, 107] and neutrosophic sets [3]. A BNS consists of two fully independent parts, which are positive membership degrees $T^+ \to [0, 1]$, $I^+ \to [0, 1]$, $F^+ \to [0, 1]$, and negative membership degrees $T^- \to [-1, 0]$, $I^- \to [-1, 0]$, $F^- \to [-1, 0]$ where the positive membership degrees T^+, I^+, F^+ represent truth membership degree, indeterminacy membership degree and false membership degree respectively of an element and the negative membership degrees T^-, I^-, F^- represent truth membership degree, indeterminacy membership degree and false membership degree respectively of an element to some implicit counter property corresponding to a BNS. Deli *et al.* [105] defined some operations namely, score function, accuracy function, and certainty function to compare BNSs and provided some operators in order to aggregate BNSs. Deli and Subas [108] defined correlation coefficient similarity measure for dealing with MADM problems under bipolar set environment. Şahin *et al*. [109] proposed Jaccard vector similarity measure for MADM problems under bipolar neutrosophic set environment. Uluçay *et al*. [110] presented Dice similarity measure, weighted Dice similarity measure, hybrid vector similarity measure, weighted hybrid vector similarity measure for BNSs and established a MADM strategy by employing the proposed similarity measures. Dey *et al*. [111] established TOPSIS strategy for MADM problems with bipolar neutrosophic information where the weights of the attributes are completely unknown to the decision maker. Pramanik *et al*. [112] defined projection, bidirectional projection and hybrid projection measures for BNSs and proved their basic properties. In the same study, Pramanik *et al*. [112], proposed three new MADM strategies based on the proposed projection, bidirectional projection and hybrid projection measures with bipoar neutrosophic information. Wang *et al.* [113] defined Frank operations of bipolar neutrosophic numbers (BNNs) and proposed Frank bipolar neutrosophic Choquet Bonferroni mean operators by combining Choquet integral operators and Bonferroni mean operators based on Frank operations of BNNs. In the same study, Wang et al. [113] developed MADM strategy based on Frank Choquet Bonferroni operators of BNNs in bipolar neutrosophic environment. Recently, many researcher has given attention to develop various strategies under bipolar neutrosophic set environment in various fields [114-117].

Opricovic [118] proposed the VIKOR strategy for a MCDM problem with conflicting attributes [119-120]. In 2015, Bausys and Zavadskas [121] proposed VIKOR strategy to solve multi criteria decision making problem in interval neutrosophic set environment. Further, Hung et al. [122] proposed VIKOR strategy for interval neutrosophic multi attribute group decision making (MAGDM). Pouresmaeil *et al.* [123] proposed a MAGDM strategy based on TOPSIS and VIKOR strategies in single valued neutrosophic set environment. Liu and Zhang [124] extended VIKOR strategy in neutrosophic hesitant fuzzy set environment. Hu *et al.* [125] proposed interval neutrosophic projection based VIKOR strategy and applied it for doctor selection. Selvakumari et al. [126] proposed VIKOR strategy for decision making problem using octagonal neutrosophic soft matrix.

VIKOR strategy in bipolar neutrosophic set is yet to appear.

Research gap:

VIKOR based MAGDM strategy in BNS environment. This study answers the following research questions:

i. Is it possible to extend VIKOR strategy in BNS environment?

ii. Is it possible to develop a new VIKOR based MAGDM strategy in BNS environment?

Motivation:

The above-mentioned analysis [118-126] describes the motivation behind proposing a novel VIKOR strategy for MAGDM in the BNS environment. This study develops a novel VIKOR strategy for MAGDM that can deal with multiple decision-makers.

The objectives of the paper are:

i. To extend VIKOR strategy in BNS environment.

ii. To develop a new MAGDM strategy based on proposed VIKOR strategy in BNS environment.

To fill the research gap, we propose VIKOR based strategy, which is capable of dealing with MAGDM problem in BNS environment.

The main contributions of this paper are summarized below:

i. We extend VIKOR strategy in bipolar neutrosophic environment.

ii. We introduce a bipolar neutrosophic weighted aggregation operator and prove its basic properties.

iii. We develop a novel VIKOR based MAGDM strategy in bipolar neutrosophic set environment to solve MAGDM problems.

iv. In this paper, we solve a MAGDM problem based on proposed VIKOR strategy.

The remainder of this paper is organized as follows: In the Section 2, we review some basic concepts and operations related to neutrosophic set, single valued neutrosophic set (SVNS), bipolar neutrosophic set. In Section 3, we propose the bipolar neutrosophic number weighted aggregation (BNNWA) operator and prove its basic properties. In section 4, we develop a novel MAGDM strategy based on VIKOR strategy to solve the MADGM problems with bipolar neutrosophic information. In Section 5, we present an example to illustrate the proposed strategy. Then in Section 6, we present the sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives.. In section 7, we present conclusion and future direction of research.

2. Preliminaries

In this section, we describe the basic definitions related to neutrosophic sets, bipolar neutrosophic sets.

Definition 2.1 Neutrosophic set

Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets [3] A in U is characterized by a truth-membership function $T_A(u)$, an indeterminacy-membership function $I_A(u)$ and a falsity-membership function $F_A(u)$,

where, $T_A(u), I_A(u), F_A(u): U \to]^-0, 1^+[$.

Neutrosophic set A can be written as:

$A = \{u, <T_A(u), I_A(u), F_A(u)>: u \in U\}$,

where, $T_A(u), I_A(u), F_A(u) \in]^-0, 1^+[$.

The sum of $T_A(u), I_A(u), F_A(u)$ is

$^-0 \le T_A(u) + I_A(u) + F_A(u) \le 3^+$.

Definition 2.2: Single valued neutrosophic set

Let U be a space of points (objects) with a generic element in U denoted by u. A single valued neutrosophic set [4] J in U is characterized by a truth-membership function $T_J(u)$, an indeterminacy-membership function $I_J(u)$ and a falsity-membership function $F_J(u)$, where,

$T_J(u), I_J(u), F_J(u): U \to [0, 1]$. A single valued neutrosophic set J can be expressed by

$J = \{u, <(T_J(u), I_J(u), F_J(u))>: u \in U\}$.

Therefore for each $u \in U$, $T_J(u), I_J(u), F_J(u) \in [0, 1]$ the sum of three functions lies between 0 and 1, i.e.

$0 \le T_J(u) + I_J(u) + F_J(u) \le 3$.

Definition 2.3: Bipolar neutrosophic set

Let U be a space of points (objects) with a generic element in U denoted by u. A bipolar neutrosophic set [105] H in U is defined as an object of the form

$H = \{u, <T_H^+(u), I_H^+(u), F_H^+(u), T_H^-(u), T_H^-(u), F_H^-(u)>: u \in U\}$, where,

$T_H^+(u), I_H^+(u), F_H^+(u): U \to [0,1]$ and

$T_H^-(u), I_H^-(u), F_H^-(u): U \to [-1, 0]$.

We denote

$H = \{u, <T_H^+(u), I_H^+(u), F_H^+(u), T_H^-(u), I_H^-(u), F_H^-(u)>: u \in U\}$ simply $H = <T_H^+, I_H^+, F_H^+, T_H^-, I_H^-, F_H^->$ as a bipolar neutrosophic number (BNN).

Definition 2.4 Containment of two bipolar neutrosophic sets [105]

Let

$H_1 = \{u, <T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)>: u \in U\}$

and

$H_2 = \{u, <T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U. Then $H_1 \subseteq H_2$ iff

$T_1^+(u) \le T_2^+(u)$, $I_1^+(u) \ge I_2^+(u)$, $F_1^+(u) \ge F_2^+(u)$ and

$T_1^-(u) \ge T_2^-(u)$, $I_1^-(u) \le I_2^-(u)$, $F_1^-(u) \le F_2^-(u)$ for all $u \in U$.

