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NEUTROSOPHIC LINEAR GOAL PROGRAMMING

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ABSTRACT

This paper proposes the framework of neutrosophic linear goal programming (NGP) approach for solving multi objective optimization problems involving uncertainty and indeterminacy. In the proposed approach, the degree of membership (acceptance), indeterminacy and falsity (rejection) of the objectives are simultaneously considered. Three neutrosophic linear goal programming models have been proposed. The drawbacks of the existing neutrosophic optimization models have been addressed and new direction of research in neutrosophic optimization problem has been proposed. The essence of the proposed approach is that it is capable of dealing with indeterminacy and falsity simultaneously.

INTRODUCTION

Goal programming can be viewed in two ways. In first consideration, it is an extension of linear programming to include multi objectives, expressed by means of attempted achievement of goal values. In second consideration, linear programming is a special case of goal programming having single objective. These two considerations reflect that goal programming lies within the paradigm of multi objective programming [1]. Goal programming may be characterized as an analytical approach devised to address multi objective decision making problems having inherent multiple conflicting objectives where targets have been assigned to all the attributes in the planning horizon and where decision making unit is mainly interested in minimizing the non-achievement of the goals.The ethos of goal programming lies in the Simon's concept [2] of satisfying of objectives. GP has appeared as robust tool for multi objective decision analysis. It appears to be an appropriate, powerful, and flexible technique in operations research for decision making problems with multiple conflicting objectives. The literature on goal programming has tremendously grown.

Goal programming is perhaps the most widely used multi criteria decision making (MCDM) approach. The idea of GP can be visualized from the concept of efficiency introduced by Koopmans [3] in the context of resource allocation planning. The roots of goal programming lie in the study of Charnes, Cooper and Ferguson [4] in 1955 in which, they deal with executive compensation methods. In 1961, Charnes and Cooper [5] offered a more explicit definition and coined the term 'goal programming'.

Thereafter, a large number of studies have been made by pioneer researchers and the significant methodological development of goal programming have been achieved by Ijiri [6], Lee [7], Ignizio [8], Schniederjans [9], Romero [10], Schniederjans [11] and other researches. The vast literature of goal programming reflects its theoretical elegance and significance.

In 1980, Narasimhan [12] employed the concept of fuzzy set theory introduced by Zadeh [13] in goal programming by incorporating fuzzy goals and constraints within the traditional goal programming model in order to add new dimension in modeling flexibility and accuracy to the goal prorgramming model for dealing with uncertainty. Thereafter, fuzzy goal progamming has been further developed by Hannan [14], Ignizio [15], Tiwari et al. [16, 17], Mohamed [18], Pramanik and Roy [19, 20], Pramanik and Dey [21,], Pramanik [22] and other researchers.

Atanassov [23, 24] incorporated the degree of non-membership (rejection) as an independent component and defined intuitionistic fuzzy set to deal uncertainty in more flexible way. In 1995 Angelov [25] presented a new

concept to optimization problem in intuitionistic fuzzy environment. In 1997, Angelov [26] restated intuitionistic fuzzy linear programming problem [25] considering maximizing membership function and minimizing the non-membership functions simultaneously by extending fuzzy linear multi- objective programmming proposed by Zimmermann [27]. In 2001, Angelov [28] also presented a general formulation of the optimization problem of an air conditioning system in the framework of intuitionistic fuzzy set theory.

Goal programming in intuitionistic fuzzy environment is called intuitionistic fuzzy goal programming. In 2005, Pramanik and Roy [29] proposed intuitionistic fuzzy goal programming (IFGP) by extending fuzzy goal programming. Pramanik and Roy [30, 31, 32] also presented intuitionistic fuzzy goal programming for quality control problem, transportation problems and bi-level programming problems respectively but these problems are numerical problems. Major success has not been achieved in intuitionistic multi-objective optimization problems.

