Entangled States and Quantum Causality Threshold in the General Theory of Relativity

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This article shows, Synge-Weber's classical problem statement about two particles interacting by a signal can be reduced to the case where the same particle is located in two different points A and B of the basic space-time in the same moment of time, so the states A and B are entangled. This particle, being actual two particles in the entangled states A and B, can interact with itself radiating a photon (signal) in the point A and absorbing it in the point B. That is our goal, to introduce entangled states into General Relativity. Under specific physical conditions the entangled particles in General Relativity can reach a state where neither particle A nor particle B can be the cause of future events. We call this specific state Quantum Causality Threshold.

1 Disentangled and entangled particles in General Relativity. Problem statement

In his article of 2000, dedicated to the 100th anniversary of the discovery of quanta, Belavkin [1] generalizes definitions assumed *de facto* in Quantum Mechanics for entangled and disentangled particles. He writes:

> "The only distinction of the classical theory from quantum is that the prior mixed states cannot be dynamically achieved from pure initial states without a procedure of either statistical or chaotic mixing. In quantum theory, however, the mixed, or decoherent states can be dynamically induced on a subsystem from the initial pure disentangled states of a composed system simply by a unitary transformation.
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> Motivated by Eintein-Podolsky-Rosen paper, in 1935 Schrödinger published a three part essay* on *The Present Situation in Quantum Mechanics*. He turns to EPR paradox and analyses completeness of the description by the wave function for the entangled parts of the system. (The word *entangled* was introduced by Schrödinger for the description of nonseparable states.) He notes that if one has pure states $\psi(\sigma)$ and $\chi(\upsilon)$ for each of two completely separated bodies, one has maximal knowledge, $\psi_1(\sigma,\upsilon) = \psi(\sigma)\chi(\upsilon)$, for two taken together. But the converse is not true for the entangled bodies, described by a non-separable wave function $\psi_1(\sigma,\upsilon)\neq\psi(\sigma)\chi(\upsilon)$: Maximal knowledge of a total system does not necessary imply maximal knowledge of all its parts, not even when these are completely separated one from another, and at the time can not influence one another at all."

In other word, because Quantum Mechanics considers particles as stochastic clouds, there can be entangled particles — particles whose states are entangled, they build a whole system so that if the state of one particle changes the state of the other particles changes immediately as they are far located one from the other.

In particular, because of the permission for entangled states, Quantum Mechanics permits quantum teleportation — the experimentally discovered phenomenon. The term "quantum teleportation" had been introduced into theory in 1993 [2]. First experiment teleporting massless particles (quantum teleportation of photons) was done five years later, in 1998 [3]. Experiments teleporting mass-bearing particles (atoms as a whole) were done in 2004 by two independent groups of scientists: quantum teleportation of the ion of Calcium atom [4] and of the ion of Beryllium atom [5].

There are many followers who continue experiments with quantum teleportation, see [6–16] for instance.

It should be noted, the experimental statement on quantum teleportation has two channels in which information (the quantum state) transfers between two entangled particles: "teleportation channel" where information is transferred instantly, and "synchronization channel" — classical channel where information is transferred in regular way at the light speed or lower of it (the classical channel is targeted to inform the receiving particle about the initial state of the first one). After teleportation the state of the first particle destroys, so there is data transfer (not data copying).

General Relativity draws another picture of data transfer: the particles are considered as point-masses or waves, not stochastic clouds. This statement is true for both mass-bearing particles and massless ones (photons). Data transfer between any two particles is realized as well by point-mass particles, so in General Relativity this process is not of stochastic origin.

In the classical problem statement accepted in General Relativity [17, 18, 19], two mass-bearing particles are con-

*Schrödinger E. *Naturwissenschaften*, 1935, Band 23, 807–812, 823–828, 844–849.

sidered which are moved along neighbour world-lines, a signal is transferred between them by a photon. One of the particles radiates the photon at the other, where the photon is absorbed realizing data transfer between the particles. Of course, the signal can as well be carried by a mass-bearing particle.