Definition 2.5 Equality of two bipolar neutrosophic sets [103]

Let

$H_1 = \{u, <T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)>: u \in U\}$

and

$H_2 = \{u, <T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u)>: u \in U\}$

be any two bipolar neutrosophic sets in U. Then, $H_1 = H_2$ iff $T_1^+(u) = T_2^+(u)$, $I_1^+(u) = I_2^+(u)$, $F_1^+(u) = F_2^+(u)$ and $T_1^-(u) = T_2^-(u)$, $I_1^-(u) = I_2^-(u)$, $F_1^-(u) = F_2^-(u)$ for all $u \in U$.

Definition 2.6 Union of any two bipolar neutrosophic sets [105]

Let $H_1 = \{u, <T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)>: u \in U\}$ and $H_2 = \{u, <T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U. Then, their union is defined as follows:

$$H_3(u) = H_1(u) \cup H_2(u) = \{u, <\max(T_1^+(u), T_2^+(u)),$$
$$\min(I_1^+(u), I_2^+(u)), \min(F_1^+(u), F_2^+(u)),$$
$$\min(T_1^-(u), T_2^-(u)), \max(I_1^-(u), I_2^-(u)),$$
$$\max(F_1^-(u), F_2^-(u))>: u \in U\}, \text{for all } u \in U.$$

Definition 2.7 Intersection of two bipolar neutrosophic sets

Let $H_1 = \{u, <T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)>: u \in U\}$ and $H_2 = \{u, <T_2^+(u), I_2^+(u), F_2^+(u), T_2^-(u), I_2^-(u), F_2^-(u)>: u \in U\}$ be any two bipolar neutrosophic sets in U. Then, their intersection [105] is defined as follows:

$$H_4(u) = H_1(u) \cap H_2(u) = \{u, <\min(T_1^+(u), T_2^+(u)),$$
$$\max(I_1^+(u), I_2^+(u)), \max(F_1^+(u), F_2^+(u)),$$
$$\max(T_1^-(u), T_2^-(u)), \min(I_1^-(u), I_2^-(u)),$$
$$\min(F_1^-(u), F_2^-(u))>: u \in U\} \text{ for all } u \in U.$$

Definition 2.8 Complement of a bipolar neutrosophic set [105]

Let $H_1 = \{u, <T_1^+(u), I_1^+(u), F_1^+(u), T_1^-(u), I_1^-(u), F_1^-(u)>: u \in U\}$ be a bipolar neutrosophic set in U. Then the complement of H_1 is denoted by H_1^c and is defined by

$$H_1^c = \{u, <1 - T_1^+(u), 1 - I_1^+(u), 1 - F_1^+(u), \{-1\} - T_1^-(u),$$
$$\{-1\} - I_1^-(u), \{-1\} - F_1^-(u)>: u \in U\}$$

for all $u \in U$.

Definition 2.13 Hamming distance measure between two BNNs [115]

Let $h_1 = <T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^->$ and $h_2 = <T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^->$ be any two BNNs in U. Then Hamming distance measure between h_1 and h_2 is denoted by $D(h_1, h_2)$ and defined as follows:

$$D(h_1,h_2) = \frac{1}{6}[|T_1^+ - T_2^+| + |I_1^+ - I_2^+| + |F_1^+ - F_2^+| + |T_1^- - T_2^-| + |I_1^- - I_2^-| + |F_1^- - F_2^-|] \quad (1)$$

Definition 2.14: Normalization procedure

In decision making situation, cost type attribute and benefit type attribute may exist simultaneously. Assume that, h_{ij} be a BNN to express the rating value of i-th alternative with respect to j-th attribute (c_j). If c_j belongs to the cost type attributes, then h_{ij} should be standardized by employing the complement of BNN h_{ij}. When the attribute c_j belongs to benefit type attributes, h_{ij} does not need to be standardized, we use the following formula of normalization as follows:

$$h_{ij}^* = <\{1\} - T_{ij}^+, \{1\} - I_{ij}^+, \{1\} - F_{ij}^+, \{-1\} - T_{ij}^-, \{-1\} - I_{ij}^-, \{-1\} - F_{ij}^- > \quad (2)$$

3. Bipolar neutrosophic number weighted aggregation operator

Let $\{h_{ij}^1, h_{ij}^2, ..., h_{ij}^t\}$ be the set of t bipolar neutrosophic numbers and $\{\beta_1, \beta_2, \beta_3, ..., \beta_t\}$ be the set of corresponding weights of t bipolar neutrosophic numbers with conditions $\beta_p \geq 0$ and $\sum_{p=1}^{t} \beta_p = 1$. Then the bipolar neutrosophic number weighted aggregation (BNNWA) operator is defined as follows:

$$h_{ij} = BNNWA_\beta(\tilde{h}_{ij}^1 \ \tilde{h}_{ij}^2 ... \ \tilde{h}_{ij}^t) =$$
$$(\beta_1 \tilde{h}_{ij}^1 \oplus \beta_2 \tilde{h}_{ij}^2 \oplus \beta_3 \tilde{h}_{ij}^3 \oplus ... \oplus \beta_p \tilde{h}_{ij}^t) =$$
$$< \left(\sum_{p=1}^{t} \beta_p \tilde{T}_{ij}^{+(p)}, \sum_{p=1}^{t} \beta_p \tilde{I}_{ij}^{+(p)}, \sum_{p=1}^{t} \beta_p \tilde{F}_{ij}^{+(p)}, \sum_{p=1}^{t} \beta_p \tilde{T}_{ij}^{-(p)}, \sum_{p=1}^{t} \beta_p \tilde{I}_{ij}^{-(p)}, \sum_{p=1}^{t} \beta_p \tilde{F}_{ij}^{-(p)} \right) > \quad (3)$$

The BNNWA operator satisfies the following properties:
1. Idempotency
2. Monotoncity
3. Boundedness

Property: 1. Idempotency

If all $h_{ij}^1, \ h_{ij}^2, ... \ , h_{ij}^t = h$ are equal, then

$$h_{ij} = BNNWA_\beta(h_{ij}^1, \ h_{ij}^2, ... \ , h_{ij}^t) = h$$

Proof:

Since $h_{ij}^1 = h_{ij}^2 = ... \ = h_{ij}^t = h$, based on the Equation (3) and with conditions, $\beta_p \geq 0$ and $\sum_{p=1}^{t} \beta_p = 1$, we obtain

$$h_{ij} = BNNWA_\beta(h_{ij}^1, h_{ij}^2, ... \ , h_{ij}^t) =$$
$$(\beta_1 h_{ij}^1 \oplus \beta_2 h_{ij}^2 \oplus \beta_3 h_{ij}^3 \oplus ... \oplus \beta_t h_{ij}^t) =$$
$$(\beta_1 h \oplus \beta_2 h \oplus \beta_3 h \oplus ... \oplus \beta_t h) =$$
$$< \left([T^+ \sum_{p=1}^{t} \beta_p, I^+ \sum_{p=1}^{t} \beta_p, F^+ \sum_{p=1}^{t} \beta_p, T^- \sum_{p=1}^{t} \beta_p, I^- \sum_{p=1}^{t} \beta_p, F^- \sum_{p=1}^{t} \beta_p] \right) >$$
$$= < (T^+, I^+, F^+, T^-, I^-, F^-) > = h.$$

Property: 3. Monotonicity

Assume that $\{h_{ij}^1, h_{ij}^2, .. \ , h_{ij}^t\}$ and $\{h_{ij}^{*1}, h_{ij}^{*2}, .. \ , h_{ij}^{*t}\}$ be any two set of collections of t bipolar neutrosophic nubers with the condition $t_{ij}^p \leq t_{ij}^{*p}$ (p = 1, 2, ..., t), then

$$BNNWA_\beta(h_{ij}^1, h_{ij}^2, ..., h_{ij}^t) \leq BNNWA_\beta(h_{ij}^{*1}, h_{ij}^{*2}, ..., h_{ij}^{*t}).$$