Smarandache [33, 34, 35, 36] introduced the concept of the degree of indeterminacy/neutrality as independent component in 1998 and defined the neutrosophic set in order to deal with uncertainty and indeterminacy involved in real world problems. The significance of Smarandache's work [33] is that it is capable of dealing with indeterminacy which is beyond the scope of fuzzy set and intuitionistic fuzzy set. The need of neutrosophic set was felt and actually discovered by Smarandache in 1995 and he wrote the manuscript in 1995 but he published it in 1998. When the new paradigm was grounded by Smarandache [33], the usual process of a paradigm shift started. The concept of neutrosophic set, derived from neutrosophy, which underlies the new paradigm, was initially ignored, ridiculed, or attacked by many [37, 38], while it was supported only by a very few, mostly young, unknown, and uninfluential researchers. Inspite of the initial lack of interest, skepticism [37, 38], or open hostility, the new paradigm persevered with virtually no support in the 1990s. Smarandache becomes the torchbearer of neutrosophy, neutrosophic set and neutrosophic logic. He has tried his level best to propagate the new paradigm by writing books, e-books, providing the free downloads of his writings in free journals and websites. The new paradigm matured significantly and gained some supports in the 2010s and started to demonstrate its superior pragmatic utility in the 2010s. The paradigm shift initiated by the concept of neutrosophy [33] and neutrosophic set and the idea of mathematics based on neutyrosophic set, which is currently ongoing, possesses similar charactewristics to other paradigm shift recognized in the history of science. The new paradigm shift covers a broad range of subjects, from philosophy to mathematics. The paradigm shift is still ongoing and it seems that it will probably take much longer time than usual to complete it. This can be concluded because of the fact that the scope of the paradigm shift is very wide and open and competitive.

In 2010, Wang et al. [39] defined singe valued neutrosophic set (SVNS) which is an instance of neutrosophic set, whose truth membership degree, indeterminacy and falsity degrees lie in the unit interval [0, 1]. It can be stated that an important point of evolution of the modern concept of uncertainty was the publication of a seminal work of Smarandache [33]. Although mathematics based on SVNSs has far greater expressive power than crisp set, fuzzy sets, intuitionistic fuzzy sets, its usefulness depends critically on one's capability to formulate appropriate membership functions, indeterminate functions and falsity functions for various given concepts in various contexts and their multiple operational rules. Union and intersections of two SVNSs can be differently defined and different results can be obtained for the same optimization problem.

Research on the theory of SVNSs has been growing steadily since its inception in 2010. The body of concepts and results pertaining to the theory of SVNS is now impressive. Research on a broad variety of applications has also been very attractive and has produced results that are perhaps even more impressive [40, 41, 42, 43, 44, 45, 46, 47].

In 2015, Roy and Das [48] presented multi-objective production planning problem based on neutrosophic linear programming approach. Das and Roy [49] presented multi-objective non-linear programming based on neutrosophic optimization technique and its application in riser design problem. Hezam et al. [50] studied Taylor series approximation to solve neutrosophic multi-objective programming problem. In 2016, Abdel-Baset et al. [51] proposed neutrosophic goal programming using deviation variables. In the studies [48, 49, 50, 51], the researchers maximize indeterminacy. But in a real management system, decision making unit does not show any interest to maximize indeterminacy. Because maximization of indeterminacy does offer any benefit to the

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management system and the organization. So it is not pragmatic to maximize indeterminacy function in the process of optimizing of the objective functions of the decision making problems. So, the techniques presented in the papers [48, 49, 50, 51] are neutrosophic in nature. Their approaches went in wrong directions. The claims of getting better optimal solutions in the studies [48, 49] are therefore not valid. However, they initiated new idea in optimization by incorporating indeterminacy. The errors committed by them occur due to the choice of defnitions of intersection of two neutrosophicsets. Therefore new methods for neutrosophic multi-objective programming problems are urgently needed.

