Cosine Similarity Measure of Interval Valued Neutrosophic Sets

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Abstract. In this paper, we define a new cosine similarity between two interval valued neutrosophic sets based on Bhattacharya's distance [19]. The notions of interval valued neutrosophic sets (IVNS, for short) will be used as vector representations in 3D-vector space. Based on the comparative analysis of the existing similarity measures for IVNS, we find that our proposed similarity measure is better and more robust. An illustrative example of the pattern recognition shows that the proposed method is simple and effective.

Keywords: Cosine Similarity Measure; Interval Valued Neutrosophic Sets .

I. INTRODUCTION

The neutrsophic sets (**NS**), pioneered by F. Smarandache [1], has been studied and applied in different fields, including decision making problems [2, 3, 4 , 5, 23], databases [6-7], medical diagnosis problems [8] , topology [9], control theory [10], Image processing [11,12,13] and so on. The character of NSs is that the values of its membership function, non-membership function and indeterminacy function are subsets. The concept of neutrosophic sets generalizes the following concepts: the classic set, fuzzy set, interval valued fuzzy set, Intuitionistic fuzzy set, and interval valued intuitionistic fuzzy set and so on, from a philosophical point of view. Therefore, Wang et al [14] introduced an instance of neutrosophic sets known as single valued neutrosophic sets (SVNS), which were motivated from the practical point of view and that can be used in real scientific and engineering application, and provide the set theoretic operators and various properties of SVNSs. However, in many applications, due to lack of knowledge or data about the problem domains, the decision information may be provided with intervals, instead of real numbers. Thus, interval valued neutrosophic sets (IVNS), as a useful generation of NS, was introduced by Wang et al [15], which is characterized by a membership function, non-membership function and an indeterminacy function, whose values are intervals rather than real numbers. Also, the interval valued neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exist in the real world. As an important extension of NS, IVNS has many applications in real life [16, 17].

Many methods have been proposed for measuring the degree of similarity between neutrosophic set, S.Broumi and F. Smarandache [22] proposed several definitions of similarity measure between NS. P.Majumdar and S.K.Samanta [21] suggested some new methods for measuring the similarity between neutrosophic set. However, there is a little investigation on the similarity measure of IVNS, although some method on measure of similarity between intervals valued neutrosophic sets have been presented in [5] recently.

Pattern recognition has been one of the fastest growing areas during the last two decades because of its usefulness and fascination. In pattern recognition, on the basis of the knowledge of known pattern, our aim is to classify the unknown pattern. Because of the complex and uncertain nature of the problems. The problem pattern recognition is given in the form of interval valued neutrosophic sets.

In this paper, motivated by the cosine similarity measure based on Bhattacharya's distance [19], we propose a new method called "cosine similarity measure for interval valued neutrosophic sets. Also the proposed and existing similarity measures are compared to show that the proposed similarity measure is more reasonable than some similarity measures. The proposed similarity measure is applied to pattern recognition

This paper is organized as follow: In section 2 some basic definitions of neutrosophic set, single valued neutrosophic set, interval valued neutrosophic set and cosine similarity measure are presented briefly. In section 3, cosine similarity measure of interval valued neutrosophic sets and their proofs are introduced. In section 4, results of the proposed similarity measure and existing similarity measures are compared .In section 5, the proposed similarity measure is applied to deal with the problem related to medical diagnosis. Finally we conclude the paper.

II. PRELIMINARIE

This section gives a brief overview of the concepts of neutrosophic set, single valued neutrosophic set, interval valued neutrosophic set and cosine similarity measure.