If there are two free mass-bering particles, they fall freely along neighbour geodesic lines in a gravitational field. This classical problem has been developed in Synge's book [20] where he has deduced the geodesic lines deviation equation (Synge's equation, 1950's). If these are two particles connected by a non-gravitational force (for instance, by a spring), they are moved along neighbour non-geodesic world-lines. This classical statement has been developed a few years later by Weber [21], who has obtained the world-lines deviation equation (Synge-Weber's equation).

Anyway in this classical problem of General Relativity two interacting particles moved along both neighbour geodesic and non-geodesic world-lines are *disentangled*. This happens, because of two reasons:

1. In this problem statement a signal moves between two interacting particles at the velocity no faster than light, so their states are absolutely separated — these are *disentangled states*;
2. Any particle, being considered in General Relativity's space-time, has its own four-dimensional trajectory (world-line) which is the set of the particle's states from its birth to decay. Two different particles can not occupy the same world-line, so they are in absolutely separated states — they are *disentangled* particles.

The second reason is much stronger than the first one. In particular, the second reason leads to the fact that, in General Relativity, *entangled* are only neighbour states of the same particle along its own world-line — its own states separated in time, not in the three-dimensional space. No two different particles could be entangled. Any two different particles, both mass-bearing and massless ones, are *disentangled* in General Relativity.

On the other hand, experiments on teleportation evident that *entanglement* is really an existing state that happens with particles if they reach specific physical conditions. This is the fact, that should be taken into account by General Relativity.

Therefore our task in this research is to introduce entangled states into General Relativity. Of course, because of the above reasons, two particles can not be in entangled state if they are located in the basic space-time of General Relativity — the four-dimensional pseudo-Riemannian space with sign-alternating label $(+---)$ or $(-+++)$. Its metric is strictly non-degenerated as of any space of Riemannian space family, namely — there the determinant $g = \det\|g_{\alpha\beta}\|$ of the fundamental metric tensor $g_{\alpha\beta}$ is strictly negative $g<0$. We expand the Synge-Weber problem statement, considering it in a *generalized space-time* whose metric can become degenerated $g=0$ under specific physical conditions. This space is one of Smarandache geometry spaces [22–28], because its geometry is partially Riemannian, partially not.

As it was shown in [29, 30] (Borissova and Rabounski, 2001), when General Relativity's basic space-time degenerates physical conditions can imply *observable teleportation* of both a mass-bearing and massless particle — its instant displacement from one point of the space to another, although it moves no faster than light in the degenerated space-time area, outside the basic space-time. In the generalized space-time the Synge-Weber problem statement about two particles interacting by a signal (see fig. 1) can be reduced to the case where the same particle is located in two different points A and B of the basic space-time in the same moment of time, so the states A and B are entangled (see fig. 2). This particle, being actual two particles in the entangled states A and B, can interact with itself radiating a photon (signal) in the point A and absorbing it in the point B. That is our goal, to introduce entangled states into General Relativity.

Moreover, as we will see, under specific physical conditions the entangled particles in General Relativity can reach a state where neither particle A nor particle B can be the cause of future events. We call this specific state *Quantum Causality Threshold*.

2 Introducing entangled states into General Relativity

In the classical problem statement, Synge [20] considered two free-particles (fig. 1) moving along neighbour geodesic world-lines $\Gamma(v)$ and $\Gamma(v+dv)$, where v is a parameter along the direction orthogonal to the geodesics (it is taken in the plane normal to the geodesics). There is $v=\text{const}$ along each the geodesic line.

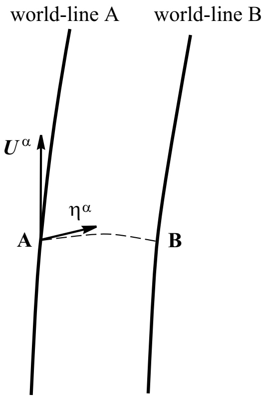

Fig. 1

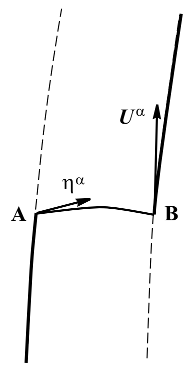

Fig. 2

Motion of the particles is determined by the well-known geodesic equation

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}\, U^\mu \frac{dx^\nu}{ds} = 0\,, \tag{1}$$

