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# Neutrosophic Parameterized Soft Relations and Their Applications

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**Abstract.** The aim of this paper is to introduce the concept of relation on neutrosophic parameterized soft set (NP- soft sets) theory. We have studied some related properties and also put forward some propositions on neutrosophic parameterized soft relation with proofs and examples. Finally the notions of symmetric, transitive, reflexive, and equivalence neutrosophic parameterized soft set relations have been established in our work. Finally a decision making method on NP-soft sets is presented.

**Keywords:** Soft set, neutrosophic parameterized soft set, NP-soft relations.

## 1. Introduction

Neutrosophic set theory was introduced in 1995 with the study of Smarandache [21] as mathematical tool for handling problem involving imprecise, indeterminacy and inconsistent data. The concept of neutrosophic set generalizes the concept of fuzzy sets [22], intuitionistic fuzzy sets [1] and so on. In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and falsity-membership are independent. Neutrosophic set theory has successfully used in logic, economics, computer science, decision making process and so on.

The concept of soft set theory is another mathematical theory dealing with uncertainty and vagueness, developed by Russian researcher [20]. The soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Many interesting results of soft set theory have been studied by embedding the ideas of fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets and so on. For example; fuzzy soft sets [3,9,17], on intuitionistic fuzzy soft set theory [10,18], on possibility intuitionistic fuzzy soft set [2], on neutrosophic soft set [19], on intuitionistic neutrosophic soft set [4,7], on generalized neutrosophic soft set [5], on interval-valued neutrosophic soft set [6], on fuzzy parameterized soft set theory [14,15,16], on intuitionistic fuzzy parameterized soft set theory [12], on IFP−fuzzy soft set theory [13], on fuzzy parameterized fuzzy soft set theory [11].

Later on, Broumi et al. [8] defined the neutrosophic parameterized soft sets (NP-soft sets) which is a generalization of fuzzy parameterized soft sets (FP-soft sets) and intuitionistic fuzzy parameterized soft sets (IFP-soft sets).

In this paper our main objective is to extend the concept relations on FP-soft sets[14] to the case of NP-soft sets. The paper is structured as follows. In Section 2, some basic definition and preliminary results are given which will be used in the rest of the paper. In Section 3, we define relations on NP-soft sets and some of its algebraic properties are studied. In section 4, we present decision making method on NP-soft relations. Finally we conclude the paper.

## 2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of neutrosophic set, soft set and neutrosophic parameterized soft set. For more details, the reader could refer to [8,20,21].

**Definition 2.1.** [21] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\},$$

where the functions $\mu$, $\nu$, $\omega$ : U→]$^{-}$0,1$^{+}$[ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^{-}0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^{+} \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of ]−0,1$^{+}$[. So instead of ]$^{-}$0,1$^{+}$[ we need to take the interval [0,1] for technical applications, because ]$^{-}$0,1$^{+}$[ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$$A_{NS} = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x)\rangle | x \in X\}$$

And

$$B_{NS} = \{\langle x, \mu_B(x), \nu_B(x), \omega_B(x)\rangle | x \in X\}$$

Then,

1. $A_{NS} \subseteq B_{NS}$ if and only if

$\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x)$ and $\omega_A(x) \geq \omega_B(x).$

2. $A_{NS} = B_{NS}$ if and only if,

$\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x)$ for any $x \in X$ .

3. The complement of $A_{NS}$ is denoted by $A^{o}_{NS}$ and is defined by

$$A^{\circ}_{NS} = \{\langle x, \omega_A(x), 1-\nu_A(x), \mu_A(x)\rangle | x \in X\}$$

4. $$A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_A(x)\}, \max\{\omega_A(x), \omega_B(x)\}\rangle : x \in X\}$$

5. $$A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_A(x)\}, \max\{\omega_A(x), \omega_B(x)\}\rangle : x \in X\}$$

As an illustration, let us consider the following example.

**Example 2.2.** Assume that the universe of discourse U=$\{x_1, x_2, x_3\}$. It may be further assumed that the values of $x_1$, $x_2$ and $\mathbf{x_3}$ are in [0, 1] Then, A is a neutrosophic set (NS) of U, such that,

$$A = \{\langle x_1, 0.7, 0.5, 0.2\rangle, \langle x_2, 0.4, 0.5, 0.5\rangle, \langle x_3, 0.4, 0.5, 0.6\rangle\}$$

**Definition 2.3.**[8] Let *U* be an initial universe, *P (U)* be the power set of *U*, *E* be a set of all parameters and *K* be a ne trosophic set over *E*. Then a neutrosophic parameterized soft sets

$$\psi_K = \{(\langle x, \mu_K(x), \nu_K(x), \omega_K(x)\rangle, f_K(x)) : x \in E\}$$

where $\mu_K : E \to [0, 1]$, $\nu_K : E \to [0, 1]$, $\omega_K : E \to [0, 1]$ and $f_K : E \to P(U)$ such that $f_K(x) = \Phi$ if $\mu_K(x) = 0$, $\nu_K(x) = 1$ and $\omega_K(x) = 1$.

