ALGEBRAIC STRUCTURES ON REAL AND NEUTROSOPHIC SEMI OPEN SQUARES

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PREFACE

Here for the first time we introduce the semi open square using modulo integers. Authors introduce several algebraic structures on them. These squares under addition modulo 'n' is a group and however under product × this semi open square is only a semigroup as under × the square has infinite number of zero divisors. Apart from + and × we define min and max operation on this square. Under min and max operation this semi real open square is a semiring.

It is interesting to note that this semi open square is not a ring under + and × since $a \times (b + c) \neq a \times b + a \times c$ in general for all a, b and c in that semi open square. So we define the new type of ring call pseudo ring. Finally we define S-vector spaces and S-pseudo linear algebras using them. This concept will in due course of time find lots of applications. Several open problems are suggested for interested researchers.

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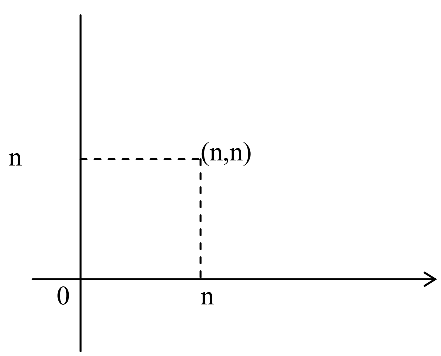
Chapter One

ALGEBRAIC STRUCTURES USING SEMI OPEN REAL SQUARE $\{(a, b) \mid a, b \in [0, n)\}$

In this chapter authors for the first time introduce algebraic structures on the square which is open in two sides and closed in the other two sides. To be more precise the real square of side n, n > 1 is of the form



The elements of the form (n, n), (n, a), (a, n) are not in the real square.

We will illustrate this situation by some examples.

Example 1.1: Let M = $\{(a, b) \mid a, b \in [0, 2)\}$ be the real semi open square of side length less than two.

Example 1.2: Let P = $\{(a, b) \mid a, b \in [0, 3)\}$ be the real semi open square of side length less than three.

Example 1.3: Let T = $\{(a, b) \mid a, b \in [0, 12)\}$ be the real semi open square whose side length is less than twelve.

Clearly $(0, 12) \notin T$. $(0, 11.999) \in T$.

We define operations on these new types of squares.

Example 1.4: Let A = $\{(a, b) \mid a, b \in [0, 11)\}$ be a real semi open square.

Now let $x = (5, 7)$ and $y = (6, 4) \in A$.

$x + y = (5, 7) + (6, 4) = (11, 11) = (0, 0)$.

x is the inverse of y with respect to the operation addition +.

Let $x = (2, 9)$ and $y = (10, 5) \in A$.
$x + y = (2, 9) + (10, 5) = (12, 14) = (1, 3) \in A$.

Further '+' is a closed operation on the semi open square A and is side length < 11. '+' is commutative for $x + y = y + x$ for all $x, y \in A$.

Let $(9, 10) \in A$ we have $(0, 0) + (9, 10) = (9, 10) \in A$.

(0, 0) is additive identity of A.

If $x = (3.1, 1.52) \in A$ then $y = (7.9, 9.48) \in A$ is such that

$$\begin{aligned} x + y &= (3.1, 1.52) + (7.9, 9.48) \\ &= (3.1 + 7.9, 1.52 + 9.48) \\ &= (11, 11) = (0, 0). \end{aligned}$$

So $x \in A$ is the inverse of $y \in A$ thus $x + y = (0, 0)$.

Example 1.5: Let $A = \{(x, y) \mid x, y \in [0, 4)\}$ be the semi open real square of side length less than four.

Let $Y = (0.378, 1.37)$ and $X = (3.622, 2.63) \in A$.

$$\begin{aligned} \text{We see } X + Y &= (0.378, 1.37) + (3.622, 2.63) \\ &= (0.378 + 3.622, 1.37 + 2.63) \\ &= (0, 0). \end{aligned}$$

$$\begin{aligned} Y + Y &= (0.378, 1.37) + (0.378 + 1.37) \\ &= (0.756, 2.74) \in A. \end{aligned}$$

$$\begin{aligned} X + X &= (3.622, 2.63) + (3.622, 2.63) \\ &= (7.244, 5.26) \\ &= (3.244, 1.26) \in A. \end{aligned}$$

A is a group under + modulo 4. A is commutative and is of infinite order.

A has also subgroups of finite order.

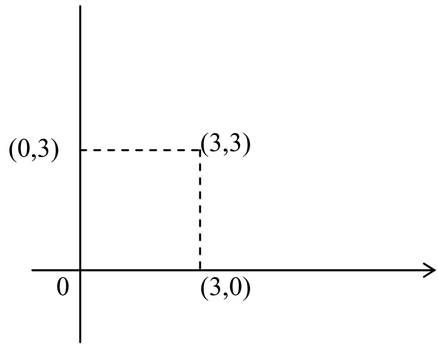
Example 1.6: Let $B = \{(a, b) \mid a, b \in [0, 7)\}$ be the semi open square of side less than 7. $(B, +)$ is a group of infinite order. $P_1 = \{(a, 0) \mid a \in [0, 7)\} \subseteq B$ is a subgroup of infinite order.

$P_2 = \{(0, b) \mid b \in [0, 7)\} \subseteq B$ is again a subgroup of infinite order.

$T = \{(a, b) \mid a, b \in Z_7\} \subseteq B$ is a subgroup of B of finite order.

$W = \{(a, b) \mid a, b \in \{0, 0.5, 1, 1.5, 2, 2.5, \ldots, 6, 6.5\} \subseteq [0, 7)$ is again a subgroup of finite order.

Example 1.7: Let $M = \{(a, b) \mid a, b \in [0, 3)\}$ be a semi open square group under + of finite order.



Let x = (0.31, 2.15) ∈ M.
x + x = (0.31, 2.15) + (0.31, 2.15)
= (0.62, 1.30) ∈ M.

Now –x = (2.69, 0.85).

This is the way operations are performed on M.

x + (–x) = (0, 0).

Inview of all these we have the following theorem.

THEOREM 1.1: *Let S = {(a, b) | a, b ∈ [0, n), 1 < n < ∞} be the real semi open square. (S, +) is an abelian group of infinite order.*

(i) S has subgroups of finite order.
(ii) S has subgroups of infinite order.

Proof is direct and hence left as an exercise to the reader.

Example 1.8: Let P = {(a, b) | a, b ∈ [0, 24)} be the semi open group under addition of infinite order. B = {(a, b) | a, b ∈ Z_{24}} ⊆ P be the finite subgroup of P.

M_1 = {(a, 0) | a ∈ Z_{24}} ⊆ P is a subgroup of finite order.

$M_2 = \{(0, a) \mid a \in Z_{24}\} \subseteq P$ is also a subgroup of finite order.

$M_3 = \{(a, 0) \mid a \in \{0, 2, 4, 6, …, 22\} \subseteq Z_{24}\} \subseteq P$ is a subgroup of finite order.

$M_4 = \{(0, a) \mid a \in \{0, 2, 4, …, 22\} \subseteq Z_{24}\} \subseteq P$ is a subgroup of finite order.

$T_1 = \{(a, 0) \mid a \in [0, 24)\} \subseteq P$ is also a subgroup of infinite order.

We see $A = \{(a, b) \mid a, b \in [0, n)\}$ is a group under +.

Consider product on $A = \{(a, b) \mid a, b \in [0, 10)\}$; the square of side less than 10.

Let $x = (5.3, 3.111)$ and $y = (2, 7) \in A$.
$x \times y = (5.3, 3.111) \times (2, 7) = (0.6, 1.777) \in P$.

Clearly × is a closed associative operation on A.

However (1, 1) acts as the multiplicative identity.

Let $A = \{(a, b) \mid a, b \in [0, 6)\}$.
Let $x = (0, 3)$ and $y = (5, 2) \in A$.
$x \times y = (0, 0)$.

Let $x = (5, 5) \in A$ we see $x^2 = (1, 1)$ is a unit in A. Thus A has zero divisors and has units.

Example 1.9: Let $S = \{(a, b) \mid a, b \in [0, 7)\}$ be the semi open square semigroup under product. S has subsemigroups of both finite and infinite order. S has zero divisors, no idempotents other than $\{(0, 0), (1, 0), (0, 1), (1, 1)\}$. S has units for $X = \{(2, 5), (1, 3), (3, 1), …\} = \{(a, b) \mid a, b \in Z_7 \setminus \{0\}\}$ are units of S.

Example 1.10: Let $M = \{(a, b) \mid a, b \in [0, 12)\}$ be the semi open square semigroup under product of infinite order. M has zero divisors and units. P = {(a, 0) |a $\in$ [0, 12)} is a subsemigroup of infinite order.

T = {(a, b) | a, b $\in$ Z_{12} $\subseteq$ [0, 12)} $\subseteq$ M is a subsemigroup of finite order. T has zero divisors of infinite order.

T has finite number of idempotents given by x = (4, 1) with x^2 = x and y = (1, 9) $\in$ M is such that y^2 = y and so on.

The study of zero divisors is an interesting feature of these semi open square semigroups under product.

Example 1.11: Let T = {(a, b) | a, b $\in$ [0, 43)} be a real semi open semigroup under product.

P = {(a, b) | a, b $\in$ Z_{43} \ {0}} $\subseteq$ T is a subsemigroup and is a subgroup under product so T is a Smarandache semigroup (S-semigroup).

Example 1.12: Let M = {(a, b) | a, b $\in$ [0, 11)} be a real semi open semigroup under product.

N = {(a, b) | a, b $\in$ Z_{11} \ {0}} $\subseteq$ M is a subsemigroup. N is a group.

Let x = (3, 8), y = (4, 7) $\in$ N;
x $\times$ y = (3, 8) $\times$ (4, 7) = (1, 1) $\in$ N.

Let x = (8, 3) and y = (7, 4) $\in$ N is such that
x $\times$ y = (1, 1) $\in$ N.

Let x = (1, 10) $\in$ N is such that x $\times$ x = (1, 1) is a unit y = (9, 2) and z = (5, 6) $\in$ N is such that
y $\times$ z = (9, 2) (5, 6) = (1, 1) $\in$ N is a unit of N.

Example 1.13: Let B = {(a, b) | a, b $\in$ [0, 13)} be the semi open square semigroup under '$\times$'.

$V = \{(x, y) \mid x, y \in Z_{13} \setminus \{0\}\} \subseteq B$; V is a group under product. Thus B is a S-semigroup. B has zero divisors, infact B has infinite number of zero divisors. No idempotents other than $\{(0, 0), (1, 0), (0, 1), (1, 1)\}$.

THEOREM 1.2: *Let $S = \{(a, b) \mid a, b \in [0, n)\}$ be semi open square semigroup under product. S is a S-semigroup if and only if Z_n is a S-semigroup.*

Proof: S is a semigroup if Z_n is a S-semigroup then there exists a subset $P \subseteq Z_n$ such that P is a group. $M = \{(a, b) \mid a, b \in P\}$ is a group, hence S is a S-semigroup. If S is a S-semigroup then S has a subset which is a group.

Hence the claim.

Example 1.14: Let $S = \{(a, b) \mid a, b \in [0, 14)\}$ be the real semi open square.

Let $(S, \times)$ be the semigroup of infinite order.

Let $P = \{2, 4, 6, 8, 10, 12\} \subseteq Z_{14}$; the table of P under product is as follows:

×	2	4	6	8	10	12
2	4	8	12	2	6	10
4	8	2	10	4	12	6
6	12	10	8	6	4	2
8	2	4	6	8	10	12
10	6	12	4	10	2	8
12	10	6	2	12	8	4

$P \setminus \{0\}$ is the group with 8 as the identity. So $[0, 14)$ is a S-semigroup.

Now $L = \{(a, b) \mid a, b \in P\}$ is a group under product.

Hence S is a S-semigroup.

THEOREM 1.3: *Let S = $\{(a, b) \mid a, b \in [0, n), n < \infty\}$ be the semi open square semigroup under product. S is always a S-semigroup.*

Proof : Follows from the fact that T = {1, n–1} is a group under product in $Z_n \subseteq [0, n)$. Hence Z_n is a S-semigroup. Now P = {(1, 1), (1, n–1), (n–1, 1), (n–1, n–1)} $\subseteq$ S is a subsemigroup which is a group.

Hence S is a S-semigroup.

Example 1.15: Let M = $\{(a, b) \mid a, b \in [0, 9)\}$ be the semigroup under product. P = {(1, 1), (1, 8), (8, 1), (8, 8)} $\subseteq$ M is a group of order four given by the following table.

×	(1,1)	(1,8)	(8,1)	(8,8)
(1,1)	(1,1)	(1,8)	(8,1)	(8,8)
(1,8)	(1,8)	(1,1)	(8,8)	(8,1)
(8,1)	(8,1)	(8,8)	(1,1)	(1,8)
(8,8)	(8,8)	(8,1)	(1,8)	(1,1)

Thus M has subgroups of finite order. Thus M is a S-semigroup.

Example 1.16: Let S = $\{(a, b) \mid a, b \in [0, Z_{12}), \times\}$ be the semi open real square semigroup. S is a S-semigroup of infinite order. S has infinite number of zero divisors but has only finite number of idempotents.

Now we can build more groups using S = $\{(a, b) \mid a, b \in [0, n)\}$ under + and more real semigroups using {S, ×}. All these will be illustrated by some examples.

Example 1.17: Let
T = $\{(a_1, a_2, a_3) \mid a_i \in \{(c_i, d_i) \mid c_i, d_i \in [0, 7)\}, 1 \leq i \leq 3\}$ be the collection of all row matrices with entries from the real semi open square of side less than 7. (T, +) is a group.

Let x = {(3, 2.1), (1.5, 2.7), (5.21, 3.12)}
and y = {(0, 5.12), (3.712, 0), (0, 6.15)} $\in$ T.

x + y = {(3, 0.22), (5.212, 2.7), (5.21, 2.27)} $\in$ T.

Now x – y = {(3, 2.1 – 5.12), (1.5 – 3.712, 2.7), (5.21, 3.12 – 6.15)}
= {(3, 2.1 + 1.88), (1.5 + 3.288, 2.7), (5.21, 3.12 + 0.85)}
= {(3, 3.98) (4.788, 2.7) (5.21, 3.97) $\in$ T.

Let x = {(3, 2.1), (1.5, 2.7), (5.21, 3.12)
–x = {(–3, –2.1), (–1.5, –2.7), (–5.21, –3.21)}
= {(4, 4.9), (5.5, 4.3), (1.79, 3.79)} $\in$ T, for x $\in$ T we have {(0, 0), (0, 0), (0, 0)} $\in$ T is such that x + {(0, 0), (0, 0), (0, 0)} = x.

Now for every x $\in$ T we have a unique –x $\in$ T such that x + (–x)
= ((0, 0), (0, 0), (0, 0))

Thus (T, +) is the real semi open square matrix group.

Example 1.18: Let M = {((a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4), (a_5, b_5), (a_6, b_6)} | a_i, b_i $\in$ [0, 10), $1 \leq i \leq 6$} be the real square row matrix real semi open square group under addition.

M is of infinite order. M has subgroups of both finite and infinite order.

Example 1.19: Let

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix} \right| a_i \in \{(a, b) \mid a, b \in [0, 16)\};\ 1 \leq i \leq 12 \right\}$$

be the semi open real square group of column matrices with entries from the square of size whose side is less than 16.

M has subgroups of finite order as well as infinite order.

Example 1.20: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 15)\}; 1 \leq i \leq 15 \right\}$$

be the real square semi open matrix group under addition. S has both finite and infinite order subgroups.

Example 1.21: Let
$T = \{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7\ a_8\ a_9\ a_{10}) \mid a_i \in [0, 24), 1 \leq i \leq 10\}$ be the real semi open square super matrix group under addition. T has subgroups of both finite and infinite order.

$P_1 = \{(a_1 \mid 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0\ 0) \mid a_1 \in Z_{24}\} \subseteq T$ is a subgroup of finite order. $P_1 \cong Z_{24}$.

$P_2 = \{(0 \mid a_2\ 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0\ 0) \mid a_2 \in Z_{24}\} \subseteq T$ is a subgroup of order 24.

$P_3 = \{(0 \mid 0\ a_3\ 0 \mid 0\ 0 \mid 0\ 0\ 0\ 0) \mid a_3 \in Z_{24}\} \subseteq T$ is a subgroup of order 24.

$P_4 = \{(0 \mid 0\ 0\ a_4 \mid 0\ 0 \mid 0\ 0\ 0\ 0) \mid a_4 \in Z_{24}\} \subseteq T$ is a subgroup of order 24.

$P_5 = \{(0 \mid 0\ 0\ 0 \mid a_5\ 0 \mid 0\ 0\ 0\ 0) \mid a_5 \in Z_{24}\} \subseteq T$ is a subgroup of order 24 so on.

$P_{10} = \{(0 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ a_{10}) \mid a_{10} \in Z_{24}\} \subseteq T$ is a subgroup of order 24.

$P_{1,2} = \{(a_1 \mid a_2\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_1, a_2 \in Z_{24}\} \subseteq T$ is a subgroup of order.

$$o\ (P_{1,2}) = 24 \times 24.$$

$P_{1,3} = \{(a_1 \mid 0\ a_3\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_1, a_3 \in Z_{24}\} \subseteq T$ is a subgroup of order.

$$o\ (P_{1,3}) = 24 \times 24 \text{ and so on.}$$

$P_{1,10} = \{(a_1 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ a_{10}) \mid a_1, a_{10} \in Z_{24}\} \subseteq T$ is a subgroup of order.

$$o\ (P_{1,10}) = 24 \times 24.$$

$P_{2,3} = \{(0 \mid a_2\ a_3\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_2, a_3 \in Z_{24}\} \subseteq T$ is a subgroup of order.

$$o\ (P_{2,3}) = 24 \times 24.$$

$P_{2,4} = \{(0 \mid a_2\ 0\ a_4 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_2, a_4 \in Z_{24}\} \subseteq T$ is a subgroup of order 24×24.

$P_{2,10} = \{(0 \mid a_2\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ a_{10}) \mid a_2, a_{10} \in Z_{24}\} \subseteq T$ is a subgroup of order 24×24.

Likewise $P_{3,4}, \ldots, P_{3,10}, P_{4,5}, P_{4,6}, \ldots, P_{4,10}, P_{5,6}, P_{5,7}, \ldots, P_{5,10}, \ldots, P_{9,10}$ are all subgroups of finite order and order of them is 24×24.

$P_{1,2,3} = \{(a_1 \mid a_2\ a_3\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_1, a_2, a_3 \in Z_{24}\} \subseteq T$ is a subgroup of order $24 \times 24 \times 24$; likewise $P_{1,2,4}, P_{1,2,5}, P_{1,2,6}, P_{1,2,7}, \ldots, P_{1,2,10}, P_{2,3,4}, P_{2,3,5}, \ldots, P_{2,3,10}$ and so on are all subgroups of order 24^3.

Likewise we can get subgroup of order $(24)^4$, $(24)^6$, $(24)^5$, … and so on $(24)^{10}$.

Thus we have several subgroups of finite order.

Let $P_{1,2,3,4} = \{(a_1 \mid a_2\ a_3\ a_4 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_i \in Z_{24}, 1 \leq i \leq 4\} \subseteq T$ is a subgroup of order $(24)^4$.

Now $B_1 = \{(a_1 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_1 \in [0, 24)\} \subseteq T$ is a subgroup of infinite order.

$B_2 = \{(0 \mid a_2\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_2 \in [0, 24)\} \subseteq T$ is a subgroup of infinite order and so on.

$B_{10} = \{(0 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ a_{10}) \mid a_{10} \in [0, 24)\} \subseteq T$ is a subgroup of infinite order.

$B_{1,2} = \{(a_1 \mid a_2\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ 0) \mid a_1, a_2 \in [0, 24)\} \subseteq T$ is a subgroup of infinite order.

$B_{1,10} = \{(a_1 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ a_{10}) \mid a_1, a_{10} \in [0, 24)\} \subseteq T$ is a subgroup of infinite order and so on.

$B_{9,10} = \{(0 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ a_9\ a_{10}) \mid a_9, a_{10} \in [0, 24)\} \subseteq T$ is a subgroup of infinite order.

Thus T has several subgroups of infinite order.

We can also write T as a direct sum of subgroups in several ways.

T has atleast ${}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9$ number of subgroups of infinite order and atleast same number of subgroups of finite order.

Now we can have L = $\{(a_1 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0\ 0\ a_{10}) \mid a_1 \in Z_{24}$ and $a_{10} \in [0, 24)\} \subseteq T$ is a also a different type of subgroup of infinite order.

Example 1.22: Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \end{array}\right] \middle|\ a_i \in [0, 19);\ 1 \leq i \leq 11 \right\}$$

be a real semi open square super column matrix group of infinite order under +.

M has several subgroups of finite order and several infinite subgroups.

However if [0, 19) is replaced by [0, 18) we have more number of subgroups of finite order.

Example 1.23: Let

$$W = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \end{array} \right] \middle| \; a_i \in [0, 24); 1 \leq i \leq 33, + \right\}$$

be the real semi open square group of super column matrices $|W| = \infty$.

W has several subgroups of finite and infinite order.

Since 24 is a composite number which is not a prime W has still more number of subgroups of finite order.

Example 1.24: Let

$$V = \left\{ \left(\begin{array}{c|cc|c} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{array} \right) \middle| \; a_i \in [0, 7); 1 \leq i \leq 8 \right\}$$

be the row super matrix group on the semi open real square. V is of infinite cardinality.

V has subgroups both of finite and infinite order.

Example 1.25: Let

$$S = \left\{ \left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ \hline a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array} \right] \right| a_i \in [0, 43); 1 \leq i \leq 43\}$$

be the super matrix group on the real semi open square under addition. S has subgroups of infinite and finite order.

As 43 is a prime S has only less number of finite subgroups.

Now having seen subgroups we proceed onto introduce and study semigroups built on the real squares of length less than n.

Example 1.26: Let S = $\{(a, b) \mid a, b \in [0, 12), \times\}$ be the semigroup. S has infinite number of zero divisors, finite number of units and only finite number of idempotents.

$\{(0, 0), (1, 1), (1, 0), (0, 1), (0, 4), (4, 0), (4, 4), (1, 4), (4, 1), (9, 0), (9, 1), (9, 9), (0, 9), (1, 9), (4, 9), (9, 4)\}$ are the idempotents of [0, 12).

P = $\{(a, 0) \mid a \in [0, 12)\}$ gives an infinite collection of zero divisors.

Infact if x = (0.32, 0) then all $y \in T = \{(0, b) \mid b \in [0, 12)\}$ is a zero divisor of x.

Thus for one x in S we have in some cases infinite number of y's in S such that $x \times y = y \times x = (0, 0)$.

S is an infinite semigroup with infinite number of zero divisors. However S has only a finite number of units given by (1, 11), (11, 1), (11, 11) and so on.

Example 1.27: Let S = $\{(a, b) \mid a, b \in [0, 6), \times\}$ be a real semi open square semigroup under product over real square.

S is of infinite order and has infinite number of zero divisors.

P = $\{(a, b) \mid a, b \in Z_6\} \subseteq S$ is a finite subsemigroup of S.

M = $\{(a, b) \mid a, b \in \{0, 2, 4\}\} \subseteq S$ is again a finite subsemigroup of S.

T = $\{(a, 0) \mid a \in [0, 6)\} \subseteq S$ is an infinite subsemigroup of S.

L = $\{(a, b) \mid a \in Z_6, b \in [0, 6)\}$ is a subsemigroup of infinite order.

E = {(0, 0), (1, 0), (0, 1), (1, 1), (3, 3), (3, 0), (3, 1), (1, 3), (0, 3), (4, 4), (4, 3), (3, 4), (4, 0), (4, 1), (1, 4), (0, 4)} are idempotents of S and they are only finite in number. W = {(1, 1), (5, 5), (5, 1), (1, 5)} are the only units of the semigroup S.

Example 1.28: Let M = $\{(a, b) \mid a, b \in [0, 12), \times\}$ be the semigroup of infinite order; M is a monoid.

Let x = (3, 6) and y = (4, 2) $\in$ M.
$x \times y = (3, 6) \times (4, 2) = (0, 0)$.

Example 1.29: Let T = $\{(a, b) \mid a, b \in [0, 53), \times\}$ be the real semi open square semigroup of infinite order.

(x, y) $\in$ T. ($x \neq 0, y \neq 0$) is not a zero divisor in T. The only type of zero divisors in T are of the form (a, 0) and (0, b); a, b $\in$ [0, 53).

Example 1.30: Let L = {(a, b) | a, b $\in$ [0, 42)} be the real semi open square semigroup of infinite order. L has zero divisors of the form (x, y) (x $\neq$ 0, y $\neq$ 0, x, y $\in$ [0, 58)).

For A = (21, 2) and B = (2, 21) in L is such that
A $\times$ B = (0, 0).

Example 1.31: Let M = {(a, b) | a, b $\in$ [0, 53)} be the semigroup under product. Let (x, y) = t and (u, v) = s $\in$ M; (x, y, u, v $\in$ [0, 53) \ {0}). Clearly st $\neq$ (0, 0).

Inview of all these we have the following theorem.

THEOREM 1.4: *Let S = {(a, b) | a, b $\in$ [0, n), $\times$} be the semi open real square semigroup under product.*

(i) If n = p, a prime then (x, y) is not a zero divisor if x, y $\in$ [0, n) \ {0}.
(ii) If n is a non prime (x, y) can be a zero divisor for some x, y $\in$ [0, n) \ {0}.
(iii) S has infinite number of zero divisors of the form (x, 0) and (0, y) even if n is a prime.
(iv) S has only finite number of units.
(v) S has no idempotents if n is a prime other than {(0, 0), (1, 0), (0, 1), (1, 1)}.

Proof is direct and hence left as an exercise to the reader.

Example 1.32: Let M = {(a, b) | a, b $\in$ [0, 49), $\times$} be the real semi open square semigroup.

M has infinite number of zero divisors but only finite number of nilpotents.

Example 1.33: Let T = {(a, b) | a, b $\in$ [0, 12), $\times$} be the real semi open square semigroup under product.

M = {(a, 0) | a $\in$ [0, 12)} $\subseteq$ T is an ideal of T.

$N = \{(0, b) \mid b \in [0, 12)\} \subseteq T$ is also an ideal of T.

However no subsemigroup of finite order is an ideal of T. Every ideal in T is of infinite order.

Example 1.34: Let $M = \{(a, b) \mid a, b \in [0, 25), \times\}$ be a semigroup under product of infinite order. M has zero divisors which are of infinite order.

M has finite number of nilpotents of order two and finite number of idempotents. Study in this direction is interesting.

Inview of this we state the following theorem.

THEOREM 1.5: *Let $S = \{(a, b) \mid a, b \in [0, n)\}$ be a real semi open square semigroup under product. S has only ideals of infinite order.*

The proof is direct and hence left as an exercise to the reader.

Example 1.35: Let $S = \{(a, b) \mid a, b \in [0, 23), \times\}$ be the real semi open square semigroup under product. S has no idempotents other than $\{(0, 0), (1, 1), (1, 0), (0, 1)\}$. S has 22×22 number of units. S has ideals of infinite order given by $P_1 = \{(0, a) \mid a \in [0, 23)\}$ and $P_2 = \{(a, 0) \mid a \in [0, 23)\}$.