which is the actual fact that the absolute differential $\mathrm{D}U^\alpha = dU^\alpha + \Gamma^\alpha_{\mu\nu}\, U^\mu dx^\nu$ of a tangential vector U^α (the velocity world-vector $U^\alpha = \frac{dx^\alpha}{ds}$, in this case), transferred along that geodesic line to where it is tangential, is zero. Here s is an invariant parameter along the geodesic (we assume it the space-time interval), and $\Gamma^\alpha_{\mu\nu}$ are Christoffel's symbols of the 2nd kind. Greek $\alpha = 0, 1, 2, 3$ sign for four-dimensional (space-time) indices.

The parameter v is different for the neighbour geodesics, the difference is dv. Therefore, in order to study relative displacements of two geodesics $\Gamma(v)$ and $\Gamma(v+dv)$, we shall study the vector of their infinitesimal relative displacement

$$\eta^\alpha = \frac{\partial x^\alpha}{\partial v}\, dv\,, \tag{2}$$

As Synge had deduced, a deviation of the geodesic line $\Gamma(v+dv)$ from the geodesic line $\Gamma(v)$ can be found as the solution of his obtained equation

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = 0\,, \tag{3}$$

that describes relative accelerations of two neighbour free-particles ($R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}$ is Riemann-Chrostoffel's curvature tensor). This formula is known as the geodesic lines deviation equation or the *Synge equation*.

In Weber's statement [21] the difference is that he considers two particles connected by a non-gravitational force Φ^α, a spring for instance. So their world-trajectories are non-geodesic, they are determined by the equation

$$\frac{dU^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}\, U^\mu \frac{dx^\nu}{ds} = \frac{\Phi^\alpha}{m_0 c^2}\,, \tag{4}$$

which is different from the geodesic equation in that the right part in not zero here. His deduced improved equation of the world lines deviation

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2} + R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}\, U^\beta U^\delta \eta^\gamma = \frac{1}{m_0 c^2}\frac{\mathrm{D}\Phi^\alpha}{dv}\, dv\,, \tag{5}$$

describes relative accelerations of two particles (of the same rest-mass m_0), connected by a spring. His deviation equation is that of Synge, except of that non-gravitational force Φ^α in the right part. This formula is known as the *Synge-Weber equation*. In this case the angle between the vectors U^α and η^α does not remain unchanged along the trajectories

$$\frac{\partial}{\partial s}(U_\alpha \eta^\alpha) = \frac{1}{m_0 c^2}\, \Phi_\alpha \eta^\alpha. \tag{6}$$

Now, proceeding from this problem statement, we are going to introduce entangled states into General Relativity. At first we determine such states in the space-time of General Relativity, then we find specific physical conditions under which two particles reach a state to be entangled.

Definition Two particles A and B, located in the same spatial section* at the distance $dx^i \neq 0$ from each other, are filled in non-separable states if the observable time interval $d\tau$ between linked events in the particles† is zero $d\tau = 0$. If only $d\tau = 0$, the states become non-separated one from the other, so the particles A and B become entangled.

So we will refer to $d\tau = 0$ as the *entanglement condition* in General Relativity.

Let us consider the entanglement condition $d\tau = 0$ in connection with the world-lines deviation equations.

In General Relativity, the interval of physical observable time $d\tau$ between two events distant at dx^i one from the other is determined through components of the fundamental metric tensor as

$$d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i, \tag{7}$$

see §84 in the well-known *The Classical Theory of Fields* by Landau and Lifshitz [19]. The mathematical apparatus of physical observable quantities (Zelmanov's theory of chronometric invariants [31, 32], see also the brief account in [30, 29]) transforms this formula to

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i, \tag{8}$$

where $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ is the gravitational potential of an acting gravitational field, and $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation.

So, following the theory of physical observable quantities, in real observations where the observer accompanies his references the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ is

$$ds^2 = c^2 d\tau^2 - d\sigma^2, \tag{9}$$

where $d\sigma^2 = \left(-g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}\right) dx^i dx^k$ is a three-dimensional (spatial) invariant, built on the metric three-dimensional observable tensor $h_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}$. This metric observable tensor, in real observations where the observer accompanies his references, is the same that the analogous built general covariant tensor $h_{\alpha\beta}$. So, $d\sigma^2 = h_{ik}\, dx^i dx^k$ is the spatial observable interval for any observer who accompanies his references.