Proof:

From the given condition $T_{ij}^{+(p)} \leq T_{ij}^{+*(p)}$, we have

$$\beta_p T_{ij}^{+(p)} \leq \beta_p T_{ij}^{+*(p)}$$
$$\Rightarrow \sum_{p=1}^{t} \beta_p T_{ij}^{+(p)} \leq \sum_{p=1}^{t} \beta_p T_{ij}^{+*(p)}.$$

From the given condition $I_{ij}^{+(p)} \geq I_{ij}^{+*(p)}$, we have

$$\beta_p I_{ij}^{+(p)} \geq \beta_p I_{ij}^{+*(p)}$$
$$\Rightarrow \sum_{p=1}^{t} \beta_p I_{ij}^{+(p)} \geq \sum_{p=1}^{t} \beta_p I_{ij}^{+*(p)}.$$

From the given condition $F_{ij}^{+(p)} \geq F_{ij}^{+*(p)}$, we have

$$\beta_p F_{ij}^{+(p)} \geq \beta_p F_{ij}^{+*(p)}$$
$$\Rightarrow \sum_{p=1}^{t} \beta_p F_{ij}^{+(p)} \geq \sum_{p=1}^{t} \beta_p F_{ij}^{+*(p)}.$$

From the given condition $T_{ij}^{-(p)} \geq T_{ij}^{-*(p)}$, we have

$$\beta_p T_{ij}^{-(p)} \geq \beta_p T_{ij}^{-*(p)}$$
$$\Rightarrow \sum_{p=1}^{t} \beta_p T_{ij}^{-(p)} \geq \sum_{p=1}^{t} \beta_p T_{ij}^{-*(p)}.$$

From the given condition $I_{ij}^{-(p)} \leq I_{ij}^{-*(p)}$, we have

$$\beta_p I_{ij}^{-(p)} \leq \beta_p I_{ij}^{-*(p)}$$
$$\Rightarrow \sum_{p=1}^{t} \beta_p I_{ij}^{-(p)} \leq \sum_{p=1}^{t} \beta_p I_{ij}^{-*(p)}.$$

From the given condition $F_{ij}^{-(p)} \leq F_{ij}^{-*(p)}$, we have

$$\beta_p F_{ij}^{-(p)} \leq \beta_p F_{ij}^{-*(p)}$$
$$\Rightarrow \sum_{p=1}^{t} \beta_p F_{ij}^{-(p)} \leq \sum_{p=1}^{t} \beta_p F_{ij}^{-*(p)}.$$

From the above relations, we obtain

$$BNNWA_\beta(h_{ij}^1, h_{ij}^2, ..., h_{ij}^t) \leq BNNWA_\beta(h_{ij}^{*1}, h_{ij}^{*2}, ..., h_{ij}^{*t}).$$

$\mathrm{NCNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le \mathrm{BNNWA}_{\beta}(h_{ij}^{*1}, h_{ij}^{*2}, ..., h_{ij}^{*t})$.

Property: 2. Boundedness

Let $\{h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}\}$ be any collection of t bipolar neutrosophic numbers.

If

$h^{+} = <\max_{p}\{T_{ij}^{+(p)}\}, \min_{p}\{I_{ij}^{+(p)}\}, \min_{p}\{F_{ij}^{+(p)}\}, \min_{p}\{T_{ij}^{-(p)}\},$

$\max_{p}\{I_{ij}^{-(p)}\}, \max_{p}\{F_{ij}^{-(p)}\}>$

$h^{-} = <\min_{p}\{T_{ij}^{+(p)}\}, \max_{p}\{I_{ij}^{+(p)}\}, \max_{p}\{F_{ij}^{+(p)}\}, \max_{p}\{T_{ij}^{-(p)}\},$

$\min_{p}\{I_{ij}^{-(p)}\}, \min_{p}\{F_{ij}^{-(p)}\}>$ (p= 1, 2, 3, ...,t).

Then, $h^{-} \le \mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le h^{+}$.

Proof:

From Property 1 and Property 2, we obtain

$\mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \ge \mathrm{BNNWA}_{\beta}(h^{-}, h^{-}, ..., h^{-}) = h^{-}$

and

$\mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le \mathrm{BNNWA}_{\beta}(h^{+}, h^{+}, ..., h^{+}) = h^{+}$.

So, we have

$h^{-} \le \mathrm{BNNWA}_{\beta}(h_{ij}^{1}, h_{ij}^{2}, ..., h_{ij}^{t}) \le h^{+}$.

4. VIKOR strategy for solving MAGDM problem under bipolar neutrosophic environment

In this section, we propose a MAGDM strategy under bipolar neutrosophic set environment. Assume that, $A = \{A_1, A_2, A_3, ..., A_r\}$ be a set of r alternatives and $C = \{c_1, c_2, c_3, ..., c_s\}$ be a set of s attributes. Assume that, $\alpha = \{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_s\}$ be the weight vector of the attributes, where $\alpha_k \ge 0$ and $\sum_{k=1}^{s}\alpha_k = 1$. Let $DM = \{DM_1, DM_2, DM_3, ..., DM_t\}$ be the set of t decision makers and $\beta = \{\beta_1, \beta_2, \beta_3, ..., \beta_t\}$ be the set of weight vector of decision makers, where $\beta_p \ge 0$ and $\sum_{p=1}^{t}\beta_p = 1$.

In this section, we describe the VIKOR based MAGDM strategy under bipolar neutrosophic set environment. The proposed strategy consists of the following steps (see Figure 1):

Step: 1. Construction of the decision matrix

Let $M^{p} = (h_{ij}^{p})_{r\times s}$ (p = 1, 2, 3, …, t) be the p-th decision matrix, where information about the alternative A_i is provided by the decision maker DM_p with respect to attribute c_j (j = 1, 2, 3, …, s). The p-th decision matrix denoted by M^p (See eq. (4)) is constructed as follows:

$$M^{p} = \begin{pmatrix} & c_1 & c_2 & \ldots & c_s \\ A_1 & h_{11}^{p} & h_{12}^{p} & \ldots & h_{1s}^{p} \\ A_2 & h_{21}^{p} & h_{22}^{p} & & h_{2s}^{p} \\ . & . & & \ldots & . \\ A_r & h_{r1}^{p} & h_{r2}^{p} & \ldots & h_{rs}^{p} \end{pmatrix} \quad (4)$$

Here p = 1, 2, 3,…, t; i = 1, 2, 3,…, r; j = 1, 2, 3,…, s.

Step: 2. Normalization of the decision matrix

Cost type attributes and benefit type attributes are generally existed in decision making process. Therefore the considered attribute values need to be normalized to aviod different physical dimensional unit. To normalize we can use the following equation:

$h_{ij}^{*} = <\{1\} - T_{ij}^{+}, \{1\} - I_{ij}^{+}, \{1\} - F_{ij}^{+},$

$\{-1\} - T_{ij}^{-}, \{-1\} - I_{ij}^{-}, \{-1\} - F_{ij}^{-}>$.

