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Ultracold Fermi and Bose Gases and Spinless Bose Charged Sound Particles

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We propose a novel approach for investigation of the motion of Bose or Fermi liquid (or gas) which consists of decoupled electrons and ions in the uppermost hyperfine state. Hence, we use such a concept as the fluctuation motion of "charged fluid particles" or "charged fluid points" representing a charged longitudinal elastic wave. In turn, this elastic wave is quantized by spinless longitudinal Bose *charged sound particles* with the rest mass m and charge e_0. The existence of spinless Bose *charged sound particles* allows us to present a new model for description of Bose or Fermi liquid via a non-ideal Bose gas of *charged sound particles*. In this respect, we introduce a new postulation for the superfluid component of Bose or Fermi liquid determined by means of *charged sound particles* in the condensate, which may explain the results of experiments connected with ultra-cold Fermi gases of spin-polarized hydrogen, ^{6}Li and ^{40}K, and such a Bose gas as ^{87}Rb in the uppermost hyperfine state, where the Bose-Einstein condensation of *charged sound particles* is realized by tuning the magnetic field.

1 Introduction

The Bose-Einstein condensation (BEC) has a wide application for investigation of superconductivity of metals and superfluidity of liquids. The primary experimental challenge to evaporative cooling of spin-polarized hydrogen was made by a dilution refrigerator, demonstrating that spin-polarized hydrogen can be confined in a statistic magnetic trap and thermally decoupled from the walls [1–3]. At the density $\frac{N}{V} \approx 10^{13}$ cm^{-3} it is observed that the gas consisting of decoupled electrons and ions in the uppermost hyperfine state is evaporatively cooled to a temperature approximately equal to 40 mK.

Here, we remark about BEC that was produced in a vapor of ^{87}Rb bosonic ions confined by magnetic fields and evaporatively cooled [4]. The condensate fraction first appeared near a temperature of 170 nanokelvin at the density $\frac{N}{V} = 2.6 \times 10^{12}$ cm^{-3}. The experiment has shown that the value of temperature 170 nK is reduced to 20 nK. In reality, the strongly interacting spin-$\frac{1}{2}$ ^{6}Li and ^{40}K fermionic gases were realized via tuning the magnetic field [5]. These experimental achievements in the field of ultra-cold Fermi gases are based mainly on the possibility of tuning the scattering length a which becomes much larger in magnitude than the mean interatomic distance by changing the external magnetic field. In this respect, the concept of Fermi surface loses its meaning due to the broadening produced by pairing of fermions, the so-called Feshbach resonances in ultracold atomic Fermi gases. However, in this letter we predict a new method of liquid cooling which is based on the formation of oscillators at every point of liquid by tuning the magnetic field, which in turn leads to vibration of "charged fluid particles". These "charged fluid particles" reproduce charged spinless quasiparticles which determine the superfluidity component of Bose or Fermi liquid by action of the static magnetic field.

In order to investigate the motion of quantum liquid (or quantum gas) in the uppermost hyperfine state, we consider the motion of "charged fluid particles" by means of a charged longitudinal elastic wave [6]. This longitudinal elastic wave is quantized by spinless Bose *charged sound particles* with the mass m and charge e_0. Further, we present a new model for description of charged Bose or Fermi liquid via a non-ideal Bose gas consisting of *charged sound particles*. As opposed to London's postulation about the superfluid component of liquid ^{4}He [7], we introduce a new postulation about the superfluid component of Bose or Fermi liquid via *charged sound particles* in the condensate. On the other hand, we estimate the zero sound speed which leads to the correct explanation of the experimental result connected with the BEC of a gas consisting of spin-polarized hydrogen.