As it is easy to see from (9), there are two possible cases where the entanglement condition $d\tau = 0$ occurs:

(1) $ds = 0$ and $d\sigma = 0$,

(2) $ds^2 = -d\sigma^2 \neq 0$, so $d\sigma$ becomes imaginary,

we will refer to them as the *1st kind and 2nd kind entanglement auxiliary conditions*.

Let us get back to the Synge equation and the Synge-Weber equation.

According to Zelmanov's theory of physical observable quantities [31, 32], if an observer accompanies his references

*A three-dimensional section of the four-dimensional space-time, placed in a given point in the time line. In the space-time there are infinitely many spatial sections, one of which is our three-dimensional space.

†Such linked events in the particles A and B can be radiation of a signal in one and its absorbtion in the other, for instance.

the projection of a general covariant quantity on the observer's spatial section is its spatial observable projection.

Following this way, Borissova has deduced (see eqs. 7.16–7.28 in [33]) that the spatial observable projection of the Synge equation is*

$$\frac{d^2\eta^i}{d\tau^2} + 2\big(D^i_k + A^{\cdot i}_{k\cdot}\big)\frac{d\eta^k}{d\tau} = 0\,, \qquad (10)$$

she called it the *Synge equation in chronometrically invariant form*. The Weber equation is different in its right part containing the non-gravitational force that connects the particles (of course, the force should be filled in the spatially projected form). For this reason, conclusions obtained for the Synge equation will be the same that for the Weber one.

In order to make the results of General Relativity applicable to practice, we should consider tensor quantities and equations designed in chronometrically invariant form, because in such way they contain only chronometrically invariant quantities — physical quantities and geometrical properties of space, measurable in real experiment [31, 32].

Let us look at our problem under consideration from this viewpoint.

As it easy to see, the Synge equation in its chronometrically invariant form (10) under the entanglement condition $d\tau = 0$ becomes nonsense. The Weber equation becomes nonsense as well. So, the classical problem statement becomes senseless as soon as particles reach entangled states.

At the same time, in the recent theoretical research [29] two authors of the paper (Borissova and Rabounski, 2005) have found two groups of physical conditions under which particles can be teleported in non-quantum way. They have been called the *teleportation conditions*:

(1) $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$, the conditions of photon teleportation;

(2) $d\tau = 0$ $\{ds^2 = -d\sigma^2 \neq 0\}$, the conditions of substantial (mass-bearing) particles teleportation.

There also were theoretically deduced physical conditions†, which should be reached in a laboratory in order to teleport particles in the non-quantum way [29].

As it is easy to see the non-quantum teleportation condition is identical to introduce here the entanglement main condition $d\tau = 0$ in couple with the 1st kind and 2nd kind auxiliary entanglement conditions!

*In this formula, according to Zelmanov's mathematical apparatus of physical observable quantities [31, 32], $D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t} = \frac{1}{2\sqrt{g_{00}}}\frac{\partial h_{ik}}{\partial t}$ is the three-dimensional symmetric tensor of the space deformation observable rate while $A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right)$ is the three-dimensional antisymmetric tensor of the space rotation observable angular velocities, which indices can be lifted/lowered by the metric observable tensor so that $D^i_k = h^{im} D_{km}$ and $A^{\cdot i}_{k\cdot} = h^{im} A_{km}$. See brief account of the Zelmanov mathematical apparatus in also [30, 33, 34, 35].

†A specific correlation between the gravitational potential w, the space rotation linear velocity v_i and the teleported particle's velocity u^i.

Taking this one into account, we transform the classical Synge and Weber problem statement into another. In our statement the world-line of a particle, being entangled to itself by definition, splits into two different world-lines under teleportation conditions. In other word, as soon as the teleportation conditions occur in a research laboratory, the world-line of a teleported particle breaks in one world-point A and immediately starts in the other world-point B (fig. 2). Both particles A and B, being actually two different states of the same teleported particle at a remote distance one from the other, are in *entangled states*. So, in this statement, the particles A and B themselves are *entangled*.