A. Neutrosophic Sets

1) Definition [1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form

A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ U}, where the functions T, I, F : U→]−0, 1+[define respectively the degree of membership (or Truth) , the degree of indeterminacy, and

the degree of non-membership (or Falsehood) of the element x ∈ U to the set A with the condition.

$$^-0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-$0, 1$^+$[. So instead of] −0, 1$^+$[we need to take the interval [0, 1] for technical applications, because]$^-$0, 1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS, $A_{NS} = \{<x, T_A(x), I_A(x), F_A(x)> \mid x \in X\}$

And $B_{NS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ If and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if, $T_A(x)=T_B(x)$, $I_A(x) =I_B(x)$, $F_A(x) =F_B(x)$

B. Single Valued Neutrosophic Sets

1) Definition [14]

Let X be a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

When X is continuous, an SVNS A can be written as

$$A=\int_X \frac{<T_A(x), I_A(x), F_A(x),>}{x}, x \in X. \qquad (2)$$

When X is discrete, an SVNS A can be written as

$$A= \sum_1^n \frac{<T_A(x_i), I_A(x_i), F_A(x_i),>}{x_i}, x_i \in X \qquad (3)$$

For two SVNS, $A_{SVNS} = \{<x, T_A(x), I_A(x), F_A(x)> \mid x \in X\}$

And $B_{SVNS} =\{ <x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)$

(2) $A_{SVNS} = B_{SVNS}$ if and only if, $T_A(x) =T_B(x)$, $I_A(x) =I_B(x)$, $F_A(x) =F_B(x)$ for any x ∈ X.

C. Interval Valued Neutrosophic Sets

1) Definition [15]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.
For two IVNS, $A_{IVNS} =\{<x, [T_A^L(x), T_A^U(x)]$,
$[F_A^L(x), F_A^U(x)]$, $[I_A^L(x), I_A^U(x)] > \mid x \in X\}$
And $B_{IVNS}= \{<x,$
$[T_B^L(x), T_B^U(x)], [F_B^L(x), F_B^U(x)], [I_B^L(x), I_B^U(x)] > \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{IVNS} \subseteq B_{IVNS}$ if and only if $T_A^L(x) \leq T_B^L(x), T_A^U(x) \leq T_B^U(x), I_A^L(x) \geq I_B^L(x), I_A^U(x) \geq I_B^U(x), F_A^L(x) \geq F_B^L(x), F_A^U(x) \geq F_B^U(x)$.

(2) $A_{IVNS} = B_{IVNS}$ if and only if, $T_A^L(x)=T_B^L(x)$, $T_A^U(x) =T_B^U(x)$, $I_A^L(x)=I_B^L(x)$, $I_A^U(x) =I_B^U(x)$, $F_A^L(x)=F_B^L(x)$, $F_A^U(x) =F_B^U(x)$ for any x ∈ X.

D. Cosine Similarity

1) Definition

Cosine similarity is a fundamental angle-based measure of similarity between two vectors of n dimensions using the cosine of the angle between them Candan and Sapino [20]. It measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them. Given two vectors of attributes, X = (x_1, x_2, … , x_n) and Y= (y_1, y_2, … , y_n), the cosine similarity, cosθ, is represented using a dot product and magnitude as

$$\mathbf{Cos\theta} = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}} \qquad (4)$$

In vector space, a cosine similarity measure based on Bhattacharya's distance [19] between two fuzzy set $\mu_A(x_i)$ and $\mu_B(x_i)$ defined as follows:

$$C_F(A, B) = \frac{\sum_{i=1}^n \mu_A(x_i) \mu_B(x_i)}{\sqrt{\sum_{i=1}^n \mu_A(x_i)^2} \sqrt{\sum_{i=1}^n \mu_B(x_i)^2}} \qquad (5)$$

The cosine of the angle between the vectors is within the values between 0 and 1.

In 2-D vector space, J. Ye [18] defines cosine similarity measure between IFS as follows:

$$C_{IFS}(A, B) = \frac{\sum_{i=1}^n \mu_A(x_i) \mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\sum_{i=1}^n \mu_A(x_i)^2+\nu_A(x_i)^2} \sqrt{\sum_{i=1}^n \mu_B(x_i)^2+\nu_B(x_i)^2}} \qquad (6)$$

III . COSINE SIMILARITY MEASURE FOR INTERVAL VALUED NEUTROSOPHIC SETS.