Using the normalized method, we obtain the following normalized decision matrix (See eq. (5)):

$$M^{p} = \begin{pmatrix} & c_1 & c_2 & \ldots & c_s \\ A_1 & \tilde{h}_{11}^{p} & \tilde{h}_{12}^{p} & \ldots & \tilde{h}_{1s}^{p} \\ A_2 & \tilde{h}_{21}^{p} & \tilde{h}_{22}^{p} & & \tilde{h}_{2s}^{p} \\ . & . & & \ldots & . \\ A_r & \tilde{h}_{r1}^{p} & \tilde{h}_{r2}^{p} & \ldots & \tilde{h}_{rs}^{p} \end{pmatrix} \quad (5)$$

Where,

$$\tilde{h}_{ij}^{p} = \begin{cases} h_{ij}^{p} & \text{if } c_j \text{ is benefit type attribute.} \\ (h_{ij}^{*})^{p} & \text{if } c_j \text{ is cost type attribute.} \end{cases}$$

Step: 3. Aggregation of the decision matrices

Using BNNWA operator in eq. (3), we obtain the aggregated decision matrix as follows:

$$M = \begin{pmatrix} & c_1 & c_2 & \ldots & c_s \\ A_1 & h_{11} & h_{12} & \ldots & h_{1s} \\ A_2 & h_{21} & h_{22} & & h_{2s} \\ . & . & & \ldots & . \\ A_r & h_{r1} & h_{r2} & \ldots & h_{rs} \end{pmatrix} \quad (6)$$

where, *i* = 1, 2, 3, …, r; *j* = 1, 2, 3, …, s; *p=1, 2, ...t.*

Step: 4. Define the positive ideal solution and negative ideal solution

$$h_{ij}^{+} = <\max_{i} T_{ij}^{+}, \min_{i} I_{ij}^{+}, \min_{i} F_{ij}^{+}, \min_{i} T_{ij}^{-}, \max_{i} I_{ij}^{-}, \max_{i} F_{ij}^{-}> \quad (7)$$

$$h_{ij}^{-} = <\min_{i} T_{ij}^{+}, \max_{i} I_{ij}^{+}, \max_{i} F_{ij}^{+}, \max_{i} T_{ij}^{-}, \min_{i} I_{ij}^{-}, \min_{i} F_{ij}^{-}>$$

(8)

Step: 5. Define and compute the value of Γ_i and Z_i (i = 1, 2, 3, ..., r)

Γ_i and Z_i represent the average and worst group scores for the alternative A_i respectively, with the relations

$$\Gamma_i = \sum_{j=1}^{s} \frac{\alpha_j \times D(h_{ij}^{+}, \tilde{h}_{ij})}{D(h_{ij}^{+}, h_{ij}^{-})} \tag{9}$$

$$Z_i = \max_j \left\{ \frac{\alpha_j \times D(h_{ij}^{+}, \tilde{h}_{ij})}{D(h_{ij}^{+}, h_{ij}^{-})} \right\} \tag{10}$$

Here, α_j is the weight of c_j.

The smaller values of Γ_i and Z_i correspond to the better average and worse group scores for alternative A_i, respectively.

Step: 6. Calculate the values of index VIKOR ϕ_i (i = 1, 2, 3, …, r) by the relation

$$\phi_i = \gamma \frac{(\Gamma_i - \Gamma^-)}{(\Gamma^+ - \Gamma^-)} + (1-\gamma) \frac{(Z_i - Z^-)}{(Z^+ - Z^-)} \tag{11}$$

Here, $\Gamma_i^- = \min_i \Gamma_i$, $\Gamma_i^+ = \max_i \Gamma_i$,
$Z_i^- = \min_i Z_i$, $Z_i^+ = \max_i Z_i$ (12)

and γ depicts the decision making mechanism coefficient. If $\gamma > 0.5$, it is for "the maximum group utility"; if $\gamma < 0.5$, it is " the minimum regret"; it has been inferred that the decision making mechanism coefficient is mostly taken as $v = 0.5$.

Step: 7. Rank the priority of alternatives

We rank the alternatives by ϕ_i, Γ_i, and Z_i according to the rule of traditional VIKOR strategy. The smaller value indicates the better alternative.

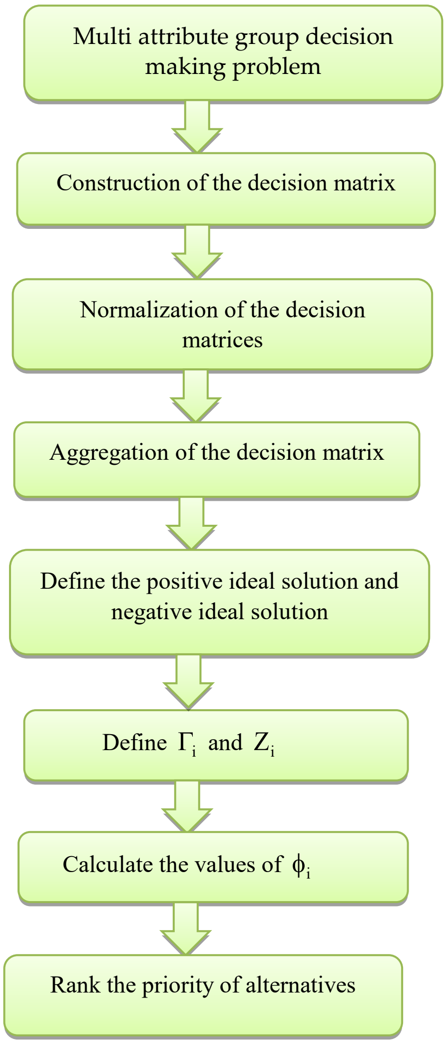

Figure 1. Decision making procedure of proposed MAGDM strategy.

5. Illustrative example

To demonstrate the applicability and fesibility of the proposed strategy, we solve a MAGDM problem adapted from [45]. We assume that an investment company wants to invest a sum of money in the best option. The investment company forms a decision making board involving of three members (DM_1, DM_2, DM_3) who evaluate the four alternatives to invest money. The alternatives are Car company (A_1), Food company (A_2), Computer company (A_3) and Arm company (A_4). Decision makers take decision to evaluate alternatives based on the criteria namely, risk factor (c_1), growth factor (c_2), environment impact (c_3). We consider three criteria as benefit type based on Zhang et al. [127]. Assume that the weight vector of attributes is

$\alpha=(0.37,0.33,0.3)^T$ and weight vector of decision makers is $\beta=(0.38,0.32,0.3)^T$. Now, we apply the proposed MAGDM strategy to solve the problem using the following steps.

Step: 1. Construction of the decision matrix

We construct the decision matrix information provided by the decision makers in terms of BNNs with respect to the criteria as follows:

Decision matrix for DM_1

$$M^1 = \begin{array}{cccc} & c_1 & C_2 & C_3 \\ A_1 & (.5,.6,.7,-.3,-.6,-.3) & (.8,.5,.6,-.4,-.6,-.3) & (.9,.4,.6,-.1,-.6,-.5) \\ A_2 & (.6,.2,.2,-.4,-.5,-.3) & (.6,.3,.7,-.4,-.3,-.5) & (.7,.5,.3,-.4,-.3,-.3) \\ A_3 & (.8,.3,.5,-.6,-.4,-.5) & (.5,.2,.4,-.1,-.5,-.3) & (.4,.2,.8,-.5,-.3,-.2) \\ A_4 & (.7,.5,.3,-.6,-.3,-.3) & (.8,.7,.2,-.8,-.6,-.1) & (.6,.3,.4,-.3,-.4,-.7) \end{array}$$

Decision matrix for DM_2

$$M^2 = \begin{array}{cccc} & c_1 & C_2 & C_3 \\ A_1 & (.6,.3,.4,-.5,-.3,-.7) & (.5,.3,.4,-.3,-.3,-.4) & (.1,.5,.7,-.5,-.2,-.6) \\ A_2 & (.7,.4,.5,-.3,-.2,-.1) & (.8,.4,.5,-.7,-.3,-.2) & (.6,.2,.7,-.5,-.2,-.9) \\ A_3 & (.8,.3,.2,-.5,-.2,-.6) & (.3,.2,.1,-.6,-.3,-.4) & (.7,.5,.4,-.4,-.3,-.2) \\ A_4 & (.3,.5,.2,-.5,-.5,-.2) & (.5,.6,.4,-.3,-.6,-.7) & (.4,.3,.8,-.5,-.6,-.5) \end{array}$$

Decision matrix for DM_3

$$M^3 = \begin{array}{cccc} & c_1 & C_2 & C_3 \\ A_1 & (.9,.6,.4,-.7,-.3,-.2) & (.7,.5,.3,-.6,-.2,-.5) & (.4,.2,.3,-.2,-.5,-.7) \\ A_2 & (.5,.3,.2,-.6,-.4,-.1) & (.5,.2,.7,-.3,-.2,-.5) & (.6,.3,.2,-.7,-.6,-.3) \\ A_3 & (.2,.5,.6,-.4,-.5,-.7) & (.3,.2,.7,-.2,-.3,-.5) & (.8,.2,.4,-.2,-.3,-.6) \\ A_4 & (.8,.5,.5,-.4,-.6,-.3) & (.9,.3,.4,-.5,-.6,-.7) & (.7,.4,.3,-.2,-.5,-.7) \end{array}$$

Step: 2. Normalization of the decision matrix

Since all the criteria are considered as benefit type, we do not need to normalize the decision matrices (M^1, M^2, M^3).