Of course, this entanglement exists in only the moment of the teleportation when the particle exists in two different states simultaneously. As soon as the teleportation process has been finished, only one particle of them remains so the entanglement disappears.

It should be noted, it follows from the entanglement conditions, that only substantial particles can reach entangled states in the basic space-time of General Relativity — the four-dimensional pseudo-Riemannian space. Not photons. Here is why.

As it is known, the interval $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$ can not be fully degenerated in a Riemannian space‡: the condition is that the determinant of the metric fundamental tensor $g_{\alpha\beta}$ must be strictly negative $g = \det\|g_{\alpha\beta}\| < 0$ by definition of Riemannian spaces. In other word, in the basic space-time of General Relativity the fundamental metric tensor must be strictly non-degenerated as $g < 0$.

The observable three-dimensional (spatial) interval $d\sigma^2 = h_{ik}\,dx^i dx^k$ is positive determined [31, 32], proceeding from physical sense. It fully degenerates $d\sigma^2 = 0$ if only the space compresses into point (the senseless case) or the determinant of the metric observable tensor becomes zero $h = \det\|h_{ik}\| = 0$.

As it was shown by Zelmanov [31, 32], in real observations where an observer accompanies his references, the determinant of the metric observable tensor is connected with the determinant of the fundamental one by the relationship $h = -\frac{g}{g_{00}}$. From here we see, if the three-dimensional observable metric fully degenerates $h = 0$, the four-dimensional metric degenerates as well $g = 0$.

We have obtained that states of two substantial particles can be entangled, if $d\tau = 0$ $\{ds^2 = -d\sigma^2 \neq 0\}$ in the space neighbourhood. So $h > 0$ and $g < 0$ in the neighbourhood, hence the four-dimensional pseudo-Riemannian space is not degenerated.

Conclusion Substantial particles can reach entangled states in the basic space-time of General Relativity (the four-dimensional pseudo-Riemannian space) under specific conditions in the neighbourhood.

‡It can only be partially degenerated. For instance, a four-dimensional Riemannian space can be degenerated into a three-dimensional one.

Although $ds^2 = -d\sigma^2$ in the neighbourhood ($d\sigma$ should be imaginary), the substantial particles remain in regular sub-light area, they do not become super-light tachyons. It is easy to see, from the definition of physical observable time (8), the entanglement condition $d\tau = 0$ occurs only if the specific relationship holds

$$\mathrm{w} + v_i u^i = c^2 \tag{11}$$

between the gravitational potential w, the space rotation linear velocity v_i and the particles' true velocity $u^i = dx^i/dt$ in the observer's laboratory. For this reason, in the neighbourhood the space-time metric is

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k, \tag{12}$$

so the substantial particles can become entangled if the space initial signature $(+---)$ becomes inverted $(-+++)$ in the neighbourhood, while the particles' velocities u^i remain no faster than light.

Another case — massless particles (photons). States of two phonos can be entangled, only if there is in the space neighbourhood $d\tau = 0$ $\{ds = 0\,,\, d\sigma = 0\}$. In this case the determinant of the metric observable tensor becomes $h = 0$, so the space-time metric as well degenerates $g = -g_{00}\, h = 0$. This is not the four-dimensional pseudo-Riemannian space.

Where is that area? In the previous works (Borissova and Rabounski, 2001 [30, 29]) a generalization to the basic space-time of General Relativity was introduced — the four-dimensional space which, having General Relativity's sign-alternating label $(+---)$, permits the space-time metric to be fully degenerated so that there is $g \leqslant 0$.