Clearly $P_i \cap P_j = (0, 0)$ if $i \neq j$, $1 \leq i, j \leq 2$.

Example 1.36: Let $S = \{(a, b) \mid a, b \in [0, 12)\}$ be the real semi open square semigroup under product. S has no ideals of finite order. $P = \{(a, b) \mid a, b \in Z_{12}\}$ is a finite subsemigroup.

$M = \{(1, 1), (1, 5), (5, 1), (5, 5), (7, 1), (1, 7), (5, 7), (7, 5), (7, 7), (1, 9), (9, 1), (9, 9), (9, 5), (9, 7), (5, 9), (7, 9), (11, 1), (1, 11), (5, 11), (11, 5), (7, 11), (11, 7), (9, 11), (11, 9), (11, 11)\}$ are the number of units in S. S has also only finite number of idempotents.

Example 1.37: Let M = {(a, b) | a, b $\in$ [0, 15), $\times$} be the real semi open square semigroup under product. M has infinite number of subsemigroups. M has only finite number of units and idempotents.

A = {(4, 4), (1, 1), (11, 11), (14, 14), (1, 4), (4, 1), (11, 1), (1, 11), (4, 11), (11, 4)} are some of the units of M.

Example 1.38: Let P = {(a, b) | a, b $\in$ [0, 81), $\times$} be the real semi open square semigroup under product. P is of infinite order. P has only finite number of nilpotents. P has infinite number of zero divisors only finite number of idempotents.

Example 1.39: Let S = {(a, b) | a, b $\in$ [0, 90), $\times$} be the real semi open square semigroup under $\times$. S has infinite number of zero divisors.

Example 1.40: Let S_1 = {(a, b) | a, b $\in$ [0, 19), $\times$} be the real semi open square semigroup under $\times$. S_1 has no idempotents other than {(0, 0), (1, 0), (0, 1), (1, 1)}. S has 18 $\times$ 18 number of units.

A = {(a, b) | a, b $\in Z_{19} \setminus \{0\}$, $\times$} is the collection of units which is also a group; so S_1 is a Smarandache semigroup and o(A) = 18 $\times$ 18.

Example 1.41: Let S = {(a, b) | a, b $\in$ [0, 41), $\times$} be the semigroup under $\times$. S has no idempotents only zero divisors. S has 40 $\times$ 40 number of units. S is a S-semigroup.

Example 1.42: Let S = {(a, b) | a, b $\in$ [0, 22), $\times$} be the S-semigroup of infinite order.

P = {(1, 1), (21, 21), (21, 1), (1, 21)} $\subseteq$ S is a subsemigroup of S which is a group of order 4. (9, 5) = x and y = (5, 9) $\in$ S are such that xy = (1, 1).

Example 1.43: Let S = $\{(a, b) \mid a, b \in [0, 72), \times\}$ be the S-semigroup. S has m units and m is less than of 71×71 units.

Example 1.44: Let S = $\{(a, b) \mid a, b \in [0, 61), \times\}$ be the S-semigroup. S has only 60×60 number of units. But S has only a few number of idempotents.

Example 1.45: Let S = $\{(a, b) \mid a, b \in [0, 31), \times\}$ be the S-semigroup. S has only 30×30 number of units. No idempotents other than $\{(0, 0), (1, 1), (1, 0), (0, 1)\}$.

Now we construct matrix semigroups using the semigroup $S = \{(a, b) \mid a, b \in [0, n), \times\}$.

Example 1.46: Let
$M = \{(x_1, x_2, x_3, x_4, x_5) \mid x_i \in \{(a, b) \mid a, b \in [0, 12), 1 \leq i \leq 5\}$ be the semigroup under $\times$. M has infinite number of zero divisors.

M has finite number of idempotents. M has only finite number of units. $X = \{(1, 1), (1, 1), (1, 1), (1, 1), (1, 1)\} \in M$ is the identity element of M.

Example 1.47: Let $M = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \mid a_i \in \{(a, b) \mid a, b \in [0, 19), 1 \leq i \leq 8\}$ be the semigroup under $\times$. M has several units.

$P = \{(a_1, a_2, a_3, \ldots, a_8) \mid a_i \in \{(a, b) \mid a, b \in Z_{19} \setminus \{0\}\}, 1 \leq i \leq 8\}$ be the group and is of finite order. Thus M is a S-semigroup.

Example 1.48: Let

$$T = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix}\right| a_i \in \{(a, b) \mid a, b \in [0, 42)\}; 1 \leq i \leq 12\right\}$$

be the semigroup under the natural product $\times_n$. T is a S-semigroup.

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix} \right| a_i \in \{(a, b) \mid a, b \in \{1, 41\}\}; 1 \leq i \leq 12\} \subseteq T$$

is a group. T is a S-semigroup and $|P| < \infty$.

Example 1.49: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \end{bmatrix} \right| a_i \in \{(a, b) \mid a, b \in [0, 23)\}; 1 \leq i \leq 28\}$$

be the semigroup under $\times_n$ (natural product).

$$A = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \end{bmatrix} \right| a_i \in \{(a, b) \mid a, b \in Z_{23} \setminus \{0\}\}; 1 \leq i \leq 28\} \subseteq M$$

is a group under $\times_n$ of finite order.

$$\begin{bmatrix} (1,1) & (1,1) & (1,1) & (1,1) \\ (1,1) & (1,1) & (1,1) & (1,1) \\ \vdots & \vdots & \vdots & \vdots \\ (1,1) & (1,1) & (1,1) & (1,1) \end{bmatrix} \in A$$

is the identity under the natural product $\times_n$. M is a S-semigroup.

Example 1.50: Let

$$M = \left\{ \left. \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix} \right| a_i \in \{(a, b) \mid a, b \in [0, 14)\}; \right.$$

$$\left. 1 \leq i \leq 30, \times_n \right\}$$

be the semigroup under natural product $\times_n$.

M has unit; $\begin{pmatrix} (1,1) & (1,1) & \ldots & (1,1) \\ (1,1) & (1,1) & \ldots & (1,1) \\ (1,1) & (1,1) & \ldots & (1,1) \end{pmatrix}$ is the identity element

under natural product $\times_n$ in M.

$$A = \left\{ \left. \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix} \right| a_i \in \{(a, b) \mid a, b \in \{1, 13\}\}; \right.$$

$$\left. 1 \leq i \leq 30 \right\}$$

is a group under natural product.

Example 1.51: Let

$$M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 43)\}; \; 1 \leq i \leq 25\right\}$$

be the semigroup under natural product $\times_n$.

$$A = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in Z_{43} \setminus \{0\}\}; \; 1 \leq i \leq 25\right\} \subseteq M$$

is group under natural product of order $(42)^{25}$.

Example 1.52: Let

$$M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 29)\}; \; 1 \leq i \leq 40\right\}$$

be the semigroup under natural product $\times_n$.

M is a S-semigroup of infinite order.

M has a group under natural product $\times_n$ of order $(28)^{40}$.

Example 1.53: Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 7)\}; 1 \leq i \leq 15 \right\}$$

be the column super matrix of semigroup under the natural product $\times_n$.

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \end{array} \right] \middle|\, a_i \in \{(a, b) \mid a, b \in Z_7 \setminus \{0\}\};\ 1 \leq i \leq 15 \right\}$$

is a group under natural product of order $(6)^{15}$.

Example 1.54: Let

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \end{array} \right] \middle|\, a_i \in \{(a, b) \mid a, b \in [0, 16)\};\ 1 \leq i \leq 27 \right\}$$

be the super column matrix semigroup under product $\times_n$.

$$P = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \end{array} \right] \middle|\ a_i \in \{(a, b) \mid a, b \in [0, 16)\};\ 1 \leq i \leq 27 \right\} \subseteq M$$

is a group under natural product.

Example 1.55: Let $S = \{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6 \mid a_7\ a_8 \mid a_9) \mid a_i \in \{(a, b) \mid a, b \in [0, 231)\};\ 1 \leq i \leq 9\}$ be a super row matrix semigroup with entries from the real square of side length less than 231.

S is a S-semigroup under natural product $\times_n$.

Example 1.56: Let

$$T = \left\{ \left(\begin{array}{ccc|c|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \end{array} \right) \middle|\ a_i \in \{(a, b) \mid a, b \in [0, 17)\};\ 1 \leq i \leq 14, \times_n \right\}$$

be the semigroup of super row matrix from the real square of side length less than 17.

T is a S-semigroup under the natural product, $\times_n$.

Example 1.57: Let

$$W = \left\{ \left[\begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \ldots & \ldots & \ldots & \ldots & a_{12} \\ \hline a_{13} & \ldots & \ldots & \ldots & \ldots & a_{18} \\ a_{19} & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & a_{30} \\ \hline a_{31} & \ldots & \ldots & \ldots & \ldots & a_{36} \\ a_{37} & \ldots & \ldots & \ldots & \ldots & a_{42} \\ \hline a_{43} & \ldots & \ldots & \ldots & \ldots & a_{48} \\ a_{49} & \ldots & \ldots & \ldots & \ldots & a_{54} \end{array}\right] \right| a_i \in \{(a, b) \mid a, b \in$$

$$[0, 53)\};\ 1 \leq i \leq 54, \times_n\}$$

be a super matrix real semi open square semigroup with entries from the real square of side length less than 54.

Example 1.58: Let

$$M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{array}\right] \right| a_i \in \{(a, b) \mid a, b \in [0, 42)\};$$

$$1 \leq i \leq 24\}$$

be a super matrix semigroup of infinite order under natural product $\times_n$.

Example 1.59: Let

$$M = \left\{\left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ \hline a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & a_{35} \\ \hline a_{36} & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & a_{45} \\ a_{46} & \cdots & \cdots & \cdots & a_{50} \end{array}\right] \right| a_i \in \{(a, b) \mid a, b \in [0, 15)\}; \; 1 \leq i \leq 50, \times_n \right\}$$

be the super matrix semigroup of infinite order.

Now having seen examples of such algebraic structures.

We now proceed onto define the notion of semirings using the real semi open square of side n.

Let S = $\{(a, b) \mid a, b \in [0, n)\}$ be the real semi open square of side length less than n. Define two binary operations min or max on S. {S, max, min} is a semiring.

Consider x = (0.37, 0.45) and y = (5.32, 0.112) $\in$ S, min{x, y} = (0.37, 0.112) and max{x, y} = (5.32, 0.45).

Thus {S, max, min} is a semiring of infinite order.

We will illustrate this situation by some examples.

Example 1.60: Let M = $\{(a, b) \mid a, b \in [0, 27)$, min, max$\}$ be the semiring. x = (0.21, 6.93) and y = (21.75, 2.13) $\in$ M.

min{x, y} = (0.21, 2.13) and max(x, y) = (21.75, 6.93). M is of infinite order.

A {(0, 0), (6.502, 10.73)} $\subseteq$ M is a subsemiring of order two.

Let x = (3.57, 2.18) and y = (9.38, 1.357) $\in$ M.

min{x, y} = (3.57, 1.357)

max{x, y} = (9.38, 2.18)

P = {(0, 0), x, y, min{x, y}, max{x, y}} is a subsemiring of order 5.

Let x = (0, 9.2), y = (7.5, 0), z = (3.5, 4.7) and u = (0.37, 10.5) $\in$ M.

min{x, y} = (0, 0)
max{x, y} = (7.5, 9.2)
min{x, z} = {(0, 4.7)};
max {x, z} = (3.5, 9.2),

min{x, u} = (0, 9.2), min(y, z) = (3.5, 0),
min{y, u} = (0.37, 0) and so on.

We see S can have subsemirings of order two, three and so on.

P = {(0, 0), (0.74, 6.251), (7.21, 14.3)} $\subseteq$ M is a subsemiring. min{(0.74, 6.251), (7.21, 14.3)} = (0.74, 6.251) and max{(0.74, 6.251), (7.21, 14.3)} = (7.21, 14.3). Thus P is a subsemiring of finite order.

Infact M has infinite number of subsemirings of order three, two and so on.

Let x = (10.3, 9.72), y = (6.7, 15) and z = (2.11, 16.3)$\in$ M.

min {x, y} = (6.7, 9.72)
min {x, z} = (2.11, 9.72)
min {y, z} = {(2.11, 15).

max {x, y} = (10.3, 15)
max {x, z} = (10.3, 16.3)
max {y, z} = {(6.7, 16.3).

So B = {(0, 0), x, y, z, min {x, y}, min {x, z}, min {y, z}. max {x, y}, max {x, z}, max {y, z}} is a subsemiring of order 10.

Let x = (9.2, 10.3) and y = (3.1, 13) ∈ M.

min{x, y} = (3.1, 10.3), max{x, y} = (9.2. 13).

T = {(0,0), x, y, min{x,y}, max{x,y}} ⊆ M is a subsemiring of order 5.

Let x = (3.6, 7), y = (8, 9.5) and z = (12.5, 17.3) ∈ M.

min {x, y} = x, min {x, z} = x
min {y, z} = y, max {x, y} = y
max {x, z} = z, max {y, z} = z

Thus {(0, 0), x, y, z} ⊆ M is a subsemiring of order four.

Likewise we can get subsemirings of any desired order.

Further every element both under max and min is an idempotent.

We see this has no maximal or the greatest element only the least element to be (0, 0).

This semiring has subsemirings of infinite order also. However all subsemirings are not of infinite order.

Example 1.61: Let S = {(a, b) | a, b $\in$ [0, 4)} be the semiring of the real semi open square of side length less than 4 under the min, max operations.

Let A = {(0, 0), x_1 = (0.2, 0.5), x_2 = (1.01, 0.3) x_3 = (0.11, 1.01)} $\subseteq$ S. Clearly A is only a set and not a subsemiring of S.

Now min (x_1, x_2) = (0.2, 0.3), min {x_1, x_3} = (0.11, 0.5) and min {x_2, x_3} = (0.11, 0.3).

We find max{x_1, x_2} = (1.01, 0.5), max{x_1, x_3} = (0.2, 1.01) and max{x_2, x_3 } = (1.01, 1.01).

Thus {(0, 0), x_1, x_2, x_3, min {x_i, x_j}, max {x_i, x_j}; $i \neq j$, $1 \leq i$, $j \leq 3$} is a subsemiring of order 10.

Hence if A = {(0, 0), x_1, x_2}, the subsemiring got by adjoining max {x_1, x_2} and min {x_1, x_2} give a subsemiring of order 5.

In general if A = {(0, 0), x_1, x_2, …, x_n} with min {x_i, x_j} $\neq$ x_k or (0, 0) for any k and max {x_i, x_j} $\neq$ x_k for any $1 \leq k \leq n$ with $1 \leq i, j \leq n$ we see the completion of A to be subsemiring Ac is of order $n^2 + 1$.

Example 1.62: Let S = {(a, b) | a, b $\in$ [0, 12)} be the semiring with max min operation.

Let A = {(0, 0), (2.6, 4), (6.1, 0.1), (2.1, 9.3), (7.5, 0.01), (0.71, 6.11)} $\subseteq$ S. Clearly A is only a subset of S and not a subsemiring. The completed subsemiring A_c has the following elements.

min {(2.6, 4), (6.1, 0.1) = {(2.6, 0.1)}
min {(2.6, 4), (2.1, 9.3)} = {(2.1, 4)}
min {(2.6, 4), (7.5, 0.2)} = {(2.6, 0.01)}
min {(2.6, 4) (7.5, 6.11)} = {(0.71, 4)}
min {(6.1, 0.1) (2.1, 9.3)} = {(2.1, 0.1)}
min {(6.1, 0.1), (7.5, 0.01)} = {(6.1, 0.01)}

min {(6.1, 0.1), (0.71, 6.11)} = {(0.71, 0.1)}
min {(2.1, 9.3), (7.5, 0.01)} = {(2.1, 0.01)}
min {(2.1, 9.3), (0.71, 6.11)} = {(0.71, 6.11)} and
min {(0.75, 0.2), (0.71, 6.11)} = {(0.71, 0.2)}.

We see all the elements are distinct.
We consider max operation on it.

max {(2.6, 4), (6.1, 0.1)} = {(6.1, 4)}
max {(2.6, 4), (2.1, 9.3)} = {(2.6, 9.3)}
max {(2.6, 4), (7.5, 0.01)} = {(7.5, 4)}
max {(2.6, 4), (0.71, 6.11)} = {(2.6, 6.11)}
max {(6.1, 0.1), (2.1, 9.3)} = {(6.1, 9.3)}
max {(6.1, 0.1), (7.5, 0.01)} = {(7.5, 0.1)}
max {(6.1, 0.1), (0.71, 6.11)} = {(6.1, 6.11)}
max {(2.1, 9.3), (7.5, 0.01)} = {(7.5, 9.3)}
max {(2.1, 9.3), (0.71, 6.11)} = {(2.1, 6.11)} and
max {(7.5, 0.01), (0.71, 6.11)} = {(7.5, 6.11)}.

Clearly A_c has 26 number of distinct elements.
Inview of this we have the following theorem.

THEOREM 1.6: *Let $S = \{(a, b) \mid a, b \in [0, n), min, max\}$ be a real semi open square semiring.*

Let $A = \{(0, 0), x_1, \ldots, x_n$ where $x_i = (a_i, b_i)$ and $a_i, b_i \in [0, n) \setminus \{0\}, 1 \leq i \leq n\}$ be such that min $\{x_t, x_j\} \neq x_k$, $1 \leq k \leq n$ and max $\{x_t, x_j\} \neq x_t$; $1 \leq t \leq n$ with $1 \leq i, j \leq n$.

A is only a subset of S but A can be completed to A_c a subsemiring of cardinality $n^2 + 1$.

Proof follows directly.

Now a natural question is suppose in A of theorem 1.5; some $\{x_i, x_j\}$ are such that min or max are in A but A only a set, can A be completed to get a subsemiring the answer is yes which is first illustrated by the following examples.

Example 1.63: Let S = {(a, b) | a, b $\in$ [0, 15), min, max} be a real semi open square semiring.

Let A = {(0, 0), x_1 = (3, 2.7), x_2 = (5, 0), x_3 = (0, 1.2)} $\subseteq$ S be a subset of A.

We complete A to a subsemiring as follows.

min {x_1, x_2} = (3, 0)
min {x_1, x_3} = (0, 1.2)
min {x_2, x_3} = (0, 0)
max {x_1, x_2} = (5, 2.7)
max {x_1, x_3} = (3, 2.7) and
max {x_2, x_3} = (5, 1.2).

A_c = {(0, 0), x_1, x_2, x_3, (3, 0), (5, 2.7), (5, 1.2)} is the subsemiring of order 7.

Let B = {(0, 0), x_1 = (3.9, 5.7), x_2 = (2.7, 12.5), x_3 = (9.3, 2.1)} be the subset of order 4.

We find max {x_1, x_2} = (3.9, 12.5)

max {x_1, x_3} = (9.3, 5.7)
max {x_2, x_3} = (9.3, 12.5)
min {x_1, x_2} = (2.7, 5.7)
min {x_1, x_3} = (3.9, 2.1) and
min {x_2, x_3} = (2.7, 2.1).

We see B_c = {(0, 0), x_1, x_2, x_3, (3.9, 12.5), (9.3, 5.7), (2.7, 5.7), (9.3, 12.5), (3.9, 2.1), (2.7, 2.1)} is a subsemiring of order 10.

We now have D = {(0, 0), x_1 = (7.3, 1.5), x_2 = (0, 4.5), x_3 = (6, 2.7), x_4 = (8.5, 0)} to be the subset of the semiring.

min {x_1, x_2} = (0, 1.5),
min {x_1, x_3} = (6, 1.5),
min {x_1, x_4} = (7.3, 0),

$$\begin{aligned}
\min \{x_2, x_3\} &= (0, 2.7),\\
\min \{x_2, x_4\} &= (0, 0) \text{ and}\\
\min \{x_3, x_4\} &= (6, 0).
\end{aligned}$$

$$\begin{aligned}
\max \{x_1, x_2\} &= (7.3, 4.5),\\
\max \{x_1, x_3\} &= (7.3, 2.7),\\
\max \{x_1, x_4\} &= (8.5, 1.5),\\
\max \{x_2, x_3\} &= (6, 4.5),\\
\max \{x_2, x_4\} &= (8.5, 4.5) \text{ and}\\
\max \{x_3, x_4\} &= (8.5, 2.7).
\end{aligned}$$

$D_c = \{(0, 0), x_1, x_2, x_3, x_4, (0, 1.5), (6, 1.5), (7.3, 0), (0, 2.7), (6, 0), (7.3, 4.5), (7.3, 2.7), (8.5, 1.5), (6, 4.5), (8.5, 4.5) (8.5, 2.7)\}$ is the completed subsemiring of the subset D.

The order of D is 16.

This is the way completion is made. It is always easy to complete any set finite or infinite subset into a subsemigroup.

Example 1.64: Let $S = \{(a, b) \mid a, b \in [0, 10), \min, \max\}$ be the real semi open square semiring of infinite order.

$A = \{(0, 0), x_1 = (2.7, 8.71), x_2 = (1.5, 9.1), x_3 = (9.2, 1.01), x_4 = (0.5, 9.52)\}$ be the subset of S.

$$\begin{aligned}
\min \{x_1, x_2\} &= (1.5, 8.71),\\
\min \{x_1, x_3\} &= (2.7, 1.01),\\
\min \{x_1, x_4\} &= (0.5, 8.71),\\
\min \{x_2, x_3\} &= (1.5, 1.01),\\
\min \{x_2, x_4\} &= (0.5, 9.1) \text{ and}\\
\min \{x_3, x_4\} &= (0.5, 1.01).
\end{aligned}$$

$$\begin{aligned}
\max \{x_1, x_2\} &= (2.7, 9.1),\\
\max \{x_1, x_3\} &= (9.2, 8.71),\\
\max \{x_1, x_4\} &= (2.7, 9.52),\\
\max \{x_2, x_3\} &= (9.2, 9.1),\\
\max \{x_2, x_4\} &= (1.5, 9.52) \text{ and}\\
\max \{x_3, x_4\} &= (9.2, 9.52).
\end{aligned}$$

Thus A_c = {(0, 0), x_1, x_2, x_3, x_4, (1.5, 8.71), (2.7, 1.01), (0.5, 8.71), (1.5, 1.01), (0.5, 9.1), (0.5, 1.01), (2.7, 9.1), (9.2, 8.71), (2.7, 9.52), (9.2, 9.1), (1.5, 9.52), (9.2, 9.52)} is a subsemiring of order 16.

Thus completion can be made for any subset of a semiring S into a subsemiring of S.

Inview of this we have the following theorem.

THEOREM 1.7: *Let S = {(a, b) | a, b ∈ [0, n), max, min} be a real semi open square semiring. Let P ⊆ S be any non empty subset of S, P can always be completed to P_c to form a subsemiring.*

If $|P| < \infty$, then $|P_c| < \infty$ and if $|P| = \infty$ then $|P_c| = \infty$.

Proof : Follows from simple computations.

We can have subsemirings also define the notion of ideals and filters in case of semirings built using the real semi open square.

Let S = {(a, b) | a, b ∈ [0, 12), min, max} be the semiring. We see I ⊆ S is an ideal of S if I is a subsemiring and for all x ∈ I and y ∈ S; min {x, y} ∈ I.

We will first provide examples of them.

Let V = {(a, 0) | a ∈ [0, 12), min, max} ⊆ S be the subsemiring. Clearly V is an ideal of S.

All subsemirings are not ideals. For take P = {(0, 0), (0.6, 0.9), (6.3, 4.2), (9.5, 6.5)} ⊆ S. P is a subsemiring and clearly P is not an ideal.

All ideals in this semiring are of infinite order. T = {(a, b) | a, b ∈ [0, 6)} ⊆ S is a subsemiring as well as ideal of S.

Let M= {(a, b) | a, b $\in$ [0, 3)} $\subseteq$ S is also a subsemiring as well as ideal of S.

Let M_1 = {(a, b) | a, b $\in$ [0, 7)} $\subseteq$ S is also a subsemiring as well as ideal of S.

Now P = {(a, b) | a, b $\in$ [3, 7)} $\subseteq$ S is also a subsemiring as well as ideal of S.

For if x = (0.2, 0.7) $\in$ S and y = (3.4, 5.2) $\in$ P then min {x, y} = (0.2, 0.7) $\notin$ P. Hence the claim.

Let V = {(a, b) | a, b $\in$ (2, 5)} $\subseteq$ S be the subsemiring of S be the subsemiring of S.

Clearly V is not an ideal of S. For let x = (0, 10) $\in$ S and y = (1, 481) $\in$ V we see min{x, y} = (0, 4.81) $\notin$ V.

Thus we can prove S has infinite number of subsemirings which are not ideals of S.

Example 1.65: Let M = {(a, b) | a, b $\in$ (0, 19)} be the semiring under max and min operation. T = {(a, b) | a, b $\in$ (5, 12)} $\subseteq$ M is a subsemiring of M and is not an ideal of M for if x = (3, 10) $\in$ T and y = (0.3, 11) $\in$ M then min {x, y} = {0.3, 10)} $\notin$ T. hence the claim.

Infact M has infinite number of subsemirings of infinite order which are not ideals of M.

Let L = {(a, b) | a, b $\in$ (7, 19)} $\subseteq$ M be a subsemiring. Let x $\in$ {(a, b) | a, b $\in$ (0, 7)} = D.

Clearly for every y $\in$ L and for every x $\in$ D; min {x, y} $\notin$ L. Thus L is not an ideal of M only a subsemiring.

Inview of all these we have the following theorem.

THEOREM 1.8: *Let $S = \{(a, b) \mid a, b \in [0, n), \min, \max\}$ be the real semi open square semiring.*

(1) All $A_t = \{(a, b) \mid a, b \in [0, t);\ 0 < t < m < n$, m and n distinct$\} \subseteq S$ are all ideals of S of infinite order.
(2) All $B_{s,t} = \{(a, b) \mid a, b \in [s, t);\ 0 < t < m < n$, m and n distinct$\} \subseteq S$ are only subsemirings which are not ideals.
(3) All $D_m = \{(a, b) \mid a, b \in [m, n);\ 0 < m\} \subseteq S$ are only subsemirings which are not ideals of S.

Proof is direct hence left as an exercise to the reader.

Example 1.66: Let $S = \{(a, b) \mid a, b \in [0, 15), \min, \max\}$ be the semiring. $M = \{(a, b) \mid a, b \in (10, 12), \min, \max\} \subseteq S$; is only a subsemiring and is not an ideal of S.

$P = \{(a, b) \mid a, b \in [0, 5), \min, \max\}$ is a subsemiring which is an ideal of infinite order.

$R = \{(a, b) \mid a, b \in (7, 15)\} \subseteq S$ is a subsemiring and is not an ideal of S.

Now we proceed onto define the notion of filter in semirings, $S = \{(a, b) \mid a, b \in [0, n), \min, \max\}$ $(n < \infty)$ $M \subseteq S$ be a subsemiring of S. M is a filter of S if for any $x \in M$ and $y \in S$ then $\max \{x, y\} \in M$.

We will first illustrate this situation by some examples.

