scilogs, I

florentin smarandache

nidus idearum

de neutrosophia

interval-defined neutrosophic elements

neutrosophic logic
neutrosophic sets
neutrosophic statistics
neutrosophic numbers
neutrosophic physics
neutrosophic quantum theory
neutrosophic probability
neutrosophic algebraic structures

neutrosophic society

pons

Florentin Smarandache

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Foreword

Welcome into my scientific lab!

My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific meditation and chats: *a nest of ideas* (**nidus idearum**, in Latin).

I called the jottings herein *scilogs* (truncations of the words *scientific*, and gr. Λόγος – appealing rather to its original meanings "ground", "opinion", "expectation"), combining the welly of both science and informal (via internet) talks (in English, French, and Romanian).

In this first books of scilogs collected from my nest of ideas, one may find new and old questions and solutions, some of them already put at work, others dead or waiting, referring to **neutrosophy** – email messages to research colleagues, or replies, notes about authors, articles or books, so on.

Feel free to budge in or just use the *scilogs* as open source for your own ideas.

F.S.

Special thanks to all my peer colleagues for incitant and pertinent instances of discussing.

Contents

The indeterminacy makes a difference.

1

Email from Chris Cornelis (Gent University, Belgium):

Concerning your submission, it contains a lot of interesting ideas that also benefit the *intuitionistic fuzzy sets* theory. Maybe I should give you some more background first about the current trends in our field: some people in fuzzy logic (mainly logicians) have reacted against *intuitionistic fuzzy sets* theory, because:

a) it is a misnomer: it is not an extension of intuitionistic logic; and
b) it is equivalent to an older domain, *interval-valued fuzzy sets*.

Both allegations are in fact, rather weak. A name is for the founder to choose; and the argument that *intuitionistic fuzzy sets* = *interval-valued fuzzy sets* holds only at syntactical level. In this sense your *neutrosophic theory* is also very important: just like *intuitionistic fuzzy sets* theory, it exploits the tripartition *true-false-indeterminate*, which is much more than to replace a crisp membership value by an interval of values.

2

Reply to Chris Cornelis:

I think the term "neutrosophic" instead of "intuitionistic fuzzy" will be better, because "neutrosophic" etymologically comes from "neutro-sophy" [French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] which means knowledge of the *neutral thought*.

It represents the main distinction between "fuzzy", "intuitionistic fuzzy", and “neutrosophic” which is the middle component, i.e. the neutral/indeterminate/unknown part (besides the "truth"/"membership" and "falsehood"/"non-membership" components that appear in fuzzy/intuitionistic fuzzy logic/set).

When I chose the term "neutrosophic" (1995), I especially thought at the *middle component* inspired from sport games (*winning, defeating,* or *tight scores*), from votes (pro, against, neither), *from positive / negative / zero numbers*, from *yes / no / undecided* in decision making, etc.

When I chose/invented the name of "neutro-sophy" [= neutral wisdom], I referred to the *middle term* (*neutral*, meaning neither true nor false, even more: something which is unknown, not precise, ambiguous, uncertain, unclear), and curiously I started from philosophy (not from math or logic)!

I started from philosophy because I saw that some philosophers proved that their theory $< T >$ was true, and other philosophers proved the opposite, that $< AntiT >$ is also true; for example, the idealists (asserting that *the idea* is the base of the world) vs. the materialists (asserting that *the matter* is the base of the world). And I observed that both groups of thinkers were true simultaneously, even more - it was possible to find a midway to reconcile both opposite theories. Then, I discovered Dr. Krasimir T. Atanassov and his *intuitionistic fuzzy logic and set*.

Introduction of non-standard analysis helped in distinguishing between *absolute truth* and *relative truth* in philosophy and logic, but doesn't have much impact in artificial intelligence.

However, letting the sum of components vary between $^-0$ and 3^+ may have an impact in artificial intelligence because the *neutrosophic logic* allows para-consistent and dialetheist (paradoxist, contradictory) information to be fusioned.

3

E-mail exchanges with Dr. W.B. Vasantha Kandasamy:

So far, we have used *neutrosophic numbers* of the form $a + bI$, where I = indeterminacy. On such numbers, many *neutrosophic algebraic structures* were defined.

But - I thought in a different way, i.e. when the set S has a determinate (known) part D and an indeterminate part (unclear, unknown) E, hence $S = D \cup E$.

For example, S can be the surface of a country, but there is an ambiguous frontier between this country and another neighboring country.

To better justify the reality of the partially determined and partially indetermined set, we can say the indeterminate zone is a buffer zone (ambiguous zone between two countries for example). For example, I know there is an unclear frontier between India and Bangladesh.

Now, we take a such space (or set) $S = D \cup E$ and we define an operation $*$ on S. We have three cases:

1) if a, b are in D, then $a * b$ should be in D?

2) if a is in D and b is in E, then $a * b$ should be in - where?

3) if a, b in E, then $a * b$ should be in - where?

We can then construct new types of *neutrosophic semigroups*, *groups*, maybe *rings*, etc. on such indeterminate/*neutrosophic sets*. Another easy example would be to consider that the length of an object is between 6 or 7 mm. We then define an interval, $[0, 7]$, and $[0,7] = [0, 6] \cup (6,7]$, where the determinate part of the length is $[0,6]$ and indeterminate part of the length is between $(6,7]$ since our measurement tools are not perfect. My question - and help required from you: what kind of algebraic structure can we build on such partially determined and partially undetermined spaces/sets?

4

Reply from Dr. W.B. Vasantha Kandasamy:

E should be the collection of indeterminantes, such that E is a semigroup. Then,

- for question one, the resultant is in D;
- for question two, it is in S;
- for question three it is in E.

This is my first impression; we will think more about it; the idea is nice. We can do lots of work.

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Reply to Dr. W.B. Vasantha Kandasamy:

1) We need a name for these new types of structures.

We already have *neutrosophic semigroups*, for example (using $a + bI$).

How should we then name another *neutrosophic semigroup* formed by a set who is partially determinate and partially indeterminate?

2) I feel we can define various laws $*$ on a partially determinate and partially indeterminate set. I mean: if a, b in D, we might get $a * b$ in E... And so on, all kind of possibilities.

But I think we need to get some examples or some justifications in the real world for this.

3) What applications in the real world can we find?

4) What connections with other theories can we get?

Maybe we can call it *strong neutrosophic semigroup*. And when we have both, partially determinate and partially indeterminate set, plus elements in the set of the form $a + bI$, we can call it *multineutrosophic semigroup* (or only *bi-neutrosophic semigroup*?).

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E-mail exchanges with Dr. W.B. Vasantha Kandasamy:

Doing a search on Internet on "neutrosophic", I found a great deal of papers published in various journals that I did not even know.

It looks that the *neutrosophic mathematics* is becoming a mainstream in applications where fuzzy mathematics do not work very well.

7

For writing a paper on FLARL it is enough to know that DSm Field and Linear Algebra is a set which is both in the same time: a field and a linear algebra (of course under corresponding defined laws).

But this approach is useful in calculating with qualitative labels (like, say: "poor, middle, good, very good" - instead of numbers: 0.2, 0.5, 0.6, 0.9).

All algebraic structures done so far in our previous books can be alternatively adjusted for qualitative algebraic structures.

8

A *neutrosophic number* has the form $a + bI$, where $I =$ indeterminate with $I^2 = I$, and a, b are integer, rational, real, or even complex numbers.

A *neutrosophic interval* of the form (aI, bI), for $a < b$, comprises *all neutrosophic numbers* of the form xI, with $a < x < b$. And similarly for $(aI, bI]$, $[aI, bI)$, and $[aI, bI]$.

But what about the *neutrosophic interval* $(a + bI, c + dI)$? There is no total order defined on the set of *neutrosophic numbers*.

If a *neutrosophic number* $x + yI$ belongs to the *neutrosophic interval* $(a + bI, c + dI)$, then what can we say about x and y?

We can define (W.B. Vasantha Kandasamy): $[a + bI, c + dI] = \{x + Iy \mid a \leq x \leq c$ or $a \geq x \geq c$ and $b \leq y \leq d$ or $b \geq y \geq d$, provide a, b, c, d are real$\}$.

This makes sense, so there is a partial order: $a + bI < c + dI$ if $a < c$ and $b < d$, or $a \leq c$ and $b < d$, or $a < c$ and $b \leq d$.

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Did you think at an even more general definition of neutrosophic numbers: $a + b \cdot I$, where I = indeterminate and $I^2 = I$, and a, b are real or even complex numbers?

Any possible application?

How should we define the multiplication: $i \cdot I$ (where $\mathrm{i} = \sqrt{-1}$ from the set of complex numbers, while I = indeterminate from the neutrosophic set)?

Excellent idea: $iI = Ii$ = complex indeterminate, while $bI = Ib$ = real indeterminate for b real.

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A more general definition is the *complex neutrosophic number*: $m + nI$, where m, n are complex numbers and I = indeterminate with $I^2 = I$, therefore if $m = a + bi$ and $n = c + di$ where $i = \sqrt{-1}$ as in the complex numbers.

Then a *complex neutrosophic number* is: $a + bi + cI + diI$, where a, b, c, d are real.

We also extend the *real neutrosophic interval* to a *complex neutrosophic interval*:

$$[a_1 + b_1 i + c_1 I + d_1 iI, a_2 + b_2 i + c_2 I + d_2 iI] = \{x + yi + zI + wiI, \text{where } a_1 \leq x < a_2, b_1 \leq y \leq b_2, c_1 \leq z \leq c_2, d_1 \leq w <= d_2\}.$$

I wonder if we can connect them to quaternions or biquaternions?

Would it be possible to consider an axis as formed by *neutrosophic numbers*?

If we design a *neutrosophic quaternion/biquaternion number*, would it be somehow applicable? I mean, any possible usefulness in physics?

What about having equations involving *neutrosophic numbers*? Since they have an indeterminate part, maybe they could be used in the quantum physics, biotechnology or in other domains where the indeterminacy plays an important role.

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What is the distinction between the increasing natural interval, let's say, [1, 3], and the decreasing natural interval [3, 1]?

What interpretation to give to a decreasing natural interval like [3, 1] in practice or in some theory?

Can you list the elements of the decreasing natural interval [3, 1] so it becomes clearer in my mind?

Can we in general define an increasing discrete set (not interval) {1, 2, 3} and also a decreasing discrete set (not interval) {3, 2, 1}?

Can we make these two different? I mean considering their orders?

Here in natural class of intervals we visualize the real line from negative infinity to positive infinity in the vertical direction with the $+\infty$ on the top and $-\infty$ on the bottom.

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The values in [1, 3] take 1, 1.0001, and so on to 3. It can be seen as a temperature increases from 1 to 3, or the market value goes from 1 to 3. On the other hand, when we say [3, 1] it is 3, 2.999999, and so on to 1. That is the temperature falls from 3 to 1, or the market value dips from 3 to 1. We feel it is a natural way of representing an increasing or a decreasing model.

When we say [1, 3] it naturally represents the increasing set {1, 2, 3} where we can have all the decimal values in between 1 and 2, and 2 and 3 - provided we take the interval on the reals; likewise, for decreasing set.

I think that we should consider that the interval $[a, b)$ converges towards $[a, a)$ when $a = b$ and that $[a, a)$ is a semipoint (so, not empty, but not a point either).

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I think it is not very good to risk everything in only one direction (for example in theoretical physics only, which has many unproven theories, many contradictories, so building your good research on an incorrect theory demolishes the whole construction as raised on sand...).

So that's why, in my opinion, attempt more fields, since if one does not bring you to the best result, another one might - so check your fortune in many domains.

That's why I believe my and Vic Christianto's "art of wag": more strategies would be like a necessary relaxation/escape from the hard science...

14

Can we combine as in *neutrosophy* $< A >, < antiA >$ and $< neutA >$ in *saivasiddantha*?

15

For an *n-multi-neutrosophic semigroup* $X_1 \cup X_2 \cup \ldots \cup X_n$, the elements in X_j are just simple numbers (not *neutrosophic numbers*).

Then the *bi-n-multineutrosophic semigroup* means a *multistructure*, $X_1 \cup X_2 \cup \ldots \cup X_n$, where each X_j is a *multi-neutrosophic semigroup*, and the elements in each X_j are of the form $a_j + b_j I$.

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In *neutrosophic cognitive graphs* we have the edges that are indeterminate; this may be considered *neutrosophic graphs of first order* (or type, or rank).

What about a graph that has some indeterminate (or unknown, unsure, ambiguous, vague) vertices?

1. I feel we may consider them *neutrosophic graphs of second order*, while a *double neutrosophic graph* would be

a graph that has both: indeterminate edges and indeterminate vertices.

2. Similarly for a matrix.

A *neutrosophic matrix of first order* would be a matrix with *neutrosophic elements* of the form $a + bI$, while a matrix with indeterminate elements (i.e. unknown, or vague, or unclear) would be a matrix of second order.

Then a *double neutrosophic matrix*: a matrix that has elements of the form $a + bI$, abut also elements that are unknown (unclear, vague).

3. Similarly for polynomials:

- polynomials with coefficients of the form $a + bI$;
- but also polynomials whose several coefficients are unknown (unclear, ambiguous, vague);
- then polynomials with both: coefficients of the form $a + bI$, and coefficients which are indeterminate (unknown, unclear, ambiguous).