As it was shown in those works, as soon as the specific condition $\mathrm{w} + v_i u^i = c^2$ occurs, the space-time metric becomes fully degenerated: there are $ds = 0$, $d\sigma = 0$, $d\tau = 0$ (it can be easy derived from the above definition for the quantities) and, hence $h = 0$ and $g = 0$. Therefore, in a space-time where the *degeneration condition* $\mathrm{w} + v_i u^i = c^2$ is permitted the determinant of the fundamental metric tensor is $g \leqslant 0$. This case includes both Riemannian geometry case $g < 0$ and non-Riemannian, fully degenerated one $g = 0$. For this reason a such space is one of Smarandache geometry spaces [22–28], because its geometry is partially Riemannian, partially not*. In the such generalized space-time the 1st kind entanglement conditions $d\tau = 0$ $\{ds = 0\,,\, d\sigma = 0\}$ (the entanglement conditions for photons) are permitted in that area where the space metric fully degenerates (there $h = 0$ and, hence $g = 0$).

Conclusion Massless particles (photons) can reach entangled states, only if the basic space-time fully degenerates $g = \det \| g_{\alpha\beta} \| = 0$ in the neighbourhood. It is permitted in the generalized four-dimensional space-time which metric can be fully degenerated $g \leqslant 0$ in that area where the degeneration conditions occur. The generalized space-time is attributed to Smarandache geometry spaces, because its geometry is partially Riemannian, partially not.

So, entangled states have been introduced into General Relativity for both substantial particles and photons.

3 Quantum Causality Threshold in General Relativity

This term was introduced by one of the authors two years ago (Smarandache, 2003) in our common correspondence [36] on the theme:

Definition Considering two particles A and B located in the same spatial section, *Quantum Causality Threshold* was introduced as a special state in which neither A nor B can be the cause of events located "over" the spatial section on the Minkowski diagram.

The term *Quantum* has been added to the *Causality Threshold*, because in this problem statement an interaction is considered between two infinitely far away particles (in infinitesimal vicinities of each particle) so this statement is applicable to only quantum scale interactions that occur in the scale of elementary particles.

Now, we are going to find physical conditions under which particles can reach the threshold in the space-time of General Relativity.

Because in this problem statement we look at causal relations in General Relativity's space-time from "outside", it is required to use an "outer viewpoint" — a point of view located outside the space-time.

We introduce a such point of outlook in an Euclidean flat space, which is tangential to our's in that world-point, where the observer is located. In this problem statement we have a possibility to compare the absolute cause relations in that tangential flat space with those in ours. As a matter, a tangential Euclidean flat space can be introduced at any point of the pseudo-Riemannian space.

At the same time, according to Zelmanov [31, 32], within infinitesimal vicinities of any point located in the pseudo-Riemannian space a *locally geodesic reference frame* can be introduced. In a such reference frame, within infinitesimal vicinities of the point, components of the metric fundamental tensor (marked by tilde)

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\alpha\beta}}{\partial \tilde{x}^\mu \partial \tilde{x}^\nu}\right)(\tilde{x}^\mu - x^\mu)(\tilde{x}^\nu - x^\nu) + \ldots \tag{13}$$

*In foundations of geometry it is known the *S-denying* of an axiom [22–25], i. e. in the same space an "axiom is false in at least two different ways, or is false and also true. Such axiom is said to be Smarandachely denied, or S-denied for short" [26]. As a result, it is possible to introduce geometries, which have common points bearing mixed properties of Euclidean, Lobachevsky-Bolyai-Gauss, and Riemann geometry in the same time. Such geometries has been called paradoxist geometries or Smarandache geometries. For instance, Iseri in his book *Smarandache Manifolds* [26] and articles [27, 28] introduced manifolds that support particular cases of such geometries.

are different from those $g_{\alpha\beta}$ at the point of reflection to within only the higher order terms, which can be neglected. So, in a locally geodesic reference frame the fundamental metric tensor can be accepted constant, while its first derivatives (Christoffel's symbols) are zeroes. The fundamental metric tensor of an Euclidean space is as well a constant, so values of $\tilde{g}_{\mu\nu}$, taken in the vicinities of a point of the pseudo-Riemannian space, converge to values of $g_{\mu\nu}$ in the flat space tangential at this point. Actually, we have a system of the flat space's basic vectors $\vec{e}_{(\alpha)}$ tangential to curved coordinate lines of the pseudo-Riemannian space. Coordinate lines in Riemannian spaces are curved, inhomogeneous, and are not orthogonal to each other (the latest is true if the space rotates). Therefore the lengths of the basic vectors may be very different from the unit.