The existing cosine similarity measure is defined as the inner product of these two vectors divided by the product of their lengths. The cosine similarity measure is the cosine of the angle between the vector representations of the two fuzzy sets. The cosine similarity measure is a classic measure used in information retrieval and is the most widely reported measures of vector similarity [19]. However, to the best of our Knowledge, the existing cosine similarity measures does not deal with interval valued neutrosophic sets. Therefore, to overcome this limitation in this section, a new cosine similarity measure between interval valued neutrosophic sets is proposed in 3-D vector space.

Let A be an interval valued neutrosophic sets in a universe of discourse X ={x}, the interval valued neutrosophic sets is characterized by the interval of membership $[T_A^L(x), T_A^U(x)]$, the interval degree of non-membership $[F_A^L(x), F_A^U(x)]$ and the interval degree of indeterminacy $[I_A^L(x), I_A^U(x)]$ which can be considered as a vector representation with the three elements. Therefore, a cosine similarity measure for interval neutrosophic sets is proposed in an analogous manner to the cosine similarity measure proposed by J. Ye [18].

E. Definition

Assume that there are two interval neutrosophic sets A and B in X $=\{ x_1, x_2 ,..., x_n\}$ Based on the extension measure for fuzzy sets, a cosine similarity measure between interval valued neutrosophic sets A and B is proposed as follows

$$C_N(A,B)= \frac{1}{n} \sum_{i=1}^{n} \frac{(T_A^L(x_i) + T_A^U(x_i))\,(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i) + I_A^U(x_i))\,(I_B^L(x_i) + I_B^U(x_i)) +(F_A^L(x_i) + F_A^U(x_i))\,(F_B^L(x_i) + F_B^U(x_i))}{\sqrt{(T_A^L(x_i) + T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i) + F_A^U(x_i))^2}\ \sqrt{(T_B^L(x_i) + T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i) + F_B^U(x_i))^2}} \quad (7)$$

F. Proposition

Let A and B be interval valued neutrosophic sets then
  i. $0 \leq C_N(A,B) \leq 1$
  ii. $C_N(A,B) = C_N(B,A)$
  iii. $C_N(A,B) = 1$ if A= B i.e

$T_A^L(x_i) = T_B^L(x_i),\ T_A^U(x_i) = T_B^U(x_i),$
$I_A^L(x_i) = I_B^L(x_i),\ I_A^U(x_i) = I_B^U(x_i)$ and
$F_A^L(x_i) = F_B^L(x_i),\ F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,...., n

Proof : (i) it is obvious that the proposition is true according to the cosine valued

(ii) it is obvious that the proposition is true.
(iii) when A =B, there are
$T_A^L(x_i) = T_B^L(x_i),\ T_A^U(x_i) = T_B^U(x_i),$
$I_A^L(x_i) = I_B^L(x_i),\ I_A^U(x_i) = I_B^U(x_i)$ and
$F_A^L(x_i) = F_B^L(x_i),\ F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,..., n , So
there is $C_N(A,B) = 1$

If we consider the weights of each element x_i, a weighted cosine similarity measure between IVNSs A and B is given as follows:

$$C_{WN}(A,B)= \frac{1}{n} \sum_{i=1}^{n} w_i \frac{(T_A^L(x_i) + T_A^U(x_i))\,(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i) + I_A^U(x_i))\,(I_B^L(x_i) + I_B^U(x_i)) +(F_A^L(x_i) + F_A^U(x_i))\,(F_B^L(x_i) + F_B^U(x_i))}{\sqrt{(T_A^L(x_i) + T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i) + F_A^U(x_i))^2}\ \sqrt{(T_B^L(x_i) + T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i) + F_B^U(x_i))^2}} \quad (8)$$

Where $w_i \in [0.1]$,i =1,2,...,n ,and $\sum_{i=1}^{n} w_i$ =1.