4. Similarly for all algebraic structures:

- algebraic structures that contain elements of the form $a + bI$ {already done, and called neutrosophic algebraic structures we can consider them of first order (or rank, or type);
- algebraic structures that contain indeterminate elements (i.e. unknown, unclear, ambiguous) that we can call neutrosophic algebraic structures of second order;

- then algebraic structures that contain both: elements of the form $a + bI$, and indeterminate elements.

We can call them double *neutrosophic algebraic structures*.

5. Similarly for other spaces and structures (not necessarily algebraic).

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I think though we better denote *neutrosophic semigroup of first order* (that of $a + bI$), then *neutrosophic semigroup of second order* (that set that is partially indeterminate).

In my opinion, it is more intuitive to say "of second order" than to say "multineutrosophic", since multi = many, which has no connection with a set being partially indeterminate.

Similarly, for other *neutrosophic algebraic structures*.

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E-mail to Dr. Doug Lefelhocz:

A general definition of the *fuzzy prime number* should be:

Let n be a positive number such that

$$p_i \leq n < p_{i+1},$$

where p_i and p_{i+1} are two consecutive primes. Then the Fuzzy Primality of n is defined as:

$$\mathrm{FP(n)= max}\{p_i/\mathrm{n}, \mathrm{n}/\mathrm{p_{i+1}}\}.$$

For example:

Since $7 \leq 7 < 11$, then:
$FP(7) = max\{7/7, 7/11\} = 1.$
$FP(8) = max\{7/8, 8/11\} = \{0.875, 0.727273\}$
$= 0.875.$
$FP(9) = max\{7/9, 9/11\} =$
$\{0.777778, 0.818182\} = 0.818182$ [9 is just in the middle between two consecutive primes 7 and 11].

We can also define a *neutrosophic prime number* in a similar way. See:
www.gallup.unm.edu/~smarandache/neutrosophy.htm
and download books on neutrosophic logic/set which are generalizations of fuzzy logic/set respectively, especially:
www.gallup.unm.edu/~smarandache/eBook-Neutrosophics4.pdf (see the second part of the book, since the first part is philosophy).

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For the *intersection of neutrosophic sets*, it is possible two ways (so that of Dr. Salama is good too): $(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (t_1 \wedge t_2, i_1 \wedge i_2, f_1 \vee f_2)$ as in Dr. Salama (more optimistic) while what I used together with Wang et al.: $(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (t_1 \wedge t_2, i_1 \vee i_2, f_1 \vee f_2)$ (more pessimistic, more prudent). So, one can use any of them.

Similarly, for the union of neutrosophic sets, it is possible two ways (so that of Dr. Salama is good too): $(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (t_1 \vee t_2, i_1 \vee i_2, f_1 \wedge f_2)$ as in Dr. Salama (more pesimistic) while what I used together with Wang et alia: $(t_1, i_1, f_1) \wedge (t_2, i_2, f_2) = (t_1 \vee t_2, i_1 \wedge i_2, f_1 \wedge f_2)$ (more optimistic, more prudent). We use any of them.

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The most general definition I gave in 1995 as: $x(T, I, F)$, where T, I, F are standard or non-standard subsets of the interval $]^{-}0, 1^{+}[$.

By making particular cases of T, I, F we get particular *neutrosophic sets*.

For example, Wang et al. considered the *interval neutrosophic set*, where T, I, F were intervals included in [0, 1]. *Hesitant neutrosophic set* (see Jun Ye) means that T, I, F are sets of finite number of elements from [0, 1], for example $T = \{0.2, 0.7\}$, etc.

Intutionistic neutrosophic set means

$$min\{T, I\} \leq 0.5, min\{T, F\} \leq 0.5, min\{I, F\} \leq 0.5.$$

So, making particularizations on T, I, F we obtain a particular case of the *neutrosophic set*, not a general case. Actually, the most general definition goes even further, by splitting T into $T_1, T_2, T_3, \ldots$, and I into $I_1, I_2, I_3, \ldots$, and F into $F_1, F_2, F_3, \ldots$ (see: Florentin Smarandache, *n-Valued Refined Neutrosophic Logic and Its Applications in Physics*, Progress in Physics, 143-146, Vol. 4, 2013, http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic.pdf about this splitting). We can further develop the *neutrosophic set* on 4 components, or 5 components, or 6, or n.

Also, we can define the *hesitant soft neutrosophic set*.

And further, we can extend to *soft n-valued refined neutrosophic set*, and *hesitant soft n-valued refined neutrosophic set*.

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Question from Haibin Wang:

Another question which gives me headache, to speak frankly, is the *order of two neutrosophic sets* A and B, where A is less than B if $t(A) < t(B)$, $i(A) > i(B)$, $f(A) > f(B)$. For t and f, I can understand. But for i, I am still not convinced myself. I worry that in the conference, they will ask me: why? Could you please clarify?

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Answer to Haibin Wang:

I agree it is a tough question with answers not completely convincing. We may also say it depends on the application.

Normally we may say that a set is bigger than another set if it provides more information and therefore less entropy (entropy means disorder, ambiguity, contradiction). Entropy is part of Indeterminacy. Hence more information means $t(B) > t(A)$, and less entropy $i(B) < i(A)$. t and f are opposite, hence from $t(B) > t(A)$ we get $f(B) < f(A)$.

But we need to find a good practical example. Do you have any idea for this containment?

Reply from Haibin Wang:

That makes sense. So, generally speaking, if $i(A) < i(B)$ that means A is less ambiguous than B. Right?

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Reply to Haibin Wang:

Yes, less ambiguous, less unknown, less contradictory. We can consider the *neutrosophic set order* not necessarily exactly like the classical inclusion, but as a relation of order regarding "more information and less entropy". We still need to get a clear example.

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Email to Dr. Madad Khan:

I think we can extend the *fuzzy Abel Grassman (AG)-groupoids* to *neutrosophic AG-groupoids, neutrosophic AG-subgroupoids, neutrosophic interior ideals of AG-groupoids, neutrosophic ideals of AG-groupoids, neutrosophic quasi-ideals of AG-groupoids, neutrosophic prime and semiprime ideals of AG-groupoids*, etc.

We can also use the *neutrosophication* in automata and in formal language.

25

The *neutrosophic set* (1,1,1) can represent (as I showed in my 1998 book) a paradox, i.e. a sentence which is true 100%, false 100%, and indeterminate 100% as well. {A paradox is true and false in the same time.}

But other cases can also be characterized by (1,1,1). It is a conflicting case, a paraconsistent case (i.e. when the sume $T + I + F > 1$).

Another case would be when: from a view point (from a criterion) a sentence is 100% true, from another view point (from another criterion) the same sentence is 100% false, and from a third view point (from a different criterion) the sentence is 100% indeterminate.

The *neutrosophic set* (0,0,0) can mean that no information at all one has about that the set, or that it is 0% true, 0% false, and 0% indeterminate.

Similarly, it would be when: from a view point (from a criterion) a sentence is 0% true, from another view point (from another criterion) the same sentence is 0% false, and from a third view point (from a different criterion) the sentence is 0% indeterminate.

This is the incomplete case, i.e. when $T + I + F < 1$.

Neither *fuzzy set* not *intuitionistic fuzzy set* could allow the sum of the components be > 1 or < 1, only *neutrosophic set/logic/probability/measure* did for the first time, which actually came against the classical probability meaning.

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We can generalize the notion of implication to *neutrosophic sets* in the following ways.

Let two *neutrosophic sets* $A(t_A, I_A, f_A)$ and $B(t_B, i_B, f_B)$. Then in *classical set theory*, we have: "$A \to B$" is equivalent to "*nonA or B*".

We extend it for the *neutrosophic sets/logic*.

$A \to B$ is equivalent to “non(t_A, i_A, f_A) or (t_B, i_B, f_B)”, which becomes "(f_A, i_A, t_A) or (t_B, i_B, f_B)", whence we get three versions of *neutrosophic implications*, depending on how we handle the indeterminacy:

1. $(max\{f_A, t_B\}, min\{i_A, i_B\}, min\{t_A, f_B\})$

or

2. $(max\{f_A, t_B\}, max\{i_A, i_B\}, min\{t_A, f_B\})$

or

3. $(max\{f_A, t_B\}, (i_A + i_B)/2, min\{t_A, f_B\})$.

We can then extend it to implication of *neutrosophic soft sets*. To write a similar article on implications of *neutrosophic soft sets*, as for *intuitionistic fuzzy set*.

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Let us make distinctions between *intuitionistic fuzzy set* and *neutrosophic set*.

First, the *intuitionistic fuzzy set* is a particular case of *neutrosophic set*, i.e. the case when $t + i + f = 1$ for the triple (t, i, f) - with single values.

Only mentioning or defining the indeterminacy (i) separately/independently from t and f, we are already in the *neutrosophic set*, no matter if the sum $t + i + f$ is 1 or not.

Second, even if $t + i + f = 1$, there is a distinction between *intuitionistic fuzzy set* and *neutrosophic set*: in *inutitionistic fuzzy set* one defines the operators (union, intersection, complement/negation, difference, etc.) for t and f only, not for "i"; while in the *neutrosophic set* the operators (union, intersection, complement/negation, difference, etc.) are defined with respect to all components t, i, f.

For example, if one defines in the *intuitionistic fuzzy set* the union this way:

$$(t_1, f_1) \vee (t_2, f_2) = (max\{t_1, t_2\}, min\{f_1, f_2\})$$

{as you see, nothing is said about the indeterminacy (i)}, in *neutrosophic set*, where the indeterminacy is independent from t and f, one defines the union as:

$$(t_1, i_1, f_1) \vee (t_2, i_2, f_2) = (max\{t_1, t_2\}, min\{i_1, i_2\}, min\{f_1, f_2\})$$

{as you see, indeterminacy (i) is involved in the definition of the operation union.

Similarly, for the *intuitionistic fuzzy set*:

if $A = (T_1, F_1)$ and $B = (I_2, F_2)$, so Indeterminacy "I" is not even mentioned, then:

$A \wedge B = (T_1 \wedge T_2, F_1 \vee F_2)$, so "$I$" (indeterminacy) is not involved in this operation;

$A \vee B = (T1 \vee T2, F1 \wedge F2)$, so "$I$" (indeterminacy) is not involved in this operation too; etc.

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The fuzzy, intuitionistic fuzzy, and *neutrosophic operators* are approximations, not exact results. And one can approximate in many ways these fuzzy/intuitionistic-fuzzy/neutrosophic interferences/operations depending on the problem to be solved.

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A semigroup S may have a proper subset S_1 which is a group (stronger structure), and another subset S_2 which is a sub-semigroup (same structure), and a third proper subset S_3 which is groupoid (weaker structure) [http://fs.gallup.unm.edu/SmarandacheStrong-WeakStructures.htm].

In general, we can propose a *neutrosophic tri-structures* in the following way:

Let M be a set endowed with a structure S defined by some axioms, which has a proper subset M_1 endowed with a stronger structure S_1, and a second proper subset M_2 endowed with a same structure $S_2 = S$, and a third proper subset M_3 endowed with a weaker structure S_3.

Because a stronger structure is in certain degree of opposition with a weaker structure, we can consider that (M_1, M_2, M_3) as a *neutrosophic tri-structure*.

We can then call them *neutrosophic tri-structures*, to distinguish them from *neutrosophic structures* based on I = Indeterminacy (defined by Vasantha & Smarandache previously).

And even more general, we can define the *neutrosophic multi-structures* (*neutrosophic n-structure*):

Let A be a set endowed with a structure S defined by some axioms, which has n proper subsets B_i each one endowed with corresponding stronger structures U_i, and n proper subsets C_i each one endowed with a same structure S, and n proper subsets D_i each one endowed with a weaker structure V_i.

Because a stronger structure is in certain degree of opposition with a weaker structure, we can consider that each (A_i, B_i, C_i) is a *neutrosophic tri-structure*, so one has n *neutrosophic structures*, or a *neutrosophic multi-structure* (*neutrosophic n-structure*): (A, S) with $((B_i, U_i), (C_i, S), (D_i, V_i))$, for $i = 1, 2, \dots, n$.

Neutrosophic Lie-algebra and *neutrosophic manifolds* can be introduced too.

30

For the future research, I think it will be good to extend the work on *refined neutrosophic set* (or *logic*).

Instead of $x(T, I, F)$ we can refine each component and get $x(T_1, T_2, \dots, T_m;\ I_1, I_2, \dots, I_p;\ F_1, F_2, \dots, F_r)$, where T_j, I_k, F_l are subsets of [0,1], and define a *hesitant refined interval neutrosophic linguistic environment* and use it in decision-making.

31

Email to Dr. Jun Ye:

We can also apply the *refined indeterminacy* to the graphs and we get *refined neutrosophic graphs*. For example, an edge *AB* can be I_l (indeterminate of type 1), another edge can be I_2 (indeterminate of type 2), etc. Or a vertex can be indeterminate of type 1 or of type 2, etc.

32

To Dr. W. B. Vasantha Kandasamy:

When they come from logic, we can use them as intersection and union, herein you're right. But they can be used in algebras too, on the sets of numbers of the form:

$a + b_1 I_1 + b_2 I_2$ (if I is split into two subcomponents only).

For example, if we want to make a refined *neutrosophic groupoid*:

Let G = groupoid, under the law $*$, then the *refined neutrosophic grupoid* generated by I_1 and I_2 under the law $*$ is $G(I_1, I_2) = G \cup I_1 \cup I_2 = \{a + b_1 I_1 + b_2 I_2\}$, where a, b_1, b_2 are in G}.