Example 1.67: Let $S = \{(a, b) \mid a, b \in [0, 22), \min, \max\}$ be the real semi open square semiring.

Consider $P = \{(a, b) \mid a, b \in [12, 22), \min, \max\} \subseteq S$, P is a subsemiring.

P is also a filter of S but P is not an ideal of S. For take $x = (9, 10) \in S$ and $y = (20, 12) \in P$.

min $\{x, y\} = (9, 10) \notin P$ but max $\{x, y\} = (20, 12) \in P$ hence P is a filter of S and is not an ideal of S.

Let $M = \{(a, b) \mid a, b \in [10, 22), \min, \max\} \subseteq S$ be the subsemiring of S. M is not an ideal of S.

$x = (5, 4.1) \in S$ and $y = (11, 10.9) \in M$

$\min\{x, y\} = (5, 4.1) \notin M$ and $\max \{x, y\} = (11, 10.9) \in M$, hence M is a filter of S.

Thus S has subsemirings which are neither ideals nor filters of S. Also S has subsemirings which are filters and not ideals of S.

Further S has subsemirings which are ideals and not filters of S. To this end we will provide some more examples.

Example 1.68: Let $S = \{(a, b) \mid a, b \in [0, 24), \min, \max\}$ be the real semi open square semiring. $P_1 = \{(a, b) \mid a, b \in [0, 5)\} \subseteq S$ is only an ideal of S and not a filter of S.

$P_2 = \{(a, b) \mid a, b \in [3, 10)\} \subseteq S$ is only a subsemiring of S and is not an ideal of S. P_2 is also not a filter of S.

$P_3 = \{(a, b) \mid a, b \in [5, 24)\} \subseteq S$ is a filter but is not an ideal.

$P_4 = \{(a, b) \mid a, b \in [7, 24)\} \subseteq S$ is also a filter which is not an ideal.

Example 1.69: Let $M = \{(a, b) \mid a, b \in [0, 190), \min, \max\}$ be the real semi open square semiring.

$T_1 = \{(a, b) \mid a, b \in [0, 19)\} \subseteq S$ is a subsemiring as well as an ideal but not a filter of M.

$T_2 = \{(a, b) \mid a, b \in [7, 11)\} \subseteq M$ is a subsemiring which is not an ideal or filter of M.

Finally $T_3 = \{(a, b) \mid a, b \in [11, 190)\} \subseteq M$ is a subsemiring which not an ideal but only a filter of M.

Inview of all these we have the following theorem.

THEOREM 1.9: *Let $S = \{(a, b) \mid a, b \in [0, n, n < \infty, \min, \max)\}$ be the semi open real square semiring.*

(1) All subsemirings of the form $P = \{(a, b) \mid a, b \in [0, t); 0 < t < m < n, \min, \max\} \subseteq S$ are ideals which are not filters of S.
(2) $M = \{(a, b) \mid a, b \in [t, s); 0 < t < m < n\} \subseteq S$ is a subsemiring and not an ideal or a filter.
(3) $N = P = \{(a, b) \mid a, b \in [t, n); 0 < t < n\} \subseteq S$ is a subsemiring which is a filter and not an ideal of S.

Proof is direct and hence left as an exercise to the reader.

We have seen subsemirings ideals and filters. S has zero divisors with respect to min operation.

For if $x = (0, t)$ and $y = (s, 0) \in S$ then $\min (x, y) = (0, 0)$ but $\max (x, y) = (s, t)$.

THEOREM 1.10: *Let $S = P = \{(a, b) \mid a, b \in [0, n); \min, \max, n < \infty\}$ be a real semi open square semiring.*

(i) S has infinite number of zero divisors under min operation.
(ii) S has subsemirings of all finite orders from 2, 3, …. and none of these subsemirings are ideals.
(iii) All ideals are of infinite order.
(iv) All filters of S are of infinite order.
(v) Every element in S is an idempotent with respect to both min and max.

The proof is direct and hence left as an exercise to the reader.

Now we using S = {(a, b) | a, b $\in$ [0, n), min, max}; built matrices and super matrices.

All these will be illustrated by examples.

Example 1.70: Let S = {(a_1, a_2, a_3, a_4) where $a_i \in$ {(a, b) | a, b $\in$ [0, 15)} $1 \leq i \leq 4$, min, max} be the semiring of infinite order.

A = {(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)} where x_i, $y_j \in$ [0, 10), $1 \leq i, j \leq 4$}. A is a subsemiring of infinite order. A is also an ideal of S.

B = {(0,0), (0,0), (0,0), (0,0)), ((0.7, 0.8), (1, 3), (4, 5), (8, 9.2)} $\subseteq$ S, B is only a subsemiring which is not an ideal of S. Infact no finite subsemiring of S can be an ideal of S.

D = {(0,0), (0,0), (0,0), (0,0),), ((x,0), (x_2,0), (x_3,0), (x_4,0)) | $x_i \in$ [0, 15), $1 \leq i \leq 4$} is a subsemiring as well as ideal of S of infinite order. Clearly D is not a filter of S.

We see S in general a filter is not an ideal and vice versa.

We take M = {(a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4) | a_i, $b_i \in$ [0, 9), $1 \leq i \leq 4$} $\subseteq$ S to be a subsemiring which is also an ideal of S. M is not a filter of S. N = {((a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4) | a_i, $b_i \in$ [9, 5), $1 \leq i \leq 4$} $\subseteq$ S is a subsemiring as well as a filter of S. N is not ideal of S.

For x = ((0.3, 1), (0, 0.2) (0.5, 0), (0.11, 0.8)) $\in$ S and for any m $\in$ N we see min {x, m} = x $\notin$ N hence N; is not an ideal only a filter of S.

We have filters in S which are not ideals and ideals in S which are not filters.

Any subsemiring A which has zero entries in any one of the co ordinates can never be a filter only an ideal.

For $B = \{((0,0), (a_1, b_1), (0, 0), (0, c_1))) \mid a_1, b_1, c_1 \in [0, 15)\} \subseteq S$ is only a subsemiring and never a filter but is always an ideal. Thus if (0, 0) or $(0, c_1)$ entries are present it can never be a filter only an ideal.

Let $P = \{((a, b), (0, 0), (0, 0), (0, 0)) \mid a, b \in [0, 15)\} \subseteq S$ be a subsemiring as well as an ideal of S. However P is not a filter of S.

Let $M = \{((a, b), (0, 0), (0, 0), (0, 0)) \mid a, b \in [7, 9)\} \subseteq S$ be only a subsemiring of S. M is not an ideal or a filter of S.

$T = \{((a, b), (0, 0), (0, 0), (0, 0)) \mid a, b \in [5, 9)\} \subseteq S$ is a subsemiring which is also a filter of S and not an ideal of S.

$M = \{((a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4))$ where $a_i, b_i \in [3, 8]$, $1 \leq i \leq 4\}$ is a subsemiring of infinite order which is not an ideal or filter of S.

S has every element in it to be an idempotent with respect to max and min operation.

Example 1.71: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \ a_i \in \{(a, b) \mid a, b \in [0, 24)\};\ 1 \leq i \leq 10, \text{min, max} \right\}$$

is a real semi open square semiring of infinite order.

M has infinite number of subsemirings of infinite order which are not ideals or filters.

M has infinite number of subsemirings of infinite order which are ideals and not filters.

M has infinite number of subsemirings of infinite order which are filters and not ideals.

$$T = \left\{ \left[\begin{array}{c} (a_1, b_1) \\ (a_2, b_2) \\ 0 \\ \vdots \\ 0 \end{array} \right] \middle| \; a_i, b_i \in [0, 24)\}; 1 \leq i \leq 2 \right\} \subseteq M$$

is a subsemiring which is an ideal and not a filter of M.

$$P = \left\{ \begin{bmatrix} (0,0) \\ (0,0) \\ \vdots \\ \vdots \\ (0,0) \end{bmatrix}, \begin{bmatrix} (5,2) \\ (3,1) \\ \vdots \\ \vdots \\ (7,0.3) \end{bmatrix} \right\} \subseteq M$$

is a subsemiring of order two which is not an ideal and not a filter of M.

$$L = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{10} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [7, 12)\}; 1 \leq i \leq 10 \right\} \subseteq M$$

is a subsemiring of infinite order which is not an ideal or filter of M.

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [5, 24)\};\ 1 \leq i \leq 10\} \subseteq M \right.$$

is a subsemiring of infinite order which is not an ideal or filter of M.

Thus we have seen some of the properties related with M.

Example 1.72: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \middle| \; \text{where } a_i \in \{(a, b) \mid a, b \in [0, 18)\}; \right.$$

$$1 \leq i \leq 24, \text{max, min}\}$$

be the semiring of infinite order.

$$P = \left\{ \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| \; a_1 \in \{(a, b) \mid a, b \in [0, 18)\} \subseteq P \right.$$

is a subsemiring of S. P is an ideal of S and is not a filter.

$$R=\left\{\begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} (8,0.3) & (0.7,1) & 0 & (0.1,0) \\ (0.9,0.1) & (9,0.3) & (0.7,0) & 0 \\ (6,9) & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ (11,2) & (0.7,8) & (0.9,1) & (8,0.7) \end{bmatrix}\right\} \subseteq S$$

is a subsemiring which is not an ideal or a filter. |R| = 2.

$$T = \left\{\left.\begin{bmatrix} [a_1,b_1] & [a_2,b_2] & [a_3,b_3] & [a_4,b_4] \\ [a_5,b_5] & [a_6,b_6] & [a_7,b_7] & [a_8,b_8] \\ \vdots & \vdots & \vdots & \vdots \\ [a_{21},b_{21}] & [a_{22},b_{22}] & [a_{23},b_{23}] & [a_{24},b_{24}] \end{bmatrix}\right| a_i, b_i \in\right.$$

$$\{(a, b) \mid a, b \in [8, 12]\}, 1 \leq i \leq 24\}.$$

T is subsemiring is not an ideal and not a filter of S. T is of infinite order.

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}\right| a_i \in \{(a, b) \mid a, b \in [0, 9)\};\right.$$

$$1 \leq i \leq 24\} \subseteq S$$

is a subsemiring which is an ideal of S and not a filter.

$$N = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \right| a_i \in \{(a, b) \mid a, b \in [9, 18)\};$$

$$1 \leq i \leq 24\} \subseteq S.$$

N is a subsemiring which is not an ideal but N is a filter.

Example 1.73: Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right| a_i \in \{(a, b) \mid a, b \in [0, 11)\};$$

$$1 \leq i \leq 16, \text{max}, \text{min}\}$$

be the semiring of infinite order.

T has subsemirings of finite order which are never ideals or filters.

T has subsemirings of infinite order which are never ideals or filters.

T has subsemirings of infinite order which are ideals never filters.

T has subsemirings which are filters but are not ideals.

Infact filters are never ideals and ideals are never filters.

Further T has all ideals and filters to be of infinite order.

Example 1.74: Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \end{array} \right] \right| a_i \in \{(a, b) \mid a, b \in [0, 9)\}; 1 \leq i \leq 15, \max, \min \right\}$$

be the semiring of infinite order.

We see M has infinite number of subsemirings of finite order say of order two, three and so on.

M has no ideals or filters of finite order all of them are of infinite order.

$$P = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline 0 \\ 0 \\ 0 \\ \hline a_3 \\ a_4 \\ \hline 0 \\ 0 \\ 0 \\ \hline a_5 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \middle| \, a_i \in \{(a, b) \mid a, b \in [0, 9)\}, 1 \leq i \leq 5 \right\} \subseteq M$$

is a subsemiring which are ideals and never a filter.

We can have several such P and these P has infinite number of elements in M such that for every $p \in P$ and $m \in M$ we have $\min \{p, m\} = (0)$.

$$R = \left\{ \left[\begin{array}{c} 0 \\ \hline 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array}\right], \left[\begin{array}{c} (0.7,8) \\ \hline (1,1) \\ \hline (6,0.9) \\ 0 \\ (5.2,7) \\ \hline (7.3,8) \\ (2.1,1.111) \\ \hline 0 \\ 0 \\ 0 \\ \hline (6.71.2.12) \\ \hline 0 \\ 0 \\ (3.7,2.11) \\ (0.15,6.12) \end{array}\right] \right\} \subseteq M,$$

R is a subsemiring and is not an ideal or filter.

Every pair of elements with (0) and $x \in M$ will be a subsemiring of M of order two.

We can also have subsemirings of order three and so on.

$$S = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ a_{15} \end{array} \right] \right| a_i \in \{(a, b) \mid a, b \in [6, 9)\}, 1 \leq i \leq 15\} \subseteq M$$

is a subsemiring which is not an ideal but a filter and S is of infinite order.

Example 1.75: Let M = $\{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8 \mid a_9\ a_{10}) \mid a_i \in \{(a, b) \mid a, b \in [0, 21)\}, 1 \leq i \leq 10\}$, max, min} be the real semi open square semiring of infinite order.

S = $\{(a_1 \mid 0\ 0\ 0\ 0 \mid a_2\ a_3 \mid 0 \mid a_4\ a_5) \mid a_i \in \{(a, b) \mid a, b \in [0, 21)\}, 1 \leq i \leq 15\} \subseteq M$; is a subsemiring which is an ideal of M.

N = $\{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8 \mid a_9\ a_{10}) \mid a_i \in \{(a, b) \mid a, b \in [8, 21)\}, 1 \leq i \leq 10\}\} \subseteq M$ is a subsemiring which is not an ideal only a filter and is of infinite order.

T = $\{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8 \mid a_9\ a_{10}) \mid a_i \in \{(a, b) \mid a, b \in [7, 14)\}, 1 \leq i \leq 10\}\} \subseteq M$ is a subsemiring and is not an ideal or a filter of M.

Example 1.76: Let

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \end{array} \right] \middle| a_i \in \{(a, b) \mid a, b \in [0, 18)\}, \right.$$

$$\left. 1 \leq i \leq 36, \min, \max \right\}$$

be a real semi open square semiring of infinite order.

$$T = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline a_7 & a_8 & a_9 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{array} \right] \middle| a_i \in \{(a, b) \mid a, b \in [0, 18)\}, 1 \leq i \leq 15 \right\}$$

be a subsemiring is an ideal but is not a filter.

$$W = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \end{array} \right] \right| a_i \in \{(a, b) \mid a, b \in [7, 8)\},$$

$$1 \leq i \leq 36\} \subseteq M;$$

W is only a filter and not an ideal. Thus M has several ideals, subsemirings which are not ideals or filters and M has filters which are not ideals.

Example 1.77: Let

$$M = \left\{ \left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array} \right] \right| a_i \in \{(a, b) \mid a, b \in [0, 21)\},$$

$1 \leq i \leq 10\}\} \subseteq M$

is a subsemiring which is not an ideal only a filter and is of infinite order.

$$T = \left\{\left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array}\right] \right| a_i \in \{(a, b) \mid a, b \in$$

$[2, 5)\}, 1 \leq i \leq 40\}\} \subseteq M$

be a subsemiring and is not an ideal only a filter.

$$W = \left\{\left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array}\right] \right| a_i \in \{(a, b) \mid a, b \in [0, 3)\},$$

$1 \leq i \leq 40\}\} \subseteq M$

be the subsemiring which is an ideal of M. W is not a filter.

We now proceed onto define pseudo rings on the real square using both the operations + and ×.

DEFINITION 1.1: *Let T = {a, b) | a, b ∈ [0, n), n < ∞, +, ×} be defined as the real semi open square pseudo ring. R is an abelian group under + and R under × is a commutative semigroup.*

But + and × does not satisfy the distributive law that is a × (b + c) ≠ a × b + a × c for all a, b, c ∈ [0, n).

Consider n = 7, a = 6.3, b = 3.1 and c = .5 ∈ [0, 7).

$$\begin{aligned} a \times (b + c) &= 6.3\ (3.1 + 5) \\ &= 6.3 + 1.1 \\ &= 6.93. \end{aligned}$$

$$\begin{aligned} \text{Now } a \times b + a \times c &= 6.3 \times 3.1 + 6.3 \times 5 \\ &= 5.53 + 31.5 \\ &= 5.53 + 3.5 \\ &= 9.03 \\ &= 2.03. \end{aligned}$$

$$a \times (b + c) \neq a \times b + a \times c.$$

Thus the distributive laws are not true for a, b, c elements in [0, 7).

Consider the interval [0, 10)
Let x = (0.3, 6.1), y = (7, 0)
and z = (8.5, 8) ∈ R = {(a, b) | a, b ∈ [0, 10), +, ×}

$x \times (y + z) = (0.3, 6.1) \times [(7.0) + (8.5, 8)]$
$= (0.3, 6.1) \times (5.5, 8)$
$= (1.65, 48.8) = (1.65, 8.8)$ … I

$x \times (y + z) = x \times y + x \times z$
$= (0.3, 6.1) \times (7, 0) + (0.3, 6.1) \times (8.5, 8)$

$= (2.1, 0) + (2.55, 48.8)$
$= (4.65, 8.8)$ … II

I and II are distinct so distributive law is not true.

$x = (6.1, 3.2)$, $y = (0.9, 0.3)$ and $z = (2.1, 9.2) \in R$.
$x \times (y + z) = (6.1, 3.2) \times [(0.9, 0.3) + (2.1, 9.2)]$
$= (6.1, 3.2) \times (3, 9.5)$
$= (18.3, 0.40)$
$= (8.3,0.4)$ … I

$x \times y + x \times z$
$= (6.1, 3.2) \times (0.9, 0.3) + (6.1, 3.2) \times (2.1, 9.2)$
$= (5.49, 0.96) + (2.81, 9.44)$
$= (8.3, 0.4)$ … II

I and II are identical.
Let $x = (6.3, 8.1)$, $y = (0.9, 0.4)$ and $z = (5.5, 8.8) \in R$.

$x \times (y + z) = (6.3, 8.1) ((0.9, 0.4) + (5.5, 8.8)$
$= (6.3, 8.1) (6.4, 9.2)$
$= (8.32, 4.52)$ … I

Now $xy + x \times z$
$= (6.3, 8.1) \times (0.9, 0.4) + (6.3, 8.1) (5.5, 8.8)$
$= (5.67, 3.24) + (4.65, 1.28)$
$= (0.32, 4.52)$ … II

(I) and (II) are distinct hence the distributive law is not true in the case.

That is why we call $S = \{(a, b) \mid a, b \in [0, n), +, \times\}$ to be a real semi open square pseudo ring of infinite order built using the real square.

Example 1.78: Let $R = \{(a, b) \mid a, b \in [0, 12), +, \times\}$ be the pseudo ring. R has infinite number of zero divisors and a finite number of idempotents.

R has subsets which are rings; for P = {(a, b) | a, b ∈ Z_{12}, +, ×} is a ring and is not a pseudo ring. o(p) < ∞. R has units; (1, 11) = x ∈ R is such that x^2 = (1, 1), x = (7, 11) ∈ R is such that x^2 = (1, 1). However R has only finite number of units.

P_1 = {(a, 0) | a ∈ [0, 12), +, ×} ⊆ R is a pseudo subring as well as pseudo ideal of R.

P_2 = {(0, a) | a ∈ [0, 12), +, ×} is a pseudo subring as well as pseudo ideal of R.

$P_1 \oplus P_2$ = R is the direct sum of pseudo subrings and $P_1 \cap P_2$ = {(0, 0)}.

Example 1.79: Let R = {(a, b) | a, b ∈ [0, 23), +, ×} be the pseudo ring of infinite order. R has only 22^2 number of units. R has infinite number of zero divisors.

R has subrings which are pseudo subrings as well as subrings which are not pseudo.

T = {(a, b) | a, b ∈ Z_{23}} ⊆ R is a subring which is not a pseudo subring.

L = {(a, 0) | a ∈ Z_{23}, +, ×} is a subring of finite order which is not a pseudo subring.

P = {(0, a) | a ∈ Z_{23}, +, ×} ⊆ R is a subring of finite order which is not a pseudo subring.

These subrings are not ideals of R.

Example 1.80: Let S = {(a, b) | a, b ∈ [0,120), +, ×} be a semi open real square pseudo ring of infinite order.

S has infinite order pseudo ideals.

T_1 = {(a, 0) | a ∈ [0,120) } ⊆ S is a pseudo subring as well as pseudo ideal of S.

$T_2 = \{(0, a) \mid a \in [0,120)\} \subseteq S$ is a pseudo subring which is a pseudo ideal of S.

$M = \{(a, b) \mid a, b \in Z_{120}, +, \times\} \subseteq S$ is a subring of S which is not a pseudo subring.

$x = (10, 12)$ and $y = (12, 10) \in S$ are such that $x \times y = (0, 0)$.

Infact M has only finite number of zero divisors however S has infinite number of zero divisors.

Both S and M have only a finite number of nontrivial idempotents. S has several pseudo subrings which are not ideals.

Example 1.81: Let $S = \{(a, b) \mid a, b \in [0,19), +, \times\}$ be a real semi open square pseudo ring.

$T = \{(a, 0 \mid a \in Z_{19}\}$ is a field isomorphic with Z_{19}. So S is a Smarandache pseudo ring. S has no idempotents. S has ideals and 18^2 number of units where (1, 1) is the identity element with respect to $\times$.

Example 1.82: Let $W = \{(a, b) \mid a, b \in [0,24), +, \times\}$ be the pseudo ring of infinite order. W has idempotents and nilpotents that are only finite in number.

Now we can build algebraic structures using the pseudo ring built using the real square. $S = \{(a, b) \mid a, b \in [0,n), n < \infty\}$.

This is illustrated by the following examples.

Example 1.83: Let
$M = \{(a_1, a_2, \ldots, a_7) \mid a_i \in \{(a, b) \mid a, b \in [0,7)\}, 1 \leq i \leq 7, +, \times\}$ is the row matrix pseudo ring built on the square of side less than 7.

M has zero divisors which are infinite in number. M has finite number of units.

x = (((1, 1), (1, 1), …, (1, 1))} is the multiplicative identity of M. M has subrings and pseudo subrings of finite and infinite order respectively. M also has pseudo ideals.

Example 1.84: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 271)\}; 1 \leq i \leq 12, +, \times_n \right\}$$

be the semi open real square pseudo ring of infinite order.

M has infinite number of zero divisors and only finite number of units.

$$\text{We see } x = \begin{bmatrix} (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \\ (1,1) \end{bmatrix} \text{ is the unit of M.}$$

M has pseudo subrings as well as pseudo ideals.

Example 1.85: Let

$$M = \left\{ \left[\begin{matrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{matrix} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 19)\}; \right.$$

$$1 \leq i \leq 40, +, \times_n\}$$

be the pseudo ring of infinite order.

M has infinite number of units and infinite number of zero divisors. All pseudo ideals of M are of infinite order. M has pseudo subrings as well as subrings of infinite and finite order respectively.

Example 1.86: Let

$$M = \left\{ \left[\begin{matrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{matrix} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 27)\}, \right.$$

$$1 \leq i \leq 25, +, \times_n\}\}$$

be the pseudo ring. M has pseudo subrings and ideals.

Example 1.87: Let A = $\{(a_1 \mid a_2\ a_3 \mid a_4 \mid a_5\ a_6\ a_7 \mid a_8) \mid a_i \in \{(a, b) \mid a, b \in [0, 23)\}, 1 \leq i \leq 8, +, \times_n\}\}$ be the pseudo ring of super row matrices of infinite order.

A has infinite number of zero divisors and finite number of units. A has pseudo subrings and pseudo ideals.

A has no idempotents other than those got by using (0, 0), (1, 1), (1, 0) and (0, 1).

A = $\{((a_1, 0) \mid (a_2, 0), (a_3, 0) \mid (a_4, 0) \mid (a_5, 0), (a_6, 0)\ (a_7, 0) \mid (a_8, 0) \mid a_i \in \{(a, b) \mid a, b \in [0, 23)\}, 1 \leq i \leq 8\}\} \subseteq$ A is a pseudo subring as well as a pseudo ideal of A.

Example 1.88: Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ a_{12} \\ a_{13} \\ \hline a_{14} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 41)\}, 1 \leq i \leq 14, +, \times_n \right\}$$

be the pseudo ring of infinite order.

M has pseudo ideals and pseudo subrings.

M has subrings of finite order which are not pseudo.

M has infinite number of zero divisors but only finite number of units.

Example 1.89: Let

$$S = \left\{ \left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array} \right] \right| a_i \in \{(a, b) \mid a, b \in [0, 235)\}, \; 1 \leq i \leq 40, +, \times_n \Bigg\}$$

be the super matrix pseudo ring of infinite order.

S has only finite number of units and finite number of idempotents but has infinite number of zero divisors.

S has several pseudo subrings which are not pseudo ideals. S has pseudo subrings which are also ideals.

Study in this direction is similar to that of usual pseudo ring got using [0, n).

Further we see we are more interested in developing these structures in case of neutrosophic squares.

We suggest the following problems.

Problems:

1. Find some stricking features enjoyed by the algebraic structures constructed using real semi open squares of the form S = {(a, b) | a, b ∈ [0, n)}.

2. Compare S with A = $\{[0, n)\}$; A has the same algebraic structures constructed on it.

3. Let P = $\{(a, b) \mid a, b \in [0, 8), +\}$ be the group using the real semi open square.

   (i) Prove o (P) = ∞.
   (ii) Can P have subgroups of finite order?
   (iii) Prove or disprove P has only finite number of subgroups of finite order.
   (iv) Can P have subgroups of infinite order?

4. Let M = $\{(a, b) \mid a, b \in [0, 7)\}$ be the group using the real semi open square of side less than 7.

   Study questions (i) to (iv) of problem 3 for this M.

5. Let T = $\{(a, b) \mid a, b \in [0, 25)\}$ be the group on the real semi open square of length less than 25.

   Study questions (i) to (iv) of problem 3 for this T.

6. Study questions (i) to (iv) of problem 3 for the real semi open square group S = $\{(a, b) \mid a, b \in [0, 24)\}$.

7. Compare the group on the real semi open square in which $[0, p)$, in which p, a prime is used with that of a group in which $[0, n)$, n not a prime is used.

8. Obtain any other special and interesting property associated with this real semi open square group.

9. Prove this group on the square X = $\{(a, b) \mid a, b \in [0, n)\}$ has two subgroups which are isomorphic with the group G = $\{[0, n), +\}$.

10. Study S = $\{(a, b) \mid a, b \in [0, n)\}$ under product.

11. Prove $\{S, \times\}$ is only a semigroup.

12. Show if $x = t$ where $t \in [0, n) \setminus Z_n$ then x has no inverse with respect to product.
13. Prove $(S, \times)$ is a semigroup of infinite order.

14. Can $(S, \times)$ have subsemigroups of infinite order which are not ideals?

15. Prove $(S, \times)$ has infinite number of zero divisors.

16. Can $(S, \times)$ have infinite number of idempotents?

17. Can $(S, \times)$ have infinite number of units?

18. Can $S = \{(a, b) \mid a, b \in [0, 82), \times\}$ the semigroup have nilpotent elements?

19. Let $S = \{(a, b) \mid a, b \in [0, 491)\}$ be the semigroup under product.

    (i) Can S have nilpotents?
    (ii) Can S have idempotents?
    (iii) Prove S has units.

20. $S = \{(a, b) \mid a, b \in [0, 42)\}$ be the real semi open square semigroup.