2 Quantization of quantum liquid or quantum gas in the uppermost hyperfine state

Now let us analyze quantization of quantum liquid (or quantum gas) in the uppermost hyperfine state. This quantum liquid (or quantum gas) consists of N Bose or Fermi positive charged ions with the charge e and mass M confined in the volume V where they are in a negative electron background since the entire system of liquid is electro-neutral. Considering quantum liquid as a continuous medium, we investigate the fluctuation motion of the number n of "charged fluid particles" on the basis of hydrodynamics (where a "charged fluid particle" is defined as a very small volume V_0 in regard to the volume V of the liquid ($V_0 \ll V$) with the mass m and charge e_0. The volume V_0 contains the number $N' = \frac{N}{n}$ of liquid ions, therefore the charge e_0 is expressed via the ion charge as $e_0 = \frac{eN}{n}$.

In accordance with the laws of hydrodynamics [6], the mass density ρ and pressure p of liquid are presented as

$$\rho = \rho_0 + \rho^{'}$$

and

$$p = p_0 + p^{'},$$

where $\rho_0 = \frac{MN}{V}$ and p_0 are, respectively, the equilibrium mass density and pressure; $\rho^{'}$ and $p^{'}$ are the relative fluctuations of the mass density and pressure.

As is known, the continuity equation has the form:

$$\frac{\partial \rho^{'}}{\partial t} = -\rho_0 \operatorname{div} \vec{v}, \tag{1}$$

which may present as:

$$\rho^{'} = -\rho_0 \operatorname{div} \vec{u}, \tag{2}$$

where $\vec{v} = \frac{\partial \vec{u}}{\partial t}$ is the speed of a charged fluid particle; $\vec{u} = \vec{u}(\vec{r}, t)$ is the displacement vector of a charged fluid particle which describes a charged longitudinal sound wave.

On the other hand, Euler's equation in the first-order-of-smallness approximation takes the reduced form:

$$\frac{\partial \vec{v}}{\partial t} + \frac{\nabla p^{'}}{\rho_0} = 0. \tag{3}$$

Hence, we consider the fluctuation motion of charged fluid particles as adiabatic, deriving the following equation:

$$p^{'} = \left(\frac{\partial p}{\partial \rho_0}\right)_S \rho^{'} = c_l^2 \rho^{'}, \tag{4}$$

where S is the entropy; $c_l = \sqrt{\left(\frac{\partial p}{\partial \rho_0}\right)_S}$ is the speed of the charged longitudinal elastic wave.

As is known, the fluctuation motion of charged fluid particles represents as a potential one:

$$\operatorname{curl} \vec{v} = \operatorname{curl} \frac{\partial \vec{u}}{\partial t} = 0. \tag{5}$$

Thus, by using the above equation we may get to the wave equation for the vector of displacement $\vec{u} = \vec{u}(\vec{r}, t)$:

$$\nabla^2 \vec{u}(\vec{r}, t) - \frac{1}{c_l^2} \frac{\partial^2 \vec{u}(\vec{r}, t)}{\partial t^2} = 0, \tag{6}$$

which in turn describes the longitudinal charged sound wave.

Now, we state that the longitudinal elastic wave consists of spinless Bose *charged sound particles* with the non-zero rest mass m. Then, the displacement vector $u(\vec{r}, t)$ is expressed via a secondary quantization vector of the wave function of spinless Bose *charged sound particles* directed along the wave vector $\vec{k}$:

$$\vec{u}(\vec{r}, t) = u_l \left(\vec{\phi}(\vec{r}, t) + \vec{\phi}^{+}(\vec{r}, t)\right), \tag{7}$$

where u_l is the normalization constant which is the amplitude of oscillations; $\vec{\phi}(\vec{r}, t)$ is the secondary quantization of vector wave functions for creation and annihilation of one longitudinal *charged sound particle* with the mass m whose direction $\vec{l}$ is directed towards the wave vector $\vec{k}$:

$$\vec{\phi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} \tag{8}$$

$$\vec{\phi}^{+}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}^{+}_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)} \tag{9}$$

with the condition

$$\int \vec{\phi}^{+}(\vec{r}, t)\, \vec{\phi}(\vec{r}, t)\, dV = n_0 + \sum_{\vec{k} \neq 0} \hat{a}^{+}_{\vec{k}}\, \hat{a}_{\vec{k}} = \hat{n}, \tag{10}$$

where $\vec{a}^{+}_{\vec{k}}$ and $\vec{a}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation for a free *charged sound particle* with the energy $\frac{\hbar^2 k^2}{2m}$, described by the vector $\vec{k}$ whose direction coincides with the direction $\vec{l}$ of a traveling charged longitudinal elastic wave; $\hat{n}$ is the operator of the total number of *charged sound particles*; $\hat{n}_0$ is the total number of *charged sound particles* at the condensate level with the wave vector $\vec{k} = 0$.