In this algebraic case [*refined neutrosophic groupoid*], what should be

$$I_1 \times I_2 = ?$$
$$I_1 \,/\, I_2 = ?$$
$$I_1 + I_2 = ?$$

33

Dr. W. B. Vasantha Kandasamy wrote:

$I_1 \times I_2$ can be defined be equal to one or the other, maybe depending on the problem we solve.

Actually, the law $*$ is done by definition: $I_1 * I_2 =$ something. The groupoid does not have inverse elements, and we can define $I_1 * I_2$ as we wish (again depending on the problem to solve).

34

Reply to Dr. W. B. Vasantha Kandasamy:

Has it be done an extension of the form:

$$a + b_1 i_1 + a_2 b_2 + \ldots + a_n i_n$$

as a generalization of the $a + bi + cj + ck$?

Of course, similar properties:

$$i_1^2 = \ldots = i_{n_n}^2 = (n-1) = i_1 i_2 \ldots i_n$$

or maybe others?

35

Questions:

How should we neutrosophically differentiate $f(x) = 2x + 3Ix$, for example?

Also, how should we neutrosophically integrate this function $f(x) = 2x + 3Ix$?

36

W. B. Vasantha Kandasamy answered:

$$df(x)/dx = 2 + 3I$$

$$\int (2x + 3Ix)dx = x^2 + 3Ix^2/2 + \text{constant}.$$

37

Other questions:

Hence we consider "I" as a constant. Hence, "I" differentiated with respect to x is equal to zero, and "I" integrated with respect to x is $Ix + C$(onstant).

Can we differentiate and/or integrate with respect to "I"? [Taking “I” as a variable, not as a constant.] Meaning $d(f(x))/dI$ =? and integral of $f(x)$ with respect to dI =?

38

W. B. Vasantha Kandasamy answered:

As $I = I^2$, we cannot go for higher degree polynomials, only linear polynomials.

However, if refined collection is taken, we can have partial derivatives.

39

Email to W. B. Vasantha Kandasamy:

Can you give me an example of partial derivative, please?

This could be interesting, especially if we involved "I".

We may advance the neutrosophic research into the derivatives and integrals. We call $a + bI$ as a *neutrosophic number*. Then, for example $2 + 3I$ as a *neutrosophic constant*. Then we call "I" as "indeterminacy" only.

40

W. B. Vasantha Kandasamy answered:

If $I_1, I_2, I_3, \dots, I_n$ are n refined neutrosophic collections with different powers for I_j^m, then we can have partial derivatives with respect to each of $I_1, I_2, \dots, I_n$.

So, here the functions variables are the refined neutrosophic $I_1, I_2, \dots, I_n$.

41

Florentin Smarandache wrote back:

Not "x", only $I_1, I_2, \dots, I_n$ are considered variables.

Okay, it makes sense for *refined neutrosophic numbers*, to have partial derivatives.

42

Email to Temur Kalanov:

About *neutrosophic numbers* of the form $a + bI$.

Let's say $\sqrt{5} = 2 + 0.23I$, where I is in [1, 1.03], meaning that $\sqrt{5}$ is in [2.23000, 2.23670].

With the calculator: $\sqrt{5}$ = 2.23607... which is in [2.23000, 2.23670]. Of course, we can re-approximate $\sqrt{5}$ in another way as well.

43

Let's say we have this *interval neutrosophic set of type 2*: {x; <[0.0, 0.1];[0.3, 0.4],[0.4, 0.5],[0.2, 0.3]>, <[0.2, 0.5];[0.2, 0.4],[0.3, 0.5],[0.1, 0.2]>, <[0.1, 0.2];[0.2, 0.3],[0.4, 0.6],[0.2, 0.4]>}. How can we interpret it?

- can we say that the truth [0.0, 0.1] for x occurs with a chance of [0.3, 0.4], and [0.4, 0.5] as indeterminate chance, and [0.2, 0.3] as non-chance?
- and the indeterminacy [0.2, 0.5] for x occurs with a chance of [0.2, 0.4], and [0.3, 0.5] as indeterminate chance, and [0.1, 0.2] as non-chance?
- and the falsehood [0.1, 0.2] for x occurs with a chance of [0.2, 0.3], and [0.4, 0.6] as indeterminate chance, and [0.2, 0.4] as non-chance?

In a similar way we can generalize the *neutrosophic set of type 2* to *neutrosophic set of type n*.

44

In the *neutrosophic cube*, one can see that each *neutrosophic element* (with three single value components) can be interpreted as a point in that cube.

Therefore, the Euclidean distance between two elements $e_1(t_1, i_1, f_1)$ and $e_2(t_2, i2, f2)$ can be interpreted as the geometric distance between the points e_1 and e_2 inside the *neutrosophic cube*, i.e.:

$$\{(t_1 - t_2)^2 + (i_1 - i^2)^2 + (f_1 - f_2)^2\}^2(1/2).$$

If we have two sets:

$$M\{a(t1, i1, f1), b(t2, i2, f2), c(t3, i3, f3)\}$$

and

$$N\{a(t4, i4, f4), b(t5, i5, f5), c(t6, i6, f6)\}$$

then the distance between the sets *M* and *N* is the sum of distances between its elements: i.e. the distance between $a(t_1, i_1, f_1)$ and $a(t_4, i_4, f_4)$, plus the distance between $b(t_2, i_2, f_2)$ and $b(t_5, i_5, f_5)$, plus the distance between $c(t_3, i_3, f_3)$ and $c(t_6, i_6, f_6)$.

The normalized distance between the sets *M* and *N* could be the total distance between its elements (as computed above) divided by the number of elements (divided by 3 in this example).

45

If an element "*a*" from the *neutrosophic set A* has the *neutrosophic values*:

$$< a, [0.1, 0.3], [0, 0.1], [0.4, 0.5] >$$

and the same element "a" in the *neutrosophic set* B has the *neutrosophic values*:

$$< a, [0.2, 0.4], [0, 0.2], [0.6, 0.8] >,$$

then <[0.1, 0.3], [0, 0.1], [0.4, 0.5]> generate a prism P_1 (an object in the real space of dimension 3, i.e. in R^3) in the *neutrosophic cube*, while <[0.2, 0.4], [0, 0.2], [0.6, 0.8]> generate another prism P_2 in R^3.

Now we need to compute the distance between two real prisms in R^3.

46

For the distance between two real sets I found two common definitions as follow:

1- version of distance between two non-empty sets is the infimum of the distances between any two of their respective points:

$$d(A, B) = \inf_{x \in A, y \in B} d(x, y).$$

2- The Hausdorf distance.

47

We can define many distances between *two interval neutrosophic sets*.

1) One would be similar to the distance between two *intuitionistic fuzzy sets*, adjusted to *neutrosophic's three components*.

2) Second is using the classical distance between two real sets.

3) Third using Hausdorf distance too.

Which one to use?

It depends on the application needed.

48

Email exchanges with Mumtaz Ali wrote:

The algebraic work in *neutrosophic codes* in the algebraic form is good, but can you give an interpretation to I = indeterminacy in the codes?

Another possibility would be to consider I = unknown symbol in the code system. Can you investigate this possibility as well? So, there would be two types of *neutrosophic codes*.

What sense can you give to 1+I for example, where I=indeterminacy? Please try to get a valid practical explanation.

This will motivate very much the *neutrosophic code study*.

We should interpret neutrosophically the old algebraic structures, taking "a", "neut(a)" (neutral element with respect to "a"), and "anti(a)" (inverse element of "a"): group, ring, etc.

49

Florentin Smarandache answered:

I thought that $1 + I = 1 + 1I$ is partially determinate and partially indeterminate.

Its determinate part is 1, and its indeterminate part is $1I$. Would it work in the code theory?

50

Mumtaz Ali wrote:

$1+I$ is an indeterminate element or unknown element? For example, $C = \{00,11\}$ is a code and we suppose that $00 = F\ (False)$ and $11 = T\ (true)$. When we send these codewords and if the errors occur due to some interruption, the receiver receives 01 or 10 which is in this case unknown or indeterminate.

So, we can assign 01 or 10 to $1+I$ or II. Consequently, the code takes the form of *neutrosophic code* as

$N(C) = \{00, 11, II, (1+I)(1+I)\}$.

51

W.B. Vasantha Kandasamy asked:

How to interpret it as a bit?

52

Florentin Smarandache wrote:

Could it be a qubit (which can be 0 and 1 in the same time)? I and Christianto have also proposed the *multibit*. Qubit means superposition of two states, 0 and 1 will be in this case. Multi-bit is a superposition of many states.

53

W.B. Vasantha Kandasamy asked:

How to interpret the + in between 1 and I?

54

Mumtaz Ali answered:

It could be a dual bit (sometimes 0 other times 1), or we label it with a different symbol and call it partially determinate.

55

Exchanging ideas with Mumtaz Ali:

A *neutrosophic triplet* is a triplet of the form: $((A, neut(A), anti(A))$, where $neut(A)$ is the neutral of A, i.e. an element different from the identity element such that $A * neut(A) = neut(A) * A = A$, while $anti(A)$ is the opposite of A, i.e. an element such that $A * anti(A) = anti(A) * A = neut(A)$.

We can develop these *neutrosophic triplet structures*, since *neutrosophy* means not only indeterminacy, but also neutral (i.e. neither true nor false). For example, we can have *neutrosophic triplet semigroups, neutrosophic triplet loops*, etc.

56

A *neutrosophic triplet group* will be, in my opinion, a set such that each element "a" has a corresponding neutral elemnt$neut(a)$, an inverse element $inv(a)$ {both defined in a neutrosophic sense that we agreed upon before}, and a law $*$ that is well defined and associative.

The $neut(a)$ is not unique, $neut(a)$ depends on each element "a". This is the main distinction between a classical group (where the neutral/identity element is unique for all elements), and a *neutrosophic triplet group*.

We can extend this type of *neutrosophic triplet structure* to other algebraic structures.

We can similarly define a *neutrosophic triplet field*, i.e. a set $(F,*,\#)$ such that $(F,*)$ is a *neutrosophic triplet group*, and $(F,\#)$ is a *neutrosophic triplet group* as well; also # is distributive with respect to $*$ { i.e. $a \# (b * c) = a \# b * a \# c$ }.

The *neutrosophic triplet structures* have many applications, since for example, in general, a country C may have many (not only one) enemy/opposite countries anti(C) and many (not only one) neutral countries neut(C). Similarly, a person P may have many enemy persons anti(P) and many neutral personals neut(P). Not like in the classical algebraic structures where there is only one neutral element for the whole set for a given operation, and each element has a unique inverse (opposite) element.

57

Two theorems on *neutrosophic triplet groups*:

Theorem 1: If $*$ is associative and commutative, then

$$neut(a) * neut(b) = neut(a * b).$$

Proof 1: Multiply to the left with "a" and to the right with "b", we get:

$$a * neut(a) * neut(b) * b = a * neut(a * b) * b$$

or

$$[a * neut(a)] * [neut(b) * b] = a * neut(a * b) * b$$

or

$$a * b = [a * b] * [neut(a * b)] = a * b.$$

Theorem 2. If $*$ is associative and commutative, then $anti(a) * anti(b) = anti(a * b)$.

Proof 2: Multiply to the left with "a" and to the right with "b", we get:

$$a * anti(a) * anti(b) * b = a * anti(a * b) * b$$

or

$$[a * anti(a)] * [anti(b) * b] = a * anti(a * b) * b$$

or

$$[neut(a)] * [neut(b)] = [a * b] * [anti(a * b)]$$

or

$$neut(a * b) = neut(a * b).$$

58

I propose the name of *neutromorphism* for the second type of homomorphism, since neutro = neutrosophic, and morphism = form.

In my opinion, the *neutromorphism* should be:

1) $f(a * b) = f(a)\#f(b)$

2) $f(neut(a)) = neut(f(a))$

3) $f(anti(a)) = anti(f(a))$, i.e.

$$a \xrightarrow{yields} f(a)$$

$$neut(a) \xrightarrow{yields} f(neut(a)) = neut(f(a))$$

$$anti(a) \xrightarrow{yields} f(anti(a)) = anti(f(a))$$

We can define as *right neutrosophic triplet numbers*:

(a, b, c) such that $a * b = a$ and $a * c = b$ and $b * c = c * b = c$.

Similarly, for *left neutrosophic triplet numbers*: (a, b, c) such that $b * a = a$ and $c * a = b$ and $b * c = c * b = c$.

These are similar to our *neutrosophic triplet definition*, with an extra condition.

59

You say that if the element "a" generates N, then N is a *neutro-cyclic triplet group*.

$$2\text{^}1 = 2 \; in \; Z10$$
$$2\text{^}2 = 4 \; in \; Z10$$
$$2\text{^}3 = 8 \; in \; Z10$$
$$2\text{^}4 = 6 \; in \; Z10.$$

So $N = \{2, 4, 5, 8\}$ is a *neutro-cyclic triplet group* generated by the element 2.

Theorems:

Let $N = < a >$ be a *neutro-cyclic triplet group*.

Then:

1) $< neut(a) >$ is always a subgroup of N.

2) $< anti(a) >$ is always a subgroup of N.

An example where the addition is distributive over multiplication, will help us to aboard the *neutrosophic triplet anti-ring*.

60

Neutrosophic Law.

What about considering on a set S a law of the following form: if a, b in S, then $a * b = c$ or d (not sure about the final result).

For example, in Z10 = {0, 1, 2, ..., 9} one defines the neutrosophic law: $a * b = a + b$ or $a \times b$.

Thus, $2 * 4 = 2 + 4$ or $2 \times 4 = 6$ or 8; so $2 * 4 = 6$ or 8.