Step: 3. Aggregated decision matrix

Using eq. (3), the aggregated decision matrix is presented as follows:

$$M = \begin{array}{cccc} & c_1 & C_2 & C_3 \\ A_1 & (.22,.17,.17,-.16,-.14,-.13) & (.22,.14,.15,-.14,-.13,-.13) & (.16,.12,.18,-.10,-.10,-.20) \\ A_2 & (.20,.10,.10,-.14,-.12,-.10) & (.21,.10,.21,-.15,-.10,-.13) & (.21,.11,.13,-.17,-.12,-.16) \\ A_3 & (.21,.12,.16,-.17,-.12,-.20) & (.13,.10,.13,-.10,-.12,-.13) & (.21,.10,.18,-.13,-.10,-.11) \\ A_4 & (.20,.17,.11,-.17,-.15,-.10) & (.24,.18,.11,-.19,-.20,-.16) & (.19,.11,.17,-.11,-.16,-.21) \end{array}$$

Step: 4. Define the positive ideal solution and negative ideal solution

The positive ideal solution h_{ij}^{+} =

$$\begin{array}{ccc} c_1 & C_2 & C_3 \\ (.22,.10,.10,-.14,-.12,-.10) & (.24,.10,.11,-.19,-.10,-.13) & (.21,.10,.13,-.17,-.10,-.11) \end{array}$$

and the negative ideal solution

h_{ij}^{-} =

$$\begin{array}{ccc} c_1 & C_2 & C_3 \\ (.20,.17,.17,-.14,-.15,-.20) & (.13,.18,.21,-.10,-.20,-.16) & (.16,.12,.18,-.10,-.16,-.11) \end{array}$$

Step: 5. Compute Γ_i and Z_i

We have computed the values of Γ_i by eq. (9) and the values of Z_i by eq. (10), the values are presented as follows:

$\Gamma_1 = 0.75$, $\Gamma_2 = 0.38$, $\Gamma_3 = 0.60$, $\Gamma_4 = 0.75$ and $Z_1 = 0.34$, $Z_2 = 0.16$, $Z_3 = 0.33$, $Z_4 = 0.34$

Step: 6. Calculate the values of ϕ_i

Using $\gamma=0.5$, and eq. (11) and eq. (12), we obtain

$\phi_1 = 1$, $\phi_2 = \mathbf{0}$, $\phi_3 = 0.77$, $\phi_4 = 1$

Step: 7. Rank the priority of alternatives

The preference order of the alternatives based on the traditional rules of the VIKOR strategy is $A_2 \succ A_3 \succ A_4 \approx A_1$.

6. The influence of parameter γ

In this section, we present sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives Figure 2 represents the graphical representation of alternatives (A_i) versus (i = 1, 2, 3, 4) for different values of γ.

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Table 1 shows that the ranking order of alternatives (A_i) with the value of γ changing from 0.1 to 0.9.

Values of γ	Values of ϕ_i	Preference order of alternatives

$\gamma = 0.1$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.915,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.2$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.880,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.3$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.845,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.4$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.810,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.5$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.770,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.6$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.740,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.7$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.700,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.8$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.670,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.
$\gamma = 0.9$	$\phi_1 = 1,\ \phi_2 = \mathbf{0},\ \phi_3 = 0.640,\ \phi_4 = 1$	$A_2 \succ A_3 \succ A_4 = A_1$.

Table 1. Values of $\phi_i\ (i = 1, 2, 3, 4)$ and ranking of alternatives for different values of $\gamma\,\gamma$.

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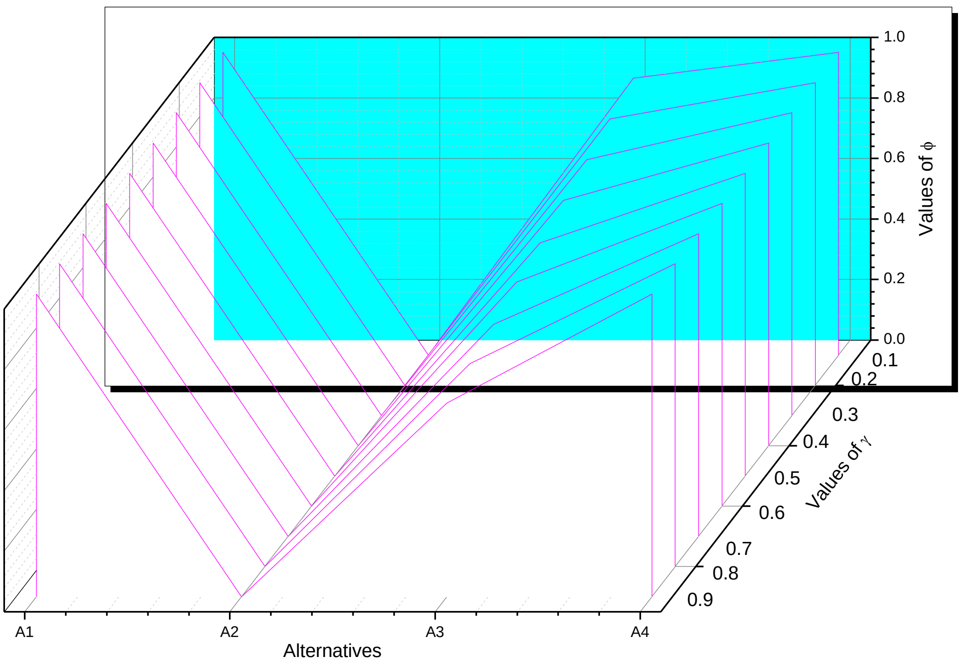

Fig 2. Graphical representation of ranking order of alternatives for different values of γ .

7. Conclusion

In this paper, we have extended the VIKOR strategy to MAGDM with bipolar neutrosophic environment. We have introduced bipolar neutrosophic numbers weighted aggregation operator and applied it to aggregate the individual opinion to one group opinion. We have developed a VIKOR based MAGDM strategy with bipolar neutrosophic set. Finally, we have solved a MAGDM problem to show the feasibility and efficiency of the proposed MAGDM strategy. We have presented a sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives. The proposed VIKOR based MAGDM strategy can be employed to solve a variety of problems such as logistics center selection [128], teacher selection [19, 129], renewable energy selection [131], fault diagnosis [132], weaver selection [14, 54], brick selection [13], school choice [130] etc.

References

[1] Zadeh, L.A. Fuzzy sets. Information Control, 8 (1965), 338–356.

[2] K. T. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1986), 87-96.

[3] F. Smarandache. A unifying field in logics: Neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability, and neutrosophic statistics, American Research Press, Rehoboth, 1998.

[4] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multi-space and Multi-structure, 4 (2010), 410-413.

[5] E. Triantaphyllou. Multi-criteria decision making methods. In *Multi-criteria decision making methods: A comparative study* (5-21). Springer US (2000).

[6] S. Opricovic, G. H. Tzeng. Extended VIKOR method in comparison with outranking methods. European journal of operational research, 178 (2) (2007), 514-529.