    Prove S has infinite number of zero divisors. Prove S is a S-semigroup.

21. Can any $S = \{(a, b) \mid a, b \in [0, n), \times\}$ for any n be not a S-semigroup?

22. $S = \{(a, b) \mid a, b \in [0, 23), \times\}$ be the semigroup. Prove S has atleast 22×22 number of units.

23. Prove $S = \{(a, b) \mid a, b \in [0, 24)\}$ the real semi open square semigroup, has only less than 23×23 number of units.

24. Let S = $\{(a_1, a_2, \ldots, a_{15}) \mid a_i \in \{(a, b) \mid a, b \in [0, 23), 1 \leq i \leq 15, +\}$ be the row matrix group under ‘+’.

(i) Show $o(S) = \infty$.
(ii) Prove S has subgroups of finite order.
(iii) Prove S has subgroups of infinite order.

25. Prove the semigroup $\{(a, b) \mid a, b \in [0, n), \times\}$ under product can never have ideals of finite order.

26. Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{19} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 28)\}; 1 \leq i \leq 19, +\right\}$$

be the real semi open square group under +.

Study questions (i) to (iii) of problem 24 for this S.

27. Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_{10} \\ a_{11} & a_{12} & a_{13} & \ldots & a_{20} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{30} \\ a_{31} & a_{32} & a_{33} & \ldots & a_{40} \\ a_{41} & a_{42} & a_{43} & \ldots & a_{50} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 42)\}; 1 \leq i \leq 50, +\right\}$$

be the group under +.

Study questions (i) to (iii) of problem 24 for this A.

28. Let

$$M = \left\{ \left[\begin{array}{cc|cccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ \hline a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{32} \\ a_{33} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \\ \hline a_{41} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{48} \\ a_{47} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{56} \\ a_{57} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{64} \end{array} \right] \right| a_i \in \{(a, b) \mid$$

$$a, b \in [0, 43)\}; 1 \leq i \leq 64, +\}$$

be the group under +.

Study questions (i) to (iii) of problem 24 for this M.

29. Let M = {(a, b) | a, b ∈ [0, 45); ×} be the semigroup on the real square of length less than 45.

(i) Find all ideals of M.
(ii) Can M have finite ideals?
(iii) Can M have infinite number of idempotents?
(iv) Prove M can have infinite number of zero divisors.
(v) Can M have subsemigroups which are of finite order?
(vi) Prove M can have only finite number of units.

30. Let N = {(a, b) | a, b ∈ [0, 19), ×} be the semigroup of the real square of length less than 19.

Study questions (i) to (vi) of problem 29 for this N.

Can N have non trivial idempotents?

31. Let P = {(a, b) | a, b $\in$ [0, 424), $\times$} be the semi open real square semigroup.

    Study questions (i) to (vi) of problem 29 for this P.

32. Let P = {($a_1, a_2, \ldots, a_{10}$) | $a_i \in$ {(a, b) | a, b $\in$ [0, 40)}, $1 \leq i \leq 10$, $\times$} be the semigroup of row matrices.

    Study questions (i) to (vi) of problem 29 for this P.

33. Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{19} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 25)\};\ 1 \leq i \leq 19, \times_n \right\}$$

    be the column matrices semigroup under $\times_n$.

    Study questions (i) to (vi) of problem 29 for this M.

34. Let

$$A = \left\{ \left[\begin{array}{cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \\ \hline a_{15} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{21} \\ a_{22} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{35} \\ \hline a_{36} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{42} \\ a_{43} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{49} \end{array}\right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 28)\};\ 1 \leq i \leq 49, \times_n \right\}$$

be a semigroup.

Study questions (i) to (vi) of problem 29 for this A.

35. Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_{20} \\ a_{21} & a_{22} & a_{23} & \dots & a_{40} \\ a_{41} & a_{42} & a_{43} & \dots & a_{60} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 45)\};\ 1 \le i \le 6, \times_n \right\}$$

be the semigroup.

Study questions (i) to (vi) of problem 29 for this B.

36. Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \dots & \dots & \dots & \dots & a_{12} \\ a_{13} & \dots & \dots & \dots & \dots & a_{18} \\ \vdots & & & & & \vdots \\ a_{67} & \dots & \dots & \dots & \dots & a_{72} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 17)\};\ 1 \le i \le 72, \times_n \right\}$$

be the semigroup.

Study questions (i) to (vi) of problem 29 for this T.

37. Let

$$W = \left\{ \left[\begin{array}{cc|ccc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{18} \\ \hline a_{19} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{27} \\ a_{28} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{36} \\ a_{37} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{45} \\ \hline a_{46} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{54} \\ \hline a_{55} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{63} \\ a_{64} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{72} \end{array} \right] \middle| \; a_i \in \right.$$

$\{(a, b) \mid a, b \in [0, 5)\}; 1 \leq i \leq 72, \times_n\}$

be the super matrix semigroup.

Study questions (i) to (vi) of problem 29 for this W.

38. Is very semigroup under product built using the real square of length less than n a Smarandache semigroup?

39. Let S = $\{(a, b) \mid a, b \in [0, 10), \min, \max\}$ be the semiring on the real square of side length less than 10.

    (i) Prove o(S) = ∞.
    (ii) Prove S has infinite number of idempotents and zero divisors.
    (iii) Find subsemirings of order 2, 3, 4, 5, 10, 18.
    (iv) Prove if A is a subset of S, A can be completed to obtain a subsemiring.
    (v) Can S have ideals of finite order?
    (vi) Can S have filters of finite order?
    (vii) Can a subsemiring be both an ideal and a filter?
    (viii) Obtain a subsemiring of infinite order which is not a filter or an ideal.

40. Let L = $\{(a, b) \mid a, b \in [0, 11), \min, \max\}$ be a semiring.

    Study questions (i) to (viii) of problem 39 for this L.

41. Let M = $\{(a, b) \mid a, b \in [0, 25), \min, \max\}$ be a semi open real square semiring of infinte order.

    Study questions (i) to (viii) of problem 39 for this M.

42. Let W = $\{(a, b) \mid a, b \in [0, 125), \min, \max\}$ be a semiring.

    Study questions (i) to (viii) of problem 39 for this W.

43. Let M = $\{(a_1, a_2, \ldots, a_{15}) \mid a_i \in (a, b) \mid a, b \in [0, 3), 1 \leq i \leq 15, \min, \max\}$ be a semiring.

Study questions (i) to (viii) of problem 39 for this M.

44. Let

$$N = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_{10} \\ a_{11} & a_{12} & a_{13} & \ldots & a_{20} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{30} \end{bmatrix}\right| a_i \in \{(a, b) \mid a, b \in [0, 220)\}; 1 \leq i \leq 30\right\}$$

be a semiring under max, min.

Study questions (i) to (viii) of problem 39 for this N.

45. Let

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{77} & a_{78} & a_{79} & a_{80} \end{bmatrix}\right| a_i \in \{(a, b) \mid a, b \in [0, 24)\}; 1 \leq i \leq 80\right\}$$

be the semiring.

Study questions (i) to (viii) of problem 39 for this M.

46. Let T = $\{(a_1 \mid a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8) \mid a_i \in \{(a, b) \mid a, b \in [0, 43)\}; 1 \leq i \leq 8, \min, \max\}$ be the semiring.

Study questions (i) to (viii) of problem 39 for this T.

47. Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 53)\}, \right.$$

$$\left. 1 \leq i \leq 12, \min, \max \right\}$$

be the semiring.

Study questions (i) to (viii) of problem 39 for this M.

48. Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid \right.$$

$$\left. a, b \in [0, 23)\}; 1 \leq i \leq 24, \min, \max \right\}$$

be the semiring.

Study questions (i) to (viii) of problem 39 for this M.

49. Let

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 7)\}, \; 1 \le i \le 3, \text{ min, max} \right\}$$

be the semiring.

Study questions (i) to (viii) of problem 39 for this M.

50. Let

$$M = \left\{ \left[\begin{array}{c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ \hline a_7 & \dots & \dots & \dots & \dots & a_{12} \\ a_{13} & \dots & \dots & \dots & \dots & a_{18} \\ a_{19} & \dots & \dots & \dots & \dots & a_{24} \\ a_{25} & \dots & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & \dots & \dots & \dots & \dots & a_{36} \\ a_{37} & \dots & \dots & \dots & \dots & a_{42} \\ a_{43} & \dots & \dots & \dots & \dots & a_{48} \\ \hline a_{49} & \dots & \dots & \dots & \dots & a_{54} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 24)\}, \; 1 \le i \le 54, \text{ min, max} \right\}$$

be the semiring.

Study questions (i) to (viii) of problem 39 for this M.

51. Find the special features enjoyed by the pseudo rings built using real squares of side length less than n.

52. Let M = $\{(a, b) \mid a, b \in [0, 42), +, \times\}$ be the pseudo ring of the square side of length less than 42.

    (i) Show M is of infinite order.
    (ii) Can finite subrings be ideals of M?
    (iii) Find pseudo subrings of finite order.
    (iv) Is it possible for M to have infinite number of idempotents?
    (v) Does M have infinite number of units?
    (vi) Find infinite pseudo subrings which are not pseudo ideals.
    (vii) Prove M has infinite number of zero divisors?

53. Prove R = $\{(a, b) \mid a, b \in [0, 19), +, \times\}$ be the pseudo ring.

    (i) Study questions (i) to (vii) of problem 52 for this R.
    (ii) Is R a Smarandache pseudo ring?

54. Let M = $\{(a, b) \mid a, b \in [0, 23), +, \times\}$ be the semi open real square pseudo ring.

    (i) Study questions (i) to (vii) of problem 52 for this R.

55. Let T = $\{(a, b) \mid a, b \in [0, 248), +, \times\}$ be the pseudo ring.

    Study questions (i) to (vii) of problem 52 for this T.

56. Let R = $\{(a_1, a_2, \ldots, a_{15}) \mid a_i \in \{(a, b) \mid a, b \in [0, 41), 1 \leq i \leq 15, +, \times\}$ be the pseudo ring.

    (i) Study questions (i) to (vii) of problem 52 for this R.
    (ii) Is R is S-pseudo ring?

(iii) Can R have S-pseudo ideals?
(iv) Can R have S-pseudo subrings which are not S-ideals?

57. Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 23)\}; \; 1 \leq i \leq 18, +, \times_n \right\}$$

be a pseudo ring.

(i) V a S-pseudo ring?
(ii) Study questions (i) to (vii) of problem 52 for this R.
(iii) Can V have S-pseudo subrings which are not S-pseudo ideals?

58. Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix} \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 271)\}; \; 1 \leq i \leq 64\}, +, \times_n \right\}$$

be a pseudo ring.

(i) Study questions (i) to (vii) of problem 52 for this R.

59. Let

$$L = \left\{\left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{array}\right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 271)\}; \right.$$

$$\left. 1 \leq i \leq 64\}, +, \times_n \right\}$$

be a pseudo ring.

(i) Study questions (i) to (vii) of problem 52 for this L.

60. Let T = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5\ a_6 \mid a_7) \mid a_i \in \{(a, b) \mid a, b \in [0, 45)\}; 1 \leq i \leq 7\}, +, \times_n\}$ be a pseudo ring.

(i) Study questions (i) to (vii) of problem 52 for this L.

61. Let

$$B = \left\{\left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array}\right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 19)\}, 1 \leq i \leq 12, +, \times \right\}$$

be the real semi open square pseudo ring of super column matrices. Study questions (i) to (vii) of problem 52 for this B.

62. Let

$$T = \left\{ \left[\begin{array}{cc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \ldots & \ldots & \ldots & \ldots & a_{12} \\ \hline a_{13} & \ldots & \ldots & \ldots & \ldots & a_{18} \\ \hline a_{19} & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & \ldots & \ldots & \ldots & \ldots & a_{36} \end{array} \right] \middle| \; a_i \in \{(a, b) \mid a, b \in [0, 148)\}, 1 \leq i \leq 36, +, \times \right\}$$

be the real semi open square pseudo ring of super column matrices. Study questions (i) to (vii) of problem 52 for this B.

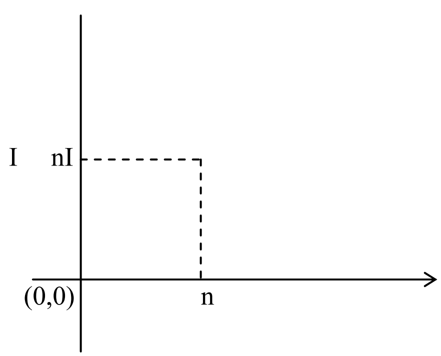
63. Distinguish between semiring and pseudo ring constructed using real squares of length less than n, n < ∞.

64. Obtain any special and stricking features about algebraic structures built using S = {(a, b) | a, b ∈ [0, n)}.

Chapter Two

ALGEBRAIC STRUCTURES USING REAL NEUTROSOPHIC SQUARES

In this chapter we first describe what we mean by the semi open neutrosophic square.



Dotted lines show that the boundary values are not taken so the point n + nI is not in the square constructed.

Elements or points will be of the form a + bI a, b $\in$ [0, n), I is the indeterminacy.

This square will be known as neutrosophic semi open; square or real semi open neutrosophic square.

We will define operations on them and compare it with the real squares at each stage.

$N_S = \{a + bI \mid a, b \in [0, n), I^2 = I\}$ is such that if
$x = a + bI$ and $y = c + dI$ then
$x + y = (a + c) + (d + b)I = s + tu$;

where $s = a + c$ if $a + c$ if $a + c < n$
$= a + c - n$ if $a + c > n$
$= 0$ if $a + c = n$.

Similarly for $t = d + b$ if $d + b < n$
$= d + b - n$ if $d + b > n$
$= 0$ if $d + b = n$.

$N_s = \{a + bI \mid a, b \in [0, n); I^2 = I\}$ is defined as the semi open real neutrosophic square.

We will illustrate this situation by some examples.

Example 2.1: Let $N_S = \{a + bI \mid a, b \in [0, 5)\}$ be the real semi open neutrosophic square.

Example 2.2: Let $N_S = \{a + bI \mid a, b \in [0, 12)\}$ be the real semi open neutrosophic square.

Example 2.3: Let $N_S = \{a + bI \mid a, b \in [0, 18)\}$ be the real semi open neutrosophic square.

We see when n = 1 we call it as the fuzzy semi open neutrosophic square which has been deal separately in.

Example 2.4: Let $N_S = \{a + bI \mid a, b \in [0, 81)\}$ be the real semi open neutrosophic square. Let $x = 79 + 5I$ and $y = 2 + 76I \in N_S$ we see $x + y = 79 + 5I + 2 + 76I = 0 + 0I$.

Let $x = 3.001 + 2.75I$ and $y = 0.815 + 0.375\,I \in N_S$.
$x + y = 3.816 + 3.125I \in N_S$.
$x + x = 3.001 + 2.75I + 3.001 + 2.75I$
$= 6.002 + 5.50I \in N_S$.

$y + y = 0.815 + 0.375\,I + 0.815 + 0.375\,I$
$= 1.630 + 0.750I \in N_S$.

This is the way operation + can be performed on N_S.

Example 2.5: Let $N_S = \{a + bI \mid a, b \in [0, 12)\}$ be the neutrosophic real half open square.

Let $0 + 0I \in N_S$.
We see $x + 0 + 0I = a + bI + 0 + 0I$
$= a + bI = x$ for every $x \in N_S$.

For every $x \in N_S$ there exists a unique $y \in N_S$ such that $x + y = 0 + 0I$.

Inview of all these results we have the following definition.

DEFINITION 2.1: *Let $N_S = \{a + bI \mid a, b \in [0, n), n < \infty, +\}$ be the semi open neutrosophic square group under addition and N_S is known as the real neutrosophic half open square group or semi open square group.*

We see N_S is of infinite order N_S has both finite and infinite order subgroups.

Example 2.6: Let $N_S = \{a + bI \mid a, b \in [0, 7), +\}$ be the real neutrosophic group of infinite order.

$P = \{a + bI \mid a, b \in Z_7\}$ is a subgroup of finite order.

$M = \{a \mid a \in [0, 7)\}$ is a subgroup of infinite order.
$N = \{bI \mid b \in [0, 7)\}$ is a subgroup of infinite order.
$T = \{a \mid a \in Z_7\}$ is a subgroup of finite order.
$L = \{aI \mid a \in Z_7\}$ is a subgroup of finite order.

Thus N_S has both subgroups of finite and infinite order.

Example 2.7: Let $N_S = \{a + bI \mid a, b \in Z_{12}, +\}$ be the semi open real neutrosophic group of infinite order.

N_S has several subgroups of finite order.

$T_1 = \{a + bI \mid a, b \in \{0, 6\}, +\} \subseteq N_S$,
$T_2 = \{a + bI \mid a, b \in \{0, 4, 8\}, +\} \subseteq N_S$,
$T_3 = \{a + bI \mid a, b \in \{0, 3, 6, 9\}, +\} \subseteq N_S$,
$T_4 = \{a + bI \mid a, b \in \{0, 2, \ldots, 10\}, +\} \subseteq N_S$,
and $T_5 = \{a + bI \mid a, b \in Z_{12}, +\} \subseteq N_S$ are finite real neutrosophic subgroups.

$M_1 = \{a \mid a \in Z_{12}, +\} \subseteq N_S$,
$M_2 = \{a \mid a \in \{0, 6\} +\} \subseteq N_S$,
$M_3 = \{a \mid a \in \{0, 2, 4, \ldots. 10\}+\} \subseteq N_S$,
$M_4 = \{a \mid a \in \{0, 3, 6, 9\}, +\} \subseteq N_S$,
and $M_5 = \{a \mid a \in \{0, 4, 8\}+\} \subseteq N_S$ are some of the real subgroups of finite order. They are not neutrosophic.

$N_1 = \{aI \mid a \in Z_{12}, +\} \subseteq N_S$,
$N_2 = \{aI \mid a \in \{0, 6\}, +\} \subseteq N_S$,
$N_3 = \{aI \mid a \in \{0, 4, 8\}, +\} \subseteq N_S$,
$N_4 = \{aI \mid a \in \{0, 2, 4, \ldots, 10\}, +\} \subseteq N_S$,
and $N_5 = \{aI \mid a \in \{0, 3, 6, 9\}, +\} \subseteq N_S$ are pure neutrosophic subgroups of finite order.

$L = \{a + bI \mid a, b \in \{0, 0.5, 1, 1.5, 2, 2.5, 3, \ldots, 11, 11.5\}, +\} \subseteq N_S$ are a real neutrosophic subgroups of finite order.

$M = \{a + bI \mid a, b \in \{0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2, \ldots, 11, 11.2, 11.4, 11.6, 11.8\}, +\} \subseteq N_S$ is a real neutrosophic subgroup of finite order and so on.

Example 2.8: Let $N_S = \{a + bI \mid a, b \in [0, 14), +\}$ be the semi open square real neutrosophic group of infinite order.

N_S has several real semi open square subgroups of both finite and infinite order. N_S has several neutrosophic semi open square subgroups of finite and infinite order.

Example 2.9: Let $N_S = \{a + bI \mid a, b \in [0, 28), +\}$ be the semi open square of real neutrosophic group of infinite order.

Since N_S happens to a commutative group of infinite order we do not have many classical properties associated with it.

However we can build real neutrosophic groups using N_S which will be illustrated by examples.

Example 2.10: Let
$S = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{a + bI \mid a, b \in [0, 10), 1 \leq i \leq 4\}$ be the real neutrosophic semi open group under +.

S is of infinite order and S has both finite and infinite subgroups.

Example 2.11: Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{16} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 32), 1 \leq i \leq 16, +\} \right.$$

be the group of column matrices of the real neutrosophic semi open square. T has several subgroups both of finite and infinite order.

Example 2.12: Let

$$T = \left\{ \left(\begin{array}{cccc} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \end{array} \right) \middle| \, a_i \in \{a + bI \mid a, b \in [0, 24), \right.$$

$$\left. 1 \leq i \leq 40, +\right\}$$

be the real neutrosophic semi open square of infinite order under +.

M have several subgroups of infinite order as well as finite order.

Example 2.13: Let

$$S = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{77} & a_{78} & a_{79} & a_{80} \end{array} \right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 43), \right.$$

$$\left. 1 \leq i \leq 80, +\right\}$$

be the real neutrosophic matrix group on semi open square.

S has several subgroups of finite and infinite order.

Example 2.14: Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ \hline a_{14} \\ \hline a_{15} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 25)\}, 1 \le i \le 15, + \right\}$$

be the real neutrosophic half open (semi open) square of super column matrices.

T has subgroups of both finite and infinite order.

Example 2.15: Let W = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5 \mid a_6) \mid a_i \in \{a + bI \mid a, b \in [0, 14)\}, 1 \le i \le 6, +\}$ be the real neutrosophic super row matrix group over the semi open square.

This W also has several subgroups of both finite and infinite order.

Example 2.16: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 45)\}, \right.$$

$$1 \leq i \leq 25, +\}$$

be the real neutrosophic semi open square matrix of matrix group under +.

V has finite order as well as infinite order subgroups.

Example 2.17: Let

$$V = \left\{ \left[\begin{array}{c|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ \hline a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 42)\}, \right.$$

$$1 \leq i \leq 42, +\}$$

be the real neutrosophic open square matrix super matrix group of infinite order. M has both subgroups of infinite and finite order.

Next we proceed onto describe $N_S = \{a + bI \mid a, b \in [0, n), n < \infty\}$ with product operation '$\times$' N_S under product '$\times$' is only a semigroup.

We will describe this situation by an example or two.

Example 2.18: Let $N_S = \{a + bI \mid a, b \in [0, 6), \times\}$ be the semigroup of real neutrosophic semi open square.

$$
\begin{aligned}
&\text{Let } x = 3 + 0.2I \text{ and } y = 0.72 + 0.18I \in N_S.\\
&x \times y = (3 + 0.2I)\,(0.72 + 0.18I)\\
&= 2.16 + 0.144I + 0.54I + 0.036I^2\\
&= 2.16 + 0.720I \in N_S \quad (I^2 = I).
\end{aligned}
$$

This is the way product on N_S is performed as $I^2 = I$.

Example 2.19: Let $N_S = \{a + bI \mid a, b \in [0, 11), \times\}$ be the semi open square semigroup of infinite order.

N_S is a commutative semigroup.

If $x = 6$ and $y = 2 \in N_S$ then $x \times y = 6 \times 2 = 1$ is a unit.

N_S has units. N_S has zero divisors.

If $x = 6 + 5I$ and $y = I$ then $xy = 0$ is a zero divisor.

If $x = 0.7101 + 10.2899I$ and $y = I \in N_S$ then $xy = (0)$ is a zero divisor in N_S.

N_S has units and zero divisor. $I^2 = I$ is atleast one idempotent of N_S.

Example 2.20: Let $N_S = \{a + bI \mid a, b \in [0, 12), \times\}$ be the neutrosophic semi open square semigroup of infinite order.

$x = 4$, $x_1 = 9$, $y = 4I$, $z = I$, $t = 9I \in Ns$ are some of the idempotents of N_S.

$g_1 = 8 + 4I$ is a zero divisor as $h_1 = I$ in N_S is such that $g_1 h_1 = 0$.

We have infinite number of zero divisors.

$x = 0.3112 + 11.6888I$ and $y = I$ are such that $xy = 0 + 0I$.

Example 2.21: Let $N_S = \{a + bI \mid a, b \in [0, 24), \times\}$ be the neutrosophic semi open square semigroup of infinite order.

$x = 5$ is such that $x^2 = 1$, $y = 7 \in N_S$ is such that $y^2 = 1$. Let $x = I$ is an idempotent in N_S.

Example 2.22: Let $N_S = \{a + bI \mid a, b \in [0, 6), \times\}$ be the semigroup of infinite order.

$x = 5+I$ and $y = I \in M$ are such that $x \times y = 0$, $y = 0.7I + 6.3$ and $x = I$ in M are such that $x \times y = yx = 0$, $x_1 = 3I$, $x_2 = 3$, $x_3 = 4$, $x_4 = 4I$ and $x_5 = 3 + 4I$ in M are such that $x_1^2 = x$, $x_2^2 = x_2$, $x_3^2 = x_3$, $x_4^2 = x_4$, $x_5^2 = (3 + 4I)\ (3 + 4I)$

$$= 9 + 12I + 12I + 16I$$
$$= 3 + 4I = x_5$$ are idempotents of M.

Let $x = 4 + 4I$ and $y = 3$ or $3I$ in M are such that $xy = 0$.

Example 2.23: Let
$M = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \{a + bI \mid a, b \in [0, 3), 1 \leq i \leq 5, \times\}$ be a semigroup of infinite order.

Let $x = (0, 0.3I, 2+I, 0, 0.7I + 1)$ and
$y = (2 + 2I, 0, I, 0.77 + 1.22I, 0) \in M$ we see $x \times y = (0, 0, 0, 0, 0)$ is a zero divisor in M.

$x = (0.2 + 2.8I, 1 + 2I, 1.3 + 1.7I, 0.5 + 2.5I, 1.5 + 1.5I)$ and
$y = (I, I, I, I, I) \in M$.

We see x ×y = (0, 0, 0, 0, 0).

Example 2.24: Let
N = $\{(a_1, a_2, \ldots, a_{12}) \mid a_i \in \{a + bI \mid a, b \in [0, 8), \times\}$ be a semi open neutrosophic square semigroup under product × of infinite order.

Example 2.25: Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 18) \; 1 \leq i \leq 18, \times_n\} \right.$$

be the neutrosophic semi open square semigroup of infinite order.

$\{T, \times_n\}$ is a commutative semigroup which has infinite number of zero divisors only finite number of units and idempotents in T.

Example 2.26: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 27)\}, \right.$$

$1 \leq i \leq 16, \times_n\}$

be the semigroup under natural product $\times_n$.

W has units, zero divisors and idempotents.

Example 2.27: Let

$$S = \left\{ \left[\begin{array}{c|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 24)\}, \right.$$

$$1 \leq i \leq 40, \times_n\}$$

be the semigroup under natural product.

S has infinite number of zero divisors and only finite number of idempotents. Only finite number of units.

Example 2.28: Let M = $\{\{(a_1\ a_2 \mid a_3 \mid a_4\ a_5\ a_6 \mid a_7) \mid a_i \in \{a + bI \mid a, b \in [0, 13)\}, 1 \leq i \leq 7, \times_n\}$ be the real neutrosophic semi open square semigroup of super row matrices under product.

M has zero divisors, idempotents, subsemigroups and ideals.

Take $P_1 = \{(a_1\ 0 \mid 0 \mid 0\ 0\ 0 \mid 0) \mid a_1 \in \{a + bI \mid a, b \in [0, 13), \times\} \subseteq M$,

$P_2 = \{(0\ a_2 \mid 0 \mid 0\ 0\ 0 \mid 0) \mid a_2 \in \{a + bI \mid a, b \in [0, 13), \times\} \subseteq M$,

$P_3 = \{(0\ 0 \mid a_3 \mid 0\ 0\ 0 \mid 0) \mid a_3 \in \{a + bI \mid a, b \in [0, 13), \times\} \subseteq M$,

$P_4 = \{(0\ 0 \mid 0 \mid a_4\ 0\ 0 \mid 0) \mid a_4 \in \{a + bI \mid a, b \in [0, 13), \times\} \subseteq M$ and so on.