Fuzzy goal programming and intuitionistic fuzzy goal programming have been developed in order to deal with uncertainty. However, these two approaches are not capable of dealing with indeterminacy. It seems, therefore that in many environments it is more realistic to endeavor achieving several objectives simultaneously involving indeterminacy and incompleteness. This observation reflects that real world problems have to be solved optimally according to criteria involving indeterminacy. Consequently, we must acknowledge the presence of several objectives which are at least contradictory, conflicting, indeterminate and often non-commensurable leading to the development of neutrosophic optimization technique.

This paper develops new framework of neutrosophic linear goal programming model.
Rest of the paper has been organized in the following way. Section 2 presents some basic definitions of neutrosophic sets, Section 3 is devoted to present the proposed framework of neutrosophic linear goal programming and intuitionistic fuzzy goal programming models. Section 4 presents the conclusion and future direction of research work.

PRELIMINARIES
We recall some basic definitions related to neutrosophic sets which are important to develop the paper.
2.1 Definition of neutrosophic set [33]
Let V be a space of points (objects) with a generic element $v \in V$. A neutrosophic set S in V is characterized by a truth membership function $T_S(v)$, an indeterminacy membership function $I_S(v)$, and a falsity membership function $F_S(v)$ and is denoted by

$$S = \{v, \langle T_S(v), I_S(v), F_S(v) \rangle \mid v \in V.\}$$

Here $T_S(v)$, $I_S(v)$ and $F_S(v)$ have been defined as follows:

$$T_S : V \rightarrow]^- 0, 1^+[$$

$$I_S : V \rightarrow]^- 0, 1^+[$$

$$F_S : V \rightarrow]^- 0, 1^+[$$

Here, $T_S(v)$, $I_S(v)$ and $F_S(v)$ are the real standard and non-standard subset of $]^- 0, 1^+[$. In general, there is no restriction on $T_S(v)$, $I_S(v)$ and $F_S(v)$. Therefore,

$$^- 0 \leq \operatorname{Inf} T_S(v) + \inf I_S(v) + \inf F_S(v) \leq \operatorname{Sup} T_S(v) + \operatorname{Sup} I_S(v) + \operatorname{Sup} F_S(v) \leq 3^+$$

2.2. Definition: Single valued neutrosophic set [39]
Let V be a space of points with generic element $v \in V$. A single valued neutrosophic set S in V is characterized by a truth-membership function $T_S(v)$, an indeterminacy-membership function $I_S(v)$ and a falsity-membership function $F_S(v)$, for each point v in V, $T_S(v)$, $I_S(v)$, $F_S(v) \in [0, 1]$, when V is continuous then single-valued neutrosophic set S can be written as
$$S = \int_V < T_S(v), I_G(v), F_G(v) > / v, v \in V.$$

When V is discrete, single-valued neutrosophic set S can be written as follows:
$$S = \sum_{i=1}^{n} < T_S(v_i), I_S(v_i), F_S(v_i) > \mid v_i, v_i \in V$$

Definition 2.3 [39]: The complement of a single valued neutrosophic set S is denoted by S^c and is defined by
$$T_{S^c}(v) = F_S(v) \, ; I_{S^c}(v) = 1 - I_S(x) \, ; F_{S^c}(v) = T_S(v)$$

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Definition 2.4 [39]: Two single valued neutrosophic sets P and Q are equal, written as P = Q, if and only if P ⊆ Q and P ⊇ Q.

Definition 2.5 [52]:The union of two single valued neutrosophic sets P and Q is a single valued neutrosophic set R, written as R= P ∪ Q, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of P and Q by $T_R(v) = \max(T_P(v), T_Q(v))$; $I_R(x) = \min(I_p(v), I_Q(v))$; $F_R(x) = \min(F_P(v), F_Q(v))$ for all v in *V*.

Definition 2.6 [52]: The intersection of two single valued neutrosophic sets P and Q is a neutrosophic set R written as R = P ∩ Q, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of P and Q by $T_R(v) = \min(T_P(v), T_Q(x))$; $I_R(v) = \max(I_P(v), I_Q(x))$; $F_R(x) = \max(F_P(v), F_Q(v))$ for all v in *V*.