Writing the world-vector of an infinitesimal displacement as $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$, we obtain $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where the components of the basic vectors $\vec{e}_{(\alpha)}$ tangential to the coordinate lines are $\vec{e}_{(0)} = \{e^0_{(0)}, 0, 0, 0\}$, $\vec{e}_{(1)} = \{0, e^1_{(1)}, 0, 0\}$, $\vec{e}_{(2)} = \{0, 0, e^2_{(2)}, 0\}$, $\vec{e}_{(3)} = \{0, 0, 0, e^2_{(3)}\}$. Scalar product of $d\vec{r}$ with itself is $d\vec{r}d\vec{r} = ds^2$ or, in another $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, so $g_{\alpha\beta} = \vec{e}_{(\alpha)} \vec{e}_{(\beta)} = e_{(\alpha)} e_{(\beta)} \cos\left(x^\alpha; x^\beta\right)$. We obtain

$$g_{00} = e^2_{(0)}\,, \qquad g_{0i} = e_{(0)} e_{(i)} \cos\left(x^0; x^i\right), \tag{14}$$

$$g_{ik} = e_{(i)} e_{(k)} \cos\left(x^i; x^k\right), \qquad i, k = 1, 2, 3\,. \tag{15}$$

Then, substituting g_{00} and g_{0i} from formulas that determine the gravitational potential $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ and the space rotation linear velocity $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$, we obtain

$$v_i = -c\, e_{(i)} \cos\left(x^0; x^i\right), \tag{16}$$

$$h_{ik} = e_{(i)} e_{(k)} \Big[\cos(x^0; x^i) \cos(x^0; x^k) - \cos(x^i; x^k)\Big]. \tag{17}$$

From here we see: if the pseudo-Riemannian space is free of rotation, $\cos\left(x^0; x^i\right) = 0$ so the observer's spatial section is strictly orthogonal to time lines. As soon as the space starts to do rotation, the cosine becomes different from zero so the spatial section becomes non-orthogonal to time lines (fig. 3). Having this process, the light hypercone inclines with the time line to the spatial section. In this inclination the light hypercone does not remain unchanged, it "compresses" because of hyperbolic transformations in pseudo-Riemannian space. The more the light hypercone inclines, the more it symmetrically "compresses" because the space-time's geometrical structure changes according to the inclination.

In the ultimate case, where the cosine reach the ultimate value $\cos\left(x^0; x^i\right) = 1$, time lines coincide the spatial section: time "has fallen" into the three-dimensional space. Of course, in this case the light hypercone overflows time lines and the spatial section: the light hypercone "has as well fallen" into the three-dimensional space.

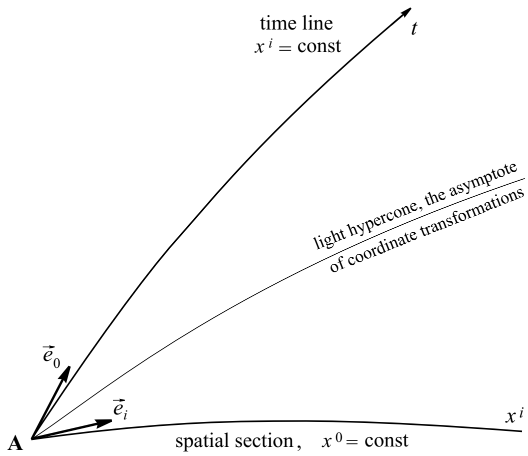

Fig. 3

As it is easy to see from formula (16), this ultimate case occurs as soon as the space rotation velocity v_i reaches the light velocity. If particles A and B are located in the space filled into this ultimate state, neither A nor B can be the cause of events located "over" the spatial section in the Minkowski diagrams we use in the pictures. So, in this ultimate case the space-time is filled into a special state called Quantum Causality Threshold.

Conclusion Particles, located in General Relativity's space-time, reach Quantum Causality Threshold as soon as the space rotation reaches the light velocity. Quantum Causality Threshold is impossible if the space does not rotate (holonomic space), or if it rotates at a sub-light speed.

So, Quantum Causality Threshold has been introduced into General Relativity.

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