[7] S. Opricovic, G. H. Tzeng. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. European journal of operational research,156 (2004), 445–455.

[8] E. Cevikcan, S. Çebi, I. Kaya. Fuzzy VIKOR and Fuzzy Axiomatic Design Versus to Fuzzy Topsis: An Application of Candidate Assessment. Multi Valued Logic and Soft Computing, 15 (2009), 181–208.

[9] S. J. Chen, and C.L. Hwang. Fuzzy multiple attribute decision making. Springer, Berlin, Heidelberg 1992.

[10] Y. J. Wang, and H. S. Lee. Generalizing TOPSIS for fuzzy multiple-criteria group decision-making. Computers and Mathematics with Applications, 53 (2007), 1762–1772.

[11] S. Pramanik, and K. Mondal. Weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. International Journal of Innovative Research in Science, Engineering and Technology, 4 (2) (2015), 158-164.

[12] T. Y. Chen. An ELECTRE-based outranking method for multiple criteria group decision making using interval type-2 fuzzy sets. Information Sciences, 263 (2014), 1–21.

[13] K. Mondal, and S. Pramanik. Intuitionistic fuzzy multi criteria group decision making approach to quality-brick selection problem. Journal of Applied Quantitative Methods 9 (2) (2014), 35–50.

[14] P. P. Dey, S. Pramanik, and B. C. Giri. Multi-criteria group decision making in intuitionistic fuzzy environment based on grey relational analysis for weaver selection in Khadi institution. Journal of Applied and Quantitative Methods, 10 (4) (2015), 1-14.

[15] K. Mondal, and S. Pramanik. Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-attribute decision making. Global Journal of Advanced Research, 2 (2) (2015), 464-471.

[16] J. Ren. Sustainability prioritization of energy storage technologies for promoting the development of renewable energy: A novel intuitionistic fuzzy combinative distance-based assessment approach. Renewable Energy, 121, (2018), 666-676.

[17] F. E. Boran, S. Genç, M. Kurt, and D. Akay. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Systems with Applications, 36 (2009), 11363–11368.

[18] X. Zhao. TOPSIS method for interval-valued intuitionistic fuzzy multiple attribute decision making and its application to teaching quality evaluation. Journal of Intelligent and Fuzzy Systems, 26 (2014), 3049–3055.

[19] S. Pramanik, and D. Mukhopadhyaya. Grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. International Journal of Computer Applications, 34(10) (2011), 21-29.

[20] S. M. Chen, and J. M. Tan. Handling multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy sets and systems*, *67*(2) (1994), 163-172.

[21] D. H. Hong, and C. H. Choi. Multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy sets and systems*, *114*(1) (2000), 103-113.

[22] P. Biswas, S. Pramanik, and B. C. Giri. Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2(2014), 102–110.

[23] P. Biswas, S. Pramanik, and B. C. Giri. A new methodology for neutrosophic multi-attribute decision making with unknown weight information. Neutrosophic Sets and Systems, 3 (2014), 42–52.

[24] P. Biswas, S. Pramanik, and B. C. Giri. TOPSIS method for multi-attribute group decision-making under single valued neutrosophic environment. Neural Computing and Applications, (2015), doi: 10.1007/s00521-015-1891-2.

[25] P. Biswas, S. Pramanik, and B. C. Giri. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12 (2016), 20-40.

[26] P. Biswas, S. Pramanik, and B. C. Giri. Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12 (2016), 127-138.

[27] P. Biswas, S. Pramanik, and B. C. Giri. Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. New Trends in Neutrosophic Theory and Applications-Vol-II. Pons Editions, Brussells (2017). In Press.

[28] P. Biswas, S. Pramanik, and B. C. Giri. Non-linear programming approach for single-valued neutrosophic TOPSIS method. New Mathematics and Natural Computation, (2017). In Press.

[29] I. Deli, and Y. Subas. A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. International Journal of Machine Learning and Cybernetics, (2016), doi:10.1007/s13042016-0505-3.

[30] P. Ji, J. Q. Wang, and H. Y. Zhang. Frank prioritized Bonferroni mean operator with single-valued neutrosophic sets and its application in selecting third-party logistics providers. Neural Computing and Applications, (2016). doi:10.1007/s00521-016-2660-6.

[31] A. Kharal. A neutrosophic multi-criteria decision making method. New Mathematics and Natural Computation, 10 (2014), 143–162.

[32] R. X. Liang, J. Q. Wang, and L. Li. Multi-criteria group decision making method based on interdependent inputs of single valued trapezoidal neutrosophic information. Neural Computing and Applications, (2016), doi:10.1007/s00521-016-2672-2.

[33] R. X. Liang, J. Q. Wang, and H. Y. Zhang. A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information. Neural Computing and Applications, (2017). Doi: 10.1007/s00521-017-2925-8.

[34] P. Liu, and Y. Wang. Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Computing and Applications, 25 (7) (2014), 2001–2010.

[35] R. Sahin, and A. Kucuk. Subsethood measure for single valued neutrosophic sets. Journal of Intelligent and Fuzzy System, (2014), doi:10.3233/IFS-141304.

[36] J. Ye. Multi criteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems,42 (2013a), 386–394.

[37] J. Ye. Single valued neutrosophic cross-entropy for multi criteria decision making problems. Applied Mathematical Modelling, 38 (3) (2013), 1170–1175.

[38] K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62-68.

[39] J. Ye. Single valued neutrosophic cross-entropy for multi criteria decision making problems. Applied Mathematical Modelling, 38 (3) (2014), 1170 –1175.

[40] J. Ye. Improved cross entropy measures of single valued neutrosophic sets and interval neutrosophic sets and their multi criteria decision making methods. Cybernetics and Information Technologies, 15 (2015), 13-26. doi: 10.1515/cait-2015-0051.

[41] S. Pramanik, S. Dalapati, S. Alam, F. Smarandache, and T. K. Roy. NS-cross entropy-based MAGDM under single-valued neutrosophic set environment. *Information*, 9 (2) (2018), 37.

[42] J. Ye. Projection and bidirectional projection measures of single valued neutrosophic sets and their decision making method for mechanical design scheme. Journal of Experimental and Theoretical Artificial Intelligence, (2016), doi:10.1080/0952813X.2016.1259263.

[43] R. Sahin, and M. Karabacak. A multi attribute decision making method based on inclusion measure for interval neutrosophic sets**.** International Journal of Engineering and Applied Sciences**,** 2(2) (2014), 13–15.

[44] H.Y. Zhang, P. Ji, J. Wang, and X.H. Chen. An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision-making problems. International Journal of Computational Intelligence Systems, 8 (2015), 1027–1043.

[45] H.Y. Zhang, J. Q. Wang, and X.H. Chen. An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. Neural Computing and Applications, 17 (2016), 615–627.

[46] Y. Huang, G. Wei, and C. Wei. VIKOR method for interval neutrosophic multiple attribute group decision-making. Information 8 (2017)**,** 144; doi:10.3390/info8040144.

[47] J. Ye. Similarity measures between interval neutrosophic sets and their applications in multi criteria decision-making. Journal of Intelligent and Fuzzy Systems, 26 (2014), 165–172.

[48] S. Pramanik, and K. Mondal. Interval neutrosophic multi-attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 9 (2015), 13-22.

[49] W. Zhao, J. G. Du, and H. J. Guan. Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator. Journal of Intelligent and Fuzzy Systems, 29 (2015), 2697–2706.

[50] H.Y. Zhang, J.Q. Wang, and X. H. Chen. Interval neutrosophic sets and their application in multi-criteria decision making problems. The Scientific World Journal, 2014. http://dx.doi.org/10.1155/2014/645953.

[51] R. Sahin. Cross-entropy measure on interval neutrosophic sets and its applications in multi criteria decision making. Neural Computing and Applications, 2015, doi 10.1007/s00521-015-2131-5.