There is indeterminacy/ambiguity (as in the neutrosophics), i.e. the result is either 6 or 8 [one does not know exactly].

61

Every idempotent element (different from the unitary element) is a *neutrosophic triplet element*.

62

We have defined as a *right neutrosophic triplet*: (a, b, c) such that $a * b = a$ and $a * c = b$ and $b * c = c * b = c$.

Similarly, for a *left neutrosophic*: (a, b, c) such that $b * a = a$ and $c * a = a$ and $b * c = c * b = c$.

And (a, b, c) will be a *neutrosophic triplet number* if it is both *left* and *right neutrosophic triplet*.

63

Can the *neutrosophic triplets* in $(R, -)$ have the general form: $(a, 2a, 3a)$, with a different from zero, since $2a - a = a$ and $3a - a = 2a$?

64

If there are more $anti(a)$'s for a given a, one takes that $anti(a) = b$ that $anti(a)$ in its turn forms a *neutrosophic triplet*, i.e. there exists $neut(b)$ and $anti(b)$.

For example, in $Z10$, if $a = 2$, then $neut(a) = 6$, and $anti(a) = 3$ or 8.

Thus, one takes the *neutrosophic triplet* $(2, 6, 8)$, because 3 does not belong to a *neutrosophic triplet* since $neut(3)$ does not exist, while $neut(8) = 6$ and $anti(8) = 2$, so its *neutrosophic triplet* is $(8, 6, 2)$.

65

We can generalize each classical algebraic structure on a set $(S,*)$ to a corresponding *neutrosophic triplet algebraic structure* on the set $(S,*)$ in the following simple way:

- the set S contains only *neutrosophic triplets* with respect to $*$;
- the set S is closed under $*$ (well-defined-ness);
- the existence of identity element in the classical algebraic structure is replaced with the existence of $neut(a)$ for each element a in the *NTAS*;

- the existence of an inverse element for each element in the classical algebraic structure is replaced with the existence of $anti(a)$ for each element a in the $NTAS$.

If there is a second law # defined on S in the classical algebraic structure, then in a corresponding *neutrosophic triplet algebraic structure* $(S, \#)$ we impose the same things for # as we did for $*$.

Well-defined-ness, associativity, commutativity, and distributivity laws remain the same in both classical and *neutrosophic-triplet structures*.

66

The main distinction between classical semigroup and *neutrosophic triplet semigroup* is that the set S is formed by *neutrosophic triplets* in *NTS*, while in classical semigroup the elements may be any.

How to define the *neutrosophic triplet monoid*? Since it looks to coincide with *the neutrosophic triplet semigroup*, since each element already has its neutral.

We can define an additive operation # which gives triplets.

For example, in Z10, for {0, 2, 4, 6, 8}, let's consider

$$a\#b = 2a + 2b \; modulo \; 10.$$

Then $neut(2) = 4 \; since \; 2x2 + 2x4 = 2;$

$$anti(2) = 0 \; since \; 2x2 + 2x0 = 4.$$

The *neutrosophic triplet* with respect with this additive law is (2, 4, 0).

67

The best would be to define a set S of *neutrosophic triplets*, such that the elements of S verify the axioms of a Boolean algebra.

To come up with such example. But different from the trivial (a, a, a).

68

We might use more specific notations: for example, $neut\_x(a)$ the neutral of "a" with respect to x operation; and $neut\_*(a)$ the neutral of "a" with respect to the $*$ operator.

Similarly, for $anti\_x(a)$ or $anti\_*(a)$.

69

The *neutrosphic set* is refined. So, Indeterminacy I is also refined, into for example I_1 (which can be uncertainty), I_2 (which can be incompleteness), etc.

Therefore, an algebraic structure, for example a field K, can be extended by *neutrosophication* to $K \cup I$ (as several scientists did), but also to $K \cup I_1 \cup I_2$.

It might bring new insides to the algebraic structures.

These would be again new structures never done before.

70

In *twisted neutrosophic algebraic structures* we take one classical algebraic structure and the other one is neutrosophic triplet structure.

We can define a new type of not-well-defined set, as *another category of neutrosophic set*.

A *neutrosphic triplet ring* is that in which $(R,+)$ is a commutative neutrosophic triplet group, and $(R,*)$ is a semi-neutrosophic triplet monoid, and $*$ is distributive over $+$.

Theorem: If

$$(\, a, neut(a), anti(a)\,)$$

form a *neutrosophic triplet*, then

$$(\, anti(a), neut(a), a\,)$$

also form a *neutrosophic triplet*, and similarly

$$(\, neut(a), neut(a), neut(a)\,).$$

Proof:

1) Of course $anti(a) * a = neut(a)$. We need to prove that: $anti(a) * neut(a) = anti(a)$.

Multiply by " a " to the left, then: $a * anti(a) * neut(a) = a * anti(a)$, or $neut(a) * neut(a) = neut(a)$.

Multiply by "a" to the left and we get: $a * neut(a) * neut(a) = a * neut(a)$, or $a * neut(a) = a$, or $a = a$, which is true.

2) To show that $(\, neut(a), neut(a), neut(a)\,)$ is a *neutrosophic triplet*, it results from the fact that $neut(a) * neut(a) = neut(a)$.

71

When we say that $(NTF,*)$ is a *neutrosophic triplet group* with respect to $*$, and $(NTF,\#)$ is also a *neutrosophic triplet group* with respect to #, then we need to have *neutrosophic triplets* with respect to $*$ and *neutrosophic triplets* with respect to # (thus *neutrosophic triplets* with respect to both operations $*$ and #).

72

We can consider as a generalization of the *neutrosophic triplet* $(a, neut(a), anti(a))$, the following:

$$\begin{pmatrix} a, \\ neut_1(a), neut_2(a), \dots, \\ neut_{m(a)}, \\ anti_1(a), anti_2(a), \dots, \\ anti_n(a) \end{pmatrix}$$

in the case we can obtain many $neut(a)$'s and many $anti(a)$'s for the same "a".

73

I agree with *neutrosophic triplet matrix*, formed by a_ij, respectively $neut(a\_ij)$, and $anti(a\_ij)$ with respect to a given law #.

$NTG = \{0, 4, 8\}$ is a *neutrosophic triplet group* in $Z12$ with respect to multiplication $*$, since for each element "a" from *NTG* there is a $neut(a)$ and $anti(a)$.

But we also can consider the NTG of triplets: $NTG2 = \{(0,0,0), (4,4,4), (8,4,8)\}$, where one defines the combination

$$(a1, a2, a3) * (b1, b2, b3) = (a1 * b1, a2 * b2, a3 * b3).$$

This will be a second type of *NTG*.

74

The *neutrosophic triplet topology*: Let X be a non-empty set and T be topology on X. Let A be in T. Then T is called a *neutrosophic topology* if $A = open\ set\ in\ T$, then

$$anti(A) = close\ set\ in\ T$$

and

$$neut(A) = neither\ open\ nor\ close\ set\ in\ T = semi\ open\ or\ semi\ close\ set\ in\ T.$$

75

Reading "Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis" paper by Wanrong Zhang, I think we can also extend it to *bipolar neutrosophic set*.

What about *multipolar neutrosophic set*?

76

I defined the *strong neutrosophic algebraic structures* in order to distinguish them from the *neutrosophic algebraic structures* - the last ones defined by Dr. Vasantha and myself in our published books.

See online at

http://fs.gallup.unm.edu/eBooks-otherformats.htm .

The *neutrosophic algebraic structures* were defined on *neutrosophic numbers* of the form $a + bI$, where I = indeterminacy and $I^n = I$, and a, b are real or complex coefficients.

But the *strong neutrosophic algebraic structures* are based on the neutrosophic numerical values t, i, f.

Some definitions of *strong neutrosophic law*, *strong neutrosophic monoid* and *strong neutrosophic hommorphism* I then presented.

77

Other ideas for *soft theory*:

- what about extending the values of attributes to infinity?; because, for example if the attribute is COLOR, then it can have infinitely many values;
- also, what about having infinitely many attributes?

78

We can do refinement of the parameters e_i as e_{i1}, e_{i2}, etc., but also we can do refinement of the *neutrosophic values* of a parameter.

For example: the *neutrosophic value* of a parameter may be: T1, T2, …; I1, I2, …; F1, F2, … .

79

1) I like the way TOPSIS considers the (Hausdorff) distance between an alternative and the positive and negative ideal solution.

Can we also use another type of distance (not only Hausdorff's)?

2) For the set of opinions, it is possible to extend to O = {agree, indeterminate, disagree}.

3) Another was of looking at the set of opinions O would be to consider for each expert (t% agreement, i% indeterminacy, and f% disagreement).

Therefore, new papers can result on expert sets.

80

We can generalize the *interval neutrosophic set of type 2* to a *subset neutrosophic set of type 3*, where each membership/indeterminacy/nonmembership is a subset of [0, 1] instead of an interval of [0, 1].

81

The degrees of membership, nonmembership, and of the so called intuitionistic fuzzy index of a hypothesis are actually the belief, disbelief, and indeterminacy (uncertainty) of a hypothesis - as in *neutrosophic set*.

IFS is a particular case of *NS*. When the sum of the components is equal to 1, then *NS* is reduced to an *IFS*.

Is it possibe to compute the degree of subsethood for two *neutrosophic sets*?

82

Email to Linfan Mao:

Two innovatory papers: *S-denying Theory* + *Neutrosophic Transdisciplinarity*. If you're interested in applying them to graph theory, combinatorics, geometry, etc. we can publish a common book: a chapter about S-denying theory's applications, and another chapter about *neutrosophic transdisciplinarity*'s applications.

83

If we have T, I, F as crisp numbers with their sum = 1, then maybe we can consider a *vague neutrosophic set* as $(T, 1-I-F), (I, 1-T-F)$, and $(F, 1-T-I)$.

More general, if T, I, F are crisp numbers, with $T + I + F = s$ in [0,3], then: we can consider $(T, s-I-F), (I, s-T-F)$, and $(F, s-T-I)$ and of course we have to fix the intervals, I mean there may be for example $(T, s-I-F)$ or $(s-I-F, T)$ - depending which one is smaller.

84

A *neutrosophic set* (T, I, F), where T, I, F are intervals in [0, 1], is represented by a prism included in the *neutrosophic cube*. Hence for the distance between two neutrosophic sets, we can consider the distance between two prisms included in the *neutrosophic cube*.

I'll think for the *vague neutrosophic set*.

85

Similarly, to the type-2 fuzzy set, we can extend it to *type-2 neutrosophic set*, i.e. a *neutrosophic set* where all three components are functions, not crisps intervals.

So $T = (T1(x), T2(x))$, where $T1$ and $T2$ are functions depending of a parameter.

Similarly, $I = (I1(x), I2(x)), F = (F1(x), F2(x))$.

So we need to do some works on them too, following what was done in Type-2 Fuzzy Set.

86

To extending from fuzzy vague set to *vague neutrosophic set*:

If $A(t, i, f)$ and $s = t + i + f$ (which can be 1, less than 1, or greater than 1), then a *vague neutrosophic set* could be: $[t, s - t], [i, s - i], [f, s - f]$.

Of course, we need to reorder, i.e. $[min\{t, s - t\}, max\ \{t, s - t\}]$, and so on for i and f.

87

I fell that "system would be ruled in the next century: Fuzzy World or Fuzzy Logic" from the article “From deterministic world view to uncertainty and fuzzy logic: a critique of artificial intelligence and classical logic”, by Ayten Yılmaz Yalçıner, Berrin Denizhan, Harun Taşkın, TJFS: Turkish Journal of Fuzzy Systems, Vol.1, No.1, pp. 55-79, 2010, can be more accurate if we say/prove that "system would be ruled in the next century: Neutrosophic World or

Neutrosophic Logic" since they are more complex and leave room for indeterminacy.

We can always extend the fuzzy analysis to *neutrosophic analysis*.

88

The definition of the *IVIFS* cannot be applied exactly to the *vague neutrosophic set*, because we have a value for I [i.e. $I = 0.3$ in our example $x(0.5, 0.3, 0.2)$], so 0.3 has to show up somewhere in the formula of VNS.

What you got I=pi(x) = [-0.2, 0.2] is not good, since we cannot have negative values.

So the formula should be:

$$[min\{t, s-t\}, max\{t, s-t\}], [min\{i, s-i\}, max\{i, s-i\}], [min\{f, s-f\}, max\{f, s-f\}];$$

if any max is > 1, it is reduced to 1.

89

It is not *IVIFS*, since we start from a crisp value (0.5, 0.3, 0.2), and then we construct a *vague neutrosophic set* ([0.5, 0.5], [0.3, 0.7], [0.2, 0.8]). There is also an *interval valued neutrosophic set* which is given from the beginning when one has uncertainty for the values of T, I, F.

Even more general was defined the *neutrosophic set* as $x(T, I, F)$ where T, I, F are not necessarily intervals but any subsets of $[0, 1]$.

So, when we transform a *crisp neutrosophic set* to a *vague neutrosophic set*, we get an *interval neutrosophic set* (associated to the *crisp neutrosophic set*).

90

Can we take the *neutrosophic score function* (following Wang, Zhang, and Liu) as:

$$S(x) = t\_x - f\_x - i\_x/2\ ?$$

91

Email exchanges with Hojjatollah Farahani:

Since you know more psychology and I know more mathematics, please send me some information about: questionnaire development, and causal relationships in psychology. Then I see what mathematical/neutrosophic models we can use.

92

Let's consider these:

1) For questionnaire.