Definition 2.7 [52]**:** Assume that { $P_j : j \in J$} be an arbitrary family of single valued neutrosophic sets in *V*, then

i) ∪P_j may be defined as follows:

$$\cup P_j = \left\langle v, \underset{j \in J}{\vee} T_{P_j}(v), \underset{j \in J}{\wedge} I_{P_j}(v), \underset{j \in J}{\wedge} F_{P_j}(v) \right\rangle$$

(ii) ∩ P_j may be defined as follows:

$$\cap P_j = \left\langle v, \underset{j \in j}{\wedge} T_{P_i}(v), \underset{j \in J}{\vee} I_{P_j}(v), \underset{j \in J}{\vee} F_{P_i}(v) \right\rangle$$

FORMULATION OF NEUTROSOPHIC LINEAR GOAL PROGRAMMING

To formulate neutrosophic goal programming, we start from multi-objective programing problem in crisp environment.

Consider an optimization problem of the form in crisp environment:

$\text{Max } \Phi_i(\bar{v})$, i = 1, 2, …, r_1 (1)

Subject to

$\psi_i(\bar{v}) \le 0$, i = r_1+1, …, r

$\bar{v} \ge \bar{0}$

where $\Phi_i(\bar{v})$ represents the i-th objective function, $\bar{v}$ is the vector of decision variables (v_1, v_2, …, v_k), $\psi_i(\bar{v})$ denotes i-th constraint, r denotes the number of objective functions and s denotes the number of constraints.

Analogous fuzzy optimization problem

In general, fuzzy optimization problem comprises of a set of objectives and constraints. The objectives and or constraints or parameters and relations are expressed by fuzzy sets which explain the degree of satisfaction of the respective condition and expressed by their membership functions [53].

Consider the analogous fuzzy optimization problem:

$\widetilde{\text{Max}} \ \Phi_i(\bar{v})$, i = 1, 2, …, r_1 (2)

Subject to

$\psi_i(\bar{v}) \ \widetilde{\le} \ 0$, i = r_1+1, …, r

$\bar{v} \ge \bar{0}$

$\widetilde{\text{Max}}$ denotes fuzzy maximization and $\widetilde{\le}$ denotes the fuzzy inequality.

To maximize the degree of membership of the objectives and constraints to the respective fuzzy sets:

$\text{Max } \mu_i(\bar{v})$, $\bar{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r   (3)

Subject to

$0 \le \mu_i(\bar{v}) \le 1$, i = 1, 2, …, r_1, r_1+1, …, r

$\bar{v} \ge \bar{0}$

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Where $\mu_i(\bar{v})$ denotes the degree of membership of i-th objective function $\Phi_i(\bar{v})$ (i = 1, 2, …r_1) and $\mu_i(\bar{v})$ denotes the degree of i-th membership function of constraint $\psi_i(\bar{v})$ (i = r_1+1, …, r).

Minimum operator of Bellman and Zadeh [54] can be applied to the optimization problem (3).

$$\mu_D(\bar{v}) = \overset{r}{\underset{i=1}{\wedge}} \mu_i(\bar{v}), \bar{v} \geq \bar{0}, i = 1, 2, …, r_1, r_1+1, …, r \tag{4}$$

Therefore, $\mu_D(\bar{v}) \leq \mu_i(\bar{v})$, i = 1, 2, …, r_1, r_1+1, …, r $\tag{5}$

According to Zimmermann [55], the problem can be solved as follows:

$$\mu_D(\bar{v}^o) = Max (min (\mu_1(\bar{v}), \mu_2(\bar{v}), …, \mu_{r_1}(\bar{v}), \mu_{r_1+1}(\bar{v}), …, \mu_r(\bar{v}))) \tag{6}$$

Subject to

$0 \leq \mu_i(\bar{v}) \leq 1$, i = 1, 2, …, r_1, r_1+1, …, r

$\bar{v} \geq \bar{0}$.