Here, the function $\mu_K$, $\nu_K$ and $\omega_K$ called membership function, indeterminacy function and non-membership function of neutrosophic parameterized soft set (NP-soft set), respectively.

**Example 2.4.** Assume that $U = \{u_1, u_2, u_3\}$ is a universal set and $E = \{x_1, x_2\}$ is a set of parameters. If

$$K = \{\langle x_1, 0.7, 0.3, 0.4\rangle, \langle x_2, 0.7, 0.5, 0.4\rangle\}$$

and

$$f_K(x_1) = \{u_2, u_5\},\ f_K(x_2) = U.$$

Then a neutrosophic parameterized soft set $\Psi_k$ is written by

$$\psi_K = \{(\langle x_1, 0.7, 0.3, 0.4\rangle, \{u_2, u_5\}), (\langle x_2, 0.7, 0.5, 0.4\rangle, U\}$$

## 3. Relations on the NP-Soft Sets

In this section, after given the cartesian products of two NP- soft sets, we define a relations on NP- soft sets and study their desired properties.

**Definition 3.1.** Let $\psi_K, \Omega_L \in NPS(U)$. Then, a Cartesian product of $\psi_K$ and $\Omega_L$, denoted by $\psi_K \hat{\times} \Omega_L$, is defined as;

$$\psi_K \hat{\times} \Omega_L = \{\langle (x, y), \mu_{K\hat{\times}L}(x, y), \nu_{K\hat{\times}L}(x, y), \omega_{K\hat{\times}L}(x, y)\rangle, f_{K\hat{\times}L}(x, y) : (x, y) \in E \hat{\times} E\}$$

Where

$$f_{K\hat{\times}L}(x, y) = f_K(x) \cap f_L(y)$$

and

$$\mu_{K\hat{\times}L}(x, y) = \min\{\mu_K(x), \mu_L(y)\}$$
$$\nu_{K\hat{\times}L}(x, y) = \max\{\nu_K(x), \nu_L(y)\}$$
$$\omega_{K\hat{\times}L}(x, y) = \max\{\omega_K(x), \omega_L(y)\}$$

Here $\mu_{K\hat{\times}L}(x, y), \nu_{K\hat{\times}L}(x, y), \omega_{K\hat{\times}L}(x, y)$ is a t-norm.

**Example 3.2.** Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}\}$,

$E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$,

$$K = \{\langle x_2, 0.5, 0.6, 0.3\rangle, \langle x_3, 0.3, 0.2, 0.9\rangle, \langle x_5, 0.6, 0.7, 0.3\rangle, \langle x_6, 0.1, 0.4, 0.6\rangle, \langle x_7, 0.7, 0.5, 0.3\rangle\}$$

and

$$L = \{\langle x_1, 0.5, 0.6, 0.3\rangle, \langle x_2, 0.5, 0.6, 0.3\rangle, \langle x_4, 0.9, 0.8, 0.1\rangle, \langle x_8, 0.3, 0.2, 0.9\rangle\}$$

be to neutrosophic sets of *E*. Suppose that

$$\begin{aligned}\psi_K = \{&(\langle x_2, 0.5, 0.6, 0.3\rangle, \{u_1, u_2, u_4, u_5, u_7, u_8, u_{10}, u_{12}, u_{14}, u_{15}\}),\\ &(\langle x_3, 0.3, 0.2, 0.9\rangle, \{u_2, u_5, u_8, u_{11}, u_{15}\}),\\ &(\langle x_5, 0.6, 0.7, 0.3\rangle, \{u_2, u_3, u_4, u_7, u_8, u_{11}, u_{12}, u_{15}\}),\\ &(\langle x_6, 0.1, 0.4, 0.6\rangle, \{u_2, u_4, u_6, u_7, u_{10}, u_{12}\}),\\ &(\langle x_7, 0.7, 0.5, 0.3\rangle, \{u_2, u_5, u_6, u_8, u_9, u_{13}, u_{15}\})\}\end{aligned}$$

and

$$\begin{aligned}\Omega_L = \{&(\langle x_1, 0.7, 0.4, 0.6\rangle, \{u_1, u_5, u_6, u_9, u_{10}, u_{13}\}),\\ &(\langle x_2, 0.5, 0.6, 0.3\rangle, \{u_1, u_2, u_4, u_5, u_7, u_8, u_{10}, u_{12}, u_{14}, u_{15}\}),\\ &(\langle x_4, 0.9, 0.8, 0.1\rangle, \{u_2, u_5, u_9, u_{10}, u_{11}, u_{14}\}),\\ &(\langle x_8, 0.4, 0.7, 0.2\rangle, \{u_2, u_5, u_8, u_{10}, u_{12}, u_{14}\})\}\end{aligned}$$