$P_7 = \{(0\ 0 \mid 0 \mid 0\ 0\ 0 \mid a_7) \mid a_7 \in \{a + bI \mid a, b \in [0, 13), \times\} \subseteq$ M are all subsemigroups which are also ideals of M.

$N_1 = \{(a_1\ 0 \mid 0 \mid 0\ 0\ 0 \mid 0) \mid a_1 \in [0, 6) \subseteq [0, 13)\ \} \subseteq M$,
$N_2 = \{(0\ a_2 \mid 0 \mid 0\ 0\ 0 \mid 0) \mid a_2 \in [0, 6) \subseteq [0, 13)\ \} \subseteq M$,
$N_3 = \{(0\ 0 \mid a_3 \mid 0\ 0\ 0 \mid 0) \mid a_3 \in [0, 6) \subseteq [0, 13)\ \} \subseteq M$ and so on;
$N_7 = \{(0\ 0 \mid 0 \mid 0\ 0\ 0 \mid a_7) \mid a_7 \in [0, 6) \subseteq [0, 13)\ \} \subseteq M$ are all subsemigroups which are not ideals.

All of them are subsemigroups of infinite order and they are not ideals.

Example 2.29: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{19} \end{bmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in [0, 27)\},\ 1 \leq i \leq 19, \times_n \right\}$$

be the real neutrosophic square of column matrix semigroup under the natural product.

$$\text{Take } P_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \, a_1 \in \{a + bI \mid a, b \in [0, 27), \times_n\} \subseteq M, \right.$$

$$P_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_2 \in \{a + bI \mid a, b \in [0, 27), \times_n\} \subseteq M \right.$$

and so on.

$$P_{19} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ a_{19} \end{bmatrix} \middle| a_{19} \in \{a + bI \mid a, b \in [0, 27), \times_n\} \subseteq M \right.$$

are all ideals of M.

$$T_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1 \in \{a + bI \mid a, b \in Z_{27}, \times_n\} \subseteq M, \right.$$

$$T_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_2 \in \{a + bI \mid a, b \in Z_{27}, \times_n\} \subseteq M, \right.$$

$$T_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ a_3 \\ \vdots \\ 0 \end{bmatrix} \middle| a_3 \in \{a + bI \mid a, b \in Z_{27}, \times_n\} \subseteq M, \right.$$

$$T_4 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_4 \\ \vdots \\ 0 \end{bmatrix} \middle| a_4 \in \{a + bI \mid a, b \in Z_{27}, \times_n\} \subseteq M \right.$$

and so on

$$T_{19} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ a_{19} \end{bmatrix} \middle| a_{19} \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \subseteq M \right.$$

are all only subsemigroups of finite order and none of them are ideals.

$$S_{1,2} = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1, a_2 \in \{a + bI \mid a, b \in Z_{27}\}, \times_n\} \subseteq M, \right.$$

$$S_{1,3} = \left\{ \begin{bmatrix} a_1 \\ 0 \\ a_3 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1, a_3 \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M,$$

$$S_{1,4} = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ a_4 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1, a_4 \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M \text{ and}$$

so on.

$$S_{1,19} = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ a_{19} \end{bmatrix} \middle| a_1, a_{19} \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M,$$

$$S_{2,3} = \left\{ \begin{bmatrix} 0 \\ a_2 \\ a_3 \\ \vdots \\ 0 \end{bmatrix} \middle| a_2, a_3 \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M$$

and so on.

$$S_{2,\,19} = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ a_{19} \end{bmatrix} \middle| \; a_2, a_{19} \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M$$

are all only subsemigroups of finite order and none of them are ideals of M.

$$S_{1,\,2,\,3} = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_1, a_2, a_3 \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M,$$

$$S_{1,\,2,\,4} = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ a_4 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_1, a_2, a_4 \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M,$$

and so on.

$$S_{1,\,18,\,19} = \left\{ \begin{bmatrix} a_1 \\ 0 \\ \vdots \\ a_{18} \\ a_{19} \end{bmatrix} \middle| \; a_1, a_{18}, a_{19} \in \{a + bI \mid a, b \in Z_{27}\}, \times_n \right\} \subseteq M$$

be the subsemigroups all of which are of finite order. None of them are ideals.

$$W_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ a_{17} \\ 0 \\ 0 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in Z_{27}\}, \times_n, 1 \leq i \leq 17 \right\},$$

$$W_2 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ a_{16} \\ 0 \\ a_{17} \\ 0 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in Z_{27}\}, \times_n, 1 \leq i \leq 17 \right\} \subseteq M,$$

$$W_3 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ a_{16} \\ 0 \\ 0 \\ a_{17} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in Z_{27}\}, \times_n, 1 \leq i \leq 17 \right\} \subseteq M$$

and so on.

$$W_4 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \\ a_{17} \\ 0 \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in Z_{27}\}, \times_n, 1 \leq i \leq 17 \right\} \subseteq M,$$

$$W_5 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ a_2 \\ a_3 \\ \vdots \\ a_{17} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in Z_{27}\}, \times_n, 1 \leq i \leq 17 \right\} \subseteq M$$

and so on.

$$W_{17} = \left\{ \begin{bmatrix} 0 \\ 0 \\ a_1 \\ a_2 \\ \vdots \\ a_{17} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in Z_{27}\}, \times_n, 1 \leq i \leq 17 \right\} \subseteq M.$$

All of them are only subsemigroups of finite order and none of them are ideals.

Example 2.30: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 193)\}, \right.$$

$$\left. 1 \leq i \leq 40, \times_n \right\}$$

be the real neutrosophic semi open square semigroup of infinite order.

Let

$$M_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \middle| \; a_1 \in \{a + bI \mid a, b \in \right.$$

$$\left. [0, 193)\}, \times_n \right\} \subseteq S,$$

$$M_2 = \left\{ \begin{bmatrix} 0 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \middle| \; a_2 \in \{a + bI \mid \right.$$

$$\left. a, b \in [0, 193)\}, \times_n \right\} \subseteq S$$

and so on.

$$M_{40} = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & a_{40} \end{bmatrix} \middle| \; a_{40} \in \{a + bI \mid \right.$$

$$a, b \in [0, 193)\}, \times_n\} \subseteq S$$

are all subsemigroups which are also ideals.

$$M_{1,2} = \left\{\left.\begin{bmatrix} a_1 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right| a_1, a_2 \in \{a + bI \mid\right.$$

$$a, b \in [0, 193)\}, \times_n\} \subseteq S$$

$$M_{8,12} = \left\{\left.\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_8 & 0 & 0 \\ 0 & a_{12} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right| a_8, a_{12} \in \{a + bI \mid\right.$$

$$a, b \in [0, 193)\}, \times_n\} \subseteq S$$

$$M_{31, 32} = \left\{\left.\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right| a_{31}, a_{32} \in \{a + bI \mid\right.$$

$$a, b \in [0, 193)\}, \times_n\} \subseteq S \text{ and}$$

$$M_{39,40} = \left\{ \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{39} & a_{40} \end{array} \right] \right| a_{39}, a_{40} \in \{a + bI \mid$$

$$a, b \in [0, 193)\}, \times_n\} \subseteq S$$

are all subsemigroups which are all ideals of S.

We see none of the finite subsemigroups of S are ideals. Further all ideals of S are of infinite order.

$$A_{1,2,5} = \left\{ \left[\begin{array}{ccccc} a_1 & a_2 & 0 & 0 & a_5 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \right| a_1\ a_2\ a_5 \in \{a + bI \mid$$

$$a, b \in Z_{193}\}, \times_n\} \subseteq S$$

is only a subsemigroups but not an ideal.

Thus S has atleast ${}_{40}C_1 + {}_{40}C_2 + \ldots + {}_{40}C_{39}$ number of subsemigroups of finite order which are not ideals in S.

Similarly S has atleast ${}_{40}C_1 + {}_{40}C_2 + \ldots + {}_{40}C_{39}$ number of subsemigroups of infinite order which are ideals of S.

$$A_{7, 21, 35, 14} = \left\{ \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & a_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{35} \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \middle| \; a_7, a_{14}, a_{21}, a_{35} \in \right.$$

$$\{a + bI \mid a, b \in [0, 193)\}, \times_n\} \subseteq S$$

is a subsemigroup of infinite order which is an ideal of S.

$$A_{10, 16, 40, 24} = \left\{ \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{10} \\ 0 & 0 & 0 & 0 & 0 \\ a_{16} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{24} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{40} \end{array} \right] \middle| \; a_{10}, a_{16}, a_{40}, a_{24} \in \right.$$

$$\{a + bI \mid a, b \in [0, 193)\}, \times_n\} \subseteq S$$

be the subsemigroup of finite order which is not an ideal.

$$B_{1,5,7,9,12,15,20,32,38} = \left\{\left[\begin{array}{ccccc} a_1 & 0 & 0 & 0 & a_5 \\ 0 & a_7 & 0 & a_9 & 0 \\ 0 & a_{12} & 0 & 0 & a_{15} \\ 0 & 0 & 0 & 0 & a_{20} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & a_{32} & 0 & 0 & 0 \\ 0 & 0 & a_{38} & 0 & 0 \end{array}\right] \middle| \; a_1, a_7,\right.$$

$$\left. a_5, a_9, a_{12}, a_{15}, a_{20}, a_{32}, a_{38} \in \{a + bI \mid a, b \in [0, 193)\}, \times_n\right\} \subseteq S$$

be a subsemigroup of infinite order which is an ideal of S.

$$x_{9,18,19,22,24,30,39,40} = \left\{\left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_9 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{18} & a_{19} & 0 \\ 0 & a_{22} & 0 & a_{24} & 0 \\ 0 & 0 & 0 & 0 & a_{30} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{39} & a_{40} \end{array}\right] \middle| \; a_{19}, a_{18}, a_{30},\right.$$

$$a_{39} \in \{a + bI \mid a, b \in Z_{193}\}, a_9, a_{22}, a_{24}, a_{40} \in \{a + bI \mid$$

$$\left. a, b \in [0, 193)\}, \times_n\right\}$$

be a subsemigroups of S of infinite order but is not an ideal of S.

Thus S has infinite order subsemigroups which are not ideals of S.

Let

$$Y_{1,40} = \left\{ \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{40} \end{bmatrix} \right| a_1 \in \{a + bI \mid a, b \in Z_{193}\},$$

$$a_{40} \in \{a + bI \mid a, b \in [0, 193)\}, \times_n\} \subseteq S$$

be subsemigroup of infinite order and $Y_{1,40}$ is not an ideal of S.

Example 2.31: Let

$$S = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 12)\},$$

$$1 \leq i \leq 10, \times_n\}$$

be the real neutrosophic semi open square matrix semigroup of infinite order.

S has subsemigroups of infinite and finite order and finite order subsemigroups are not ideals. However all infinite subsemigroups are not ideals.

Let

$$P = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 12)\},$$

$$1 \leq i \leq 5, \times_n\} \subseteq S$$

be a subsemigroup of S which is also an ideal of S and P is of infinite order.

Let

$$Y = \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix} \middle| a_i \in \{a + bI \mid a, b \in Z_{12}\}, \right.$$

$$\left. 1 \leq i \leq 10, \times_n \right\} \subseteq S$$

be a subsemigroup of S of finite order and Y is not an ideal of S.

Let

$$X = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 12)\}, \right.$$

$$\left. 1 \leq i \leq 5\} \text{ and } a_j \in \{a + bI \mid a, b \in [0, 12)\},\ 5 \leq j \leq 10, \times_n \right\}$$

be a subsemigroup of infinite order, clearly $\times_n$ is not an ideal of S.

We have several subsemigroups of infinite order which are not ideals.

X is one such case

$$L_{1,2} = \left\{ \begin{pmatrix} a_1 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| a_1 \in \{a + bI \mid a, b \in Z_{12}\} \right.$$

and $a_2 \in \{a + bI \mid a, b \in [0, 12)\} \times_n\} \subseteq S$ is only a subsemigroup of infinite order which is not an ideal of S.

$$L_{1,9} = \left\{ \begin{pmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_9 & 0 \end{pmatrix} \middle| a_1 \in \{a + bI \mid a, b \in Z_{12}\} \right.$$

and $a_9 \in \{a + bI \mid a, b \in [0, 12)\}, \times_n\} \subseteq S$ is a subsemigroup of infinite order and is not an ideal of S.

Thus we can have several such ones. S has also infinite number of zero divisors.

$$\text{Let } M_1 = \left\{\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| a_1 \in [0, 12), \times_n\right\} \subseteq S$$

be a subsemigroup of infinite order which is real.

$$N_1 = \left\{\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| a_1 \in Z_{12}I, \times_n\right\}$$

is a finite neutrosophic subsemigroup which is not an ideal.

$$W_1 = \left\{\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \middle| a_1 = bI \text{ where } b \in \{[0, n)\}, \times_n\right\} \subseteq S$$

is a subsemigroup of infinite order which is also an ideal.

W_1 is a pure neutrosophic subsemigroup.

$$\text{If } A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix} \in S \text{ and}$$

$$w_1 = \begin{pmatrix} 6.31I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in W_1 \text{ then}$$

$$Aw_1 = \begin{pmatrix} 6.31Ia + b6.31I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} (a+b)6.31I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \in W_1.$$

Hence the claim.

S has atleast ${}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9$ number of pure neutrosophic subsemigroups of infinite order which are also pure neutrosophic ideals of S.

All these matrices have pure neutrosophic ideals also.

Inview of all these we have the following theorem.

THEOREM 2.1: *Let $S = \{m \times n$ neutrosophic matrices with entries from $S_N = \{a + bI \mid a, b \in [0, s), 1 < s < \infty\}\}$ be the real neutrosophic square matrix semigroup under natural product $\times_n$.*

(i) S has atleast ${}_{m \times n}C_1 + \ldots + {}_{m \times n}C_{m \times n-1}$ number of subsemigroups of finite order which are not ideals.

(ii) S has subsemigroups of infinite order which are not ideals.

(iii) All ideals of S are of infinite order.

(iv) S has atleast ${}_{m \times n}C_1 + {}_{m \times n}C_2 + \ldots + {}_{m \times n}C_{m \times n-1}$ number of pure neutrosophic subsemigroups of finite order which are not ideals.

(v) S has atleast ${}_{m \times n}C_1 + \ldots + {}_{m \times n}C_{m \times n-1}$ number of real subsemigroups of finite order which are not ideals.

(vi) S has atleast ${}_{m \times n}C_1 + {}_{m \times n}C_2 + \ldots + {}_{m \times n}C_{m \times n-1}$ number of real subsemigroups of infinite order which are not ideals.

(vii) S has atleast ${}_{m \times n}C_1 + {}_{m \times n}C_2 + \ldots + {}_{m \times n}C_{m \times n-1}$ number of pure neutrosophic subsemigroups of infinite order which are ideals.

Proof is left as an exercise to the reader.

Example 2.32: Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \end{array}\right] \middle|\; a_i \in \{a + bI \mid a, b \in [0, 43)\}, 1 \leq i \leq 14, \times_n \right\}$$

be the real neutrosophic square column super matrix semigroup of infinite order.

All properties mentioned in the theorem are true for this M.

Example 2.33: Let N = $\{(a_1\ a_2 \mid a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9)$ where $a_i \in \{a + bI \mid a, b \in [0, 12)\}, 1 \leq i \leq 9, \times_n\}$ be the real neutrosophic super row matrix semigroup of infinite order.

Let $P_1 = \{(a_1\ 0 \mid 0 \mid 0\ 0\ 0 \mid 0\ 0 \mid 0) \mid a_1 \in \{a + bI \mid a, b \in Z_{12}\} \subseteq N$ be the subsemigroup of finite order. P_1 has zero divisors but P_1 is not an ideal. Units and idempotents in P_1 are only finite in number.

We have atleast ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of such subsemigroups of finite order which are not ideals satisfying the above said property.

Example 2.34: Let

$$W = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} \\ \hline a_{40} & a_{41} & a_{42} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 5)\}, \right.$$

$$1 \leq i \leq 42, \times_n\}$$

be the real neutrosophic semigroup of a super column matrices.

W is of infinite order and satisfies all the properties mentioned in theorem 2.

Example 2.35: Let

$$P = \left\{ \left[\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ \hline a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{array}\right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 5)\}, 1 \leq i \leq 45, \times_n \right\}$$

be the pure neutrosophic super matrix semigroup of infinite order.

P has atleast ${}_{45}C_1 + \ldots + {}_{45}C_{44}$ number of real subsemigroups of finite order which are not ideals.

P has atleast ${}_{45}C_1 + {}_{45}C_2 + \ldots + {}_{45}C_{44}$ number of real subsemigroups of infinite order which are not ideals.

P has zero divisors and units.

Example 2.36: Let

$$M = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \end{pmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in [0, 23)\}, 1 \leq i \leq 24, \times_n \right\}$$

be the semigroup of infinite order. M has infinite number of zero divisors, finite number of idempotents and units.

Next we define using the real neutrosophic semi open square $S_N = \{a + bI \mid a, b \in [0, n)\}$, the semigroup using min (or max) operation (or used in the mutually exclusive sense.

Let $S_N = \{a + bI \mid a, b \in [0, n)\}$ define min $\{a + bI, c + Id\}$ = min (a, c) + min {b, d}I.

Clearly min $\{a + bI, a + bI\} = a + bI$.

Thus every element under min operation is an idempotent.

Example 2.37: Let $S_N = \{a + bI \mid a, b \in [0, 7), \min\}$ be the semigroup. S_N is an idempotent semigroup of infinite order.

$x = 0.3 + 17I$ and $y = 6.1 + 0.1I \in S_N$.
min $\{x, y\} = \{0.3 + 0.1\}$.
min $\{x, x\} = 0.3 + 1.7I = x$.
min $\{y, y\} = 6.1 + 0.1I = y$.

If $x = 0.7$ and $y = 0.8I$ then min $\{x, y\} = 0$ for min $\{x, y\}$ = min $\{0.7, 0\}$ + min $\{0, 0.8I\} = 0 + 0I$.

Thus S_N under min operation has infinite number of zero divisors and every element is an idempotent hence in S_N under the min operation every x is a subsemigroup.

Thus if $P = \{x\}$ then min $\{x, x\} = x$ hence P is a subsemigroup.

Let $W = \{0.7 + 2.1I, 5.3 + 0.7I\}$
$= \{x, y\}$ $(x, y \in S_N)$.

min $\{x, x\} = x$, min $\{y, y\} = y$ and min $\{x, y\} = 0.7 + 0.7I \notin W$.

Thus $W = \{0.7 + 2.1I, 5.3 + 0.7I, 0.7 + 0.7I\}$ is a subsemigroup of order three.

Let P = {0.3 + 0.4 I = x, y = 0.6 + 0.003I, z = 7 + 0.00002I} $\subseteq$ S_N. We see min {x, y} = 0.003I + 0.3 = s

min {x, z} = 0.3 + 0.00002I = t
min {y, z} = 0.6 + 0.00002I = u

P_c = {x, y, z, s, t, u} is the completed subsemigroup of S_N of order six.

Example 2.38: Let M = {a + bI | a, b $\in$ [0, 27), min} be the semigroup. M has infinite number of zero divisors if x = a and y = bI, a, b $\in$ [0, 27) then min {x, y} = 0 + 0I.

Let x = 0.3 + 8I and y = 7 + 0.5I $\in$ M.

min {x, y} = {0.3 + 0.5I}.

Thus {x, y, 0.3 + 0.5I} $\subseteq$ M is a subsemigroup of order three in M. M has infinite number of subsemigroups which are of finite order.

Let W = {a + bI | a, b $\in$ [0, 12) $\subseteq$[0, 27)}. W is a subsemigroup of infinite order.

Infact W is an ideal of M. Given any subset A of the semigroup M, we can complete that subset say to A_c and A_c will be subsemigroup of M.

In general A_c is not an ideal of M.

Example 2.39: Let B = {a + bI | a, b $\in$ [0, 12), min} be the semigroup of infinite order.

$P_{0.5}$ = {a + bI | a, b $\in$ [0, 0.5)},
$P_{2.5}$ = {a + bI | a, b $\in$ [0, 2.5)},
$P_{3.7}$ = {a + bI | a, b $\in$ [0, 3.7)},

and so on.

$P_t = \{a + bI \mid a, b \in [0, t), t < 11\}$ are all subsemigroups of B which are ideals of B.

Let $M_1 = \{0.3 + 0.8I, 0.7 + 0.5I, 9 + 0.2I, 0.1 + 6.5I\} \subseteq B$. M_1 is only a subset of B and is not a subsemigroup.

However M_1 can be completed to get a subsemigroup. Infact there are infinite number of subsets in B which are not subsemigroups and all of them can be completed to form a subsemigroup.

Infact there are infinite number of subsets in B which are not subsemigroups and all of them can be completed to form a subsemigroup. This is the main advantage of using the min operation.

Now we give examples of semigroups constructed using $S_N = \{a + bI \mid a, b \in [0, n), \min\}$ the semigroup.

Example 2.40: Let $P = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \{a + bI \mid a, b \in [0, 7)\}\ 1 \leq i \leq 5, \min\}$ be the real neutrosophic square row matrix semigroup of infinite order.

P has subsemigroups of finite and infinite order. All ideals of P are of infinite order.

$T = \{(0.7I, 0.8 + 6I, 0.7 + 0.5I, 0, 1.8 + 3I)\} \subseteq P$ is a subsemigroup of order one.

Let $W = \{(0.3, 0.4 + 2I, 3I, 4, 6+2I)\ y = (0.7 + 3I, 0.2+ 0.5I, 4+2I, 3+I, 3+I)\} \subseteq P$

W is only a subset and not a subsemigroup.

For $\min\{x, y\} = (0.3, 0.2 + 0.5I, 2I, 3, 3+I)$ is not in W.

But $W_c = \{x, y, \min\{x, y\}\} \subseteq P$ is a subsemigroup of P.

W_c is the completion of W.

Let M = {(0, 0, 0.3I, 0.2 + I, 1) = x,
y = (0.7, 0.8I, 4, 0, 6I)} $\subseteq$ P; we see M is only a subset of P.

M is completed to M_c = {x, y, min {x, y} = (0, 0, 0, 0, 0)} $\subseteq$ P which is a subsemigroup of order three in P.

Clearly if N is a subset of cardinality two which is not a subsemigroup then N_c the completion of N into a subsemigroup is a subsemigroup of order three.

This is universally true for all subset with a pair of distinct elements which is not a subsemigroup.

Example 2.41: Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_9 \end{array}\right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 6)\}, \text{min}, 1 \leq i \leq 9 \right\}$$

be the real neutrosophic square semigroup of column matrices. T is of infinite order.

T has subsemigroups of both infinite and finite order.

The number of finite subsemigroups are infinite in number.

Similarly the number of subsemigroups of infinite order is also infinite in number.

T has infinite number of zero divisors. T has only finite number of units and infinite number of idempotents in it.

All proper subsets of T can be completed into a subsemigroup of T.

If $X = \{x_1, x_2, \ldots, x_n\}$ is a subset such that $\min \{x_i, x_j\} \notin X$ if $i \neq j$, $1 \leq i, j \leq n$ then $X_c = \{x_1, x_2, \ldots, x_n, \min \{x_i, x_j\}\}$ $1 \leq i, j \leq n$ is a subsemigroup of T.

Example 2.42: Let

$$S = \left\{ \left[\begin{array}{c|cc|c} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 8)\}, \min, 1 \leq i \leq 28 \right\}$$

be the semigroup built using the real neutrosophic square under min operation.

S has infinite number of subsemigroups which are ideals and some of them are not ideals of S.

S has infinite number of zero divisors and idempotents. Of course every element is a subsemigroup.

S has subsemigroups of order one, two, three etc.

Infact S has subsemigroup of order n; $n \in Z^+$.

Example 2.43: Let

$$M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ \hline a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} \end{array}\right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 9)\}, \right.$$

$$\left. \min, 1 \leq i \leq 52 \right\}$$

be the semigroup of column super matrix of infinite order. M has infinite number of zero divisors. M has subsemigroups of finite and infinite order.

Now we build on $S_N = \{a + bI \mid a, b \in [0, n)\}$ with max operation on it semigroups of infinite order.

Example 2.44: Let $S_N = \{a + bI \mid a, b \in [0, 24), \max\}$ be the real neutrosophic square semigroup under max of infinite order.

S_N has every element to be an idempotent. Every singleton element is a subsemigroup.

Clearly S_N has no zero divisors or units.

Let $x = 5 + 0.9I$ and $y = 0.7I + 8 \in S_N$, $\max \{x, y\} = 8 + 0.9I$.

This is the way max operation is performed on S_N.

Let x = {0 + 0I} and y = a + bI (a, b ∈ [0, 24) \ {0} max {y, x} = a + bI = y.

Let x = 6.3 + 7.2I and y = 5.8 + 12I ∈ S_N. Max {x, y} = 6.3 + 12I ∈ S_N.

Thus P_c = {x, y, 6.3 + 12I} is a subsemigroup of S_N.

Let P = {(0 + 0I), x = 0.7 + 11I, y = 8I, z = 5 + 15I} ⊆ S_N. P is only a subset of S_N.

P can be completed to get at the subsemigroup.

P_c = {0.7 + 11I, 5 + 15I, 0.7 + 11I, 8I, 0 + 0I} ⊆ S_N is the subsemigroup of S_N.

B = {9I, 8, 4 + 12I, 0.3, 17I, 21 + 5I} ⊆ S_N be the subset of S_N. We can complete B by using max operation on it.

B_c = {8 + 9I, 12I + 4, 0.3 + 17I, 21 + 9I, 8 + 12, 8 + 17I, 4 + 17I, 21 + 12I, 21 + 17I} is a subsemigroup of order 9.

|B| = 5.

But max {8, 21 + 5I} = 21 + 5I

max {9I, 4 + 12I} = 4 + 12I
max {9I, 0.3 + 17I} = 0.3 + 17I.

That is why $o(B_c)$ = 9.

Let V = {6.3 + 7.5I, 9 + 3.4I, 1.2I + 19, 14I + 0.3} ⊆ S_N is not a subsemigroup of S_N.

V_c = {6.3 + 7.5I, 9 + 7.5I, 19 + 7.5I, 14I + 6.3, 19 + 3.4I, 14I + 9, 19 + 14I, 9 + 3.4I, 1.2I + 19, 14I + 0.3} is a subsemigroup of finite order in S_N.

$o(V) = 4$ and $o(V_c) = 10$.

This is the way subsemigroups are completed.

Any subset finite or infinite in S_N can be completed to subsemigroups.

We see min $\{x, 0\} = 0$ and max $\{x, 0\} = x$.

These semigroups under max has elements which is the greatest one but in case of semigroups under min $0 + 0I = 0$ is the least element.

Example 2.45: Let $W = \{a + bI \mid a, b \in [0, 19), \max\}$ be the semigroup of infinite order. W has no zero divisors. Every element in W is an idempotent under max operation.

Let $x = 3 + 12I$ and $y = 12 + 3I \in W$; max $\{x, y\} = 12 + 12I$.

Let $P = \{a + bI \mid a, b \in [0, 12) \subseteq [0, 19)\} \subseteq W$, P is a subsemigroup of W under max operation.