The problem (6) is equivalent to the following problem:

Max α $\tag{7}$

$\alpha \leq \mu_i(\bar{v})$, i = 1, 2, …, r_1, r_1+1, …, r

$\bar{v} \geq \bar{0}$.

An analogous intuitionistic fuzzy optimization (IFO) problem

An analogous intuitionistic fuzzy optimization problem can be represented as follows:

To maximize the degree of acceptance of intuitionistic fuzzy objective functions and constraints, and to minimize the degree of rejection of intuitionistic fuzzy objective functions and constraints we can write:

Max $\mu_i(\bar{v})$, $\bar{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r $\tag{8}$

Min $\nu_i(\bar{v})$, $\bar{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r $\tag{9}$

Subject to

$\mu_i(\bar{v}) + \nu_i(\bar{v}) \leq 1$ i = 1, 2, …, r_1, r_1+1, …, r,

$\nu_i(\bar{v}) \in [0, 1]$, i = 1, 2, …, r_1, r_1+1, …, r,

$\mu_i(\bar{v}) \in [0, 1]$, i = 1, 2, …, r_1, r_1+1, …, r,

$\bar{v} \geq \bar{0}$

Here $\mu_i(\bar{v})$ denotes the degree of membership of i-th objective function $\Phi_i(\bar{v})$ (i = 1, 2, …r_1) and $\mu_i(\bar{v})$ denotes the degree of i-th membership function of constraint $\psi_i(\bar{v})$ (i = r_1+1, …, r).

Here $\nu_i(\bar{v})$ denotes the degree of non-membership of i-th objective function $\Phi_i(\bar{v})$ (i = 1, 2, …r_1)and $\nu_i(\bar{v})$ denotes the degree of i-th non-membership function of constraint $\psi_i(\bar{v})$ (i = r_1+1, …, r).

Conjunction of intuitionistic fuzzy sets can be defined as follows:

$G \cap C = \{\langle \bar{v}, \mu_G(\bar{v}) \wedge \mu_C(\bar{v}), \nu_G(\bar{v}) \vee \nu_C(\bar{v})\rangle | \bar{v} \in \Re^k\}$, $\tag{10}$

where G represents an intuitionistic fuzzy objectives and C represents constraints. This conjunction operator can be easily generalized and applied to the IFO problem.

Here,

$$D = \{\langle \bar{v}, \mu_D(\bar{v})), \nu_D(\bar{v})\rangle | \bar{v} \in \Re^k\}, \mu_D(\bar{v}) = \overset{r}{\underset{i=1}{\wedge}} \mu_i(\bar{v}), \nu_D(\bar{v}) = \overset{r}{\underset{i=1}{\vee}} \nu_i(\bar{v}) \tag{11}$$

where D represents an intuitionistic fuzzy set based representation of the decision.

Min-operator can be used for conjunction and max-operator for disjunction.

$$\mu_D(\bar{v}) = \overset{r}{\underset{i=1}{\wedge}} \mu_i(\bar{v}), \bar{v} \in \Re^k, i = 1, 2, …, r_1, r_1+1, …, r, \tag{12}$$

$$\nu_D(\bar{v}) = \overset{r}{\underset{i=1}{\vee}} \nu_i(\bar{v}), \bar{v} \in \Re^k, i = 1, 2, …, r_1, r_1+1, …, r, \tag{13}$$

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Therefore, $\mu_D(\bar{v}) \leq \mu_i(\bar{v}), v_D(\bar{v}) \geq v_i(\bar{v})$, i = 1, 2, …, r_1, r_1+1, …, r, $\qquad$ (14)

The above intuitionistic fuzzy optimization problem can be transformed into intuitionistic fuzzy goal programming problem as follows: To maximize the degree the acceptance of intuitionistic fuzzy objectives and constraints, and to minimize the degree of rejection of intuitionistic objectives and constraints, we can write

Max $\mu_i(\bar{v})$, $\bar{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r, $\qquad$ (15)

Min $v_i(\bar{v})$, $\bar{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r , $\qquad$ (16)

Subject to

$v_i(\bar{v}) \geq 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$\mu_i(\bar{v}) + v_i(\bar{v}) \leq 1$ i = 1, 2, …, r_1, r_1+1, …, r,

$\bar{v} \geq \bar{0}$.