The questionnaire has questions and answers. Instead of classical answers yes/no, we can consider answers with yes/unknown/no.

Another type of neutrosophic answer is: p% yes, r% indeterminate, and s% false at a given question. For example:

Q: Do you like movie X?

A: 50% I like it for its actors; 20% I do not like it because of its director; 40% I am undecided because because some movie scenes are neither good nor bad.

2) For relationship.

The interpersonal relation between A and B is: +1 (means directly proportional), -1 (means inversely proportionally), I (meaning indeterminate).

Orfor example the relation of friendship between C and D is 70% true (from one point of view they can be friends), 20% false (from another point of view they hate each other; for example because of a common girl friend), and 30% unclear (vague, unknown) from other points of view. So, we need to find interesting psychological examples of questionnaires and relationships that can be described neutrosophically as said above.

3) For causal relationships, like A -> B and B -> C, we use the neutrosophic implication.

A -> B has a neutrosophic value (t, i, f). We combine them as A->B and B->C give A->C, i.e. $(t_1,i_1,f_1) \wedge (t_2,i_2,f_2) = (t_1 \wedge t_2, i_1 \vee i_2, f_1 \vee f_2)$.

93

I think that most psychologists are not familiar with this method. This method is can be used for all psychological research. Every questionnaire consisting of items in Likert scale for example (very high, high, middle, low and very low), we can use Netrosophic logic for them.

94

Can you then please provide questionnaire consisting of items in Likert scale, etc. to me? If we connect them with neutrosophy, it would be a pioneering work in psychology.

95

About *Godel's Incompleteness Theorem*: I agree with the content related to the distinctions between Human and Computer. I think that the differences (Love, God, Own mistakes, Repentance, Ethical) between Human and Calculator will be in the future little by little diminished, since it would be possible to train a computer at least for partial adjustments in each of them.

96

I think we can define more types of neutrosophic rings that are soft or not, then also *neutrosophic soft set + group*, or *neutrosophic set + ring*, or *neutrosophic set + semigroup* maybe.

97

Email exchanges with Mumtaz Ali:

I think we can define *neutrosophic triplet matrix*. For example, let aij be a matrix, then $neut(aij)$ and $anti(aij)$ matrices such that $aij * neut(aij) = aij$ and $aij * anti(aij) = neut(aij)$.

Then, the triplet $(aij, neut(aij), anti(aij))$ is called *neutrosophic triplet matrix*.

Indeed, there are *left neutrosophic triplet matrix* and *right neutrosophic triplet matrix*.

$neut(aij)$ will be different from the identity matrix.

98

Is *multiset* well defined or not?

If *multiset* is not well defined, then it is an example of a *neutrosophic set* because it is not consistent.

Florentin Smarandache answered:

Multiset is well defined.

99

If we take all the $neut(a)$'s and $anti(a)$'s in a set, then that set will be a *multiset*. So, a *neutrosophic triplet* forms a *multiset*.

100

Example. Consider $(Z_{10},\times)$. Then

$(0,0,0),(2,6,8),(4,6,4),(6,6,6),(8,6,2)$

are neutrososphic triplets in $(Z_{10},\times)$.

After taking all these elements in a set, we have

$NTMset = \{0,0,0,2,2,4,4,6,6,6,6,6,6,8,8\}$.

Then cleary NTM is a *multiset*.

Theorem. Every NTM (*neutrosophic triplet multiset*) is a multiset, but the converse is not true.

101

Suggest a name for this newly born *multiset*!

Florentin Smarandache answered:

Neutrosophic triplet multiset.

102

We can extend all the properties of a *multiset* to this newly born *multiset*. So, we can do a lot of work on *neutrosophic triplet multisets*.

103

We can define *neutrosophic triplet relational algebra* where the relational algebra is based on multisets. It has a lot of applications in physics, philosophy, computer science, database systems etc.

104

Do you know about relational algebra which is used in relational database system? Since relational algebra is an algebra on multisets.

105

A *neutrosophic triplet* "a" can be $neut(b)$ for some element b and at the same time a can be $anti(a)$ for some other element c.

This is true for all *neutrosophic triplet* in a *neutrosophic triplet group*, while in a classical group, not all element can do this.

Using this property of *neutrosophic triplets*, we can find its applications.

106

Let $f: A \to B$ be a function. Then f is called *neutrosophic triplet function* if it satisfies the following conditions:

1). $f(x * y) = f(x)$, and

2). $f(x * z) = f(y)$ for some x, y, z belongs to A.

What should we call the following function?

Let $f: A \to B$ be a function. Then f is said to be *triplet function of type 2* if $f(a * b) = a$ and $f(a * c) = b$, where a, b, c are in A.

107

I think we can find a link between these two newly *neutrosophic triplet functions*. We can also link fixed point to these two definitions.

That is a fixed point of a function is an element of the function's domain that is mapped to itself by the function. For example, if a function f is defined by $f(\mathrm{x}) = x^2 - 3x + 4$, then 2 is a fixed point of f because $f(2) = 2$.

108

I want to connect *neutrosophic triplets* with fixed point and then using this connection, we establish the relation between *neutrosophic triplet theory* and *fixed point theory*.

109

We can define a *neutrosophic sequence* in the following way: A sequence is called *neutrosophic sequence* if it has some kind of indeterminacy.

110

Example. Consider the sequence $(-1)^n$ for $n = 0,1,2,3,4,\ldots\ldots$ is an example of a *neutrosophic sequence* because we are not certain about its convergent. It is divergent and this divergentness is an indeterminacy.

Florentin Smarandache answered:

I do not like this example. We might consider a sequence whose certain terms (or many of them, or all of them) are indeterminate. For example: $1, 4, 3, x, y, 7, 24, 19, \ldots$ where x, y, z are unknown.

111

Theorem. Every divergent sequence is a *neutrosophic sequence* because the divergent sequence has no convergent point. / We don't know about it.

112

Let's start a founadation of a new mathematics called *neutrosophic mathematics,* which is the generalization of classical mathematics as well because in classical mathematics wherever the indeterminacy occurs, it is left over there but in *neutrosophic mathematics* we study the indeterminacy as well.

113

In classical mathematics the set which is not well defined is not studied, but here, in our *neutrosophic mathematics*, we can study this kind of set - because such sets occur in our reality. In fact, a set which is not well defined is a *neutrosophic set*.

114

I think we should define a new space called *neutrosophic space*. It should be in terms of Euclidean space.

Florentin Smarandache answered:

Then it should be *neutrosophic Euclidean space* (its name).

115

I have found some operations due to which we can find *neutrosophic triplet groups, neutrosophic triplet rings, neutrosophic triplet fields*. See the following:

Example. Consider Z10. Let $NTG = \{0,2,4,6,8\} \subset Z_{10}$. If we define an operation $*$ by the following way as $a * b = 5a + b(\text{mod}\,10)$. Then, the *neutrosophic triplets* with respect to this operation are the following:

$(0,0,0),(2,2,2),(4,4,4),(6,6,6),(8,8,8)$.

It is also associative. i.e.

$(a*b)*c = a*(b*c)$

$(5a+b)*c = a*(5b+c)$

$5(5a+b)+c = 5a+(5b+c)$

$25a+5b+c = 5a+5b+c$

$5a+5b+c = 5a+5b+c$

Thus, $(NTG,*) = \{0,2,4,6,8\}$ is a *neutrosophic triplet group* with respect to $*$. But $a*b \neq b*a$. So $(NTG,*)$ is not a *commutative neutrosophic triplet group*.

116

Example. Again, consider $(Z_{10},\#)$, where $\#$ is defined as $a\#b = 3ab(\text{mod}10)$. Then $(Z_{10},\#)$ is a *commutative neutrosophic triplet group* with respect to $\#$ and the *neutrosophic triplets* are as follows:

$(0,0,0),(1,7,9),(2,2,2),(3,7,3),(4,2,6),(5,5,5),(6,2,4),$

$(7,7,7),(8,2,8),(9,7,1)$.

It is also associative. That is,

$(a\#b)\#c = a\#(b\#c)$

$(3ab)\#c = a\#(3bc)$

$3(3ab)c = 3a(3bc)$

$9abc = 9abc$

This $(Z_{10}, \#)$ is a *neutrosophic triplet group* with respect to $\#$.

117

Similarly as going in physics from a microsystem to a macrosystem, or vice-versa, we do in neutrogeometry from 2D to 3D and in general to n-D(imensional) space R^n.

118

Note: $\#$ is also distributive over $*$.

In fact, $(Z_{10}, *, \#)$ is a *neutrosophic triplet field* if we exclude the commutativity of $(Z_{10}, *)$ because $*$ is not commutative.

Florentin Smarandache answered:

We can call it non-commutative field.

119

The *neutrosophic triplets* of Z_{10} with respect to $*$ generate the following *neutrosophic triplet multiset*,

$$NTMset = \begin{Bmatrix} 0,0,0,0,0,0,0,0,1,2,2,2,3,3,4,4,4,5,5,5,5,5,6,6,6 \\ 7,8,8,8,9 \end{Bmatrix}$$

The *neutrosophic triplets* of Z_{10} with respect to $\#$ generates the following *neutrosophic triplet multiset*:

$$NTMset = \begin{Bmatrix} 0,0,0,1,1,1,2,2,2,2,2,2,3,3,4,4,5,5,5,6 \\ 7,7,7,7,7,7,8,8,9,9 \end{Bmatrix}$$

120

We will now define the *neutrosophic triplet multigroups* like *multigroups*. I think we can define all the *multiset algebraic structures* in terms of *neutrosophic triplet multiset algebraic structures*.

This is another big and vast field for the study and research in *neutrosophic triplets*.

121

For *neutrosophic logic,* we have T, I, F.

But "I" can be split for example in: true and false (=contradiction), and true or false (=uncertainty).

We get a generalization of Belnap's four-values logic (since the sum of components can be different from 1).

We can split further "I" as: contradiction (true and false), uncertainty (true or false), and unknown.

We get a logic on five-values.

Even more refinement can be done (of course if we get nice examples to show its usefulness): split all three components: $T_1, T_2, \dots, T_m$, $I_1, I_2, \dots, I_n$, $F_1, F_2, \dots, F_p$, not only I.

For example, we can split T into T_1 and T_2, where T_1 = percentage of truth coming from a truthful source and T_2 = percentage of truth coming from a less truthful source.

Surely, we can do such splitting if necessary and if justified with some practical use.

122

In my opinion, the *neutrosophic probability* is a virgin domain since no study has been done so far.

I only defined it and tried to extend the classical probability's axioms to *neutrosophic probability*: that chance that an event E will occur is T, I, F...

123

Some people on the web (from India) consider that a *neutrosophic number* is a *neutrosophic set* (as a fuzzy number is a fuzzy set).

What notation and name should we use to distinguish between *neutrosophic number* as a *neutrosophic set*, and *neutrosophic number*s as $a + bI$, where $I^2 = I$ and $I + I = 2I$?

124

I saw a subject called *fuzzy linear equations*. One might extend it to *neutrosophic linear equations*.

125

Vasantha & Smarandache defined in 2003:

Neutrosophic number has the form $a + bI$, where I = indeterminacy and it is different from the imaginary root $i = \sqrt{-1}$; we have $I^2 = I$ and $I + I = 2I$, while a, b are real or complex numbers.

$\mathbb{R}(I)$ is the *real neutrosophic field*, where $\mathbb{R}$ is the set of real numbers.

$\mathbb{C}(I)$ is the *complex neutrosophic field*, where $\mathbb{C}$ is the set of complex numbers.

Using the indeterminacy "*I*" we have also defined the *neutrosophic group, neutrosophic field, neutrosophic vector space*, etc.

Neutrosophic matrix, M = a_{ij}, where a_{ij} are *neutrosophic numbers*.

126

A *neutrosophic graph* is a graph in which at least one edge is an indeterminacy denoted by dotted lines.

The indeterminacy of a path connecting two vertices was never in vogue in mathematical literature.

127

Two graphs G and H are *neutrosophically isomorphic* if:

a) They are isomorphic;

b) If there exists a one to one correspondence between their point sets which preserve indeterminacy adjacency.

128

A *neutrosophic walk* of a *neutrosophic graph* G is a walk of the graph G in which at least one of the lines is an indeterminacy line. The *neutrosophic walk* is *neutrosophic closed* if $V_o = V_n$ and is *neutrosophic open* otherwise.

A *neutrosophic bigraph*, G is a bigraph, whose point set V can be partitioned into two subsets V_1 and V_2 such that at least a line of G which joins V_1 with V_2 is a line of indeterminacy.

A *neutrosophic cognitive map* (*NCM*) is a *neutrosophic directed graph* with concepts like policies, events etc., as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts.

129

Let Ci and Cj denote the two nodes of the *neutrosophic cognitive map*. The directed edge from Ci to Cj denotes the causality of Ci on Cj called connections. Every edge in the *neutrosophic cognitive map* is weighted with a number in the set $\{-1, 0, 1, I\}$. Let eij be the weight of the directed edge $CiCj$, $eij \in \{-1, 0, 1, I\}$. $eij = 0$ if Ci does not have any effect on Cj, $eij = 1$ if increase (or decrease) in Ci causes increase (or decreases) in Cj, $eij = -1$ if increase (or decrease) in Ci causes decrease (or increase) in Cj. $eij = I$ if the relation or effect of Ci on Cj is an indeterminate.

Neutrosophic cognitive maps with edge weight from $\{-1, 0, 1, I\}$ are called *simple neutrosophic cognitive maps*.