If we take $w_i = \frac{1}{n}$, i =1,2,...,n , then there is $C_{WN}(A,B) = C_N(A,B)$.

The weighted cosine similarity measure between two IVNSs A and B also satisfies the following properties:
  i. $0 \leq C_{WN}(A,B) \leq 1$
  ii. $C_{WN}(A,B) = C_{WN}(B,A)$
  iii. $C_{WN}(A,B) = 1$ if A= B i.e

$T_A^L(x_i) = T_B^L(x_i),\ T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i),\ I_A^U(x_i) = I_B^U(x_i)$ and $F_A^L(x_i) = F_B^L(x_i),\ F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,..., n

G. Proposition

Let the distance measure of the angle as **d(A,B)= arcos** $(C_N(A,B))$ then it satisfies the following properties.

  i. $d(A, B) \geq 0$, if $0 \leq C_s(A,B) \leq 1$
  ii. $d(A, B) = \textbf{arcos(1) = 0, if } C_N(A,B) = 1$
  iii. $d(A, B) = d(B, A)$ if $C_N(A,B) = C_N(B,A)$
  iv. $d(A, C) \leq d(A, B) + d(B, C)$ if $A \subseteq B \subseteq C$ for any interval valued neutrosophic sets C.

Proof : obviously, d(A,B) satisfies the (i) – (iii). In the following , d(A,B) will be proved to satisfy the (iv).

For any $C = \{ x_i\}$, $A \subseteq B \subseteq C$ since Eq (7) is the sum of terms. Let us consider the distance measure of the angle between vectors:

$d_i(A(x_i), B(x_i)) = \textbf{arcos}(C_N(A(x_i), B(x_i))),$
$d_i(B(x_i), C(x_i)) = \textbf{arcos}(C_N(B(x_i), C(x_i)))$ **and**
$d_i(A(x_i), C(x_i)) = \textbf{arcos}(C_N(A(x_i), C(x_i)))$, for i=1, 2, .., n, where

$$C_N(A,B)= \frac{1}{n} \sum_{i=1}^{n} \frac{(T_A^L(x_i) + T_A^U(x_i))\,(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i) + I_A^U(x_i))\,(I_B^L(x_i) + I_B^U(x_i)) +(F_A^L(x_i) + F_A^U(x_i))\,(F_B^L(x_i) + F_B^U(x_i))}{\sqrt{(T_A^L(x_i) + T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i) + F_A^U(x_i))^2}\ \sqrt{(T_B^L(x_i) + T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i) + F_B^U(x_i))^2}} \quad (9)$$

$$C_N(B,C)= \frac{1}{n} \sum_{i=1}^{n} \frac{(T_B^L(x_i) + T_B^U(x_i))\,(T_C^L(x_i)+T_C^U(x_i))+(I_B^L(x_i) + I_B^U(x_i))\,(I_C^L(x_i) + I_C^U(x_i)) +(F_B^L(x_i) + F_B^U(x_i))\,(F_C^L(x_i) + F_C^U(x_i))}{\sqrt{(T_B^L(x_i) + T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i) + F_B^U(x_i))^2}\ \sqrt{(T_C^L(x_i) + T_C^U(x_i))^2+(I_C^L(x_i)+I_C^U(x_i))^2+(F_C^L(x_i) + F_C^U(x_i))^2}} \quad (10)$$

$$C_N(A,C)= \frac{1}{n} \sum_{i=1}^{n} \frac{(T_A^L(x_i) + T_A^U(x_i))\,(T_C^L(x_i)+T_C^U(x_i))+(I_A^L(x_i) + I_A^U(x_i))\,(I_C^L(x_i) + I_C^U(x_i)) +(F_A^L(x_i) + F_A^U(x_i))\,(F_C^L(x_i) + F_C^U(x_i))}{\sqrt{(T_A^L(x_i) + T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i) + F_A^U(x_i))^2}\ \sqrt{(T_C^L(x_i) + T_C^U(x_i))^2+(I_C^L(x_i)+I_C^U(x_i))^2+(F_C^L(x_i) + F_C^U(x_i))^2}} \quad (11)$$