[52] Z. P. Tian, H. Y. Zhang, J. Wang, J. Q. Wang, and X. H. Chen. Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets. International Journal of Systems Science, 2015. doi:10.1080/00207721.2015.1102359.

[53] S. Dalapati, S. Pramanik, S. Alam, F. Smarandache, and T. K. Roy. IN-cross entropy based MAGDM strategy under interval neutrosophic set environment. Neutrosophic Sets and Systems, 18 (2017), 43-57.

[54] S. Pramanik, P. Biswas, and B. C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, 28 (2017), 1163–1176. doi:10.1007/s00521-015-2125-3.

[55] P. P. Dey, S. Pramanik, and B. C. Giri. Extended projection-based models for solving multiple attribute decision making problems with interval valued neutrosophic information. New Trends in Neutrosophic Theories and Applications. Pons-Editions, Brussels, 2016, 127-140.

[56] P. P. Dey, S. Pramanik, and B. C. An extended grey relational analysis based interval neutrosophic multi-attribute decision making for weaver selection. Journal of New Theory, 9 (2015), 82-93.

[57] K. Mondal, and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on grey relational analysis. Neutrosophic Sets and Systems, 7 (2015), 8-17.

[58] S. Pramanik, and K. Mondal. Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Global Journal of Advanced Research, 2 (1) (2015), 212-220.

[59] S. Pramanik, and K. Mondal. Some rough neutrosophic similarity measure and their application to multi attribute decision making. Global Journal of Engineering Science and Research Management, 2 (7) (2015), 61-74.

[60] K. Mondal, and S. Pramanik. Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. Neutrosophic Sets and Systems, 8 (2015), 16-22.

[61] S. Pramanik, and K. Mondal. Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Journal of New Theory, 4 (2015), 90-102.

[62] K. Mondal, S. Pramanik, and F. Smarandache. Several trigonometric hamming similarity measures of rough neutrosophic sets and their applications in decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 93-103). Brussels: Pons Editions, 2016.

[63] S. Pramanik, R Roy, T. K. Roy, and F. Smarandache. Multi criteria decision making using correlation coefficient under rough neutrosophic environment. Neutrosophic Sets and Systems, 17 (2017), 29-38. http://doi.org/10.5281/zenodo.1012237.

[64] S. Pramanik, R. Roy, and T. K. Roy. Multi criteria decision making based on projection and bidirectional projection measures of rough neutrosophic sets. In F. Smarandache, & S. Pramanik (Eds.), New Trends in Neutrosophic Theory and Applications, Vol. II. Brussels: Pons Editions. In Press.

[65] K. Mondal, S. Pramanik, and F. Smarandache. Multi attribute decision making based on rough neutrosophic variational coefficient similarity measure. Neutrosophic Sets and Systems, 13 (2016), 3-17.

[66] K. Mondal, S. Pramanik, and F. Smarandache. Rough neutrosophic TOPSIS for multi-attribute group decision making. Neutrosophic Sets and Systems,13 (2016), 105-117.

[67] K. Mondal, and S. Pramanik. Tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. Critical Review, 11 (2015), 26-40.

[68] S. Broumi, and F. Smarandache. Interval neutrosophic rough sets. Neutrosophic Sets and Systems, 7 (2015), 23-31.

[69] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Interval neutrosophic sets and logic: theory and applications in computing. Hexis, Phoenix, 2005.

[70] S. Pramanik, R. Roy, T. K. Roy, and F. Smarandache. Multi attribute decision making strategy on projection and bidirectional projection measures of interval rough neutrosophic sets. Neutrosophic Sets and Systems, 19 (2018). Accepted for publication.

[71] K. Mondal, S. Pramanik, and B. C. Giri. Rough neutrosophic aggregation operators for multi-criteria decision-making. Fuzzy Multicriteria Decision Making Using Neutrosophic Sets Springer International Publishing, (2018). Accepted for publication.

[72] S. Pramanik, R. Roy, T. K. Roy, and F. Smarandache. Multi attribute decision making based on several trigonometric hamming similarity measures under interval rough neutrosophic environment. Neutrosophic Sets and Systems, 19 (2018).

[73] S. Pramanik, R. Roy, T. K. Roy, and F. Smarandache. Multi attribute decision making strategy on projection and bidirectional projection measures of interval rough neutrosophic sets. Neutrosophic Sets and Systems, 19 (2018).

[74] P. K. Maji. Neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics, 5 (2012), 157–168.

[75] P. K. Maji. Neutrosophic soft set approach to a decision-making problem. Annals of Fuzzy Mathematics and Informatics, 3 (2013), 313–319.

[76] R. Sahin, and A. Kucuk. Generalized neutrosophic soft set and its integration to decision-making problem. Applied Mathematics and Information Science, 8 (2014), 2751–2759.

[77] P. P. Dey, S. Pramanik, and B.C. Giri. Neutrosophic soft multi-attribute decision making based on grey relational projection method. Neutrosophic Sets and Systems, 11 (2016), 98-106.

[78] P. P. Dey, S. Pramanik, and B.C. Giri. Neutrosophic soft multi-attribute group decision making based on grey relational analysis method. Journal of New Results in Science, 10 (2016), 25-37.

[79] P. P. Dey, S. Pramanik, and B. C. Giri. Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. Critical Review, 11 (2015), 41-55.

[80] S. Pramanik, and S. Dalapati. GRA based multi criteria decision making in generalized neutrosophic soft set environment. Global Journal of Engineering Science and Research Management, 3 (2016), 153–169.

[81] M. Şahin, S. Alkhazaleh, and V. Uluçay. Neutrosophic soft expert sets. Applied Mathematics, 6 (2015), 116-127.

[82] S. Pramanik, P. P. Dey, and B. C. Giri. TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems. Neutrosophic Sets and Systems, 10 (2015), 88-95.

[83] J. Ye. Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment. Journal of Intelligence Systems, 24 (2015), 23–36.

[84] R. Sahin, and P. D. Liu. Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision-making. Neural Computing and Applications, 2016, DOI 10.1007/s00521-015-2163-x.

[85] R. Sahin, and P. D. Liu. Distance and similarity measures for multiple attribute decision making with single-valued neutrosophic hesitant fuzzy information. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 35-54). Brussels: Pons Editions, 2016.

[86] P. Biswas, S. Pramanik, and B. C. Giri. Some distance measures of single valued neutrosophic hesitant fuzzy sets and their applications to multiple attribute decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 27-34). Brussels: Pons Editions, 2016.

[87] P. Biswas, S. Pramanik, and B. C. GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set Information. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 55-63). Brussels: Pons Editions, 2016.

[88] S. Broumi, and F. Smarandache. Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets and Systems, 6 (2014), 42-48.

[89] S. Broumi, and I. Deli. Correlation measure for neutrosophic refined sets and its application in medical diagnosis. Palestine Journal of Mathematics, 5 (2016), 135–143.

[90] K. Mondal, and S. Pramanik. Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. Journal of New theory, 8 (2015), 41-50.

[91] K. Mondal, and S. Pramanik. Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making. Global Journal of Advanced Research, 2 (2) (2015), 486-494.

[92] S. Pramanik, D. Banerjee, and B.C. Giri. Multi criteria group decision making model in neutrosophic refined set and its application. Global Journal of Engineering Science and Research Management, 3 (6) (2016), 1- 10.

[93] S. Pramanik, D. Banerjee, and B.C. Giri. TOPSIS approach for multi attribute group decision making in refined neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 79-91). Brussels: Pons Editions, 2016.

[94] M. Ali, I. Deli, and F. Smarandache. The theory of neutrosophic cubic sets and their applications in pattern recognition. Journal of Intelligent and Fuzzy System, 30 (2016), 1957–1963.

[95] Y. B. Jun, F. Smarandache, C. S. Kim. Neutrosophic cubic sets. New Mathematics and Natural Computation, 13 (2017), 41–54.