Thus, as is seen, the displacement vector $\vec{u}(\vec{r}, t)$ satisfies wave-equation (6) and in turn takes the form:

$$\vec{u}(\vec{r}, t) = \vec{u_0} + \frac{u_l}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} + \vec{a}^{+}_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)}\right). \tag{11}$$

While investigating a superfluid liquid, Bogoliubov [8] separated the atoms of helium in the condensate from those atoms filling the states above the condensate. In an analogous manner, we may consider the vector operator $\vec{a}_0 = \vec{l}\sqrt{n_0}$ and $\vec{a}^{+}_0 = \vec{l}\sqrt{n_0}$ as c-numbers (where $\vec{l}$ is the unit vector in the direction of propagation of the sound wave) within the approximation of a macroscopic number of *sound particles* in the condensate $n_0 \gg 1$. These assumptions lead to a broken Bose-symmetry law for *sound particles* in the condensate. To extend the concept of a broken Bose-symmetry law for *sound particles* in the condensate, we apply the definition of BEC of *sound particles* in the condensate as was postulated by the Penrose-Onsager for the definition of BEC of helium atoms [9]:

$$\lim_{n_0, n \to \infty} \frac{n_0}{n} = const. \tag{12}$$

On the other hand, we may observe that presence of *charged sound particles* filling the condensate level with the wave vector $\vec{k} = 0$ leads to the appearance of the constant displacement $\vec{u}_0 = \frac{2u_l \vec{l}\sqrt{n_0}}{\sqrt{V}}$ of *charged sound particles*.

To find the normalization constant u_l, we introduce the following condition which allows us to suggest that at absolute zero all *sound particles* fill the condensate level $\vec{k} = 0$.

This reasoning implies that at $n_0 = n$ the constant displacement takes the maximal value $2d = \sqrt{|\vec{u}_0|^2}$ which represents the maximal distance between two neighboring *charged sound particles*. On the other hand, this distance is determined by the formula $d = \left(\frac{3V}{4\pi n}\right)^{\frac{1}{3}}$, which is in turn substituted into the expression $2d = \sqrt{|\vec{u}_0|^2}$. Then, consequently, we get to the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$.

The condition for conservation of density at each point of liquid stipulates that

$$\rho_0 = \frac{MN}{V} = \frac{mn}{V}, \tag{13}$$

which represents a connection of the mass and density of the *charged sound particles* with the mass and density of the ions. Thus, we argue that liquid (or gas) can be described by the model of an ideal gas of *n charged sound particles* with the mass m and charge e_0 in the volume V. Hence, we remark that the Coulomb scattering between *charged sound particles* is neglected in the considered theory.

3 "Charged fluid particles" in trapped static magnetic field

Now, we consider the Hamiltonian operator $\hat{H}_l$ of liquid [6] in a trapped static magnetic field [10]:

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{1}{2}\int\left(\frac{c_l\rho^{'}}{\sqrt{\rho_0}}\right)^2 dV + \\ + \frac{\rho_0}{2}\int(\Omega\vec{u}_l)^2\, dV, \tag{14}$$

where $\Omega = \frac{e_0 H}{mc}$ is the trapping frequency of a "charged fluid particle"; e_0 is the charge of a "fluid particle"; H is the absolute value of the magnetic strain; c is the velocity of light in vacuum. Hence, we note that the charge of a fluid particle equals $e_0 = eN^{'} = \frac{Ne}{n}$, where $N^{'}$ is the number of ions in a small volume V_0 of one charged fluid particle.