For three vectors

$A(x_i) = <x_i,\ [\ T_A^L(x_i),\ T_A^U(x_i)],\ [\ I_A^L(x_i), I_A^U(x_i)],\ [F_A^L(x_i),\ F_A^U(x_i)]>$

$B(x_i) = < x_i,\ [\ T_B^L(x_i),\ T_B^U(x_i)\ ],[\ I_B^L(x_i),\ I_B^U(x_i)\ ],\ [F_B^L(x_i),\ F_B^U(x_i)]>$

$C(x_i) = <x_i,\ [\ T_C^L(x_i),\ T_C^U(x_i)],\ [\ I_C^L(x_i),\ I_C^U(x_i)\ ],\ [F_C^L(x_i),\ F_C^U(x_i)]>$, in a plane ,

If A $(x_i) \subseteq$ B $(x_i) \subseteq$ C (x_i) (I =1, 2,…, n), then it is obvious that d(A(x_i), C(x_i)) ≤ d(A(x_i), B(x_i)) + d(B(x_i), C(x_i)), According to the triangle inequality. Combining the inequality with Eq (7), we can obtain d(A, C) ≤ d(A, B) + d(B, C)

Thus, d(A,B) satisfies the property (iv). So we have finished the proof.

IV. COMPARISON OF NEW SIMILARITY MEASURE WITH THE EXISTING MEASURES.

Let A and B be two interval neutrosophic set in the universe of discourse X={$x_1, x_2,..,x_n$}. For the cosine similarity and the existing similarity measures of interval valued neutrosophic sets introduced in [5, 21], they are listed as follows:

Pinaki's similarity I [21]

$$S_{PI}= \frac{\sum_{i=1}^{n}\{min\{T_A(x_i),T_B(x_i)\}+min\{I_A(x_i),I_B(x_i)\}+ min\{F_A(x_i),F_B(x_i)\}\}}{\sum_{i=1}^{n}\{max\{T_A(x_i),T_B(x_i)\}+max\{I_A(x_i),I_B(x_i)\}+ max\{F_A(x_i),F_B(x_i)\}\}} \quad (12)$$

Also ,P. Majumdar [21] proposed weighted similarity measure for neutrosophic set as follows:

$$S_{PII} = \frac{\sum_{i=1}^{n} w_i (T_A(x_i) \cdot T_B(x_i)+(I_A(x_i) \cdot I_B(x_i)+(F_A(x_i) \cdot F_B(x_i))}{max(w_i \sqrt{T_A(x_i)^2+I_A(x_i)^2+F_A(x_i)^2}, w_i\sqrt{T_B(x_i)^2+I_B(x_i)^2+F_B(x_i)^2}))}$$
(13)

Where, S_{PI}, S_{PII} denotes Pinaki's similarity I and Pinaki's similarity II

Ye's similarity [5] is defined as the following:

$S_{ye}(A, B) = 1 - \frac{1}{6}\sum_{i=1}^{n} w_i [|infT_A(x_i) - infT_B(x_i)| + |supT_A(x_i) - supT_B(x_i)| + |infI_A(x_i) - infI_B(x_i)| + |supI_A(x_i) - supI_B(x_i)| + |infF_A(x_i) - infF_B(x_i)| + |supF_A(x_i) - supF_B(x_i)|]$ (14)

Example 1:

Let A = {<x, (0.2, 0.2 0.3)>}   and B= {<x, (0.5, 0.2 0.5)>}

Pinaki similarity I = 0.58

Pinaki similarity II (with w_i =1) = 0.29

Ye similarity (with w_i =1) = 0.83

Cosine similarity $C_N(A, B)$ = 0.95

Example 2:

Let A= {<x, ([0.2, 0.3], [0.5, 0.6] ,[0.3, 0.5])>} and B{<x, ([0.5, 0.6], [0.3, 0.6] ,[0.5, 0.6])>}