[96] D. Banerjee, B. C. Giri, S. Pramanik, and F. Smarandache. GRA for multi attribute decision making in neutrosophic cubic set environment. Neutrosophic Sets and Systems, 15 (2017), 60–69.

[97] S. Pramanik, S. Dalapati, S. Alam, T. K. Roy, and F. Smarandache. Neutrosophic cubic MCGDM method based on similarity measure. Neutrosophic Sets and Systems, 16 (2017), 44–56.

[98] S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache. An Extended TOPSIS for Multi-Attribute Decision Making Problems with Neutrosophic Cubic Information. Neutrosophic Sets and Systems, 17 (2017), 20–28.

[99] J. Zhan, M. Khan, and M. Gulistan. Applications of neutrosophic cubic sets in multi-criteria decision-making. International Journal for Uncertainty Quantification, 7 (2017), 377–394.

[100] J. Ye. Linguistic neutrosophic cubic numbers and their multiple attribute decision-making method. *Information* 8 (2017), 110.

[101] Z. Lu, and J. Ye. Cosine measures of neutrosophic cubic sets for multiple attribute decision-making. Symmetry, 9 (2017), 121.

[102] S. Pramanik, S. Dalapati, S. Alam, and T. K. Roy. NC-TODIM-based MAGDM under a neutrosophic cubic set environment. Information, 8 (2017), 149. doi:10.3390/info8040149.

[103] L. Shi, and J. Ye. Dombi aggregation operators of neutrosophic cubic sets for multiple attribute decision-making. Algorithms 11 (2018), 29. doi:10.3390/a11030029.

[104] S. Pramanik, S. Dalapati, S. Alam, F. Smarandache, and T. K. Roy. NC-cross entropy based madm under neutrosophic cubic set environment, 2018 - preprints.org.

[105] I. Deli, M. Ali, and F. Smarandache. Bipolar neutrosophic sets and their application based on multi-criteria decision making Proceedings of the 2015 International Conference on Advanced Mechatronic Systems, Beiging, China, August, 249-254.

[106] W. R. Zhang. Bipolar fuzzy sets. In Proceedings of the IEEE World Congress on Computational Science (FuzzIEEE), Anchorage, AK, USA, 4–9 May 1998; pp. 835–840.

[107] W. R. Zhang. Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis. In Proceedings of the IEEE Industrial Fuzzy Control and Intelligent Systems Conference, and the NASA Joint Technology Workshop on Neural Networks and Fuzzy Logic, Fuzzy Information Processing Society Biannual Conference, San Antonio, TX, USA, 18–21 December 1994; pp. 305–309.

[108] I. Deli, and Y. A. Subas. Multiple criteria decision making method on single valued bipolar neutrosophic set based on correlation coefficient similarity measure. In *International conference on mathematics and mathematics education (ICMME2016)* (12-14).

[109] M. Sahin, I. Deli, and V. Ulucay. Jaccard vector similarity measure of bipolar neutrosophic set based on multi-criteria decision making. In: International conference on natural science and engineering (ICNASE'16), March 19–20, Kilis.

[110] V. Uluçay, I. Deli, and M. Şahin. Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. Neural Computing and Applications, 29 (3) (2018), 739-748.
[111] P. P. Dey, S. Pramanik, and B. C. Giri. TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 65-77). Brussels: Pons Editions, 2016.
[112] S. Pramanik, P. P. Dey, B. C. Giri, and F. Smarandache. Bipolar Neutrosophic Projection Based Models for Solving Multi-attribute Decision Making Problems. Neutrosophic Sets and Systems, 15 (2017), 70-79.
[113] L. Wang, H. Zhang,and J. Wang. Frank Choquet Bonferroni operators of bipolar neutrosophic sets and their applications to multi-criteria decision-making problems. International Journal of Fuzzy Systems, 2017.
[114] M. Akram, and M. Sarwar. Novel multiple criteria decision making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs. Italian Journal Of Pure And Applied Mathematics, 38 (2017), 368-389.
[115] S. Pramanik, S. Dalapati, S. Alam, and T. K. Roy.TODIM Method for Group Decision Making under Bipolar Neutrosophic Set Environment. New Trends in Neutrosophic Theory and Applications-Vol-II. Pons Editions, Brussells (2017). In Press.
[116] M. Ali, L. H. Son, I. Deli, and N. D. Tien. Bipolar neutrosophic soft sets and applications in decision making. Journal of Intelligent and Fuzzy Systems, 33(6) (2017), 4077-4087.
[117] S. Pramanik, and K. Mondal. Rough Bipolar Neutrosophic Set. Global Journal of Engineering Science and Research Management, 3 (6) (2016), 71-81. doi. 10.5281/zenodo.55966.
[118] S. Opricovic, G. H. Tzeng. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. European journal of operational research, 156 (2004), 445–455.
[119] S. Opricovic, G. H. Tzeng. Extended VIKOR method in comparison with outranking methods. European journal of operational research, 178 (2) (2007), 514-529.
[120] E. Cevikcan, S. Çebi, I. Kaya. Fuzzy VIKOR and Fuzzy Axiomatic Design Versus to Fuzzy Topsis: An Application of Candidate Assessment. Multi Valued Logic and Soft Computing, 15 (2009), 181–208.
[121] R. Bausys, and E. K. Zavadskas. Multicriteria decision making approach by VIKOR under interval neutrosophic set environment. Economic Computation and Economic Cybernetics Studies and Research, 4 (2015), 33-48.
[122] Y. Huang, G. Wei, and C. Wei. VIKOR method for interval neutrosophic multiple attribute group decision-making. Information 8 (2017), 144. doi:10.3390/info8040144.
[123] H. Pouresmaeil, E. Shivanian, E. Khorram, and H. S. Fathabadi. An extended method using TOPSIS and VIKOR for multiple attribute decision making with multiple decision makers and single valued neutrosophic numbers. Advances and Applications in Statistics, 50 (4) (2017), 261.
[124] P. Liu, L. Zhang. The extended VIKOR method for multiple criteria decision making problem based on neutrosophic hesitant fuzzy set, (2014).
[125] J. Hu, L. Pan, and X. Chen. An interval neutrosophic projection-based VIKOR method for selecting doctors. Cognitive Computation, 9(6) (2017), 801-816.
[126] K. Selvakumari, and M. Ajitha Priyadharshini. VIKOR method for decision making problem using octagonal neutrosophic soft matrix. International Journal of Latest Engineering Research and Applications, 2 (7) (2017), 41-45.
[127] M. Zhang, P. Liu, and L. Shi. An extended multiple attribute group decision-making TODIM method based on the neutrosophic numbers. Journal of Intelligent and Fuzzy Systems, 30 (2016), 1773–1781. doi: 10.3233/IFS-151889.
[128] S. Pramanik, S. Dalapati, and T. K. Roy. Logistics center location selection approach based on neutrosophic multi-criteria decision making. In F. Smarandache, & S. Pramanik (Eds), New trends in neutrosophic theory and applications (pp. 161-174). Brussels: Pons Editions, 2016.
[129] K. Mondal, and S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28-34.
[130] K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62-68.
[131] J. R. San Cristóbal. Multi-criteria decision-making in the selection of a renewable energy project in Spain: The VIKOR method. Renewable Energy, 36 (2011), 498-502.
[132] J. Ye. Fault diagnoses of steam turbine using the exponential similarity measure of neutrosophic numbers. Journal of Intelligent and Fuzzy System 30 (2016), 1927–1934.
[133] Abdel-Basset, M., Mohamed, M., Smarandache, F., & Chang, V. (2018). Neutrosophic Association Rule Mining Algorithm for Big Data Analysis. *Symmetry*, *10*(4), 106.
[134] Abdel-Basset, M., & Mohamed, M. (2018). The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems. Measurement. Volume 124, August 2018, Pages 47-55
[135] Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 1-11.
[136] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.
[137] Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, *86*, 12-29.

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