Substituting $\rho^{'}$ from (2) into (14), we obtain

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{\rho_0}{2}\int(c_l\,\mathrm{div}\,\vec{u})^2\, dV + \\ + \frac{\rho_0}{2}\int(\Omega\vec{u}_l)^2\, dV. \tag{15}$$

Using Dirac's approach in [11] for quantization of the electromagnetic field, we have:

$$\frac{\partial\vec{u}(\vec{r},t)}{\partial t} = -\frac{ic_l\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}} k\left(\vec{a}_{\vec{k}}\, e^{-ikc_l t} - \vec{a}^{+}_{-\vec{k}}\, e^{ikc_l t}\right)e^{i\vec{k}\vec{r}}, \tag{16}$$

as well as

$$\mathrm{div}\,\vec{u}(\vec{r},t) = \frac{i\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}}\vec{k}\left(\vec{a}_{\vec{k}}\, e^{-ikc_l t} + \vec{a}^{+}_{-\vec{k}}\, e^{ikc_l t}\right)e^{i\vec{k}\vec{r}}. \tag{17}$$

Now, introducing (16) and (17) into (15) and using

$$\frac{1}{V}\int e^{i(\vec{k}_1+\vec{k}_2)\vec{r}} = \delta^3_{\vec{k}_1+\vec{k}_2},$$

we obtain the terms in the right side of the Hamiltonian of the system presented in (15):

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = -\frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k}} - \vec{a}^{+}_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} - \vec{a}^{+}_{\vec{k}}\right),$$

$$\frac{\rho_0}{2}\int(\mathrm{div}\,\vec{u})^2\, dV = \frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k}} + \vec{a}^{+}_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} + \vec{a}^{+}_{\vec{k}}\right)$$

and

$$\frac{\rho_0}{2}\int(\Omega\vec{u}_l)^2\, dV = \frac{\rho_0\Omega^2 u_l^2}{2}\sum_{\vec{k}}\left(\vec{a}_{\vec{k}} + \vec{a}^{+}_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} + \vec{a}^{+}_{\vec{k}}\right).$$

These expressions determine the reduced form of the Hamiltonian operator $\hat{H}_l$ by the form:

$$\hat{H}_l = \sum_{\vec{k}}\left(2\rho_0 u_l^2 c_l^2 k^2 + \rho_0\Omega^2 u_l^2\right)\vec{a}^{+}_{\vec{k}}a_{\vec{k}} + \\ + \frac{\rho_0\Omega^2 u_l^2}{2}\sum_{\vec{k}}\left(\vec{a}^{+}_{-\vec{k}}\vec{a}^{+}_{\vec{k}} + \vec{a}_{\vec{k}}\vec{a}_{-\vec{k}}\right), \tag{18}$$

where u_l^2 is defined by the first term in the right side of (18) which represents the kinetic energy of a *charged sound particle* $\frac{\hbar^2k^2}{2m}$, if we suggest:

$$2\rho_0 u_l^2 c_l^2 k^2 = \frac{\hbar^2 k^2}{2m}. \tag{19}$$

Then,

$$u_l^2 = \frac{\hbar^2}{4c_l^2 m\rho_0},$$

which allows one to determine the mass m of a *charged sound particle* using the value of the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$ and (13):

$$m = \frac{\hbar}{c_l}\left(\frac{n}{V}\right)^{\frac{1}{3}}. \tag{20}$$

Thus, the main part of the Hamiltonian operator $\hat{H}_l$ takes the form:

$$\hat{H}_l = \sum_{\vec{k}\neq 0}\left(\frac{\hbar^2k^2}{2m} + mv^2\right)\vec{a}^{+}_{\vec{k}}\,a_{\vec{k}} + \frac{mv^2}{2}\sum_{\vec{k}\neq 0}\left(\vec{a}^{+}_{-\vec{k}}\,\vec{a}^{+}_{\vec{k}} + \vec{a}_{\vec{k}}\,\vec{a}_{-\vec{k}}\right), \tag{21}$$

where we denote $v = \frac{\hbar\Omega}{\sqrt{2}mc_l}$, which in turn is the speed of charged sound in a Bose or Fermi liquid excited by static magnetic field; n_0 is the number of *charged sound particles* in the condensate.

