New Trends in Neutrosophic Algebraic Structures and Hyperstructures

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Abstract

The objective of this paper is to present new trends in neutrosophic algebraic structures and hyperstructures. New research areas in neutrosophic algebraic structures and hyperstructures are also presented.

1. Classical Algebraic Structures

Definition

Let X be a nonempty set called a universe of discourse and let $*, \star, \circ : X \times X \to X$ be binary operations defined on X. The couple $(X, *)$ or the triple $(X, *, \star)$ or the quadruple $(X, *, \star, \circ)$ is(are) called algebraic structure(s) or abstract system(s). X is named according to the properties or axioms satisfied by the binary operation(s) on X. A mapping on X that preserves the binary operation(s) on X is called a homomorphism.

Example

Some well known algebraic structures are:

(i) Groupoids, $(\mathbb{N}, +)$.
(ii) Semigroups, $(\mathbb{N}, .)$.
(iii) Monoids, $(\mathbb{Z}, .)$.
(iv) Groups, $(\mathbb{Z}_n, +)$.
(v) Loops, $(\mathbb{R}, .)$.

Example

More examples of well known algebraic structures are:

(vi) Rings, $(\mathbb{Z}_n, +, .)$.

(vii) Fields, $(\mathbb{Q}, +, .)$.

(viii) Integral domains, $(\mathbb{Z}, +, .)$.

(ix) Modules, $(\mathbb{R}, +, .)$.

(x) Vector spaces, $(\mathbb{R}^n, +, .)$.

Example

Let H be a normal subgroup of the the group $(G, *)$. A mapping $\phi : G \to G/H$ defined by

$$\phi(x) = x * H \quad \forall x \in G$$

is a group homomorphism.

1. Classical Algebraic Structures

Example

Let J be an ideal of the the ring $(R, +, .)$. A mapping $\psi : R \to R/J$ defined by

$$\psi(x) = x + J \quad \forall x \in R$$

is a ring homomorphism.

Definition

Let $(X,*)$ be any classical algebraic structure. A *neutrosophic* algebraic structure is a structure $(NX,*)$ generated by X and I that is $NX =< X \cup I >$ defined by

$$< X \cup I >= \{(a, bI) : a, b \in X\}.$$

I is called the *neutrosophic* element with the property:

$$\begin{aligned} I + I + \cdots + I &= nI, \\ 0.I &= 0, \\ I^n &= I, \quad \text{where} \;\; n \in \mathbb{N}. \end{aligned}$$

I^{-1}, the inverse of I is not defined and hence does not exist. $(X(I),*)$ usually derives its name from the name of X. For example if X is a group, then $X(I)$ is called a *neutrosophic* group.

2. Neutrosophic Algebraic Structures

Definition

A mapping $\phi : (X(I), *) \to (Y(I), \star)$ is called a *neutrosophic* homomorphism if it preserves $*, \star$ and I that is

$$\begin{aligned} \phi(x * y) &= \phi(x) \star \phi(y) \ \forall\, x, y \in X(I), \\ \phi(I) &= I. \end{aligned}$$

Remark: It should be remarked that some properties of X may not hold in $(X(I)$.

Example

Some examples of already established *neutrosophic* algebraic structures are:

(i) Neutrosophic groupoids, $(\mathbb{N}(I), +)$.
(ii) Neutrosophic semigroups, $(\mathbb{N}(I), .)$.
(iii) Neutrosophic monoids, $(\mathbb{Z}(I), .)$.
(iv) Neutrosophic groups, $(\mathbb{Z}_n(I), +)$.
(v) Neutrosophic loops, $(\mathbb{R}(I), .)$.
(vi) Neutrosophic rings, $(\mathbb{Z}_n(I), +, .)$.
(vii) Neutrosophic fields, $(\mathbb{Q}(I), +, .)$.
(viii) Neutrosophic integral domains, $(\mathbb{Z}(I), +, .)$.
(ix) Neutrosophic modules, $(\mathbb{R}(I), +, .)$.
(x) Neutrosophic vector spaces, $(\mathbb{R}^n(I), +, .)$.