For the defined membership function $\mu_i(\bar{v})$, the flexible membership goals having the aspired level unity can be presented as follows:

$\mu_i(\bar{v}) + d_{i1}^- - d_{i1}^+ = 1$, i = 1, 2, …, r_1, r_1+1, …, r $\qquad$ (17)

For the case of rejection (non-membership), we can write

$v_i(\bar{v}) + d_{i2}^- - d_{i2}^+ = 0$, i = 1, 2, …, r_1, r_1+1, …, r $\qquad$ (18)

Since decision making unit wants to minimize the degree of rejection and maximize the degree of acceptance, IFGP can be formulated as:

IFGP model-1

Min λ $\qquad$ (19)

Subject to

$\mu_i(\bar{v}) + d_{i1}^- - d_{i1}^+ = 1$, i = 1, 2, …, r_1, r_1+1, …, r,

$v_i(\bar{v}) + d_{i2}^- - d_{i2}^+ = 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$\mu_i(\bar{v}) + v_i(\bar{v}) \leq 1$ i = 1, 2, …, r_1, r_1+1, …, r,

$\lambda \geq d_{i1}^-$, i = 1, 2, …, r_1, r_1+1, …, r,

$\lambda \geq d_{i2}^+$, i = 1, 2, …, r_1, r_1+1, …, r,

$d_{i1}^- \times d_{i1}^+ = 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$d_{i2}^- \times d_{i2}^+ = 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$d_{i1}^- \geq 0$, $d_{i1}^+ \geq 0$, $d_{i2}^- \geq 0$, $d_{i2}^+ \geq 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$\bar{v} \geq \bar{0}$.

Model (IIa)

The minimization of the sum of the weighted deviation form:

Min $\eta = \sum_{i=1}^{r} w_{i1}^- d_{i1}^- + \sum_{i=1}^{r} w_{i2}^+ d_{i2}^+$ $\qquad$ (20)

Subject to

$\mu_i(\bar{v}) + d_{i1}^- - d_{i1}^+ = 1$, i = 1, 2, …, r_1, r_1+1, …, r,

$v_i(\bar{v}) + d_{i2}^- - d_{i2}^+ = 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$\mu_i(\overline{v}) + v_i(\overline{v}) \le 1$ i = 1, 2, …, r_1, r_1+1, …, r,

$d_{i1}^{-} \times d_{i1}^{+} = 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$d_{i2}^{-} \times d_{i2}^{+} = 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$w_{i1}^{-} \ge 0$, $w_{i2}^{+} \ge 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$d_{i1}^{-} \ge 0$, $d_{i1}^{+} \ge 0$, $d_{i2}^{-} \ge 0$, $d_{i2}^{+} \ge 0$, i = 1, 2, …, r_1, r_1+1, …, r,

$\overline{v} \ge \overline{0}$.

Model (IIb)
The minimization of the sum of the deviation form:

Min $\zeta = (\sum\limits_{i=1}^{r} d_{i1}^{-} + \sum\limits_{i=1}^{r} d_{i2}^{+})$ (21)

Subject to the same set of constraints (20).

Here, d_{i1}^{-}, and d_{i2}^{+}, are deviational variables. The numerical weights w_{i1}^{-}, w_{i2}^{+} associated with d_{i1}^{-}, d_{i2}^{+} represent the relative importance of achieving the aspired level of the respective intuitionistic fuzzy goal subject to the given set of constraints. To assess the relative importance of the intuitionistic fuzzy goals, the weighting scheme suggested by Pramanik and Roy [29] can be used to assign the values of w_{i1}^{-}, w_{i2}^{+}.