Then, the Cartesian product of $\psi_K$ and $\Omega_L$ is obtained as follows;

$$\begin{aligned}\psi_K \hat{\text{x}} \Omega_L = \{&(\langle (x_2,x_1),0.5,0.6,0.6\rangle,\{u_1,u_5,u_{10}\}),\\ &(\langle (x_2,x_2),0.5,0.6,0.3\rangle,\\ &\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\})\\ &(\langle (x_2,x_4),0.5,0.8,0.3\rangle,\{u_2,u_5,u_{10},u_{14}\}),\\ &(\langle (x_2,x_8),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8,u_{10},u_{12},u_{14}\}),\\ &(\langle (x_3,x_1),0.3,0.4,0.9\rangle,\{u_5\}),\\ &(\langle (x_3,x_2),0.3,0.6,0.9\rangle,\{u_2,u_5,u_8,u_{15}\}),\\ &(\langle (x_3,x_4),0.3,0.8,0.9\rangle,\{u_2,u_5,u_{11}\}),\\ &(\langle (x_3,x_8),0.3,0.7,0.9\rangle,\{u_2,u_5,u_8\}),\\ &(\langle (x_5,x_1),0.6,0.7,0.6\rangle,\varnothing),\\ &(\langle (x_5,x_2)0.5,0.7,0.3\rangle,\{u_2,u_4,u_7,u_8,u_{12},u_{15}\}),\\ &(\langle (x_5,x_4),0.6,0.8,0.3\rangle,\{u_2,u_{11}\}),\\ &(\langle (x_5,x_8),0.4,0.7,0.3\rangle,\{u_2,u_8,u_{12}\}),\\ &(\langle (x_6,x_1),0.1,0.4,0.6\rangle,\{u_6,u_{10}\}),\\ &(\langle (x_6,x_2),0.1,0.6,0.6\rangle,\{u_2,u_4,u_7,u_{10},u_{12}\}),\\ &(\langle (x_6,x_4),0.1,0.8,0.6\rangle,\{u_2,u_{10}\}),\\ &(\langle (x_6,x_8),0.1,0.7,0.6\rangle,\{u_2,u_{10},u_{12}\}),\\ &(\langle (x_7,x_1),0.7,0.5,0.6\rangle,\{u_5,u_6,u_9,u_{13}\}),\\ &(\langle (x_7,x_2),0.5,0.6,0.3\rangle,\{u_2,u_5,u_8,u_{15}\}),\\ &(\langle (x_7,x_4),0.7,0.8,0.3\rangle,\{u_2,u_5,u_9\}),\\ &(\langle (x_7,x_8),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8\})\}\end{aligned}$$

**Definition 3.3.** Let $\psi_K, \Omega_L \in NPS(U)$. Then, a *NP*-soft relation from $\psi_K$ to $\Omega_L$, denoted by $R_N$, is a *NP*-soft subset of $\psi_K \hat{\text{x}} \Omega_L$. Any *NP*-soft subset of $\psi_K \hat{\text{x}} \Omega_L$ is called a *NP*-soft relation on $\psi_K$.

Note that if $\alpha = (\langle x, \mu_K(x), \nu_K(x), \omega_K(x)\rangle,\ f_K(x)) \in \psi_K$ and $\beta = (\langle y, \mu_L(y), \nu_L(y), \omega_L(y)\rangle,\ f_L(y)) \in \Omega_L$, then

$$\alpha R_N \beta \Leftrightarrow \Big(\Big\langle (x,y), \mu_{K\hat{X}L}(x,y), \nu_{K\hat{X}L}(x,y), \omega_{K\hat{X}L}(x,y)\Big\rangle,\ f_{K\hat{X}L}(x,y) \in R_N\Big)$$

Where $f_{K\hat{x}L}(x,y) = f_K(x) \cap f_L(y)$.