Pinaki similarty I = NA

Pinaki similarty II(With w_i =1) = NA

Ye similarity (with w_i =1) =0.81

Cosine similarity $C_N(A, B)$ = 0.92

On the basis of computational study. J.Ye [5] have shown that their measure is more effective and reasonable .A similar kind of study with the help of the proposed new measure based on the cosine similarity, has been done and it is found that the obtained results are more refined and accurate. It may be observed from the example 1 and 2 that the values of similarity measures are more closer to 1 with $C_N(A, B)$,the proposed similarity measure. This implies that we may be more deterministic for correct diagnosis and proper treatment.

V. APPLICATION OF COSINE SIMILARITY MEASURE FOR INTERVAL VALUED NEUTROSOPHIC NUMBERS TO PATTERN RECOGNITION

In order to demonstrate the application of the proposed cosine similarity measure for interval valued neutrosophic numbers to pattern recognition, we discuss the medical diagnosis problem as follows:

For example the patient reported temperature claiming that the patient has temperature between 0.5 and 0.7 severity /certainty, some how it is between 0.2 and 0.4 indeterminable if temperature is cause or the effect of his current disease. And it between 0.1 and 0.2 sure that temperature has no relation with his main disease. This piece of information about one patient and one symptom may be written as:

(patient , Temperature) = <[0.5, 0.7], [0.2 ,0.4], [0.1, 0.2]>
(patient , Headache) = < [0.2, 0.3], [0.3 ,0.5], [0.3, 0.6]>
(patient , Cough) = <[0.4, 0.5], [0.6 ,0.7], [0.3, 0.4]>

Then, P= {< x_1, [0.5, 0.7], [0.2 ,0.4], [0.1, 0.2] >, < x_2, [0.2, 0.3], [0.3, 0.5], [0.3, 0.6] > ,< x_3, [0.4, 0.5], [0.6 ,0.7], [0.3, 0.4]>}

And each diagnosis A_i (i=1, 2, 3) can also be represented by interval valued neutrosophic numbers with respect to all the symptoms as follows:

A_1= {< x_1, [0.5, 0.6], [0.2 ,0.3], [0.4, 0.5] >, < x_2 , [0.2 , 0.6], [0.3 ,0.4], [0.6 , 0.7]>,< x_3, [0.1, 0.2], [0.3 ,0.6], [0.7, 0.8]>}

A_2= {< x_1, [0.4, 0.5], [0.3, 0.4], [0.5, 0.6] >, < x_2 , [0.3, 0.5], [0.4 ,0.6], [0.2, 0.4]> ,< x_3 , [0.3, 0.6], [0.1, 0.2], [0.5, 0.6]>}

A_3= {< x_1, [0.6, 0.8], [0.4 ,0.5], [0.3, 0.4]>, < x_2 , [0.3, 0.7], [0.2, 0.3], [0.4, 0.7]> ,< x_3, [0.3, 0.5], [0.4, 0.7], [0.2, 0.6]>}

Our aim is to classify the pattern P in one of the classes A_1, A_2, A_3 .According to the recognition principle of maximum degree of similarity measure between interval valued neutrosophic numbers, the process of diagnosis A_k to patient P is derived according to

k = arg Max{ $C_N(A_i , P)$)}

from the previous formula (7) , we can compute the cosine similarity between A_i (i=1, 2, 3) and P as follows;

$C_N(A_1 , P)$=0.8988, $C_N(A_2 , P)$=0.8560, $C_N(A_3 , P)$ =0.9654

Then, we can assign the patient to diagnosis A_3 (Typoid) according to recognition of principal.

VI. Conclusions.

In this paper a cosine similarity measure between two and weighted interval valued neutrosophic sets is proposed. The results of the proposed similarity measure and existing similarity measure are compared. Finally, the proposed cosine similarity measure is applied to pattern recognition.

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