Let D be the domain space and R be the range space with $D1, \dots, Dn$ the conceptual nodes of the domain space D and $R1, \dots, Rm$ be the conceptual nodes of the range space R such that they form a disjoint class i.e. D ∩ R = ϕ. Suppose there is a *fuzzy relational maps* relating D and R and if at least an edge relating a $Di\ Rj$ is an indeterminate then we call the *fuzzy relational maps* as the *neutrosophic relational maps*, i.e. NRMs.

Thus, to the best of our knowledge indeterminacy models can be built using *neutrosophy*.

One model already discussed is the *neutrosophic cognitive model*. The other being the *neutrosophic relational maps* model, which are a further generalization of *fuzzy relational maps*.

It is not essential when a study/prediction/ investigation is made we are always in a position to find a complete answer. This is not always possible (sometimes or many times); almost all models are built using unsupervised data, we may have the factor of indeterminacy to play a role. Such study is possible only by using the *neutrosophic logic*.

130

Email to Dr. Emil Dinga:

Logica neutrosofică (LN) este o generalizare a logicii trivalente a lui Lukasievicz, pentru că fiecare componentă poate avea o infinitate de valori.

La Lukasievicz era: 1 (adevărat), 0 (fals), și 1/2 (nedeterminat).

În *logica neutrosofică* valoarea de adevăr a unei propoziții este (T, I, F), unde T, I, F sunt în [0, 1] (am simplificat-o, fără analiza nonstandard).

De exemplu: șansa ca peste cinci zile va ploua la București este: (0.4, 0.1, 0.5), adică 40% șanse să plouă, 50% șanse să nu plouă, și 10% nedeterminat/neștiut.

Sau poate să fie (0.22, 0.67, 0.11) etc.

LN este triplu infinită.

Logica lui Lupașcu se referă la Terțiul Inclus (care în *logica neutrosofică* este componentă nedeterminată).

LN este o generalizare a logicii fuzzy, dar și a terțului inclus al lui Lupașcu.

131

Neutrosophic quantum theory (NQT) is the study of the principle that certain physical quantities can assume *neutrosophic values*, instead of discrete values as in *quantum theory*.

These quantities are thus neutrosophically quantized.

A *neutrosophic value* (*neutrosophic amount*) is expressed by a set (mostly an interval) that approximates (or includes) a discrete value.

An oscillator can lose or gain energy by some neutrosophic amount (we mean neither continuously nor discretely, but as a series of integral sets: *S, 2S, 3S, …,* where *S* is a set).

In the most general form, one has an *ensemble of sets of sets*, i.e. R_1S_1, R_2S_2, R_3S_3, ..., where all R_n and S_n are sets that may vary in function of time and of other parameters. Several such sets may be equal, or may be reduced to points, or may be empty.

{The multiplication of two sets *A* and *B* is classically defined as: $AB = \{ab, a \in A \text{ and } b \in B\}$. And similarly a number *n* times a set *A* is defined as: $nA = \{na, a \in A\}$.}

132

The *unit of neutrosophic energy* is Hv, where H is a set (in particular an interval) that includes Planck constant h, and v is the frequency. Therefore, an oscillator could change its energy by a *neutrosophic number of quanta*: Hv, $2Hv$, $3Hv$, etc.

For example, when H is an interval $[h_1, h_2]$, with $0 \le h_1 \le h_2$, that contains Planck constant h, then one has: $[h_1v, h_2v]$, $[2h_1v, 2h_2v]$, $[3h_1v, 3h_2v]$,..., as series of intervals of energy change of the oscillator.

The most general form of the units of neutrosophic energy is H_nv_n, where all H_n and v_n are sets that similarly as above may vary in function of time and of other oscillator and environment parameters.

Neutrosophic quantum theory is a combination of classical mechanics of Newton and quantum theory. Instead of continuous or discrete energy change of an oscillator, one has a series of sets (and, in particular case, a series of intervals) of energy change.

And in the most general form one has an ensemble of sets of sets of energy change.

133

Neutrosophic quantum statistics consists in the study, among the neutrosophic quantized energy levels, of the approximate equilibrium distribution of each specific type of elementary particles.

Instead of quantum numbers, which take certain discrete values, we consider neutrosophic quantum numbers, which take certain set values (and, as a particular case: certain interval values). We mean that a discrete value is neutrosophically approximated by a small set (neighborhood) that includes it. In such a way, the quantized energy levels are extended to neutrosophic quantized energy levels.

a. According to the Neutrosophic Fermi-Dirac Statistics, in the same neutrosophic quantum mechanical state there cannot be two identical fermions.
b. According to the Neutrosophic Bose-Einstein Statistics, in the same neutrosophic quantum mechanical state there can be any number of identical bosons.

134

As an example of application of *neutrosophy* in information fusion in finance for example there are some papers by Dr. Mohammad Khoshnevisan and Dr. Sukanto

Bhattacharya, where the fuzzy theory doesn't work because fuzzy theory has only two components, while the *neutrosophy* has three components: truth, falsehood, and indeterminacy (or <A>, <Anti-A>, and <Neut-A>), i.e. about investments which are: Conservative and security-oriented (risk shy), Chance-oriented and progressive (risk happy), or Growth-oriented and dynamic (risk neutral). Other applications are in voting process, for example: FOR, AGAINST, and NEUTRAL (about a candidate) (<A>, <Anti-A>, and <Neut-A>).

But new ideas always face opposition...

135

Email to Dr. Gheorghe Săvoiu:

Cred că ați putea lega *economia mesonică* (plasarea între antinomii) cu *neutrosofia*, bazată pe <A>, <antiA> și <neutA>. <A> este o entitate, <antiA> este opusul ei, iar <neutA> este neutralul dintre antonimiile <A> si <antiA>.

Până acum nu am aplicat *neutrosofia* în economie; deci, ați fi primul făcând această legatură.

Astfel, <neutA> poate fi format din <A> și <antiA>, sau poate fi vag, nedeterminat.

În *logica neutrosofică*, o propoziție are un procent de adevăr, un procent de falsitate, și un procent de nedeterminare. De exemplu: "F.C. Argeș va câștiga în meciul cu Dinamo" poate fi 50% adevărată (șansa de câștig), 30% falsă (șansa de pierdere), și 20% nedeterminată (șansa de meci egal).

136

Email to Mirela Teodorescu:

Neutrosofia nu înseamnă numai studiul neutralitaților <neutA>, dar și a conecțiilor dintre <A> și <antiA> (ca dialectica), și conecțiile dintre <A> și <neutA>, conecțiile dintre <neutA> și <antiA>, și chiar conecțiile dintre toate trei împreuna <A>, <neutA>, <antiA>.

Neutrosofia este o generalizare a dialecticii, care studiază numai conecțiile dintre <A> și <antiA>.

137

Am convenit cu domnul Ștefan Vlăduțescu să facem o culegere de aplicații ale neutrosofiei (combinații de idei opuse, ori idei opuse și neutralele dintre ele) în literatură și artă.

Se întâmplă ca prin combinarea de urât și frumos să iasă ceva neutru, sau ambiguu, sau nedeterminat.

Se poate depista în aceeași operă (artistică sau literară) atât părți frumoase, cât și părți urâte, dar și părți ambigue din punct de vedere ontologic.

Interpretând o operă artistică/literară din puncte de vedere diferite, puteți obține opinii contradictorii sau ambigue (nedeterminate).

De pildă, vizionați un film. Dar filmul poate fi bun din punct de vedere al interpretării unor actori, însa prost din punct de vedere al regiei, sau neclar din punct de vedere al acțiunii filmului.

138

Email from Mirela Teodorescu:

Ce bine că primesc apă la moară!

Am văzut un film: *Dracula Untold*. Producție 2014, efecte moderne, actori buni, scenariu interesant pentru cei care nu cunosc istoria Țărilor Române.

Tema are ceva adevar istoric. Se respectă numele românești: Vlad, Dumitru, Vasile, Ion, Mihai...

Denumiri geografice: Cozia, Pasul Tihuța, Muntele Dintele..., numai ca juxtapunerea lor este neconformă. Da, aici e multă neclaritate și confuzie.

Așadar, este o producție comercială.

Apreciez valoarea estetică și nu valoarea de adevăr istoric.

La final, morala: Vlad Țepeș a fost un erou care și-a salvat poporul cu prețul de a deveni călător în timp.

139

Email to Mirela Teodorescu:

Este exact ceea ce ziceam neutrosofic: bun dintr-o parte, rău din altă parte, nedecis din alt unghi de vedere.

Desigur, depinde de "definiția" frumosului sau/și urâtului.

Același obiect poate fi frumos dintr-un punct de vedere, urât din alt punct de vedere, și nici frumos, nici urât din al treilea punct de vedere (“neutralul” din *neutrosofie*).

Există și modele instabile de frumos sau urât.

140

To Ovidiu Șandru:

Nu știu dacă ești interesat de sisteme inconsistente, contradictorii?

Sunt conectate și cu Extensica.

Poate te-ar interesa în dinamica sistemelor, în care am vazut ca ești preocupat. Sau, dându-se un sistem consistent de axiome, putem lua una și o nega în mai multe feluri. Punem ambele axiome ("A" și "nonA") împreună în același sistem de axiome.

141

Email from Mirela Teodorescu:

În următoarea etapă voi scrie un alt articol legat de *neutrosofie* în procesul de producție, cum apar incertitudinile și cum se soluționează practic.

142

Alexandru Gal și Luige Vlădăreanu au folosit *Uncertainty* și *Contradiction* în diagrame.

Am următoarea idee despre *logica neutrosofică*, ceea ce ar face subiectul altor cercetari pe viitor, și anume: [v. și explicația din http://fs.gallup.unm.edu/neutrosophy.htm]:

- componentele (T, I, F) au proprietatea ca indeterminacy I se poate descompune în multe subcomponente (caracterizând partea neclară, neexactă), și anume I = (U, C), în acest caz pentru roboți, unde U = uncertainty = T \/ F (truth or falsity), iar C = contradiction = T /\ F (truth and falsity);

- deci, se poate lucra direct pe patru componente neutrosofice (T, U, C, F); nu s-au făcut cercetări pe această logică neutrosofică având patru componente, și nici operatorii de inferență nu au fost definiți, dar acest lucru se poate face;
- componenta I=indeterminacy se poate descompune și în trei sau mai multe subcomponente dacă este nevoie în vreo aplicație.

De exemplu, *logica neutrosofică* pe cinci componente: Truth, Uncertainty, Contradiction, Notknown, Falsity = (T, C, U, N, F) în cazul când avem, ca indeterminare, pe lângă U și C, și N=Notknown (Necunoscut), ș.a.m.d.

Totul depinde de ceea ce este nevoie în aplicații.

143

The sum t+i+f can be 3 when the components are independent, but if they all are dependent, then t+i+f = 1.

We only utilize min/max in the inference for the neutrosophic set/logic. We can go more general in the following way: instead of "min" we can use any t-norm from fuzzy set/logic (i.e. the AND fuzzy operator, or CONJUNCTION operator), and instead of "max" we can use a t-conorm from the same fuzzy set/logic (i.e. the OR fuzzy operator, or DISJUNCTION operator). For example, we have used the dual min/max, but we can also use the

$xy/x+y-xy$, i.e. (tA, iA, fA)/\(tB, iB, fB) = (tAtB, iA+iB-iAiB, fA+fB-fAfB) while (tA, iA, fA)\/(tB, iB, fB) = (tA+tB-tAtB, iAiB, fAfB). Other dual is: $max\{0, x+y-1\}/min\{1, x+y\}$. I agree that min/max is the most used and much easier, especially if we have the t, i, f as intervals.

144

For *refined neutrosophic numbers* of the form:

$$a + b_1I_1 + b_2I_2 + \ldots + b_nI_n,$$

where $I1, I2, \ldots, In$ are types of indeterminacy.

Maybe it looks artificial until one can find any application.

145

Since we work with approximations in fuzzy and neutrosophic theories, we can take for delta-equalities of *neutrosophic sets*:

- either <=, >=, >=;
- or <=, <=, >=;
- or <=, <=, <=.

depending on the problem to solve.

146

The quantum calculators can be extended to *neutrosophic quantum computers*, where one has 1 (true), 0 (false), and 0 and 1 overlapping (as indeterminacy).

147

More algebraic structures on *neutrosophic triplets* can be developed: *neutrosophic triplets' ring*, *neutrosophic triplets' semigroup*, *neutrosophic triplets' vectorspace* (of course, we have to make sure the axioms of each algebraic structure are verified).

148

We have the possibility to neutrosophically extend the Set of Experts O = {agree, disagree} to the *Neutrosophic Set of Experts* NO = {agree, indeterminate, disagree}, considering F: E×X×NO --> P(U).

What also about extending O in another way: the experts do not only say agree, or disagree, or indeterminate/pending/unknown, but a percentage of agreement, a percentage of indeterminacy, and a percentage of disagreement - as in neutrosophic logic, considering the following:

$$F: E \times X \times NO(t, i, f) \geq P(U).$$

149

To extend from the *neutrosophic triangular number* to the *refined neutrosophic triangular number*, and similarly from *neutrosophic trapezoidal number* to the *refined neutrosophic trapezoidal number*.

150

Jun Ye defined the *neutrosophic trapezoidal number*, but not the *refined neutrosophic trapezoidal number*. So one can write a paper on <T1, T2, ...; I1, I2, ...; F1, F2, ...> generalizing Jun's result (and we cite him) on *refined neutrosophic trapezoidal number*.