Formulation of the neutrosophic goal programming
Neutrosophic optimization problem can be represented as follows:
To maximize the degree of acceptance (truth) of neutrosophic objectives and constraints, to minimize the degree of indeterminacy and to minimize the degree of rejection (falsity) of neutrosophic objectives and constraints:

Max $\mu_i(\overline{v})$, $\overline{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r,   (22)

Min $\omega_i(\overline{v})$, $\overline{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r,

Min $v_i(\overline{v})$, $\overline{v} \in \Re^k$, i = 1, 2, …, r_1, r_1+1, …, r,

Subject to

$\mu_i(\overline{v}) + \omega_i(\overline{v}) + v_i(\overline{v}) \le 3$, i = 1, 2, …, r_1, r_1+1, …, r,

$\mu_i(\overline{v}) \in [0, 1]$, i = 1, 2, …, r_1, r_1+1, …, r,

$\omega_i(\overline{v}) \in [0, 1]$, i = 1, 2, …, r_1, r_1+1, …, r,

$v_i(\overline{v}) \in [0, 1]$, i = 1, 2, …, r_1, r_1+1, …, r,

$\overline{v} \ge \overline{0}$.

where $\mu_i(\overline{v})$ denotes the degree of membership of $\overline{v}$ to the i-th SVNS and $v_i(\overline{v})$ denotes the degree of rejection of functions $\overline{v}$ from the i-th SVNS.
Conjunction of SVNSs is defined by

$G \cap C = \{\langle \overline{v}, \mu_G(\overline{v}) \wedge \mu_C(\overline{v}), \omega_G(\overline{v}) \vee \omega_C(\overline{v}), v_G(\overline{v}) \vee v_C(\overline{v}) \rangle | \overline{v} \in \Re^k\}$,   (23)

Here G represents a neutrosophic objective function and C represents neutrosophic constraint. This conjunction operator can be easily generalized and applied to the neutrosophic optimization problem:

$D = \{\langle \overline{v}, \mu_D(\overline{v})), v_D(\overline{v}) \rangle | \overline{v} \in \Re^k\}$, $\mu_D(\overline{v}) = \mathop{\wedge}\limits_{i=1}^{r} \mu_i(\overline{v})$, $\omega_D(\overline{v}) = \mathop{\vee}\limits_{i=1}^{r} \omega_i(\overline{v})$,

$v_D(\overline{v}) = \mathop{\vee}\limits_{i=1}^{r} v_i(\overline{v})$ (24)

where D represents a single valued neutrosophic set based representation of the decision.
Min-operator is used for conjunction and max-operator for disjunction:

$\mu_D(\overline{v}) = \mathop{\wedge}\limits_{i=1}^{r} \mu_i(\overline{v})$, $\overline{v} \in \Re^k$, $\omega_D(\overline{v}) = \mathop{\vee}\limits_{i=1}^{r} \omega_i(\overline{v})$, $\overline{v} \in \Re^k$, $v_D(\overline{v}) = \mathop{\vee}\limits_{i=1}^{r} v_i(\overline{v})$, $\overline{v} \in \Re^k$.

(25)

Therefore, $\mu_D(\bar{v}) \leq \mu_i(\bar{v}), \omega_D(\bar{v}) \geq \omega_i(\bar{v}), \nu_D(\bar{v}) \geq \nu_i(\bar{v}), i = 1, 2, \ldots, r_1, r_1+1, \ldots, r$. (26)

where $\mu_i(\bar{v})$ denotes the degree of membership of $\bar{v}$ to the i-th SVNS, $\omega_i(\bar{v})$ denotes the degree of indeterminacy, and $\nu_i(\bar{v})$ denotes the degree of rejection of functions $\bar{v}$ from the i-th SVNS.