However P is not an ideal of W for if $x = 14 + 15I \in W$ and $y = a + bI \in P$ we see max $\{x, y\} = x \notin P$ hence the claim.

Let $M = \{a + bI \mid a, b \in [7, 19) \subseteq [0, 19)\} \subseteq W$ be a subsemigroup of W. M is an ideal for any $x \in M$ and $y \in W$ we see max $\{x, y\} \in M$. Thus M is an ideal infact W has infinite number of subsemigroups which are ideals of W.

Let $T = \{a + bI \mid a, b \in [6, 12) \subseteq [0, 19\} \subseteq W$. T is a subsemigroup and is not an ideal of W.

Example 2.46 : Let $P = \{a + bI \mid a, b \in [0, 12), \max\}$ be a semigroup of infinite order.

$T_1 = \{a + bI \mid a, b \in [1, 6)\} \subseteq P$ is a subsemigroup of P and is not an ideal of P.

$T_2 = \{a + bI \mid a, b \in [6, 12)\} \subseteq P$ is a subsemigroup of P and is not an ideal of P.

and $T_3 = \{a + bI \mid a, b \in [3, 6)\} \subseteq P$ is a subsemigroup of P and is not an ideal of P.

In view of all these we have the following theorem.

THEOREM 2.2: *Let $S_N = \{a + bI \mid a, b \in [0, n), max\}$ be a neutrosophic semi open square semigroup of infinite order.*

(i) S_N *has no least element.*
(ii) S_N *has subsemigroups of order one, two, ...*
(iii) *All ideals of* S_N *are of infinite order.*
(iv) $P_1 = \{a + bI \mid a, b \in [0, t); t < n - 1\} \subseteq S_N$ *is only a subsemigroup and not an ideal of* S_N.
(v) $T_1 = \{a + bI \mid a, b \in [t, n)\} \subseteq S_N$ *is a subsemigroup which is also an ideal of* S_N.
(vi) $W_1 = \{a + bI \mid a, b \in [t, m), 0 \neq t \text{ and } m < n - 1\} \subseteq S_N$ *is a subsemigroup and not an ideal of* S_N.
(vii) S_N *has no zero divisors.*

The proof is direct and hence left as an exercise to the reader.

Now we proceed onto give the theorem in case of min operation.

THEOREM 2.3: *Let $S_N = \{a + bI \mid a, b \in [0, n), n < \infty, min\}$ be a neutrosophic semi open square semigroup of infinite order.*

(i) S_N *has infinite number of zero divisors.*
(ii) $0 + 0I$ *is the least element in* S_N.
(iii) *Every element is an idempotent in* S_N.
(iv) S_N *has subsemigroups of order one, two, three etc.*
(v) S_N *has no ideals of finite order.*
(vi) $T = \{a + bI \mid a, b \in [0, t), t < n-1\} \subseteq S_N$ *are ideals of* S_N.

(vii) *$P = \{a + bI \mid a, b \in [s, t)\ 0 < s$ and $t < n-1\} \subseteq S_N$ are only subsemigroups and not ideals of S_N.*

(viii) *$B = \{a + bI \mid a, b \in [t, n), 0 < t\} \subseteq S_N$ are only subsemigroups and not ideals of S_N.*

Now we proceed onto construct semigroups under max operation using the real neutrosophic square.

Example 2.47: Let $M = \{(a_1, a_2, a_3, a_4, \ldots, a_{20}) \mid a_i \in \{a + bI \mid a, b \in [0, 22)\}\ 1 \leq i \leq 20, \max\}$ be a semigroup of infinite order. M has no zero divisors.

Every element in M is an idempotent. Every singleton is a subsemigroup of order one.

M has subsemigroups of order two, three and so on but none of the finite ordered subsemigroups are ideals of M.

Let $P_1 = (a_1, 0, \ldots, 0) \mid a_1 = \{a + bI \mid a, b \in [0, 22)\} \subseteq M$; be a subsemigroup of infinite order. However P_1 is not an ideal of M.

For if $x = (9 + 8.1I, 0, 0, \ldots, 0) \in P_1$ and $y = (a_1, a_2, \ldots, a_{20})$ is S atleast some of $a_i \neq 0.2 \leq i \leq 20$ then $\max \{x, y\} \notin P_1$ hence the claim.

P_1 is only a subsemigroup of infinite order.

Similarly we can have several subsemigroups of infinite order none of which are ideals of S_N.

Let $B = \{(a_1, a_2, \ldots, a_{20}) \mid a_i \in \{a + bI \mid a, b \in [0, 11)\}, 1 \leq i \leq 20, \max\} \subseteq M$ be the subsemigroup of M.

Clearly B is not an ideal of M.

On the other hand $D = \{(a_1, \ldots, a_{20}) \mid a_i \in \{a + bI \mid a, b \in [1, 1, 22)\}, \max, 1 \leq i \leq 20\} \subseteq M$ is a subsemigroup of M which is also an ideal of M.

Let M = $\{(a_1, a_2, \ldots, a_{20}) \mid a_i \in \{a + bI \mid a, b \in [4, 12)\}\ 1 \leq i \leq 20, \max\} \subseteq M$ be subsemigroup of M which is not an ideal of M.

Thus we have infinite ordered subsemigroups of M which are not ideals of M.

M also has finite subsemigroups which are not ideals of M.

Infact no subsemigroup of finite order is an ideal of M.

Example 2.48: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{15} \end{bmatrix} \middle|\ a_i \in \{a + bI \mid a, b \in [0, 6)\}, \max, 1 \leq i \leq 15 \right\}$$

be a semi open neutrosophic square semigroup of column matrix with entries from the real neutrosophic square. Clearly S has all the properties mentioned in theorem 2.2.

Example 2.49: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle|\ a_i \in \{a + bI \mid a, b \in [0, 45)\}, \max, 1 \leq i \leq 15 \right\}$$

be a semi open neutrosophic square semigroup with no zero divisors.

$$\text{If } A = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_1, a_2, a_3 \in \{a + bI \mid a, b \in [0, 45)\}, \right.$$

$$\max\} \subseteq M$$

can only be subsemigroup and never an ideal of M under max operation.

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [7, 12)\}, \right.$$

$$\max, 1 \leq i \leq 15\} \subseteq M$$

is a subsemigroup which is not an ideal of M.

$$N = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & 0 & 0 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 12)\}, \right.$$

$$\max, 1 \leq i \leq 13\} \subseteq M$$

is only a subsemigroup of M of infinite order and is not an ideal of M.

Example 2.50: Let

$$T = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \vdots & \vdots & \vdots & \vdots \\ a_{57} & a_{58} & a_{59} & a_{60} \end{array} \right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 25)\}, \right.$$

$$\left. \max, 1 \leq i \leq 60 \right\}$$

be the semigroup. T has ideals T has subsemigroups of finite and infinite order which are not ideals. T has no zero divisors.

Now we have also super matrix semigroups built using the real neutrosophic squares under max operation which are described by the following examples.

Example 2.51: Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array} \right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 43)\}, \max, 1 \leq i \leq 12 \right\}$$

be the real neutrosophic semi open square column super matrix semigroup.

B has subsemigroup which are not ideals. Further B has no zero divisors.

Every singleton set in B is a subsemigroup which is not an ideal of B.

Example 2.52: Let W = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6\ a_7\ a_8 \mid a_9\ a_{10} \mid a_{11}\ a_{12}\ a_{13}\ a_{14}\ a_{15}) \mid a_i \in \{a + bI \mid a, b \in [0, 32)\}, \max, 1 \leq i \leq 15\}$ be the real neutrosophic square super matrix semigroup of infinite order under max operation.

W has finite subsemigroup of order one, two, three, four and so on.

All these finite subsemigroups of W are not ideals of W.

Example 2.53: Let

$$M = \left\{ \left[\begin{array}{c|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \end{array}\right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 43)\}, \right.$$

$$\left. \max, 1 \leq i \leq 12 \right\}$$

be the real neutrosophic square super matrix semigroup under max operation.

M has ideals and has no zero divisors.

Example 2.54: Let

$$P = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} \\ a_{40} & a_{41} & a_{42} \end{array}\right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 40)\}, \text{max}, \; 1 \leq i \leq 42 \right\}$$

be the real neutrosophic square super column matrix semigroup under max operation.

P has infinite number of subsemigroups which are not ideals. P has no zero divisors or units but every element is an idempotent. Every singleton element is a subsemigroup.

Example 2.55: Let

$$P = \left\{ \left(\begin{array}{c|cc|ccc|cccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \end{array}\right) \middle| \; a_i \in \{a + bI \mid a, b \in [0, 27)\}, \text{max}, \; 1 \leq i \leq 36 \right\}$$

be the semigroup under max. V has infinite number of idempotents and V has no zero divisors.

V has infinite number of subsemigroups which are not ideals.

Example 2.56: Let

$$P = \left\{ \left[\begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \dots & \dots & \dots & \dots & a_{12} \\ \hline a_{13} & \dots & \dots & \dots & \dots & a_{18} \\ a_{19} & \dots & \dots & \dots & \dots & a_{24} \\ a_{25} & \dots & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & \dots & \dots & \dots & \dots & a_{36} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 47)\}, \max, 1 \leq i \leq 36 \right\}$$

be the semigroup. P has ideals which are infinite in order.

P has also subsemigroups of infinite order which are not ideals. Every singleton element in P is a subsemigroup but is not an ideal.

P has subsemigroups of order two, three, four and so on.

Now we propose the following problems for this chapter.

Problems:

1. Obtain some special and interesting features enjoyed by the algebraic structures built using the semi open real neutrosophic square.

2. What is the speciality about the group

   $N_S = \{a + Ib \mid a, b \in [0, n), n < \infty, +\}$?

3. Let $M = \{(a_1, a_2, \ldots, a_8) \mid a_i \in \{a + bI \mid a, b \in [0, 24)\}, 1 \leq i \leq 8, +\}$ be the group.

   (i) Find all subgroups of finite order.
   (ii) Is the number of finite subgroups of M finite?
   (iii) Can M have infinite number of infinite order subgroups?
   (iv) How many subgroups in M are isomorphic to $N_S = \{(a + bI \mid a, b \in [0, 24), +\}$?

4. Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{18} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 43)\}, +, 1 \leq i \leq 18 \right\}$$

be the neutrosophic semi open square group of column matrices.

Study questions (i) to (iv) of problem 3 for this T.

Does the structure have any change in n if n is a composite number or a prime number.

5. Let

$$M = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{pmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 23)\}, +, 1 \leq i \leq 15 \right\}$$

be real neutrosophic semi open square matrix group under +.

Study questions (i) to (iv) of problem 3 for this M.

6. Let

$$S = \left\{ \left(\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{array} \right) \middle| \, a_i \in \{a + bI \mid a, b \in [0, 24)\}, +, 1 \le i \le 25 \right\}$$

be the real neutrosophic semi open square matrix group.

Study questions (i) to (iv) of problem 3 for this S.

7. Let

$$S = \left\{ \left[\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{96} & \dots & \dots & \dots & a_{100} \end{array} \right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 28)\}, +, 1 \le i \le 100 \right\}$$

be the real neutrosophic semi open square matrix group.

Study questions (i) to (iv) of problem 3 for this S.

8. Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 43)\}, 1 \leq i \leq 14, + \right\}$$

be the real neutrosophic semi open square super column matrix group.

Study questions (i) to (iv) of problem 3 for this M.

9. Let N= $\{(a_1 \mid a_2 \mid a_3\ a_4 \mid a_5 \mid a_6\ a_7\ a_8\ a_9) \mid a_i \in \{a + bI \mid a, b \in [0, 27)\}, 1 \leq i \leq 9, +\}$ be the real neutrosophic semi open square super row matrix group.

Study questions (i) to (iv) of problem 3 for this N.

10. Let

$$T = \left\{\left(\begin{array}{c|ccc|cc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{27} \\ a_{28} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \end{array}\right) \middle| \; a_i \in \{a + bI \mid a, b \in [0, 27)\}, 1 \leq i \leq 36, +\right\}$$

be the real neutrosophic semi open square super row matrix group.

Study questions (i) to (iv) of problem 3 for this T.

11. Let

$$W = \left\{\left(\begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ \hline a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{49} \\ \hline a_{50} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{63} \\ \hline a_{64} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{70} \end{array}\right) \middle| \; a_i \in \{a + bI \mid a, b \in [0, 83)\}, 1 \leq i \leq 70, +\right\}$$

be the real neutrosophic semi open square super matrix group.

Study questions (i) to (iv) of problem 3 for this W.

12. Let

$$B = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ \hline a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} \end{array}\right] \right| a_i \in \{a + bI \mid a, b \in [0, 57)\},$$

$$\text{min}, 1 \leq i \leq 56\}$$

be the real neutrosophic semi open square column super matrix group.

Study questions (i) to (iv) of problem 3 for this B.

13. Let M = $\{(a_1, \ldots, a_{12}) \mid a_i \in \{a + bI \mid a, b \in [0, 43)\}, +, 1 \leq i \leq 12\}$ be the real neutrosophic semi open square group.

Study questions (i) to (iv) of problem 3 for this M.

14. Let S = $\{(a + bI) \mid a, b \in [0, 40)\}, \times\}$ be a semigroup of real neutrosophic semi open square.

(i) Prove S is of infinite order.
(ii) Find finite and infinite order subsemigroups.
(iii) Prove S has infinite number of zero divisors.

(iv) Can S have infinite number of idempotents?

(v) Prove S has only finite number of units.

(vi) Can S have ideals of finite order?

(vii) Can S have ideals only of infinite order?

(viii) Is S a S-semigroup?

(ix) Can S have S-ideals?

(x) Does S contain S-subsemigroups which are not S-ideals?

15. Let $N_S = \{(a + bI) \mid a, b \in [0, 17)\}, \times\}$ be the real neutrosophic semi open square semigroup.

Study questions (i) to (x) of problem 14 for this S.

16. Let $M = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in \{a + bI \mid a, b \in [0, 23)\}, \times, 1 \leq i \leq 10\}$ be the semigroup of infinite order.

Study questions (i) to (x) of problem 14 for this P.

17. Let $M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{15} \end{bmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in [0, 22)\}, \times, \right.$

$1 \leq i \leq 15\}$ be the real neutrosophic semi open square semigroup.

Study questions (i) to (x) of problem 14 for this M.

18. Let

$$S = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 43)\}, \right.$$

$$\times, 1 \leq i \leq 40\}$$

be the real neutrosophic semi open square semigroup.

Study questions (i) to (x) of problem 14 for this S.

19. Let

$$T = \left\{ \left. \begin{bmatrix} a_1 & a_2 & \cdots & a_{10} \\ a_{11} & a_{12} & \cdots & a_{20} \\ a_{21} & a_{22} & \cdots & a_{30} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 48)\}, \times, \right.$$

$$1 \leq i \leq 30\}$$

be the real neutrosophic semi open square semigroup.

Study questions (i) to (x) of problem 14 for this T.

20. Let

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{array} \right] \right| a_i \in \{a + bI \mid a, b \in [0, 42)\}, \times,$$

$1 \leq i \leq 30\}$ be the semigroup.

Study questions (i) to (x) of problem 14 for this M.

21. Let $L = \{(a_1 \mid a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8) \mid a_i \in \{a + bI \mid a, b \in [0, 24)\}, \times, 1 \leq i \leq 8\}$ be the semigroup.

Study questions (i) to (x) of problem 14 for this L.

22. Let

$$M = \left\{ \left(\begin{array}{cc|c|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{27} \end{array} \right) \right| a_i \in$$

$\{a + bI \mid a, b \in [0, 40)\}, 1 \leq i \leq 27, \times\}$ be the semigroup.

Study questions (i) to (x) of problem 14 for this M.

23. Let

$$B = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ \hline a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} \end{array}\right] \right| a_i \in \{a + bI \mid a, b \in [0, 148)\}, \times,$$

$1 \leq i \leq 39\}$ be the semigroup.

Study questions (i) to (x) of problem 14 for this M.

24. Let $S_N = \{a + bI \mid a, b \in [0, 24), \min\}$ be the real neutrosophic square semigroup.

(i) Prove $o(S_N) = \infty$.

(ii) Prove S_N has infinite number of zero divisors.

(iii) Prove S_N has infinite number of idempotents.

(iv) Prove S_N has subsemigroups of order one, two, three and so on.

(v) Show S_N has no ideals of finite order.

(vi) S_N has infinite order subsmigroups which are not ideals?

(vii) Is $P = \{a + bI \mid a, b \in [0, 12)\}$ the subsemigroup an ideal of S_N?s

(viii) Let $M = \{a + bI \mid a, b \in [12, 24)\} \subseteq S_N$ be the subsemigroup of S_N, M is not an ideal of S_N.

(ix) Prove $T = \{a + bI \mid a, b \in [16, 18)\} \subseteq S_N$ is only subsemigroup which are not ideal of S_N.

25. Let M be the super matrix semigroup given in the following.

$$M = \left\{ \left[\begin{array}{cc|cccc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{50} \\ a_{51} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{60} \\ \hline a_{61} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{70} \\ a_{71} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{80} \\ \hline a_{81} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{90} \end{array} \right] \right| a_i \in$$

$\{a + bI \mid a, b \in [0, 480)\}, \times, 1 \le i \le 90\}$.

Study questions (i) to (x) of problem 14 for this M.

26. Let $B = \{a + bI \mid a, b \in [0, 29), \min\}$ be the semigroup.

    Study questions (i) to (ix) of problem 25 for this B.

27. Let $M = \{a + bI \mid a, b \in [0, 40), \min\}$ be the semi open neutrosophic square semigroup.

    Study questions (i) to (ix) of problem 25 for this M.

28. Let $N = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in \{a + bI \mid a, b \in [0, 12), 1 \leq i \leq 10, \min\}$ be the semigroup.

    Study questions (i) to (ix) of problem 25 for this N.

29. Let $L = \left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{36} \end{pmatrix}\right| a_i \in \{a + bI \mid a, b \in\right.$

    $[0, 489)\}, 1 \leq i \leq 36, \min\}$ be the semigroup.

    Study questions (i) to (ix) of problem 25 for this L with appropriate modifications.

30. Let

$$P = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{27} \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 4)\}, 1 \leq i \leq 27, \min\right\}$$

be the semigroup under min.

Study questions (i) to (ix) of problem 25 for this P with appropriate modifications in the questions.

31. Let

$$S = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{57} & a_{58} & a_{59} & a_{60} & a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix} \right| a_i \in \right.$$

$\{a + bI \mid a, b \in [0, 14)\}$, $1 \leq i \leq 64$, min$\}$ be the semigroup. Study questions (i) to (ix) of problem 25 for this S with appropriate modifications in the questions.

32. Let M = $\{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6 \mid a_7\ a_8\ a_9\ a_{10}\ a_{11} \mid a_{12} \mid a_{13}) \mid a_i \in \{a + bI \mid a, b \in [0, 40)\}, 1 \leq i \leq 13, \text{min}\}$ be the super row matrix semigroup.

Study questions (i) to (ix) of problem 25 for this S with appropriate modifications in the questions.

33. Let $S = \left\{ \left. \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ a_{12} \\ a_{13} \\ \hline a_{14} \end{array}\right] \right| a_i \in \{a + bI \mid a, b \in [0, 42)\}, 1 \leq i \leq 14, \right.$

min$\}$ be the semigroup of super column matrix.

Study questions (i) to (ix) of problem 25 for this S.

34. Let

$$M = \left\{ \left[\begin{array}{c|ccc|cc|c|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \end{array} \right] \middle| \; a_i \in \right.$$

$\{a + bI \mid a, b \in [0, 251)\}$, $1 \leq i \leq 40$, min$\}$ be the super matrix semigroup.

Study questions (i) to (ix) of problem 25 for this S with appropriate changes.

35. Let $M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ \hline a_{45} & a_{46} & a_{47} & a_{48} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in \right.$

$[0, 15)\}$, min, $1 \leq i \leq 48\}$

be the super column matrix semigroup.

Study questions (i) to (ix) of problem 25 for this M.

36. Let $M = \left\{\left[\begin{array}{cc|ccc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{47} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{64} \\ \hline a_{65} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \\ a_{73} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{80} \end{array}\right] \middle|\ a_i \in \right.$

$\{a + bI \mid a, b \in [0, 6)\}$, $1 \leq i \leq 80$, min$\}$ be the super matrix semigroup.

Study questions (i) to (ix) of problem 25 for this M.

37. Let $S_N = \{a + bI \mid a, b \in [0, 24)\}$, max$\}$ be the semigroup of real neutrosophic semi open square.

(i) Show $o(S_N) = \infty$.

(ii) Prove S_N has no zero divisors.

(iii) Show S_N has subsemigroups of order one two three and so on.

(iv) Show S_N has infinite order subsemigroups which are not ideals of S_N.

(v) Show S_N has no ideals of finite order.

(vi) Prove every element in S_N is an idempotents.

38. Let M = {a + bI | a, b $\in$ [0, 24)} be the semigroup under max .

    Study questions (i) to (vi) of problem 37 for this M.

39. Let N = {a + bI | a, b $\in$ [0, 127)} be the semigroup under max .

    Study questions (i) to (vi) of problem 37 for this N.

40. Let M = {(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) | $a_i \in$ {a + bI | a, b $\in$ [0, 16)}, $1 \leq i \leq 8$, max} be the row matrix semigroup.

    Study questions (i) to (vi) of problem 37 for this M.

41. Let L = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_{19} \\ a_{20} & a_{21} & \ldots & a_{38} \\ a_{39} & a_{40} & \ldots & a_{57} \end{pmatrix}\right|\right.$ $a_i \in$ {a + bI | a, b $\in$

    [0, 6)}, $1 \leq i \leq 57$, min} be the semigroup.

    Study questions (i) to (vi) of problem 37 for this S.

42. Let M = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{36} \\ a_{37} & a_{38} & \ldots & a_{48} \\ a_{49} & a_{50} & \ldots & a_{60} \\ a_{61} & a_{62} & \ldots & a_{72} \end{pmatrix}\right|\right.$ $a_i \in$ {a + bI | a, b $\in$

    [0, 25)}, $1 \leq i \leq 72$, max} be the semigroup.

    Study questions (i) to (vi) of problem 37 for this M.

43. Let

$$S = \left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_6 \\ a_7 & a_8 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{36} \end{pmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 12)\}, 1 \leq i \leq 36, \max\right\}$$

be the semigroup.

Study questions (i) to (vi) of problem 38 for this S.

44. Let W = $\{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8\ a_9\ a_{10} \mid a_{11}) \mid a_i \in \{a + bI \mid a, b \in [0, 42)\}, 1 \leq i \leq 16, \max\}$ be the semigroup.

Study questions (i) to (vi) of problem 38 for this T.

45. Let

$$S = \left\{\left.\left[\begin{array}{c|ccc|cc|c|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \end{array}\right]\right| a_i \in \{a + bI \mid a, b \in [0, 40)\}, 1 \leq i \leq 40, \max\right\}$$

be the semigroup.

Study questions (i) to (vi) of problem 37 for this M.

46. Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ \hline a_{15} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 42)\}, 1 \leq i \leq 16, \max \right\}$$

be the semigroup.

Study questions (i) to (vi) of problem 37 for this T.

47. Let

$$S = \left\{ \left[\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ \hline a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \\ \hline a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \\ a_{56} & a_{57} & a_{58} & a_{59} & a_{60} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} \\ \hline a_{66} & a_{67} & a_{68} & a_{69} & a_{70} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 14)\}, 1 \leq i \leq 70, \max \right\}$$

be the semigroup of super column matrix.

Study questions (i) to (vi) of problem 37 for this S.

48. Let

$$S = \left\{ \left[\begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{47} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{64} \\ \hline a_{65} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \end{array} \right] \right| a_i \in$$

$$\{a + bI \mid a, b \in [0, 24)\}, 1 \leq i \leq 72, \max\}$$

be the super matrix semigroup.

Study questions (i) to (vi) of problem 37 for this S.

Chapter Three

SEMIRINGS AND PSEUDO RINGS USING REAL NEUTROSOPHIC SQUARES

In this chapter we for the first time introduce the concept of semiring and pseudo rings built using the real neutrosophic semi open square $S_N = \{a + bI \mid a, b \in [0, n)\}$.

DEFINITION 3.1: *Let $S_N = \{a + bI \mid a, b \in [0, n), \max, \min\}$ be the semiring of the real neutrosophic semi-open square.*

We will represent this by some examples.

Example 3.1: Let $S = \{a + bI \mid a, b \in [0, 12), \min, \max\}$ be the semiring of infinite order.

Let $P = \{0 + 0I, 6 + 8I\} \subseteq S$; P is a subsemiring of order two.

$M = \{0 + 0I, 0.3I, 0.2, 0.7I + 0.1\} \subseteq S$.

M is not a subsemiring for $\min \{0.3I, 0.2\} = 0$ and max {0.3I, 0.2} = 0.2 + 0.3I and min {0.3I, 0.7I + 0.1} = {0.3I}, max {0.3I, 0.7I + 0.1} = {0.1 + 0.7I}.

min {0.2, 0.7I + 0.1}
= 0.1 and
max {0.2, 0.7I + 0.1}
= 0.7I + 0.2 and so on.

Thus M_c = {0 + 0I, 0.3I, 0.2, 0.1 + 0.7I, 0.2 + 0.3I, 0.1, 0.7I + 0.2} $\subseteq$ S is a subsemiring.

Let P = {a + bI | a, b $\in$ [0, 6)} be a subsemiring.

Clearly P is an ideal of S. T = {a + bI | a, b $\in$ [5, 12)} $\subseteq$ S; T be a subsemiring but T is not an ideal, but T is a filter. Likewise P is not a filter.

Let W = {a + bI | a, b $\in$ [3, 9)} $\subseteq$ S; W is a subsemiring and W is not a filter or an ideal of S.

Example 3.2: Let M = {a + bI | a, b $\in$ [0, 27), max, min } be the semiring. M has several subsemirings of finite and infinite order. Some of the subsemirings are ideals and some of them are not.

We see M has zero divisors. M has also filters all of them are of infinite order and none of the filters are ideals and vice versa.

P_1 = {a + bI | a, b $\in$ [0, 9)} $\subseteq$ M is an ideal and is not a filter of M; where as T_1 = {a + bI | a, b $\in$ [9, 27)} $\subseteq$ M is not an ideal only a filter of M.

B_1 = {a + bI | a, b $\in$ [9, 20)} $\subseteq$ M is a subsemiring of M which is not an ideal or filter in M.

P_2 = {a + bI | a, b $\in$ [0, 10)} $\subseteq$ M is only an ideal and not a filter of M.

$T_2 = \{a + bI \mid a, b \in [12, 27)\} \subseteq M$ is a filter of M and is not an ideal of M.

$B_2 = \{a + bI \mid a, b \in [10, 15)\}$ is not a filter or an ideal of M only a subsemiring.

Thus M has infinite order subsemirings which are not filters or ideals. Further M has subsemirings or order two, three and so on and none of them are ideals.

For take $N_1 = \{0 + 0I, 5 + 0.2I\} \subseteq M$ is a subsemiring which is not a filter or ideal of M and $o(N_1) = 2$.

Let $N_2 = \{ 0 + 0I, 0.3 + 0.7I, 0.9 + I\} \subseteq M$ is a subsemiring of order three and is not an ideal or filter of M.