151

One can extend the *bipolar neutrosophic set* to *m-polar neutrosophic set* - in a similar way as it is *m-polar fuzzy set*.

151

Between <A> and <antiA> there is a *multiple-included middle law*. That means that between two opposites, white and black, there is a *multitude of neutralities* (an infinite spectrum of colors between white and black). Always the number of *neutralities* between <A> and <antiA> depend on the entity <A>.

152

Email to Elemer Rosinger:

We can get a system in between the Cartesian system and Quantum system, as in *neutrosophy*, why not? Even various degrees of included multiple-middles, I mean a system which is partially Cartesian and partially Quantum.

153

The un-existence and un-reality could be the dream status, or even coma.

While the Taoism connects $< A >$ with $< antiA >$, the neutrosophy connects $< A >, < antiA >$, and

$< neutA >$ [here $< neutA >$ is $< un\text{A} >$].

154

Je propose quelque chose de nouveau dans la fusion, qui vient de la logique neutrosophique: introduire l'element "ni A ni B", qui est opposé à "A ou B" = A\/B.

Je veux dire, on aura:

A/\(nonB), (nonA)/\B, (nonA)/\(nonB) = ni A ni B.

155

Thinking at including somehow the indeterminacy "I" in the coordinates.

In general, for a Minkovski space-time (x, y, z, t), we can define: $x = x_1 + x_2 I, y = y_1 + y_2 I, z = z_1 + z_2 I$, and time $t = t_1 + t_2 I$, where x, y, z, t are now *neutrosophic numbers*.

It would be interesting to get some applications and to study how well-known equations from math, physics, etc. become in such a *neutrosophic system of coordinates*.

For example, the equation of a line $ax + by = c$ in 2D would become $ax_1 + by_1 + (ax_2 + by_2)I = c_1 + c_2 I$, or $ax_1 + by_1 = c_1$ and the indeterminacy part $ax_2 + by_2 = c_2$.

How should we interpret these? The real part and respectively indeterminacy part of the linear equation?

Any practical example?

It would be innovatory to use this *neutrosophic system of coordinates* in physics for certain equations and to find a good interpretation.

156

A *neutrosophic interpretation* of the Hindu philosophy (*Upanishads*, *Vedas*, the universal law and order *Dharma* and *Rta, Vedanta*, etc.) can be done.

Or a comparison of various philosophies (I mean one which asserts <*A*> and another philosophy which asserts the opposite <*antiA*>).

157

In a similar way to, and an extension of, the Antonym Test in psychology, it would be a verbal test where the subject must supply as many as possible synonyms of a given word within as short as possible a period of time.

How to measure it?

The spectrum of supplied synonyms (s), within the measured period of time (t), shows the subject's level of *linguistic neutrosophy*: *s/t.*

158

Email to Fu Yuhua:

Although *yin-yang* is part of the Taoism, what we did already, maybe we can write something about only what is in between *yin* and *yang* (I mean we need to find the neutral which is neither *yin* nor *yang*, or something which is both of them, *yin+yang* in the same time).

I mean to complement *yin+yang* with what is none of them, and what is both of them simultaneously.

For example, there are persons whose sex is indeterminate (neither male nor female), etc.

We can write another book, maybe named "Neither Yin, Nor Yang", or another title. We can take each *yin-yang* philosopher and complement him/her.

159

I extended the T-norm and T-conorm from the fuzzy set/logic to N-norm and N-conorm to neutrosophic set/logic - see page 228 and section 8.31 in the book:

http://www.gallup.unm.edu/~smarandache/DSmT-book2.pdf

where I try to use the neutrosophic belief in information fusion.

N-norm and N-conorm are *classes of neutrosophic operators*, similarly to fuzzy operators.

So you can define neutrosophic operators different from mine from the book "A Unifying Field in Logics..." .

Did you check the connectives defined in the book "Interval Neutrosophic Set and Logic":

http://www.gallup.unm.edu/~smarandache/INSL.pdf ?

So I focused more on applications [a student from Australia, Sukanto Bhattacharya, got his PhD using neutrosophics in finance - and I was an outside evaluator for his thesis].

But you're a philosopher, so you can use very well the *neutrosophy* in Italian philosophy, or in any other thinking - since the neutrosophic axiomatization is a little tricky due to the three components instead of two. {By the way, can you send me an article or book with fuzzy set/logic axiomatization? This might give me some inspiration to help you in *neutrosophic axiomatization*.}

160

Email to Umberto Riviecci:

Neutrosophy is a generalization of dialectics. As you know, dialectics studies the opposites and their interactions. Neutrosophy studies the opposites together with the neutrals (those who are neither for nor against an idea), because in a dynamic process the neutrals can become either pro- or contra- an idea, so the neutrals influence too the evolution of an idea.

What I mean, you might be interested in using neutrosophy in studying some philosophical schools, see for example such studies in Chinese philosophy:

http://www.gallup.unm.edu/~smarandache/NeutrosophicDialogues.pdf

or Arabic philosophy:

http://www.gallup.unm.edu/~smarandache/ArabicNeutrosophy-en.pdf

which was translated into Arabic:

http://www.gallup.unm.edu/~smarandache/ArabicNeutrosophy-ar.pdf

and published in Alexandria, Egypt.

I started the *neutrosophy* from reading philosophy, i. e. I observed that some philosophers asserted an idea <A> and proved it was true, while other philosophers asserted the opposite idea <antiA> and proved it was true as well.

So in philosophy it was possible to have opposite ideas true both of them in the same time! This kind of study we can do in Italian philosophy - if interested.

161

The *neutrosophic probability* and *statistics* is a virgin domain since no study has been done so far.

I only defined it and tried to extend the classical probability's axioms to *NP*: that chance that an event *E* will occur is *T, I, F*. But one can redefine the axioms in a different way.

I have defined the *neutrosophic probability* and I gave examples of easy sample spaces with indeterminacy (called neutrosophic probability spaces).

Neutrosophic statistics can be developed on such spaces with indeterminacies.

162

One can consider that n individuals of a population (a sample) may belong to the population (or sample) in the following way: each individual A_j, $j = 1, 2, \ldots, n$, as degree of membership to the population T_j , degree of indeterminacy (not knowing if membership or nonmembership) I_j, and the degree on nonmembership F_j.

163

Neutrosophic probability allows the characterization of a middle component called "indeterminacy" (i.e. the event neither occurring, nor not-occurring, but unknowns part of the event which might be because of hidden parameters we are not aware of) - that's the main distinction between the classical and imprecise probabilities with respect to *NP*. I see its definition: http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf .

But it does not mean the the middle component, indeterminacy, should be all the time. For example, when tossing a die there is no indeterminacy (this is objective probability, i.e. probability that can be computed exactly). But in subjective probability [which means probability that can not be computed exactly, for example the probability that a soccer team will win a game: it may win, it may loose, or it may have tied game (neither winning nor loosing), but we can not exactly compute this probability].

In neutrosophy, a generalization of dialectics, between an entity <*A*> and <*antiA*> (its opposite) there are <*neutA*> (neutralities). But this does not apply for all entities. *NP* is based on neutrosophy.

For example, between <*White*> and <*Black*> there are many colors (neutralities, neither *White* not *Black*). Between <*Good*> and <*Bad*> there also are *neutralities* (say half good and half bad, etc.).

But between <1+1=2> and <1+1 different from 2> it is not [sure, herein we may come up with say: 1+1=2 in base 10, but 1+1=10 in base two, hence 1+1 is equal and is not equal to 2].

164

In order to apply the probability theory, you have to know the probability space.

There are two types of probabilities, objective where the probability space is known and you can exactly compute the probability of an event (say tossing a die), and subjective probability where the probability space is only partially known due to hidden parameters that influence the outcome and we are not aware of.

In the subjective probability we can not exactly compute the chance of an event to occur.

So in a soccer game you can not compute exactly the probability of a team to win since more unexpected parameters may be involved in the outcome: say some player(s) may get sick or have an accident, the weather may change, game cancelled as you said, etc. This is the indeterminacy that occur in *neutrosophic logic* and not in classical probability.

165

In many social, political, humanistic subjective events we don't have an exact probability space to compute the chance of an event to occur.

In classical probability, you don't have room for paraconsistent outcome as in *NL* or *NP* (sum of components > 1).

For example, you can have *NL* (John is a good student) = (0.7, 0.2, 0.8) meaning that John is 70% a good student (considering his math skills), 80% a bad student (considering his English skills) and 20% indeterminate (not sure about his skills in other fields), but you can not have them all together in classical probability, classical logic, or in fuzzy logic.

166

For decision making in robotics, etc. one computes the entropy - there are special procedures for decision making.

Again, using *neutrosophic logic* you can get the option 1) to take a decision, 2) or not to take it, or 3) pending (indeterminate) when you wait for more information to come in.

167

The introductory part in *neutrosophic logic* uses elementary calculations, you're right. But the problems become more complicated with the quantifiers, see the next book:

www.gallup.unm.edu/~smarandache/INSL.pdf

168

I see no problem with a soccer game in classical logic or classical probability.

There is a set of three outcomes in a game between A and B.

$$\{A\_wins, B\_wins, AB\_Tie\}$$

169

You can NOT have a tri-dimensional vector in classical logic or probability.

But in *neutrosophic probability* you may directly have for example NP(A) = (0.6, 0.1, 0.3), which means the probability that team A wins is 60%, that team A looses is 30%, and that team A has a tight game is 10%.

170

Assuming there are no other possible outcomes (game cancelled ? ...), then these describe the situation. If there is another (independent) set of outcomes, say a soccer game between teams C and D with outcomes $\{C\_wins, D\_wins, CD\_Tie\}$.

Then Probability (A_wins and C_wins) is computed with 1 multiplication.

All "and" combinations can be computed with $3 \times 3 = 9$ multiplications.

171

In *neutrosophic probability* you only combine, using a *neutrosophic probability operator*, the two probabilities:

NP(A) = (0.6, 0.1, 0.3) and NP(C),

where let's say for example $NL(C) = (0.4, 0.4, 0.2)$.

172

Q: A proposition (team A wins) is either true or false?

A: Not necessarily. It may also be neither winning not loosing, i.e. tight game, or cancelled game, or postponded game.

Therefore, something in between (included middle principle DOES apply herein).

173

Another proposition, (team *A* wins or ties) is either true or false. There is no excluded middle necessary.

In this case generally speaking the excluded middle applies, i.e. it is not possible to have another alternative besides wining, tight, or loosing; yet, there might be a small possibility that the game is cancelled, or postponed (hence there might be some room for indeterminacy).

Not for all propositions the included middle principle applies.

174

I am looking for an example that would have the following outline.

A situation is given (say, a soccer tournament of many games). I am able to bet money on the result.

Using *neutrosophic logic*, can I make a decision that will make more money, on average, than if I use probability, and perhaps predictions of transitivity of A>B, B>C> ==> A>C?

175

Neutrosophic logic is a tool to measure a possible objective or subjective outcome.

When the probability space is known (as in tossing a die) then the NL is reduced to classical probability (since not indeterminacy exist).

But in many subjective outputs the probability space can not be exactly computed with the classical probability.

How can you use the classical probability to calculate if team A wins?

You don't have an exact probability space, you don't know all parameters (physical, psychological, unhonest refferrees, etc.) which will influence the final result.

NL or NP better measure the subjective probability.

176

Given a more complex situation, can we compute better with NCM say, "Should the US send $100,000,000 to the government of Niger to alleviate starvation?"

Can you get a demonstrably better result? (Note, this decision requires a yes/no answer, not "This is a medium-good idea").

177

Since we get aware of possible hidden parameters, we have a reserve (indeterminancy - pending, when we can wait for more information to come in) in taking a decision.

It is possible and for good to be undecided and wait, than taking a wrong decision.

178

Or a robot, given contradictory information

"Visual sensors detect incoming bullets. Retreat."

"The goal is in the forward direction. Continue forward."

It must move.

Perhaps I am missing something, but

I do not see how computer algebra systems need to be changed to handle any of the mechanisms needed for NCM or competitors which so far as I can tell include simple arithmetic, interval arithmetic, arithmetic on distributions, and perhaps logic with symbols (indeterminates), representation of graphs and sets.

179

Neutrosophic Cubic Set.

Jun et al. (2012) have defined the (Fuzzy) Cubic Set as follows:

Let *X* be a non-empty set. A Cubic Set in *X* is a structure of the form:

$$\underline{A} = \{x, A(x), \lambda(x) \mid x \in X\}$$

where *A* is an interval-valued fuzzy set in *X* and λ is a fuzzy set in *X*.

Then one can extend the (Fuzzy) Cubic Set to a *neutrosophic cubic set* in the following way:

$$\underline{N} = \{x, < A_1(x), A_2(x), A_3(x) >, < \lambda_1(x), \lambda_2(x), \lambda_3(x) > \mid x \in X\}$$

where $<A_1(x), A_2(x), A_3(x)>$ is an *interval-valued neutrosophic set* in *X* and

$<\lambda_1(x), \lambda_2(x), \lambda_3(x)>$ is a *neutrosophic set* in *X*.

Reference: Y. B. Jun, C. S. Kim, and K. O. Yang, *Cubic Sets*, Annals of Fuzzy Mathematics and Informatics, 4(1), 83-98, 2012.

180

I remember I said in a previous email that instead of using non-standard analysis, which is more difficult to implement and not necessary for technical problems but for philosophical proposal only in the case when needed to make a distinction between "absolute" and "relative" truth/falsehood/indeterminacy,

I said to use the simple real subunitary intervals (not non-standard ones).