**Example 3.4.** Let us consider the Example 3.2. Then, we define a *NP*-soft relation, from $\psi_K$ to $\Omega_L$, as follows

$$\alpha R_N \beta \Leftrightarrow \Big(\Big\langle (x_i,y_j), \mu_{K\hat{X}L}(x_i,y_j), \nu_{K\hat{X}L}(x_i,y_j), \omega_{K\hat{X}L}(x_i,y_j)\Big\rangle,\ f_{K\hat{X}L}(x_i,y_j)\Big)(1 \le i, j \le 8)$$

Such that

$$\mu_{K\hat{X}L}(x_i,y_j) \ge 0.3$$
$$\nu_{K\hat{X}L}(x_i,y_j) \le 0.5$$
$$\omega_{K\hat{X}L}(x_i,y_j) \le 0.7$$

Then

$$\begin{aligned}R_N = \{&(\langle (x_2,x_1),0.5,0.6,0.6\rangle,\{u_1,u_5,u_{10}\}),\\ &(\langle (x_2,x_2),0.5,0.6,0.3\rangle,\\ &\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\})\\ &(\langle (x_2,x_4),0.5,0.8,0.3\rangle,\{u_2,u_5,u_{10},u_{14}\}),\\ &(\langle (x_2,x_8),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8,u_{10},u_{12},u_{14}\}),\\ &(\langle (x_5,x_2),0.5,0.7,0.3\rangle,\{u_2,u_4,u_7,u_8,u_{12},u_{15}\}),\\ &(\langle (x_5,x_4),0.6,0.8,0.3\rangle,\{u_2,u_{11}\}),\\ &(\langle (x_5,x_8),0.4,0.7,0.3\rangle,\{u_2,u_8,u_{12}\}),\\ &(\langle (x_7,x_1),0.7,0.5,0.6\rangle,\{u_5,u_6,u_9,u_{13}\}),\\ &(\langle (x_7,x_2),0.5,0.6,0.3\rangle,\{u_2,u_5,u_8,u_{15}\}),\\ &(\langle (x_7,x_4),0.7,0.8,0.3\rangle,\{u_2,u_5,u_9\}),\\ &(\langle (x_7,x_8),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8\})\}.\end{aligned}$$

**Definition 3.5.** Let $\psi_K$ , $\Omega_L \in NPS(U)$ and $R_N$ be *NP*-soft relation from $\psi_K$ to $\Omega_L$. Then domain and range of $R_N$ respectively is defined as;

$$D(R_N)=\{\alpha\in\psi_K:\alpha R_N\beta\}$$
$$R(R_N)=\{\beta\in\Omega_L:\alpha R_N\beta\}.$$

**Example 3.6.** Let us consider the Example 3.4

$$D(R_N)=\{(\langle x_2,0.5,0.6,0.3\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\}),(\langle x_3,0.3,0.2,0.9\rangle,\{u_2,u_5,u_8,u_{11},u_{15}\}),(\langle x_5,0.6,0.7,0.3\rangle,\{u_2,u_3,u_4,u_7,u_8,u_{11},u_{12},u_{15}\}),(\langle x_6,0.1,0.4,0.6\rangle,\{u_2,u_4,u_6,u_7,u_{10},u_{12}\}),(\langle x_7,0.7,0.5,0.3\rangle,\{u_2,u_5,u_6,u_8,u_9,u_{13},u_{15}\})\}$$

$$R(R_N)=\{(\langle x_1,0.7,0.4,0.6\rangle,\{u_1,u_5,u_6,u_9,u_{10},u_{13}\}),(\langle x_2,0.5,0.6,0.3\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\}),(\langle x_4,0.9,0.8,0.1\rangle,\{u_2,u_5,u_9,u_{10},u_{11},u_{14}\}),(\langle x_8,0.4,0.7,0.2\rangle,\{u_2,u_5,u_8,u_{10},u_{12},u_{14}\})\}$$

**Definition 3.7.** Let $R_N$ be a *NP*-soft relation from $\psi_K$ to $\Omega_L$. Then, inverse of $R_N$ , $R_N^{-1}$ from $\psi_K$ to $\Omega_L$ is a *NP*-soft relation defined as;