3. Refined Neutrosophic Algebraic Structures

Consider the split of the indeterminacy factor I into two indeterminacies I_1 and I_2 defined as follows:

$$\begin{aligned} I_1 &= \text{contradiction (true (T) and false (F))}, & (1)\\ I_2 &= \text{ignorance (true (T) or false) (F)}. & (2) \end{aligned}$$

It can be shown from (1) and (2) that:

$$\begin{aligned} I_1^2 &= I_1, & (3)\\ I_2^2 &= I_2, & (4)\\ I_1 I_2 &= I_2 I_1 = I_1. & (5) \end{aligned}$$

Now, let X be a nonempty set and let I_1 and I_2 be two indeterminacies. Then the set

$$X(I_1, I_2) =< X, I_1, I_2 >= \{(x, yI_1, zI_2) : x, y, z \in X\} \quad (6)$$

is called a refined *neutrosophic* set generated by X, I_1 and I_2, and, (x, yI_1, zI_2) is called a refined *neutrosophic* element of $X(I_1, I_2)$. If $''+''$ and $''.''$ are ordinary addition and multiplication, I_k with $k = 1, 2$ have the following properties:

3. Refined Neutrosophic Algebraic Structures

(i) $I_k + I_k + \cdots + I_k = nI_k$.

(ii) $I_k + (-I_k) = 0$.

(iii) $I_k.I_k.\cdots.I_k = I_k^n = I_k$ for all positive integer $n > 1$.

(iv) $0.I_k = 0$.

(v) I_k^{-1} is undefined and therefore does not exist.

If $* : X(I_1, I_2) \times X(I_1, I_2) \to X(I_1, I_2)$ is a binary operation defined on $X(I_1, I_2)$, then the couple $(X(I_1, I_2), *)$ is called a refined *neutrosophic* algebraic structure and it is named according to the laws (axioms) satisfied by $*$. If $(X(I_1, I_2), *)$ and and $(Y(I_1, I_2), *')$ are two refined *neutrosophic* algebraic structures, the mapping $\phi : (X(I_1, I_2), *) \to (Y(I_1, I_2), *')$ is called a *neutrosophic* homomorphism if the following conditions hold:

(i) $\phi((a, bI_1, cI_2) * (d, eI_1, fI_2)) = \phi((a, bI_1, cI_2)) *' \phi((d, eI_1, fI_2))$.

(ii) $\phi(I_k) = I_k \ \forall (a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$ and $k = 1, 2$.

3. Refined Neutrosophic Algebraic Structures

Definition

Let $(X(I_1, I_2), +, .)$ be any refined *neutrosophic* algebraic structure where $''+''$ and $''.''$ are ordinary addition and multiplication respectively. For any two elements $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define

$$\begin{aligned} (a, bI_1, cI_2) + (d, eI_1, fI_2) &= (a + d, (b + e)I_1, (c + f)I_2), && (7) \\ (a, bI_1, cI_2).(d, eI_1, fI_2) &= (ad, (ae + bd + be + bf + ce)I_1, \\ &\quad (af + cd + cf)I_2). && (8) \end{aligned}$$

Remark: It should be remarked that some algebraic properties of X may not hold in $X(I_1, I_2)$.

Example

Some examples of already established refined *neutrosophic* algebraic structures are:

(i) Refined neutrosophic groupoids, $(\mathbb{N}(I_1, I_2), +)$.
(ii) Refined neutrosophic semigroups, $(\mathbb{N}(I_1, I_2), .)$.
(iii) Refined neutrosophic monoids, $(\mathbb{Z}(I_1, I_2), .)$.
(iv) Refined neutrosophic groups, $(\mathbb{Z}_n(I_1, I_2), +)$.
(v) Refined neutrosophic loops, $(\mathbb{R}(I_1, I_2), .)$.
(vi) Refined neutrosophic rings, $(\mathbb{Z}_n(I_1, I_2), +, .)$.
(vii) Refined neutrosophic fields, $(\mathbb{Q}(I_1, I_2), +, .)$.
(viii) Refined neutrosophic integral domains, $(\mathbb{Z}(I_1, I_2), +, .)$.
(ix) Refined neutrosophic modules, $(\mathbb{R}(I_1, I_2), +, .)$.
(x) Refined neutrosophic vector spaces, $(\mathbb{R}^n(I_1, I_2), +, .)$.

4. Neutrosophic Algebraic Hyperstructures

Definition

Let $(H, \star)$ be any algebraic hyperstructure and let $< H \cup I >= \{x = (a, bI) : a, b \in H\}$. Then the couple $H(I) = (< H \cup I >, \star)$ is called a *neutrosophic* algebraic hyperstructure generated by H and I under the hyperoperation $\star$. The part a is called the non-*neutrosophic* part of x and the part b is called the *neutrosophic* part of x.
If $x = (a, bI)$ and $y = (c, dI)$ are any two elements of $H(I)$, where $a, b, c, d \in H$, the composition of x and y is defined by

$$\begin{aligned} x \star y &= (a, bI) \star (c, dI) \\ &= \{(u, vI) : u \in a \star c,\ v \in a \star d \cup b \star c \cup b \star d\} \\ &= (a \star c, (a \star d \cup b \star c \cup b \star d)I). \end{aligned}$$

It should be noted that $a \star c \subseteq H$ and $(a \star d \cup b \star c \cup b \star d) \subseteq H$.