Hence do not stress using the non-standard analysis for computer algebra systems, but simple real intervals.

Therefore, I tried to simplify as much as possible the definition of neutrosophics.

181

Yet, despite Dr. Fateman opinion, I think the most general valuable logic as today is to considered a three-values logic for each proposition: truth value, falsehood value, and indeterminacy value [hence *neutrosophic logic*].

When we analyze the proposition "Next year John will be sick", you can not use classical probability, neither classical logic, but a logic on three components: say 40% John will be sick since he had a history of diseases which occured to him periodically, 35% he will not be sick since today he is in a good helth, 25% indeterminant since he might have an accident or he might contact a virus from a foreign country he will be visiting, etc.

What about if next year some months he will be sick and other months he will be healthy? How would you classify this, sickness or good helth? I think something both of them, sick and healthy (which belongs to indeterminacy).

You'd not be able to use classical probability or classical logic for this.

Hence, in computer algebra systems this logic would be the best to calculate the logical values.

182

Sure there are cases when the indeterminacy is zero for a scalar, or empty set for interval-valued logic. In this case neutrosophic logic is reduced to fuzzy logic.

In the case when working with exact scientific proposition, then there is no included-middle principle, hence the *neutrosophic logic* is reduced to the classical logic.

But in many subjective, psychological, biological cases we have three possible components (truth, falsehood, indeterminacy) for a proposition.

183

Neutrosophic statistical mechanics is the theory in which, using the *neutrosophic statistical behavior* of the constituent particles of a macroscopic system, are predicted the approximate properties of this macroscopic system.

Neutrosophic statistics means statistical analysis of population or sample that has indeterminate (imprecise, ambiguous, vague, incomplete, unknown) data.

For example, the population or sample size might not be exactly determinate because of some individuals that partially belong to the population or sample, and partially they do not belong, or individuals whose appurtenance is completely unknown.

Also, there are population or sample individuals whose data could be indeterminate.

{Depending on the type of indeterminacy one can define various types of *neutrosophic statistics*.}

184

Clan capitalism is like *neutrosophic logic* (neither neoliberalism, nor keyesian - but in between).

185

The effect of clan groups is like a democracy institution, but they are on the negative side, that is they can deteriorate democracy institutions, that is why: neoliberalism proponents who always think that less state-regulation is better, actually make those clans can grow bigger. that is how neoliberalism is very wrong, but i don't investigate yet if they do that by purpose (less state regulation, in order those clan groups really can stir things to their advantages).

Neoliberalism has to be controlled. Regulation has also to be controlled. What happens is that regulation will limit the neoliberalism, but if regulation is too harsh then neoliberalism should fight. So, always a mutual fight between the opposites. The truth should be in between.

So, each economy should have a percentage n% of neoliberalism and another percentage of regulation r%, where $n + r = 100$. They are flexible and vary from a period to another, I mean when one increases a little the other decreases a little.

Actually the fluctuation of neoliberalism percentage should vary between $[n_1, n_2]$% and the regulation between

$[r_1, r_2]$%. I think should be our economical mathematical theory. Of course, the question is: how to find n_1,n_2 and r_1, r_2?

There should always be an equilibrium between neoliberalism and regulation - as if one increases too much, the other should fight for re-balancing.

186

Okay, then like in *neutrosophic logic*: three components: we should also include anticlan (ac) law, so:

$$n + r + ac = 100.$$

187

In *neutrosophic logic* and *set* one has three possibilities related to the relationships between the *neutrosophic components* T, I, and F as single numbers in the interval [0, 1]:

1) If T, I, F are all dependent of each other, then 0 <= $T + I + F \leq 1$;
2) If among T, I, F there are two components which are dependent of each other, but the third one is independent of them, then $0 \leq T + I + F \leq 2$;
3) And, if T, I, F are all independent two by two of each other, then $0 \leq T + I + F \leq 3$.

188

To introduce the *Unipolar/Bipolar/Tripolar Neutrosophic Set*.

We generalize the bipolar valued fuzzy set to a *tripolar valued neutrosophic set*, where an element x from a neutrosophic set A has a positive and negative membership T^+ and T^- , a positive and negative indeterminacy-membership I^* and I^-, and a positive and negative non-membership F^+ and T^-, where T^+, I^+, F^+ are subsets of [0, 1], while T^-, I^-, F^- are subsets of [-1, 0],

But we also considered a *bipolar valued neutrosophic set*, when for each element x from a neutrosophic set A one has for the three components T, I, F only two positive and negative components, while the third component will be only positive (or only negative), for example:

- if only T and F are positive and negative components, while I is only positive component;
- or if only T and F are positive and negative components, while I is only negative component;
- or if only T and I are positive and negative, while F is only positive;
- or if only T and I are positive and negative, while F is only negative;
- or if only I and F are positive and negative, while T is only positive;
- or if only I and F are positive and negative, while T is only negative.

Or a *unipolar neutrosophic set*, when only one component among T, I, F is positive and negative, while the others are only positive or only negative. For example:

- only T is positive and negative, while I and F are both only positive;
- only T is positive and negative, while I and F are both only negative;
- only T is positive and negative, while I is only positive and F is only negative;
- only T is positive and negative, while I is only negative and F is only positive;
- similarly, if one considers only I as positive and negative, while T and F are only either positive or negative (one has 4 sub-cases as above);
- and again similarly for the case when only F is positive and negative, while T and I are only either positive or negative (one has 4 sub-cases as above).

189

The *quaternion number* is a number of the form:

$Q = a \cdot 1 + b \cdot i + c \cdot j + d \cdot k,$

where $i^2 = j^2 = k^2 = i \cdot j \cdot k = -1$, and a, b, c, d are real numbers.

The *octonion number* has the form:

$O = a + b_0 i_0 + b_1 i_1 + b_2 i_2 + b_3 i_3 + b_4 i_4 + b_5 i_5 + b_6 i_6$, where $a, b_0, b_1, b_2, b_3, b_4, b_5, b_6$ are real numbers, and each of the triplets (i_0, i_1, i_3), (i_1, i_2, i_4), (i_2, i_3, i_5), (i_3, i_4, i_6), (i_4, i_5, i_0), (i_5, i_6, i_1), (i_6, i_0, i_2) bears like the quaternions (i, j, k).

We extend now for the first time the octonion number to a *n-nion number*, for integer $n \geq 4$, in the following way: $N = a + b_1i_1 + b_2i_2 + \ldots + b_{n-1}i_{n-1} + b_ni_n$, where $a, b_1, b_2, \ldots, b_{n-1}, b_n$ are real numbers, and each of the triplets, $f(i_{k(\text{mod}\,n)}, i_{k+1(\text{mod}\,n)}, i_{k+3(\text{mod}\,n)})$ or $k \in \{1, 2, \ldots, n\}$, bears like the quaternions (i, j, k).

We also introduce for the first time the *neutrosophic n-nion number* as follows:

$NN = (a_1+a_2I) + (b_{11}+b_{12}I)i_1 + (b_{21}+b_{22}I)i_2 + \ldots + (b_{n-1,1}+b_{n-1,2}I)i_{n-1} + (b_{n1}+b_{n2}I)i_n$ where all $a_1, a_2, b_{11}, b_{12}, b_{21}, b_{22}, \ldots, b_{n-1,1}, b_{n-1,2}, b_{n1}, b_{n2}$ are real or complex numbers, I = indeterminacy, and each of the triplets $(i_{k(\text{mod}\,n)}, i_{k+1(\text{mod}\,n)}, i_{k+3(\text{mod}\,n)})$, for $k \in \{1, 2, \ldots, n\}$, bears like the quaternions (i, j, k).

See: Weisstein, Eric W. "Octonion." From *MathWorld* -- A Wolfram Web Resource. http://mathworld.wolfram.com/Octonion.html

190

In any society, there are three categories of people:

a. Those that support the society [*the Supporters*],
b. Those that do not care about it [*the Ignorants*],
c. Those that are against the society [*the Revolters*],

– as in the *neutrosophic set* and *logic*.

These categories are dynamic: they are in continuous change during a period of time.

Some supporters may become disappointed about the society and switch to the revolters' side, while other supporters may become careless and thus joining the ignorants' side. Similary, for the categories of Ignorants and Revolters, that can change sides.

When the number and force of Revolters increase considerably, passing a certain threshold, riots, revolts, or even revolutions start, trying to change the society.

This *neutrosophic cycle* and *dinamicity* (S, I, R) is in permanent struggle with each other.

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Applications of *neutrosophics* in biology: Besides males (M) and females (F), one has gelded or *neutered beings* (N).

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Email to A.A.A. Agboola:

Since it is possible to split indeterminacy "I" a following particular case can be used in *neutrosophic algebraic structures*.

Let's consider two types of indeterminacies,

- I_1 = contradiction (i.e. True and False)
- and I_2 = ignorance (i.e. True or False).

We may consider the same thing, as I^2 = I, that:

I1^2 = I1 and I2^2 = I2.

But for multiplication $I_1 I_2$ (i.e. I_1 multiplied with I_2) = I_1 because:

$$I_1 I_2 = (T \text{ and } F) \text{ and } (T \text{ or } F) = (T/\backslash F)/\backslash(T\backslash/F)$$
$$= T/\backslash F = I_1.$$

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We can now develop *refined neutrosophic algebraic structures* on sets of *neutrosophic refined numbers* of the form: $a + b1I1 + b2I2$, where a, $b1$, $b2$ are real numbers (or complex numbers).

The addition, subtraction, multiplication will be similar.

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We may go further and split " *I* " into three subcomponents:

I_1 and I_2 as before, and I_3 = uncertainty (i.e. either True or False).

Even the fact I1^ 2 = I_1 is justified because:

$I_1 I_1 = (T/\backslash F)/\backslash(T/\backslash F) = T/\backslash F = I_1$,

and similarly I2^2 = I2 is justified in the same way, because:

$I_2 I_2 = (T\backslash/F)/\backslash(T\backslash/F) = T\backslash/F = I_2$.

These examples justify the 2003 definition that I^2 = I, where I is indeterminacy.

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Email to Victor Christianto:

There is no clear frontier/boundary between quantum level and macro level.

When the frontiers between <A> and <nonA>, or between <A> and <anti> is unclear, such paradoxes are called *Sorites paradoxes*.

Or, when the frontier between <A> and <neutA>, or between <neutA> and <antiA> is unclear, one has a Sorites paradox. {Recall that <nonA> = <antiA> ∪ <neutA>.}

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Email from Hojjatollah Farahani:

I found that the *neutrosophic theory* can come over all domains. It is so useful in *psychological* research. I have categorized the main problem in three sections.

The first section that we can work on it is related to assessment and questionnaire development (such as *neutrosophic Likert scale, neutrosophic validity, neutrosophic reliability*) and the second section is related to causal relationships (*neutrosophic cognitive maps*), and the last one is related to *neutrosophic statistics*. I worked on fuzzy assessment and fuzzy and *neutrosophic cognitive maps* but I am ready to put a lot of effort on section one under your supervision. Please let me know your ideas and give me some tips.

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Email to W. B. Vasantha Kandasamy:

We can extend the *neutrosophic cognitive maps* (NCM), whose edge values were $\{0, 1, -1, I\}$, to $(t, i, f) - NCM$ whose edges get the values (t, i, f), I means the casualty between two graph nodes A and B can be (0.5, 0.3, 0.5), and so on. And similarly for (t, i, f) *−neutrosophic relational maps*.

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Email to Dr. Haibin Wang:

I sent you three messages with files on Description Logic that I got from Internet, although you are very much aware about. If they were not needed I apologize.

In a similar way you can build ontology on *neutrosophic logic*.

After your dissertation, please feel free to do research on building ontology on *neutrosophic logic*, and I'll try to help. Or you can propose to your postdoc advisor to do such research and then publish the research.

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Email to Dr. John Mordeson:

T and F are complementary in fuzzy set and in intuitionistic fuzzy set.

Indeed, T and F look complementary in neutrosophic set too, but they are not in general.

While in fuzzy set and intuitionistic fuzzy set T and F are dependent of each other, in neutrosophic set all three components T, I, F are independent.

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Email to Clifford Chafin:

I did not get yet to partial differential equations in *neutrosophic calculus*. My book on *Neutrosophic Calculus* (2015) that you mentioned before stops at the *first order neutrosophic derivative* and *neutrosophic integral*.

I also observed that in *neutrosophic calculus* there are limits, continuity, derivatives, and integrals that are not complete, I mean there are *neutrosophic functions* that at a given point may have a degree of a limit, or may be continuous in a certain degree (not 100%), or may be differentiable or integrable in a certain degree (not 100%). These occur because of indeterminacies...

I expect in *neutrosophic partial differential equations* there would also be "partial solutions", i.e. solutions that do not completely satisfy the PDE from a classical point of view.

Index

Welcome into my scientific lab!

My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific chats.

I called the jottings herein *scilogs* (truncations of the words *scientific*, and gr. Λόγος – appealing rather to its original meanings "ground", "opinion", "expectation"), combining the welly of both science and informal (via internet) talks.

In this book, one may find new and old questions and ideas, some of them already put at work, others dead or waiting, referring to various fields of research (e.g. from *neutrosophic algebraic structures* to *Zhang's degree of intersection*, or from *Heisenberg uncertainty principle* to *neutrosophic statistics*) – email messages to research colleagues, or replies, notes about authors, articles or books, so on.

Feel free to budge in the lab or use the *scilogs* as open source for your own ideas. *F. S.*

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