$$\alpha R_N^{-1}\beta=\beta R_N\alpha$$

**Example 3.8.** Let us consider the Example 4.4. Then $R_N^{-1}$ is from $\psi_K$ to $\Omega_L$ is obtained by

$$R_N^{-1}=\{(\langle(x_1,x_2),0.5,0.6,0.6\rangle,\{u_1,u_5,u_{10}\}),(\langle(x_2,x_2),0.5,0.6,0.3\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8,u_{10},u_{12},u_{14},u_{15}\})(\langle(x_4,x_2),0.5,0.8,0.3\rangle,\{u_2,u_5,u_{10},u_{14}\}),(\langle(x_8,x_2),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8,u_{10},u_{12},u_{14}\}),(\langle(x_2,x_5),0.5,0.7,0.3\rangle,\{u_2,u_4,u_7,u_8,u_{12},u_{15}\}),(\langle(x_4,x_5),0.6,0.8,0.3\rangle,\{u_2,u_{11}\}),(\langle(x_8,x_5),0.4,0.7,0.3\rangle,\{u_2,u_8,u_{12}\}),(\langle(x_1,x_7),0.7,0.5,0.6\rangle,\{u_5,u_6,u_9,u_{13}\}),(\langle(x_2,x_7),0.5,0.6,0.3\rangle,\{u_2,u_5,u_8,u_{15}\}),(\langle(x_4,x_7),0.7,0.8,0.3\rangle,\{u_2,u_5,u_9\}),(\langle(x_8,x_7),0.4,0.7,0.3\rangle,\{u_2,u_5,u_8\})\}.$$

**Proposition 3.9.** Let $R_{N_1}$ and $R_{N_2}$ be two NP-soft relation. Then

1. $\left(R_{N_1}^{-1}\right)^{-1}=R_{N_1}$
2. $R_{N_1}\subseteq R_{N_2}\Rightarrow R_{N_1}^{-1}\subseteq R_{N_2}^{-1}$

**Proof:**

1. $\alpha\left(R_{N_1}^{-1}\right)^{-1}\beta=\beta R_{N_1}^{-1}\alpha=\alpha R_{N_1}\beta$
2. $\alpha R_{N_1}\beta\subseteq\alpha R_{N_2}\beta\Rightarrow\beta R_{N_1}^{-1}\alpha\subseteq\beta R_{N_2}^{-1}\alpha$
$\Rightarrow R_{N_1}^{-1}\subseteq R_{N_2}^{-1}$

**Definition 3.10.** If $R_{N_1}$ **and** $R_{N_2}$ are two NP- soft relation from $\psi_K$ to $\Omega_L$ , then a compostion of two *NP*-soft relations $R_{N_1}$ and $R_{N_2}$ is defined by

$$\alpha\left(R_{N_1}oR_{N_2}\right)\gamma=\left(aR_{N_1}\beta\right)\wedge\left(\beta R_{N_2}\gamma\right)$$

**Proposition 3.11.** Let $R_{N_1}$ and $R_{N_2}$ be two *NP*-soft relation from $\psi_K$ to $\Omega_L$.

Then, $\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}{}^{-1} o R_{N_1}{}^{-1}$

**Proof:**

$$\begin{aligned}\alpha\left(R_{N_1} o R_{N_2}\right)^{-1}\gamma = \gamma\left(R_{N_1} o R_{N_2}\right)\alpha \\ =\left(\gamma R_{N_1}\beta\right)\wedge\left(\beta R_{N_2}\alpha\right) \\ =\left(\beta R_{N_2}\alpha\right)\wedge\left(\gamma R_{N_1}\beta\right) \\ =\left(\alpha R_{N_2}{}^{-1}\beta\right)\wedge\left(\beta R_{N_1}{}^{-1}\gamma\right) \\ =\alpha\left(R_{N_2}{}^{-1} o R_{N_1}{}^{-1}\right)\gamma\end{aligned}$$

Therefore we obtain

$$\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}{}^{-1} o R_{N_1}{}^{-1}$$

**Definition 3.12.** A $NP-soft$ relation $R_N$ on $\psi_K$ is said to be a NP-soft symmetric relation

If $\alpha R_{N_1}\beta \Rightarrow \beta R_{N_1}\alpha,\ \forall \alpha,\beta\in\psi_K.$

**Definition 3.13.** A $NP-soft$ relation $R_N$ on $\psi_K$ is said to be a *NP*-soft transitive relation

if $R_N o R_N \subseteq R_N$,

that is,

$\alpha R_N \beta$ and $\beta R_N \gamma \Rightarrow \alpha R_N \gamma, \forall \alpha,\beta,\gamma\in\psi_K.$

**Definition 3.14.** A $NP-soft$ relation $R_N$ on $\psi_K$ is said to be a *NP*-soft reflexive relation

If $\alpha R_N \alpha,\ \forall\alpha\in\psi_K.$

**Definition 3.15.** A *NP*-soft relation $R_N$ on $\psi_K$ is said to be a *NP*-soft equivalence relation if it is symmetric, transitive and reflexive.