$N_3 = \{0 + 0I, 3 + 0.4I, 0.7 + 5I\} \subseteq M$ is only a subset which is not a subsemiring.

For min $\{3 + 0.4I, 0.7 + 5I\} = \{0.7 + 0.4I\} \notin N_3$.

$$\max \{3 + 0.4I, 0.7 + 5I\}$$
$$= 3 + 5I \notin N_3.$$

Thus N_3 can be completed to $L = \{0 + 0I, 0.4I + 3, 0.7 + 5I, 3 + 5I, 0.7 + 0.4I\}$ is a subsemiring of order 5 which is not an ideal or filter of M.

$B = \{0 + 0I, 0.7 + I, 1.3 + 4I, 7 + 9.5I\} \subseteq M$ is a subsemiring which is not an ideal or filter of M. $o(B) = 4$.

We see one can have subsemirings of all possible orders say one, two, three and so on.

Let $x = 0.4$ and $y = 5I \in M$ we see $\min\{x, y\} = 0 + 0I$ where as $\max \{x, y\} = 0.4 + 5I$. Thus M has infinite number of zero divisors and every element is an idempotent.

In view of all these facts we have the following theorem.

THEOREM 3.1: *Let $S = \{a + bI \mid a, b \in [0, n), \min, \max\}$ be the real neutrosophic semi open square semiring of infinite order.*

1. *S has infinite number of zero divisors.*
2. *S has subsemirings of order two, three, four, etc.*
3. *S has ideals of infinite order.*
4. *S has filters of infinite order.*
5. *S has subsemirings of infinite order which are not ideals or filters.*
6. *No ideal in S can be a filter of S.*
7. *Every element in S is an idempotent.*

The proof of the theorem is direct and hence left as an exercise to the reader.

We will built semirings using real neutrosophic semiring using min and max operation.

Example 3.3: Let
$M = \{(a_1, a_2, a_3, a_4) \mid a_i \in \{a + bI \mid a, b \in [0, 20)\}; 1 \leq i \leq 4\}$ be the real neutrosophic open semi square row semiring. M has subsemirings of finite and infinite order.

$P_1 = \{(a_1, 0, 0, 0) \mid a_1 \in \{a + bI \mid a, b \in [0, 20)\} \subseteq M$,
$P_2 = \{(0, a_2, 0, 0) \mid a_2 \in \{a + bI \mid a, b \in [0, 20)\} \subseteq M$,
$P_3 = \{(0, 0, a_3, 0) \mid a_3 \in \{a + bI \mid a, b \in [0, 20)\} \subseteq M$,
$P_4 = \{(0, 0, 0, a_4) \mid a_4 \in \{a + bI \mid a, b \in [0, 20)\} \subseteq M$ are also subsemirings which are ideals of M and none of them are filters of M.

Let $T_1 = \{(a_1, a_2, a_3, a_4) \mid a_i \in [10, 20), 1 \leq i \leq 4\} \subseteq M$,
$T_2 = \{(a_1, a_2, a_3, a_4) \mid a_i \in [5, 20), 1 \leq i \leq 4\} \subseteq M$,
$T_3 = \{(a_1, a_2, a_3, a_4) \mid a_i \in [15, 20), 1 \leq i \leq 4\} \subseteq M$ and
$T_4 = \{(a_1, a_2, a_3, a_4) \mid a_i \in [19, 20), 1 \leq i \leq 4\} \subseteq M$ be four subsemirings of infinite order.

Clearly T_1, T_2, T_3 and T_4 are not ideals only filters of M.

For each T_i the zero element is different. For (10, 10, 10, 10) is the zero element as

min $\{(x_1, x_2, x_3, x_4), (10, 10, 10, 10)\}$ = (10, 10, 10, 10) for all $(x_1, x_2, x_3, x_4) \in T_1$ and so on.

So in case of these semirings of semi open square; the zero element will vary form subsemiring to subsemiring.

For in case of subsemirings which are filters we do not assume (0, 0, 0, 0) to be present in it.

Example 3.4: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{20} \end{bmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in [0, 43)\}, 1 \leq i \leq 20, \min, \max \right\}$$

be the semiring. P has several ideals which are not filters and filters which are not ideals.

P also has finite order subsemirings which are not ideals. P also has infinite order subsemirings which are not ideals.

$$\text{Let } T_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \, a_1 \in \{a + bI \mid a, b \in [0, 43), \min, \max\} \right\}$$

be a subsemiring of P which is also an ideal of P and T_1 is not a filter of P.

Let

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{20} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in [14, 43), 1 \leq i \leq 20\} \subseteq P \right.$$

be a subsemiring which is not an ideal only a filter of P.

Now consider

$$N_1 = \left\{ \left. \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right| a_1 \in \{a + bI \mid a, b \in Z_{43}\} \subseteq P. \right.$$

N_1 is a subsemiring of finite order which is not an ideal of P.

$$L_1 = \left\{ \begin{bmatrix} 3+4I \\ 0.7+5I \\ 2+5I \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \right\} \subseteq P$$

is a subsemiring of order two which is not an ideal of P.

Finally

$$B_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_1\ a_2 \in Z_{43},\ a_3, a_4 \in [0, 43) \right\} \subseteq P$$

is a subsemiring which is not an ideal. However B_1 is of infinite order. B_1 is not a filter of P.

Example 3.5: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 12)\},\ \min, \max,\ 1 \leq i \leq 9 \right\}$$

be a semiring M has ideals of infinite order, subsemirings of finite and infinite order. M has no filters of finite order however has filters of infinite order.

M has infinite number of zero divisors.

Example 3.6: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{89} & a_{90} & a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 5)\},\ \min, \max,\ 1 \leq i \leq 96 \right\}$$

is a semiring. S has subsemirings of order two, three and so on. We can also have subsemirings of infinite order some of which are ideals and some of them are only filters.

Example 3.7: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ a_{12} \\ \hline a_{13} \end{array}\right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 12)\}, \min, \max, 1 \leq i \leq 13 \right\}$$

be the real neutrosophic semi open square semiring of super column matrices. W has subsemirings of order two, three and so on.

Example 3.8: Let P = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5 \mid a_6\ a_7\ a_8\ a_9 \mid a_{10}\ a_{11}\ a_{12}) \mid a_i \in \{a + bI \mid a, b \in [0, 4)\}, \min, \max, 1 \leq i \leq 12\}$ be a semiring. P has ideals and filters of infinite order.

P has no filters of finite order. However P has finite subsemirings. P is the real neutrosophic semi open square super row matrix semiring.

Example 3.9: Let

$$W = \left\{ \left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{array} \right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 2)\}, \right.$$

$$\text{min, max, } 1 \leq i \leq 24\}$$

be the real neutrosophic semi open square super matrix semiring. W has subsemirings of order two, three etc but no ideals or filters of finite order. W has infinite number of zero divisors.

Now we proceed onto develop the notion of pseudo ring built using the real neutrosophic semi open square.

DEFINITION 1.2: *Let $R = \{a + bI \mid a, b \in [0, n), n < \infty, +, \times\}$ be defined as the real neutrosophic semi open square pseudo ring.*

Let x = 3 + 2.1I, y = 1.2 + 1.6I and z = 2.5 + 2I be three real neutrosophic numbers.

x, y, z ∈ {a + bI | a, b ∈ [0, 4)}.

Consider x × (y + z)

= (3 + 2.1I) × [1.2 + 1.6I + 2.5 + 2I]
= (3 + 2.1I) × [3.7 + 3.6I]
= 11.1 + 10.8I + 7.77I + 7.56I
= 3.1 + 2.13I … I

Now x × y + x × z
= 3 + 2.1I × 1.2 + 1.6I + 3 + 2.1I × 2.5 + 2I
= 3.6 + 4.8I + 2.52 I + 3.36I

$= 3.6 + 2.68I$ … II

I and II are distinct as the distributive laws in general is not true in case of pseudo rings built using real neutrosophic semi open squares.

We will illustrate this situation by some examples.

Example 3.10: Let $M = \{a + bI \mid a, b \in [0, 12), +, \times\}$ be the real neutrosophic semi open square pseudo ring. M is of infinite order M has infinite number of zero divisors and only finite number of units and idempotents.

$x = 4I \in M$; $x^2 = 4I$, $x = 9 \in M$ is such that $x^2 = 9$ are two non trivial idempotents of M. M has finite pseudo subrings as well infinite pseudo subring.

$P = \{a + bI \mid a, b \in Z_{12}\}$ is finite pseudo subring of M.

Example 3.11: Let $S = \{a + bI \mid a, b \in [0, 13), +, \times\}$ be the real neutrosophic semi open square pseudo ring. S is of infinite order. S has infinite number of zero divisors and only finite number of units.

S is a S-pseudo ring as S has a subset which is a field. Let $x = 3.1 + 9.9I$ and $I \in S$.

$x \times y = 0 + 0I$; hence S has zero divisors. S has only finite number of idempotents which is non trivial.

Example 3.12: Let $B = \{a + bI \mid a, b \in [0, 25), +, \times\}$ be the real neutrosophic semi open square pseudo ring. B has zero divisors which are infinite in number $x = 5I$ in B is such that $x^2 = 0$ $y = 5 + 5I \in B$ is such that $y^2 = (5 + 5I)(5 + 5I) = 25 + 25I + 25I + 25I = 0$.

Thus x and y are nilpotent elements of order two.

Further $x = 10I + 5$ and $y = 5I$ in B are such that $x \times y = (10I + 5) \times 5I = 50I + 25I = 0$ is a zero divisor.

Example 3.13: Let $R = \{a + bI \mid a, b \in [0, 48); +, \times\}$ be the real neutrosophic semi open square pseudo ring of infinite order.

R has infinite number of zero divisors but only a finite number of units and idempotents. R has pseudo subrings of finite order.

An open conjecture is.

Can $R = \{a + bI \mid a, b \in [0, n), n < \infty$ n prime, $+, \times\}$ have pseudo subrings of infinite order?

Inview of all these we have the following theorem.

THEOREM 3.2: *Let $R = \{a + bI \mid a, b \in [0, n), n < \infty, +, \times\}$ be the real neutrosophic square pseudo ring. The following are true.*

(i) R is of infinite order.
(ii) R has infinite number of zero divisors.
(iii) R has subrings of finite order.
(iv) R has finite number of units and idempotents.

The proof is direct and hence left as an exercise to the reader.

We can build pseudo rings using the real neutrosophic squares which are illustrated by the following.

Example 3.14: Let
$M = \{(a_1, a_2, a_3) \mid a_i \in \{a + bI \mid a, b \in [0, 5)\}, 1 \leq i \leq 3, +, \times\}$ be the real neutrosophic square row matrix pseudo ring. M has pseudo subrings of infinite and finite order.

$P_1 = \{(a_1, 0, 0) \mid a_1 \in \{a + bI \mid a, b \in [0, 5)\} \subseteq M,$

$P_2 = \{(0, a_2, 0) \mid a_2 \in \{a + bI \mid a, b \in [0, 5)\} \subseteq M$ and
$P_3 = \{(0, 0, a_3) \mid a_3 \in \{a + bI \mid a, b \in [0, 5)\} \subseteq M$ are pseudo subrings of M which are ideals of M and are of infinite order.

Consider $T_1 = \{(a_1, a_2, 0) \mid a_1 \in \{a + bI \mid a, b \in Z_5\}$ and $a_2 \in \{a + bI \mid a, b \in [0, 5), +, \times\}$ be the pseudo subring of M. T_1 is of infinite order but T_1 is not an ideal of M.

Now we proceed onto describe the zero divisors in M. We have two types of zero divisors in M.

If x = (a, 0, 0) and y = (0, b, 0) where a1, b1 $\in \{a + bI \mid a, b \in [0, 5)\}$ in M are such that $x \times y = (0, 0, 0)$.

Also if $x = a + bI$ with $a + b \equiv 0 \pmod 5$ and $y = I$ we see $x \times y = 0$. These are the two types of zero divisors which are infinite in number. M has only finite number of idempotents of some special form.

Example 3.15: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 4)\}, 1 \le i \le 11, +, \times_n \right\}$$

be the real neutrosophic pseudo ring of infinite order.

S has infinite number of zero divisors, finite number of units and idempotents.

Example 3.16: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 19)\}, +, \times_n, \right.$$

$$\left. 1 \leq i \leq 30 \right\}$$

be the real neutrosophic semi open square pseudo ring. W has zero divisors, units and has subrings pseudo ideals and pseudo subrings.

Example 3.17: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_7 \\ a_8 & a_9 & a_{10} & \ldots & a_{14} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{43} & a_{44} & a_{45} & \ldots. & a_{49} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 42)\}, \right.$$

$$\left. 1 \leq i \leq 49, +, \times_n \right\}$$

be the real neutrosophic semi open square pseudo ring of infinite order.

M has infinite number of zero divisors, finite number of pseudo ideals.

Example 3.18: Let $M = \{(a_1 \mid a_2\ a_3 \mid a_4 \mid a_5 \mid a_6\ a_7 \mid a_8\ a_9\ a_{10} \mid a_{11}) \mid a_i \in \{a + bI \mid a, b \in [0, 10)\}, 1 \leq i \leq 11, +, \times\}$ be the real neutrosophic semi open square pseudo ring of super row matrices.

M has zero divisors. M has at least ${}_{11}C_1 + {}_{11}C_2 + \ldots + {}_{11}C_{10}$ number pseudo subrings which are pseudo ideals.

M has $3({}_{11}C_1 + {}_{11}C_2 + \ldots + {}_{11}C_{10})$ number of subrings of finite order which are not pseudo subrings for all these subrings satisfy the distributive law is true.

Example 3.19: Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ a_{12} \\ \hline a_{13} \end{array}\right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 11)\}, 1 \leq i \leq 13, +, \times_n \right\}$$

be the real neutrosophic semi open square pseudo ring of column super matrices.

This has atleast ${}_{13}C_1 + \ldots + {}_{13}C_{12}$ number of pseudo ideals and atleast ${}_{13}C_1 + \ldots + {}_{13}C_{12}$ number of subrings which not pseudo subrings.

Example 3.20: Let

$$T = \left\{ \left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \end{array}\right] \middle| \; a_i \in \{a + bI \mid a, b \in [0, 3)\}, \; 1 \leq i \leq 28, +, \times \right\}$$

be the pseudo ring of super matrices. This T has pseudo subrings and pseudo ideals.

Now we proceed onto define the notion of pseudo strong vector spaces over pseudo rings as well as vector spaces over fields $Z_p \subseteq [0, [0, p))$.

All these will be described by examples in the following.

Example 3.21: Let $V = \{(a_1, a_2, a_3) \mid a_i \in \{a + bI \mid a, b \in [0, 19)\}, 1 \leq i \leq 3, +\}$ be the real neutrosophic semi open square vector space over the field Z_{19}.

If V is defined over the neutrosophic ring $\langle Z_{19} \cup I \rangle$.

V will be known as the S-vector space over $\langle Z_{19} \cup I \rangle$.

If on the other hand V is defined over the pseudo ring $R = \{a + bI \mid a, b \in [0, 19)\}$ then V is defined as the pseudo vector space over the pseudo ring R.

Example 3.22: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in [0, 17)\}, \right.$$

$$\left. 1 \leq i \leq 16, + \right\}$$

be the vector space over the field Z_{17}.

S-vector space over the S-neutrosophic ring $\langle Z_{17} \cup I \rangle$ and pseudo real neutrosophic semi open square vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 17)\}, +, \times\}$.

We see V is infinite dimensional over Z_{17} or $\langle Z_{17} \cup I \rangle$ and R.

Further V has several vector subspaces over Z_{17} or $\langle Z_{17} \cup I \rangle$ or R.

$$P_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| \, a_1 \in \{a + bI \mid a, b \in [0, 17)\}, + \right\} \subseteq V,$$

$$P_2 = \left\{ \begin{bmatrix} 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| \, a_1 \in \{a + bI \mid a, b \in [0, 17)\}, + \right\} \subseteq V$$

and so on.

$$P_{16} = \left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{16} \end{bmatrix} \middle| a_{16} \in \{a + bI \mid a, b \in [0, 17)\}, +\right\} \subseteq V$$

are 16 subspaces of V such that

$$P_i \cap P_j = \left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \text{ if } i \neq j,\ 1 \leq i, j \leq 16.$$

Further $V = P_1 + P_2 + \ldots + P_{16}$. We see the same is true over $\langle Z_{17} \cup I\rangle$ or $[0, 17)$.

We can have also subspaces which are not direct sum.

We see

$$T_1 = \left\{\begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| a_1 \in \{a + bI \mid a, b \in Z_{17}\}\right.$$

is a finite dimensional subspace over Z_{17} infact dimension of T_1 over Z_{17} is 2.

However T_1 is not a finite dimensional subspace over $R = \{a + bI \mid a, b \in [0, 17), +, \times\}$.

Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in Z_{17}\}, +, \right.$$

$$\left. 1 \leq i \leq 16 \right\}$$

be a vector subspace of V over Z_{17} and over $\langle Z_{17} \cup I \rangle$.

Example 3.23: Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \, a_i \in \{a + bI \mid a, b \in [0, 43), 1 \leq i \leq 8, +\} \right.$$

be a vector space over Z_{43} or a S-vector space over $\langle Z_{43} \cup I \rangle$ or a pseudo vector space over the real neutrosophic semi open square pseudo ring over $R = \{a + bI \mid a, b \in [0, 43)\}, +, \times\}$.

In all the three cases M has subspaces. M has also finite dimensional subspaces.

In case of pseudo vector spaces M does not contain pseudo vector subspaces of finite dimension over R.

Now we proceed onto describe the uses of each of these spaces. We can as in case of vector spaces $V = \{a + bI \mid a, b \in [0, p); p < \infty\}$ over the field Z_p define linear transformations and linear operators however the notion of linear functionals cannot be carried out.

We first give examples of these situations also.

Example 3.24: Let

$$S_1 = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 47),$$

$$1 \leq i \leq 25, +\}$$

and

$$S_2 = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 47)\},$$

$$1 \leq i \leq 18, +\}$$

be two vector spaces defined over the field Z_{47}.

We define $T : S_1 \to S_2$ as follows:

$$T\left(\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \end{bmatrix} \right) = \begin{bmatrix} a_1 + a_2 & a_3 + a_4 & a_3 \\ a_6 + a_7 & a_8 + a_9 & a_{10} \\ a_{11} + a_{12} & a_{13}a_{14} & a_{15} \\ a_{16} + a_{19} & a_{17} & a_{18} \\ a_{20} & a_{22} & a_{23} \\ a_{21} & a_{24} & a_{25} \end{bmatrix}$$

It is easily verified that T is a linear transformation.

We can define M : $S_2 \to S_1$

$$M \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \right\} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & 0 \\ a_5 & 0 & a_6 & a_7 & a_8 \\ 0 & a_9 & 0 & 0 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & 0 & 0 & a_{18} \end{bmatrix};$$

Cleary M is a linear transformation from S_2 to S_2.

This is the way linear transformations on the vector space over the field is defined. This is as in case of usual vector spaces.

Example 3.25: Let

$$M_1 = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \\ a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 7)\}, 1 \leq i \leq 16, +\right\}$$

and

$$M_2 = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 7)\}, \right.$$

$1 \leq i \leq 16, +\}$

be two S-vector spaces over the S-ring $\langle Z_7 \cup I \rangle$.

$T : M_1 \rightarrow M_2$ is defined by

$$T\left(\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \\ a_{15} & a_{16} \end{bmatrix}\right) = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}.$$

It is easily verified T is a S-linear transformation of M_1 to M_2. We can have several such S-linear transformations.

Clearly we are not in a position to define linear functionals in a natural way.

Example 3.26 : Let

$$W = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 43)\},\right.$$

$1 \leq i \leq 15, +\}$

be the pseudo real neutrosophic semi open square vector space defined over the pseudo ring $R = \{a + bI \mid a, b \in [0, 43)\}, +, \times\}$.

We define $f : W \rightarrow R$ to be the pseudo linear functional as follows:

$$f \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \right\} = a_1 + a_3 + a_5 + a_7 + \ldots + a_{15} \pmod{43}.$$

It is easily verified that f is a linear functional on W.

Define $f_1 : W \to R$ by

$$f_1 \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \right\} = a_2 + a_4 + a_8 \,;$$

f_1 is again a linear functional on W.

Several linear functionals can be defined on W.

Example 3.27: Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_9 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 23)\}, 1 \leq i \leq 9, +\right\}$$

be a pseudo special vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 23)\}, +, \times\}$.

Define $f : V \to R$ by

$$f(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_9 \end{bmatrix}) = \sum_{i=1}^{9} a_i \,(\text{mod}\, 23)$$

f is a linear functional on W. We have very many different linear functionals on V.

However if we take a vector space V built using [0, n) over the field Z_n or over the neutrosophic S-ring $\langle Z_n \cup I \rangle$ we see we will not be in a position to define linear functional in a natural way.

Only on pseudo vector spaces we are in a position to built linear functionals as these are defined over the pseudo ring $R = \{a + bI \mid a, b \in [0, n)\}$.

Likewise we can define standard inner product in a natural way only when we take the pseudo ring R. For in case of vector spaces and S-vector spaces the standard inner product is not in Z_p or $\langle Z_p \cup I \rangle$ hence we can have the concept of inner product only in case of pseudo vector spaces alone we can have the concept of standard inner product spaces.

We will illustrate it by some examples.

Example 3.28: Let
$A = \{(a_1, a_2) \mid a_1, a_2 \in \{a + bI \mid a, b \in [0, 11), +\}$ be the vector space over Z_{11}.

We cannot have the standard product for u, v $\in$ A.

$$u.v = (u_1, u_2) \,.\, (v_1\ v_2)$$
$$= u_1v_1 + u_2v_2$$
for $u.v \notin Z_{11}$.

For consider

$$u = (0.7 + 10.5I, 5 + 3I)$$
$$\text{and } v = (8.1 + 2.1I, 0.9 + 0.6I) \in A.$$

$u.v = (0.7 + 10.5I, 5 + 3I) \times (8.1 + 2.1I, 0.9 + 0.6I)$

$= (0.7 \times 8.1 + 10.5I \times 8.1 + 2.1I \times 0.7 + 10.5I \times 2.1I, 5 \times 0.9 + 5 \times 0.6I + 3I \times 0.9 + 3I \times 0.6I)$

$= (5.67 + 8.05I + 1.47I + 0.05I, 4.5 + 3I + 2.7I + 0.18I)$

$= (5.67 + 9.57I, 4.5 + 5.25I) \notin Z_{11}.$

Thus inner product cannot be defined in the usual way.

Hence we were forced to define pseudo vector spaces over pseudo ring.

However even in the case of S-rings we cannot define inner products or linear functionals.

This is illustrated by the following.

Example 3.29: Let $V = \{(a_1, a_2, a_3) \mid a_i \in \{a + bI \mid a, b \in [0, 13)\}, 1 \leq i \leq 3, +\}$ be the S-vector space over the S-ring $S = \langle Z_{13} \cup I\rangle$.

$$\text{Let } x = (0.7 + 0.6I, 0.11, 0.2I)$$
$$\text{and } y = (0.5I, 0.9I, 2) \in V.$$

$$\begin{aligned}\langle x, y\rangle &= (0.7 + 0.6I, 0.11, 0.2I)\,(0.5I, 0.9I, 2) \\ &= (0.7 + 6I \times 0.5I, 0.11 \times 0.9I, 0.2I \times 2) \\ &= (0.35I + 3I, 0.99I, 0.4I) \\ &= (3.36I, 0.099I, 0.4I) \notin \langle Z_{13} \cup I\rangle.\end{aligned}$$

Thus on S-vector spaces also we cannot define inner product or linear functionals in a natural way.

Example 3.30: Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 7)\}, 1 \leq i \leq 7, + \right\}$$

be the pseudo real neutrosophic semi open square vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 7)\}, +, \times\}$.

Now for A, B $\in$ V we define the pseudo inner product as follows:

$$\text{Let } A = \{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} \in V.$$

$$\langle A, B \rangle = A \times_n B = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \times_n \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

$= (a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 + a_5b_5 + a_6b_6) \bmod 7.$

is the way the inner product is defined on V.

This is illustrated for

$$A = \begin{bmatrix} 0.7+5I \\ 6+2I \\ 0.91+0.21I \\ 3+0.12I \\ 0.4+0.7I \\ 0.12+0.11I \end{bmatrix} \text{ and } B = \begin{bmatrix} 3+0.7I \\ 0.8+0.8I \\ 2+0.5I \\ 2.1+2I \\ 0.53+6.81I \\ 6.5+3I \end{bmatrix} \in V.$$

$$\langle A, B\rangle = A \times_n B = \begin{bmatrix} 0.7+5I \\ 6+2I \\ 0.91+0.21I \\ 3+0.12I \\ 0.4+0.7I \\ 0.12+0.11I \end{bmatrix} \times \begin{bmatrix} 3+0.7I \\ 0.8+0.8I \\ 2+0.5I \\ 2.1+2I \\ 0.53+6.81I \\ 6.5+3I \end{bmatrix}$$

$$= \begin{bmatrix} 2.1+15I+0.49I+3.5I \\ 4.8+1.6I+4.8I+1.6I \\ 1.82+0.42I+4.55I+1.05I \\ 6.3+6I+0.252I+0.24I \\ 0.212+2.724I+0.371I+4.767I \\ 0.780+0.36I+0.33I+0715I \end{bmatrix}$$

$$= \begin{matrix} 2.1 + 4.99I + \\ 4.8 + I + \\ 1.82 + 6.02I + \\ 6.3 + 6.492I + \\ 0.212 + 0.862I + \\ 0.78 + 1.405I \end{matrix} .$$

$= 2.012 + 0.269\ I \in R.$

This is the way the pseudo inner product on V is defined.

Example 3.31: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \middle|\ a_i \in \{a + bI \mid a, b \in [0, 5)\}, \right.$$

$$1 \le i \le 12, +\}$$

be the pseudo inner product space defined on the pseudo ring $R = \{a + bI \mid a, b \in [0, 5)\} +, \times\}$.

$$\text{Let } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \\ b_{10} & b_{11} & b_{12} \end{bmatrix} \in V.$$

$$\langle A, B\rangle = A \times B = \sum_{i=1}^{12} a_i b_i \in R.$$

$$\text{We see if } A = \begin{pmatrix} 0.2+I & 0.7+0.2I & 0 \\ 0 & 0.4 & 0.7I \\ 2+0.6I & 0 & 0.12 \\ 0.8I & 0.6+0.3I & 0 \end{pmatrix} \text{ and}$$

$$B = \begin{pmatrix} 0 & 0.8+0.8I & 0.5I+0.2I \\ 0.6+4.2I & 0.2I & 0.2+2I \\ 0 & 0.9+3.4I & 0.6+4I \\ 2+I & 0 & 3.2+4.3I \end{pmatrix} \in M.$$

$A \times_n B = 0 + (0.7 + 0.2I)(0.8 + 0.7I) + 0 + 0 + 0.4 \times 0.2I + 0.7I \times 0.2 + 2I + 0 + 0 + 0.12 \times 0.6 + 4I + 0.8I \times 2 + I + 0 + 0$
$= 0.56 + 0.16\,I + 0.49I + 0.14I + 0.08I + 0.14I + 0.14I + 0.72 + 4.8I + 1.6I + 0.8I$
$= 1.28 + 3.21I \in R.$

This is the way pseudo inner product is defined on M.