4. Neutrosophic Algebraic Hyperstructures

Example

Some established *neutrosophic* algebraic hyperstructures are:

(i) Neutrosophic hypergroups.
(ii) Neutrosophic hyperrings.
(iii) Neutrosophic hypervector spaces.
(iv) Neutrosophic hypermodules.
(v) Refined neutrosophic hypergroups.
(vi) Refined neutrosophic hyperrings.
(vii) Refined neutrosophic hypervector spaces.
(viii) Refined neutrosophic hypermodules.
(ix) Neutrosophic Quadruple hypervector spaces.

5. NeutroAlgebraicStructures and AntiAlgebraicStructures

In any classical algebraic structure $(X, *)$, the law of composition of the elements of X otherwise called a binary operation $*$ is well defined for all the elements of X that is, $x * y \in X \ \ \forall x, y \in X$; and axioms like associativity, commutativity, distributivity, etc. defined on X with respect to $*$ are totally true for all the elements of X. The composition of elements of X this way is restrictive which does not reflect the reality. It does not not give room for composition that is either partially defined, or partially undefined, or totally undefined with respect to $*$. However in the domain of knowledge, science and reality, the law of composition and axioms defined on X may either be only partially defined (partially true), or partially undefined (partially false), or totally undefined (totally false) with respect to $*$.

5. NeutroAlgebraicStructures and AntiAlgebraicStructures

In an attempt to model the reality by allowing the law of composition on X to be either partially defined and partially undefined or totally undefined, Smarandache in 2019 introduced the notions of NetroDefined and AntiDefined laws, as well as the notions of NeutroAxiom and AntiAxiom which has given birth to new fields of research called NeutroStructures,AntiStructures, NeutroAlgebras and AntiAlgebras. For any classical algebraic law or axiom defined on X, there correspond neutrosophic triplets $<$ Law, NeutroLaw, AntiLaw $>$ and $<$ Axiom, NeutroAxiom, AntiAxiom $>$ respectively.

5. NeutroAlgebraicStructures and AntiAlgebraicStructures

Definition

(i) A classical operation is an operation well defined for all the set's elements.

(ii) A NeutroOperation is an operation partially well defined, partially indeterminate, and partially outer defined on the given set.

(iii) An AntiOperation is an operation that is outer defined for all set's elements.

(iv) A classical law/axiom defined on a nonempty set is a law/axiom that is totally true (i.e. true for all set's elements).

5. NeutroAlgebraicStructures and AntiAlgebraicStructures

Definition

(v) A NeutroLaw/NeutroAxiom (or Neutrosophic Law/Neutrosophic Axiom) defined on a nonempty set is a law/axiom that is true for some set's elements [degree of truth (T)], indeterminate for other set's elements [degree of indeterminacy (I)], or false for the other set's elements [degree of falsehood (F)], where $T, I, F \in [0, 1]$, with $(T, I, F) \neq (1, 0, 0)$ that represents the classical axiom, and $(T, I, F) \neq (0, 0, 1)$ that represents the AntiAxiom.

(vi) An AntiLaw/AntiAxiom defined on a nonempty set is a law/axiom that is false for all set's elements.

(vii) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements, and false for other elements).

(viii) An AntiAlgebra is an algebra endowed with at least one AntiOperation or at least one AntiAxiom.

5. NeutroAlgebraicStructures and AntiAlgebraicStructures

Example

Some of the newly established NeutroAlgebraicStructures and AntiAlgebraicStructures are:

(i) NeutroBE-Algebra.

(ii) AntiBE-Algebra.

(iii) NeutroGroup.

(iii) AntiGroup.

(iv) NeutroRing.

6. New Research areas in Neutrosophic Algebraic Structures and Hyperstructures

Some new areas of research in neutrosophic algebraic Structures and hyperstructures are:

(i) AntiRing.
(ii) NeutroVectorSpaces.
(iii) AntiVectorSpaces.
(iv) NeutroModules.
(v) AntiModules.
(vi) NeutroHypergroups.
(vii) AntiHypergroups.
(viii) NeutroHyperrings.
(ix) AntiHyperrings.

Appreciation

I THANK YOU ALL FOR YOUR RAPT ATTENTION

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