**Example 3.16.** Let

$U=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$,

$E=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$ and

$X=\{\langle x_1,0.5,0.4,0.7\rangle,\langle x_2,0.6,0.8,0.4\rangle,$
$\langle x_3 0.2,0.5,0.1\rangle\}.$

Suppose that

$$\begin{aligned}\psi_K=\{&(\langle x_1,0.5,0.4,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\&(\langle x_2 0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\&(\langle x_3,0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

Then, a cartesian product on $\psi_K$ is obtained as follows.

$$\begin{aligned}\psi_K\hat{\mathrm{x}}\psi_K=\{&(\langle(x_1,x_1),0.5,0.4,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\&(\langle(x_1,x_2),0.5,0.8,0.7\rangle,\{u_2,u_6,u_8\}),\\&(\langle(x_1,x_3),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\&(\langle(x_2,x_1),0.5,0.8,0.7\rangle,\{u_2,u_6,u_8\}),\\&(\langle(x_2,x_2),0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\&(\langle(x_2,x_3),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\&(\langle(x_3,x_1),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\&(\langle(x_3,x_2),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\&(\langle(x_3,x_3),0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

Then, we get a neutrosophic parameterized soft relation $R_N$ on $\psi_K$ as follows

$$\alpha R_N \beta \Leftrightarrow \Big(\Big\langle (x_i, y_j), \mu_{K\hat{X}L}(x_i, y_j), \nu_{K\hat{X}L}(x_i, y_j), \omega_{K\hat{X}L}(x_i, y_j)\Big\rangle, f_{K\hat{X}L}(x_i, y_j)\Big)(1 \le i, j \le 8)$$

Where

$\mu_{K\hat{X}L}(x_i, y_j) \ge 0.3$

$\nu_{K\hat{X}L}(x_i, y_j) \le 0.5$

$\omega_{K\hat{X}L}(x_i, y_j) \le 0.7$

Then

$$\begin{aligned} R_N = \{ & (\langle (x_1, x_1), 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\}), \\ & (\langle (x_1, x_2), 0.5, 0.8, 0.7\rangle, \{u_2, u_6, u_8\}), \\ & (\langle (x_2, x_1), 0.5, 0.8, 0.7\rangle, \{u_2, u_6, u_8\}), \\ & (\langle (x_2, x_2), 0.6, 0.8, 0.4\rangle, \{u_2, u_6, u_8\})\}. \end{aligned}$$

$R_N$ on $\psi_K$ is an NP-soft equivalence relation because it is symmetric, transitive and reflexive.

**Proposition 3.17.** If $R_N$ is symmetric, if and only if $R_N^{-1}$ is so.

**Proof:** If $R_N$ is symmetric, then

$aR_N^{-1}\beta = \beta R_N \alpha = \alpha R_N \beta = \beta R_N^{-1}\alpha.$ So, $R_N^{-1}$ is symmetric.

Conversely, if $R_N^{-1}$ is symmetric,

then

$\alpha R_N^{-1}\beta = \alpha (R_N^{-1})^{-1}\beta = \beta (R_N^{-1})\alpha = \alpha (R_N^{-1})\beta = \beta R_N \alpha$

So, $R_N$ is symmetric.

**Proposition 3.18.** $R_N$ is symmetric if and only if $R_N^{-1} = R_N$.

**Proof:** If $R_N$ is symmetric, then

$\alpha R_N^{-1}\beta = \beta R_N \alpha = \alpha R_N \beta.$ So, $R_N^{-1} = R_N$.

Conversely, if $R_N^{-1} = R_N$, then

$\alpha R_N \beta = \alpha R_N^{-1}\beta = \beta R_N \alpha.$ So, $R_N$ is symmetric .

**Proposition 3.19.** If $R_{N_1}$ and $R_{N_2}$ are symmetric relations on $\psi_K$, then $R_{N_1} o R_{N_2}$ is symmetric on $\psi_K$ if and only if $R_{N_1} o R_{N_2} = R_{N_2} o R_{N_1}$

**Proof:** If $R_{N_1}$ and $R_{N_2}$ are symmetric, then it implies $R_{N_1}^{-1} = R_{N_1}$ and $R_{N_2}^{-1} = R_{N_2}$. We have

$\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}^{-1} o R_{N_1}^{-1}.$

Then $R_{N_1} o R_{N_2}$ is symmetric. It implies

$R_{N_1} o R_{N_2} = \left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}^{-1} o R_{N_1}^{-1} = R_{N_2} o R_{N_1}.$

Conversely,

$\left(R_{N_1} o R_{N_2}\right)^{-1} = R_{N_2}^{-1} o R_{N_1}^{-1} = R_{N_2} o R_{N_1} = R_{N_1} o R_{N_2}.$

So, $R_{N_1} o R_{N_2}$ is symmetric.