Example 3.32: Let

$$A = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{pmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 31)\}, \right.$$

$$\left. 1 \leq i \leq 15, + \right\}$$

be the pseudo real neutrosophic semi open square vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 31)\} +, \times\}$.

$$\text{Let } X = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ x_6 & x_7 & x_8 & x_9 & x_{10} \\ x_{11} & x_{12} & x_{13} & x_{14} & x_{15} \end{pmatrix} \text{ and}$$

$$Y = \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 & y_9 & y_{10} \\ y_{11} & y_{12} & y_{13} & y_{14} & y_{15} \end{pmatrix} \in A.$$

$$\langle X, Y\rangle = X \times_n Y = \sum_{i=1}^{15} x_i y_i \in R.$$

Thus if

$$X = \begin{pmatrix} 3I & 0 & 0.3+0.7I & 0 & 0.5+2I \\ 0 & 7I & 0 & 0.5+2I & 0 \\ 4+3.1I & 0 & 4.1 & 0.7I & 3I+21 \end{pmatrix} \text{ and}$$

$$Y = \begin{pmatrix} 0.7 & 8I+7 & 0 & 9+10.1I & 0.51 \\ 7I+4.21 & 0.8 & 4I+0.8 & 0 & 9.1+21I \\ 0 & 0.71+8I & 0.8 & 0.51 & 0 \end{pmatrix} \in R.$$

We see $X \times_n Y = \sum_{i=1}^{15} x_i y_i$

$= 2.1I + 0 + 0 + 0 + 0.204 + 1.02I + 0 + 5.6I + 0 + 0 + 0 + 0 + 0 + 3.28 + 0.357I + 0$

$= (0.204 + 3.28) + (2.1 + 1.02 + 5.6 + 0.357)I$

$= 3.484 + 9.077I \in R.$

We see V is a pseudo inner product space over the pseudo ring R.

Example 3.33: Let $V = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6) \mid a_i \in \{a + bI \mid a, b \in [0, 11)\}, 1 \leq i \leq 6\}$ be the pseudo real neutrosophic semi open square vector space over the pseudo ring R = {a + bI | a, b ∈ [0, 11)} +, ×}.

We can define pseudo inner product on V as follows. For $X = (x_1\ x_2 \mid x_3\ x_4\ x_5 \mid x_6)$ and

$Y = (y_1\ y_2 \mid y_3\ y_4\ y_5 \mid y_6) \in V$ define $\langle X, Y\rangle = \sum_{i=1}^{6} x_i y_i \in R$.

Let X = (0.7 + 6I 0.8I | 3.2 + 0.4I 10.2 + 4I 0.6I | 0) and
Y = (0, 0.2 + 0.7I | 0 0 0.2 + 4I | 7I + 8) ∈ V

$\langle X, Y\rangle = 0 + 0.16I + 0.56I + 0 + 0.12I + 2.4I + 0$
$= 3.24I \in R.$

This is the way pseudo inner product is defined on V.

Example 3.34: Let

$$V = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \end{array} \right] \middle|\ a_i \in \{a + bI \mid a, b \in [0, 23)\}, 1 \le i \le 7, +\} \right.$$

be the pseudo real neutrosophic semi open square column super matrix vector space over the pseudo ring
R = {a + bI | a, b ∈ [0, 23)}, +, ×}.

We define inner product $X \times_n Y = \sum_{i=1}^{7} x_i y_i$ in the following way.

$$\text{Let } X = \begin{bmatrix} 0.7I + 3.1 \\ \hline 12I + 4.8 \\ 0.3 + 4I \\ \hline 0.8I + 0.7 \\ 0.5I + 0.8 \\ 0.9I + 0.9 \\ \hline 11 + 3.3I \end{bmatrix} \in V;$$

$X \times_n X = (0.7 + 3.1)^2 + (12I + 4.8)^2 + (0.3 + 4I)^2 + (0.8I + 0.7)^2 + (0.5I + 0.8I)^2 + (0.9 + 0.9I)^2 + (11 + 3.3I)^2$

$= 0.49I + 4.34I + 9.61 + 144I + 2 \times 57.6I + 0.09 + 16I + 23.04 + 2.4I + 0.64I + 0.49 + 1.12I + 0.64I + 0.25I + 0.64 + 0.8 + 0.81 + 0.81I + 1.62I + 121 + 72.6I + 10.89I$

$= 0.4 + 9.61 + 0.49 + 0.64 + 0.8 + 0.81 + 6) + (0.49 + 4.34 + 16 + 0.64 + 144 + 115.2 + 2.4 + 1.12 + 0.64 + 0.25 + 0.81 + 1.62 + 72.6 + 10.89)\ I \in R.$

This is the way pseudo inner product is defined on V. V on which pseudo inner product is defined is called as the pseudo inner product space over R.

As in case of usual spaces we in case of pseudo vector spaces also define the notion of dual pseudo space or pseudo dual space.

However in all cases the dimension of pseudo vector spaces are of infinite order.

Example 3.35: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 17)\}, 1 \leq i \leq 4, + \right\}$$

be the pseudo vector space over the pseudo ring
$R = \{a + bI \mid a, b \in [0, 17)\}, +, \times\}$.

We see if $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ and $Y = \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix} \in V$; then the inner product

$$\langle X, Y\rangle = \sum_{i=1}^{4} x_i y_i \in R.$$

$$\text{Let } A = \begin{bmatrix} 0.7+5I & 0.8+0.4I \\ 6.2+1.1I & 4+2.5I \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} 0 & 4+0.2I \\ 7.5+9I & 0 \end{bmatrix} \in V.$$

$$\langle A, B\rangle = A \times B = \sum_{i=1}^{4} x_i y_i \,.$$

$= 0 + 3.2 + 1.6I + 0.16I + 0.8I + 9.9I + 55.8I +$
$8{,}25I + 46.50$
$= 15.70 + (1.6 + 0.16 + 0.8 + 9.9 + 55.8 + 8.25)\, I$
$= 15.70\ (76.51)\, I$
$= 15.70 + 8.51I \in R.$

We can as in case of usual inner product spaces define describe and develop all other properties.

As the distributive laws do not work out we see it is difficult to build linear algebras even if they are pseudo linear algebras still the problem of solving equations etc happen to be difficult.

However we suggest we can built pseudo linear algebras over Z_p or $\langle Z_p \cup I\rangle$ or R = $\{a + bI \mid a, b \in [0, p)\}, +, \times\}$ or T = $\{[0, p), +, \times\}$.

Just for the sake of completeness a few examples are illustrated.

Example 3.36: Let P = $\{a + bI \mid a, b \in [0, 23)\}, +, \times\}$ be the pseudo linear algebras over the field Z_{23}.

We see in P we do not in general have
$x \times (y + z) = x \times y + x \times z$ for all x, y, z $\in$ P.

However dimension of P over the field Z_{23} is infinite.

Example 3.37: Let
V = $\{(a_1, a_2, a_3) \mid a_i \in \{a + bI \mid a, b \in [0, 7)\}, 1 \leq i \leq 3, +, \times\}$ is a pseudo linear algebra over the field Z_7.

Example 3.38: Let

$$V = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 43)\}, 1 \leq i \leq 9, +, \times_n\}$$

be the pseudolinear algebra over the S-ring F = or $\langle Z_{43} \cup I\rangle$.

Example 3.39: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 61)\}, 1 \leq i \leq 15, +, \times_n \right\}$$

be the pseudo linear algebra over the S-ring F = Z_{61} or

V is a S-pseudo linear algebra over the S-ring R = $\langle Z_{61} \cup I\rangle$ or

V is a strong pseudo linear algebra over the pseudo ring R = {[0, 61), +, ×} or

V is a pseudo linear algebra over the pseudo ring K = {a + bI | a, b ∈ [0, 61), +, ×}.

Example 3.40: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 43)\}, 1 \leq i \leq 15, +, \times_n \right\}$$

be the pseudo linear algebra over
B = {a + bI | a, b ∈ [0, 41), +, ×}; the pseudo ring.

Example 3.41: Let V = $\{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \{a + bI \mid a, b \in [0, 31)\}, 1 \leq i \leq 5, +, \times\}$ be a pseudo-pseudo linear algebra over the pseudo S-ring R = $\{a + bI \mid a, b \in [0, 31), +, \times\}$.

Example 3.42: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{23} & a_{24} \end{bmatrix} \middle| \; a_i \in \{a + bI \mid a, b \in [0, 37)\}, 1 \leq i \leq 24, +, \times_n \right\}$$

be the pseudo linear algebra over the pseudo S-ring
R = $\{a + bI \mid a, b \in [0, 37), +, \times\}$.

We can for all these new pseudo linear algebras define the notion of pseudo linear transformation, pseudo linear operator, pseudo linear functional and pseudo linear inner product.

All these work can be carried out as a matter of routine.

It is interesting however to research on pseudo linear algebras as the distributive law is not true. The study to over come this will be innovative.

We suggest the following problems for this chapter.

Problems:

1. Find some special and interesting features enjoyed by the real neutrosophic square semiring
M = $\{a + bI \mid a, b \in [0, n), \min, \max, n < \infty\}$.

2. Let S = $\{a + bI \mid a, b \in [0, 43), \min, \max\}$ be the real neutrosophic square semiring.

    (i) Show S has infinite number of zero divisors.

(ii) Show S has finite order subsemirings say of order 2, 3 and so on.
(iii) Show S is of order ∞.
(iv) Prove S has ideals of infinite order.
(v) Can S have ideals of finite order?
(vi) Prove S has filters of infinite order.
(vii) Can S have filters of finite order?
(viii) Prove S has subsemirings of infinite order which are not ideals.

3. Let $M = \{a + bI \mid a, b \in [0, 25), \min, \max\}$ be the real neutrosophic semi open squares semiring of infinite order.

Study questions (i) to (viii) of problem 2 for this M.

4. Let $W = \{a + bI \mid a, b \in [0, 2), \min, \max\}$ be the real neutrosophic squares semiring of infinite order.

Study questions (i) to (viii) of problem 2 for this W.

5. Let $M = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in \{a + bI \mid a, b \in [0, 50), 1 \leq i \leq 5, +, \times\}$ be the real neutrosophic semiring.

Study questions (i) to (viii) of problem 2 for this M.

6. Let $S = \left\{\left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{array}\right] \middle|\, a_i \in \{a + bI \mid a, b \in [0, 12)\}, 1 \leq i \leq 15,\right.$ $\left.\min, \max\right\}$ be the real neutrosophic semi open squares column matrix semiring.

Study questions (i) to (viii) of problem 2 for this S.

7. Let L = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 16)\}, 1 \leq i \leq 30, \text{min, max}\right\}$ be the real neutrosophic semi open square column matrix semiring.

   (i) Study questions (i) to (viii) of problem 2 for this L.

   (ii) Can L have any other properties other than the ones mentioned in problem 2.

8. Let W = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{97} & a_{98} & a_{99} & a_{100} \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 14)\}, 1 \leq i \leq 100\right\}$ be the real neutrosophic semi open square semiring of matrices.

   Study questions (i) to (viii) of problem 2 for this W.

9. Let L = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 44)\}, 1 \leq i \leq 9, \text{max, min}\right\}$ be the real neutrosophic semi open square semiring of matrices.

   Study questions (i) to (viii) of problem 2 for this L.

10. Let X = $\{(a_1\ a_2 \mid a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9) \mid a_i \in \{a + bI \mid a, b \in [0, 18)\}, 1 \leq i \leq 9\}$ be the real neutrosophic semi open square super row matrix semiring.

Study questions (i) to (viii) of problem 2 for this X.

11. Let Y = $\left\{\left(\begin{array}{cccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \end{array}\right) \middle| \, a_i \in \{a + bI \mid a, b \in [0, 20)\}, 1 \leq i \leq 24, \max, \min\right\}$ be the real neutrosophic semi open square row matrix semiring.

Study questions (i) to (viii) of problem 2 for this Y.

12. Let T = $\left\{\left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ \hline a_{15} \end{array}\right] \middle| \, a_i \in \{a + bI \mid a, b \in [0, 12)\}, 1 \leq i \leq 15, \max, \min\right\}$ be the real neutrosophic semi open square column matrix semiring.

Study questions (i) to (viii) of problem 2 for this T.

13. Let $M = \left\{\left(\begin{array}{c|cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \end{array}\right)\right|$

$a_i \in \{a + bI \mid a, b \in [0, 27)\}$, $1 \leq i \leq 24$, max, min$\}$ be the real neutrosophic semi open square row super matrix semiring.

Study questions (i) to (viii) of problem 2 for this M.

14. Let $M = \left\{\left[\begin{array}{c|cc|c} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \end{array}\right]\right| a_i \in \{a + bI \mid a, b \in$

$[0, 43)\}$, $1 \leq i \leq 16$, max, min$\}$ be the real neutrosophic semi open square row super matrix semiring.

Study questions (i) to (viii) of problem 2 for this M.

15. Let $N = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ \hline a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} \\ \hline a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} \\ a_{40} & a_{41} & a_{42} \\ \hline a_{43} & a_{44} & a_{45} \end{array} \right] \right| a_i \in \{a + bI \mid a, b \in [0, 15)\},$

$1 \leq i \leq 45$, max, min$\}$ be the real neutrosophic semi open square row super matrix semiring.

Study questions (i) to (viii) of problem 2 for this W.

16. Find some special and interesting features enjoyed by the pseudo ring $R = \{a + bI \mid a, b \in [0, n), n < \infty, +, \times\}$ where R is built over the real neutrosophic semi open square.

17. Can R have a pseudo subring of infinite order?

18. Find all pseudo subrings which are ideals.

19. Let $R = \{a + bI \mid a, b \in [0, 10), +, \times\}$ be the pseudo ring on real neutrosophic semi open square.

(i) Find all subrings of finite order.
(ii) Can R have finite pseudo ideals?

(iii) Can R have zero divisors?
(iv) Find all idempotents of R.
(v) Can R have units?
(vi) Can R have infinite number of units and idempotents?
(vii) Can R have pseudo ideals of infinite order?
(viii) Can R have pseudo subrings of infinite order?

20. Let S = $\{a + bI \mid a, b \in [0, 143), +, \times\}$ be the real neutrosophic semi open square pseudo ring.

Study questions (i) to (viii) of problem 19 for this S.

21. Let M = $\{a + bI \mid a, b \in [0, 12), +, \times\}$ be the real neutrosophic semi open square pseudo ring.

Study questions (i) to (viii) of problem 19 for this M.

22. Let T = $\{(a_1, a_2, \ldots, a_{10}) \mid a_i \in \{a + bI \mid a, b \in [0, 23)\}, 1 \leq i \leq 10, +, \times\}$ be the pseudo ring.

Study questions (i) to (viii) of problem 19 for this T.

23. Let M = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0, 120)\}, 1 \leq i \leq 18,\right.$ $+, \times\}$ be the pseudo ring.

Study questions (i) to (viii) of problem 19 for this M.

24. Find some special and interesting features enjoyed by pseudo rings built using the interval [0, n), n a prime.

25. Let M = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{21} & \dots & \dots & \dots & a_{25} \end{bmatrix} \right|$ $a_i \in \{a + bI \mid a, b \in$ $[0, 24)\}, 1 \leq i \leq 25, +, \times\}$ be the pseudo ring.

Study questions (i) to (viii) of problem 19 for this M.

26. Find S-ideals if any in M of problem 25.

27. Can M in problem 25 have S-zero divisors and S-idempotents?

28. Let S = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5 \mid a_6\ a_7\ a_8 \mid a_9) \mid a_i \in \{a + bI \mid a, b \in [0, 44)\}, 1 \leq i \leq 9, +, \times\}$ be the real neutrosophic semi open square super row matrix pseudo ring.

Study questions (i) to (viii) of problem 19 for this S.

29. Let W = $\left\{\left(\begin{array}{cc|ccc|c|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{18} \\ a_{19} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{27} \end{array}\right) \right|$ $a_i \in \{a + bI \mid a, b \in [0, 47); 1 \leq i \leq 27, +, \times_n\}$ be the real neutrosophic semi open square super row matrix pseudo ring.

Study questions (i) to (viii) of problem 19 for this W.

30. Let S = $\left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \end{array} \right] \right|$ $a_i \in \{a + bI \mid a, b \in [0, 24); 1 \leq i \leq 14, +,$

$\times_n\}$ be the real neutrosophic semi open square column super matrix pseudo ring.

Study questions (i) to (viii) of problem 19 for this S.

31. Let V = $\left\{ \left[\begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ \hline a_{49} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{64} \\ \hline a_{65} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \end{array} \right] \right|$ $a_i \in \{a$

$+ bI \mid a, b \in [0, 124); 1 \leq i \leq 72, +, \times\}$ be the real neutrosophic semi open square super matrix pseudo ring.

(i) Study questions (i) to (viii) of problem 19 for this V.
(ii) Can V has S-idempotents?
(iii) Can V have S-ideals?
(iv) Can V have S-zero divisors?
(v) Can V have S-units?

32. Find some special features enjoyed by vector spaces built using the group $V = \{(a_1, \ldots, a_n) \mid a_i \in \{a + bI \mid a, b \in [0, p)\}$, p a prime, $1 \leq i \leq n\}$ over the field Z_p.

33. Study the same problem 32 if Z_p is replaced by $\langle Z_p \cup I\rangle$.

34. Study problem 32 where V is defined over the pseudo ring $R = \{a + bI$ where $a, b \in [0, p), +, \times\}$.

35. Obtain any other special and interesting features enjoyed by the three types of vector spaces.

36. Suppose V and W are two vector spaces built using the real neutrosophic semi open square over the field Z_p, p a prime.

(i) What is the algebraic structure enjoyed by Hom (V, V)?
(ii) What is the algebraic structure enjoyed by Hom (V, V) and Hom (W, W)?

37. Let V and W be two S-vector spaces built using the semi open real neutrosophic square over the S-ring $\langle Z_p \cup I\rangle = F$.

(i) What is the algebraic structure enjoyed by $Hom_F(V, W)$?
(ii) Find the algebraic structure enjoyed by $Hom_F(V, V)$ and $Hom_F(W, W)$.
(iii) Compare this with the structures in problem 36.

38. Let V and W be any two pseudo vector spaces defined over the pseudo ring
$R = \{a + bI \mid a, b \in [0, p), \text{p a prime}, +, \times\}$.

   (i) Find the algebraic structure enjoyed by $Hom_R(V, W)$.
   (ii) What is the algebraic structure enjoyed by $Hom_R(V,V)$ and $Hom_R(W,W)$.
   (iii) Compare this V and W with the pseudo vector spaces $Hom_R(V,W)$, $Hom_R(V,V)$ and $Hom_R(W,W)$.

39. Let $M = \{(a_1, a_2, \ldots, a_{12}) \mid a_i \in \{a + bI \mid a, b \in [0, 43); 1 \leq i \leq 12, +\}$ be the vector space over the field Z_{43}.

   (i) Find dimension of M over Z_{43}.
   (ii) Is M finite dimensional over Z_{43}?
   (iii) Can M have subspaces of finite dimension?
   (iv) Find $Hom_{Z_{43}}(M, M)$.
   (v) Write M as a direct sum.

40. Let $L = \{(a_1\ a_2 \mid a_3 \mid a_4\ a_5 \mid a_6) \mid a_i \in \{a + bI \mid a, b \in [0, 41); 1 \leq i \leq 6, +\}$ be a vector space over Z_{41}.

   Study questions (i) to (v) of problem 39 for this L.

41. Let $S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{14} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 23)\}, 1 \leq i \leq 14, \right.$
$+\}$ be a S-vector space over the S-ring $\langle Z_{23} \cup I \rangle$.

   Study questions (i) to (v) of problem 39 for this S.

42. Let T = $a_i \in \{a + bI \mid a, b$

$\in [0, 127)\}, 1 \leq i \leq 36, +\}$ be a pseudo vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 127), +, \times\}$.

Study questions (i) to (v) of problem 39 for this T.

43. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{40} & a_{41} & a_{42} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 47)\}, \right.$ $1 \leq i \leq 42, +\}$ be the S-vector space over the S-ring $\langle Z_{47} \cup I \rangle$.

Study questions (i) to (v) of problem 39 for this M.

44. Let $M = \left\{ \left[\begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ \hline a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \\ \hline a_{43} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{49} \\ \hline a_{50} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \end{array} \right] \middle| a_i \in \{a + bI \mid \right.$ $a, b \in [0, 53)\}, 1 \leq i \leq 56, +\}$ be the pseudo vector space over the pseudo vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 56), +, \times\}$.

Study questions (i) to (v) of problem 39 for this S.

45. Let $V = \{(a_1\ a_2\ a_3\ \ldots\ a_{12}) \mid a_i \in \{a + bI \mid a, b \in [0, 17)\},$ $1 \leq i \leq 12, +\}$ be a vector space over the field Z_{17} and

$$W = \left\{ \left. \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in [0, 17)\}, 1 \leq i \leq \right.$$

12, +} be the vector space over the field Z_{17}.

(i) What is the algebraic structure enjoyed by $\text{Hom}_{Z_{17}}(V, W)$?

(ii) Find the algebraic structure enjoyed by Hom(V,V) and Hom(W,W).

(iii) Find a basis of V and W over Z_{17}.

(iv) If Z_{17} is replaced by $\langle Z_{17} \cup I\rangle$ the S-ring. What is the basis for V and W as S-vector spaces?

(v) If V and W are defined over the pseudo ring R = {a + bI | a, b ∈ [0, 17), +, ×} what are the dimensions of V and W over R?

(vi) Is dim V = dim W over Z_{17} or $\langle Z_{17} \cup I\rangle$ or R?

46. Let $V = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{21} & \dots & \dots & \dots & a_{25} \end{bmatrix} \right| a_i \in \{a + bI \mid a, b \in \right.$

[0, 61)}, $1 \leq i \leq 25$, +} be the vector space over Z_{61} and

$$W = \left\{ \left. \begin{pmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \end{pmatrix} \right| a_i \in \{a + bI \mid a, b \in \right.$$

[0, 61)}, $1 \leq i \leq 30$, +} be the vector space over the field Z_{61}.

Study questions (i) to (vi) of problem 46 for this V and W.

47. Let $M = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{32} \end{pmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 5)\}, 1 \leq i \leq 32, + \right\}$ be the pseudo vector space over the pseudo ring $R = \{a + bI \mid a, b \in [0, 5)\}, +, \times\}$.

   (i) Define linear functional from $M \to R$.
   (ii) What is the algebraic structure enjoyed by $S = \{$all linear functional from M to R$\}$?
   (iii) Can M be an inner product space?

48. Obtain some special features enjoyed by the pseudo linear algebras defined over the field Z_p (p a prime).

49. Let $M =\{(a_1\ a_2\ a_3 \ldots a_{10}) \mid a_i \in \{a + bI \mid a, b \in [0, 41)\}, 1 \leq i \leq 10, +, \times\}$ be a pseudo linear algebra over the field $F = Z_{41}$.

   (i) Find sublinear algebras of M.
   (ii) Can M have finite dimensional pseudo algebras?
   (iii) Find $\text{Hom}_{Z_{41}}(V, V)$.
   (iv) What is the dimension of V over Z_{41}?
   (v) Find a basis of V over Z_{41}.
   (vi) Can V have more than one basis?
   (vii) Can V be written as direct sum of pseudo sublinear algebras?

50. Let $M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0, 23)\}, 1 \leq i \leq 12, +, \times \right\}$ be a pseudo linear algebra over Z_{23}.

Study questions (i) to (vii) of problem 49 for this M.

51. Let $W = \left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix}\right| a_i \in \{a + bI \mid a, b \in [0,\right.$ $41)\}, 1 \leq i \leq 30, +, \times_n\}$ be a pseudo linear algebra over the S-ring $R = \langle Z_{41} \cup I\rangle$.

(i) Study questions (i) to (vii) of problem 49 for this W.
(ii) Compare this W with M of problem 50.

52. Let $S = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0,\right.$ $7)\}, 1 \leq i \leq 40, +, \times n\}$ be a S-pseudo linear algebra over the S-ring $R = \langle Z_7 \cup I\rangle$.

Study questions (i) to (vii) of problem 49 for this S.

53. Let $V = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}\right| a_i \in \{a + bI \mid a, b \in [0,\right.$ $53)\}, 1 \leq i \leq 16, +, \times_n\}$ be the pseudo strong pseudo linear algebra over the pseudo ring $R = \{[0, 53), +, \times\}$.

Study questions (i) to (vii) of problem 49 for this V.

54. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_8 \\ a_9 & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & a_{24} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in\right.$ $[0, 13)\}, 1 \leq i \leq 24, +, \times_n\}$ be a pseudo strong pseudo linear algebra over the pseudo ring P = $\{a + bI \mid a, b \in [0, 13), +, \times\}$.

Study questions (i) to (vii) of problem 49 for this W.

55. Let P = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \ldots & \ldots & \ldots & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & a_{15} \\ a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \\ a_{26} & \ldots & \ldots & \ldots & a_{30} \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in\right.$ $[0, 17)\}, 1 \leq i \leq 30, +, \times_n\}$ be the pseudo strong pseudo linear algebra over the pseudo real neutrosophic open semi square R= $\{a + bI \mid a, b \in [0, 17), +, \times\}$.

Study questions (i) to (vii) of problem 49 for this P.

56. Let L = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \ldots & \ldots & \ldots & \ldots & a_{12} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{55} & a_{56} & a_{57} & a_{58} & a_{59} & a_{60} \end{bmatrix} \middle| a_i \in \{a + bI \mid a,\right.$ $b \in [0, 19)\}, 1 \leq i \leq 60, +, \times_n\}$ be the strong pseudo linear algebra over the pseudo ring M = $\{a + bI \mid a, b \in [0, 19), +, \times\}$.

(i) Study questions (i) to (vii) of problem 49 for this L.

(ii) Now if in L M is replaced by $R_1 = \{[0, 19), +, \times n\}$ compare them.

(iii) Similarly if M is replaced by $N = \{\langle Z_{19} \cup I\rangle\}$ compare both the linear algebras.

(iv) Finally if M is replaced by the field Z_{19} compare it with the rest of the three pseudo linear algebras.

57. Study pseudo inner product spaces defined over pseudo ring $R = \{a + bI \mid a, b \in [0, p), +, \times; p \text{ a prime}\}$.

58. Give examples of pseudo linear functionals defined on pseudo strong linear algebras defined over ring R mentioned in problem 57.

59. Let $V = \{(a, b) \mid a, b \in \{c, dI \mid c, d \in [0, 29)\}, +, \times\}$ be the pseudo strong pseudo linear algebra over $R = \{a + bI \mid a, b \in [0, 29), +, \times\}$.

(i) Find Hom $(V, R) = V^*$.

(ii) Is $V^* \cong V$?

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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Here for the first time we introduce the semi open squares using modulo integers. This square under addition modulo 'n' is a group, and however under product 'x' this semi open square is only a semigroup as under 'x' the square has infinite number of zero divisors.We define the new type of rings called pseudo rings. Several open problems are suggested.

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