NGP Model (I).
Minimize λ (27)
Subject to

$\mu_i(\bar{v}) + d_{i1}^- - d_{i1}^+ = 1, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\omega_i(\bar{v}) + d_{i2}^- - d_{i2}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\nu_i(\bar{v}) + d_{i3}^- - d_{i3}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\lambda \geq d_{i1}^-, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\lambda \geq d_{i2}^+, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\lambda \geq d_{i3}^+, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\mu_i(\bar{v}) + \omega_i(\bar{v}) + \nu_i(\bar{v}) \leq 3, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i1}^- \geq 0, d_{i2}^- \geq 0, d_{i3}^- \geq 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i1}^- \times d_{i1}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i2}^- \times d_{i2}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i3}^- \times d_{i3}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\mu_i(\bar{v}) \in [0, 1], \quad i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\omega_i(\bar{v}) \in [0, 1], i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\nu_i(\bar{v}) \in [0, 1], i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\bar{v} \geq \bar{0}$.

NGP Model (IIa)

Min $\eta = \sum_{i=1}^{r} w_{i1}^- d_{i1}^- + \sum_{i=1}^{r} w_{i2}^+ d_{i2}^+ + \sum_{i=1}^{r} w_{i3}^+ d_{i3}^+$ (28)

Subject to

$\mu_i(\bar{v}) + d_{i1}^- - d_{i1}^+ = 1, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\omega_i(\bar{v}) + d_{i2}^- - d_{i2}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\nu_i(\bar{v}) + d_{i3}^- - d_{i3}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\mu_i(\bar{v}) + \omega_i(\bar{v}) + \nu_i(\bar{v}) \leq 3, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i1}^- \times d_{i1}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i2}^- \times d_{i2}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i3}^- \times d_{i3}^+ = 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$d_{i1}^- \geq 0, d_{i1}^+ \geq 0, d_{i2}^- \geq 0, d_{i2}^+ \geq 0, d_{i3}^- \geq 0, d_{i3}^+ \geq 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$w_{i1}^- \geq 0, w_{i2}^+ \geq 0, w_{i3}^+ \geq 0, i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\mu_i(\bar{v}) \in [0, 1], i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$\omega_i(\bar{v}) \in [0, 1], i = 1, 2, \ldots, r_1, r_1+1, \ldots, r,$

$v_i(\bar{v}) \in [0, 1]$, $i = 1, 2, \ldots, r_1, r_1+1, \ldots, r$,

$\bar{v} \geq \bar{0}$.

NGP Model (IIb). (29)

Min $\eta = \sum\limits_{i=1}^{r} d_{i1}^{-} + \sum\limits_{i=1}^{r} d_{i2}^{+} + \sum\limits_{i=1}^{r} d_{i3}^{+}$

Subject to the same set of constraints (28).

Here d_{i1}^{-}, d_{i2}^{-}, d_{i3}^{-}, d_{i1}^{+}, d_{i2}^{+}, d_{i3}^{+} are deviational variables. The numerical weights w_{i1}^{-}, w_{i2}^{+}, w_{i3}^{+} associated with

d_{i1}^{-}, d_{i2}^{+}, d_{i3}^{+} represent the relative importance of achieving the aspired level of the respective neutrosophic goal subject to the given set of constraints. To assess the relative importance of the neutrosophic goals, the weighting scheme suggested by Pramanik and Roy [29] can be used to assign the values of w_{i1}^{-}, w_{i2}^{+}, w_{i3}^{+}.

CONCLUSION

This paper presents framework of neutrosophic linear goal programming problem. Three new intuitionistic fuzzy goal programming models have been presented. The proposed intuitionistic fuzzy goal programming models have been also extended to neutrosophic linear goal programming models. The essence of the proposed neutrosophic linear goal programming is that it is capable of dealing with indeterminacy and falsity simultaneously. Abdel-Baset et al. [51] presented goal programming models in 2016. However, in their study they maximize indeterminacy which is not realistic in decision making context. In this paper the definition of intersection of two single valued neutrosophic sets due to Salama and Alblowi [52] has been employed and direction of research in neutrosophic optimization problem has been proposed. The author hopes that the proposed framework of neutrosophic linear goal programming will open up new avenue of research in the field of optimization problems in neutrosophic environment. Many areas need to be explored and developed in neutrosophic goal programming especially priority structure of neutrosophic goals and priority based neutrosophic linear goal programming.

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