**Corollary 3.20. If** $R_N$ is symmetric, then $R^n{}_N$ is symmetric for all positive integer n, where $R^n{}_N = \underbrace{R_N o R_N o \ldots . o R_N}_{n \text{ times}}$.

**Proposition 3.21.** If $R_N$ is transitive, then $R_N^{-1}$ is also transitive.

**Proof:**

$$\begin{aligned} \alpha R_N^{-1}\beta = \beta R_N \alpha &\supseteq \beta\left(R_N o R_N\right)\alpha \\ &= \left(\beta R_N \alpha\right) \wedge \left(\gamma R_N \alpha\right) \\ &= \left(\gamma R_N \alpha\right) \wedge \left(\beta R_N \alpha\right) \\ &= \left(\alpha R_N^{-1}\gamma\right) \wedge \left(\gamma R_N^{-1}\beta\right) \\ &= \alpha\left(R_N^{-1} o R_N^{-1}\right)\beta \end{aligned}$$

So, $R_N^{-1} o R_N^{-1} \subseteq R_N^{-1}.$ The proof is completed.

**Proposition 3.22.** If $R_N$ is reflexive, then $R_N^{-1}$ is so.

**Proof:** $aR_N^{-1}\beta = \beta R_N \alpha \subseteq \alpha R_N \alpha = \alpha R_N^{-1}\alpha$ and

$\beta R_N^{-1}\alpha = aR_N \beta \subseteq \alpha R_N \alpha = \alpha R_N^{-1}\alpha.$

The proof is completed.

**Proposition 3.23.** If $R_N$ is symmetric and transitive, then $R_N$ is reflexive.

**Proof:** Its clearly.

**Definition 3.24.** Let $\psi_K \in NPS(U)$, $R_N$ be an $NP-soft$ equivalence relation on $\psi_K$ and $\alpha \in R_N$. Then, an equivalence class of $\alpha$, denoted by $[\alpha]_{R_N}$, is defined as

$$[\alpha]_{R_N} = \{\beta : \alpha R_N \beta\}.$$

**Example 3.25.** Let us consider the Example 3. 16 Then an equivalence class of $(\langle x_1, 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\})$ will be as follows.

$$[(\langle x_1, 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\})]_{R_N} =$$
$$\{(\langle x_1, 0.5, 0.4, 0.7\rangle, \{u_2, u_3, u_5, u_6, u_7, u_8\}),$$
$$(\langle x_2 0.6, 0.8, 0.4\rangle, \{u_2, u_6, u_8\}),$$
$$(\langle x_3, 0.2, 0.5, 0.1\rangle, \{u_1, u_2, u_4, u_5, u_7, u_8\})\}$$

## 4. Decision Making Method

In this section, we construct a soft neutrosophication operator and a decision making method on $NP-soft$ relations.

**Definition 4.1.** Let $\psi_K \in NPS(U)$ and $R_N$ be a NP-soft relation on $\psi_K$. The neutrosophication operator, denoted by $sR_N$, is defined by

$$sR_N : R_N \to F(U),\ sR_N(X \times X, U) = \{\langle u, \mu_{R_N}(u), \nu_{R_N}(u), \omega_{R_N}(u)\rangle, u \in U\}$$

Where

$$\mu_{R_N}(u) = \frac{1}{|X \times X|}\sum_i \sum_j \mu_{R_N}(x_i, x_j)\chi(u)$$
$$\nu_{R_N}(u) = \frac{1}{|X \times X|}\sum_i \sum_j \nu_{R_N}(x_i, x_j)\chi(u)$$
$$\omega_{R_N}(u) = \frac{1}{|X \times X|}\sum_i \sum_j \omega_{R_N}(x_i, x_j)\chi(u)$$

and where

$$\chi(u) = \begin{cases} 1, & u \in f_{R_N}(x_i, x_j) \\ 0, & u \notin f_{R_N}(x_i, x_j) \end{cases}$$

Note that $|X \times X|$ is the cardinality of $X \times X$.

**Definition 4.2.** Let $\Psi_K \in$ NP-soft set and $sR_N$ a neutrosophication operator, then a reduced fuzzy set of $\tilde{\psi}_K$ is a *fuzzy* set over U denoted by

$$\tilde{\psi}_K(u) = \left\{ \frac{\mu_{sR_N}(u)}{u} : u \in U \right\}$$

Where $\mu_{sR_N} : U \to [0,1]$ and

$$\mu_{sR_N}(u) = \frac{|\mu_K(u) + \nu_K(u) - \omega_K(u)|}{2}$$

Now; we can construct a decision making method on $NP-soft$ relation by the following algorithem;

1. construct a feasible neutrosophic subset $X$ over $E$,
2. construct a $NP-soft\ set\ \psi_K$ over $U$,
3. construct a $NP-soft$ relation $R_N$ over $\psi_K$ according to the requests,
4. calculate the neutrosophication operator $sR_N$ over $R_N$,
5. calculate the reduced fuzzy set $\tilde{\psi}_K$
6. select the objects from $\tilde{\psi}_K$, which have the largest membership value.

**Example 4.3.** A customer, Mr. *X*, comes to the auto gallery agent to buy a car which is over middle class. Assume that an auto gallery agent has a set of different types of car $U=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8\}$, which may be characterized by a set of parameters $E=\{x_1,x_2,x_3,x_4\}$. For $i=1,2,3,4$ the parameters $x_i$ stand for ty", "cheap", "modern" and "large", respectively. If Mr. *X* has to consider own set of parameters, then we select a car on the basis of the set of customer parameters by using the algorithm as follows.

1. Mr. *X* constructs a neutrosophic set *X* over *E*,
X= {<x$_1$,0.5, 0.4 ,0.7>,<x$_2$,0.6, 0.8 ,0.4>,<x$_3$,0.2, 0.5 ,0.1>}
2. Mr. *X* constructs a $NP-soft$ set $\psi_K$ over $U$,

$$\psi_K=\{(\langle x_1,0.5,0.4,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\ (\langle x_2,0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\ (\langle x_3,0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}$$

3. The neutrosophic parameterized soft relation $R_N$ over $\psi_K$ is calculated according to the Mr X's request (The car must be a over middle class, it means the membership degrees are over 0.5 ),

$$\begin{aligned}R_N=\{&(\langle(x_1,x_1),0.4,0.6,0.7\rangle,\{u_2,u_3,u_5,u_6,u_7,u_8\}),\\ &(\langle(x_1,x_2),0.4,0.6,0.7\rangle,\{u_2,u_6,u_8\}),\\ &(\langle(x_1,x_3),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\ &(\langle(x_2,x_1),0.4,0.6,0.7\rangle,\{u_2,u_6,u_8\})\\ &(\langle(x_2,x_2),0.6,0.8,0.4\rangle,\{u_2,u_6,u_8\}),\\ &(\langle(x_2,x_3),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\ &(\langle(x_3,x_1),0.2,0.5,0.7\rangle,\{u_2,u_5,u_7,u_8\}),\\ &(\langle(x_3,x_2),0.2,0.8,0.4\rangle,\{u_2,u_8\}),\\ &(\langle(x_3,x_3),0.2,0.5,0.1\rangle,\{u_1,u_2,u_4,u_5,u_7,u_8\})\}\end{aligned}$$

4. The soft neutrosophication operator $s_{R_N}$ over $R_N$ calculated as follows

$$\begin{aligned}sR_N=\{&\langle u_1,0.022,0.055,0.011\rangle,\\ &\langle u_2,0.311,0.633,0.533\rangle,\langle u_3,0.044,0.066,0.077\rangle,\\ &\langle u_4,0.022,0.055,0.011\rangle,\langle u_5,0.111,0.233,0.244\rangle,\\ &\langle u_6,0.2,0.288,0.277\rangle,\langle u_7,0.133,0.322,0.288\rangle,\\ &\langle u_8,0.311,0.633,0.533\rangle\}\end{aligned}$$

5. Reduced fuzzy set $\tilde{\psi}_K$ calculated as follows

$$\tilde{\psi}_K(u)=\left\{\frac{0.033}{u_1},\frac{0.205}{u_2},\frac{0.016}{u_3},\frac{0.033}{u_4},\frac{0.05}{u_5},\frac{0.105}{u_6},\frac{0.083}{u_7},\frac{0.205}{u_8};\ u\in U\right\}$$

6. Now, Mr. *X* select the optimum car u$_2$ and u$_8$ which have the biggest membership degree 0.205 among the other cars.

## 5. Conclusion

In this work, we have defined relation on NP-soft sets and studied some of their properties. We also defined symmetric, transitive, reflexive and equivalence relations on the

NP-soft sets. Finally, we construct a decision making method and gave an application which shows that this method successfully works. In future work, we will extend this concept to interval valued neutrosophic parameterized soft sets.

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