For the evolution of the energy level, it is necessary to diagonalize the Hamiltonian $\hat{H}_l$, which can be accomplished by introducing the vector Bose-operators $\vec{b}^+_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ [12]:

$$\vec{a}_{\vec{k}} = \frac{\vec{b}_{\vec{k}} + L_{\vec{k}}\vec{b}^+_{-\vec{k}}}{\sqrt{1 - L^2_{\vec{k}}}}, \tag{22}$$

where $L_{\vec{k}}$ is the unknown real symmetrical function of the wave vector $\vec{k}$.

By substituting (22) into (21), we obtain

$$\hat{H}_l = \sum_{\vec{k}\neq 0} \varepsilon_{\vec{k}} \vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}, \tag{23}$$

where $\vec{b}^+_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ are the creation and annihilation operators of charged Bose quasiparticles with the energy:

$$\varepsilon_{\vec{k}} = \left[\left(\frac{\hbar^2 k^2}{2m}\right)^2 + \hbar^2 k^2 v^2\right]^{1/2}. \tag{24}$$

In this context, the real symmetrical function $L_{\vec{k}}$ of the wave vector $\vec{k}$ is found to be

$$L^2_{\vec{k}} = \frac{\frac{\hbar^2 k^2}{2m} + mv^2 - \varepsilon_{\vec{k}}}{\frac{\hbar^2 k^2}{2m} + mv^2 + \varepsilon_{\vec{k}}}. \tag{25}$$

Thus, the average energy of the system takes the form:

$$\overline{\hat{H}_l} = \sum_{\vec{k}\neq 0} \varepsilon_{\vec{k}} \overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}}, \tag{26}$$

where $\overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}}$ is the average number of charged Bose quasiparticles with the wave vector $\vec{k}$ at the temperature T:

$$\overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}} = \frac{1}{e^{\frac{\varepsilon_{\vec{k}}}{kT}} - 1}. \tag{27}$$

Thus, we have found the spectrum of free charged spinless quasiparticles excited in a Bose or Fermi liquid which is similar to Bogoliubov's one [8]. In fact, the Hamiltonian of system (24) describes an ideal Bose gas consisting of charged spinless phonons at a small wave number $k \ll \frac{2mv}{\hbar}$ but at $k \gg \frac{2mv}{\hbar}$ the Hamiltonian operator describes an ideal gas of *charged sound particles*. This reasoning implies that the tuning magnetic field forms the superfluidity component of a Bose or Fermi liquid which is been in the uppermost hyperfine state.

4 BEC of *charged sound particles*

As opposed to London's postulation concerning BEC of atoms [7], we state that *charged sound particles* in the condensate define the superfluid component of Bose and Fermi liquids. Consequently, statistical equilibrium equation (10) takes the following form:

$$n_{0,T} + \sum_{\vec{k}\neq 0} \overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}} = n, \tag{28}$$

where $\overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}}$ is the average number of *charged sound particles* with the wave vector $\vec{k}$ at the temperature T.

To find the form $\overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}}$, we use the linear transformation presented in (22):

$$\overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}} = \frac{1 + L^2_{\vec{p}}}{1 - L^2_{\vec{p}}} \overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}} + \frac{L_{\vec{k}}}{1 - L^2_{\vec{k}}} \left(\overline{\vec{b}^+_{\vec{k}} \vec{b}^+_{-\vec{k}}} + \overline{\vec{b}_{\vec{k}} \vec{b}_{-\vec{k}}}\right) + \frac{L^2_{\vec{k}}}{1 - L^2_{\vec{k}}}.$$

According to the Bloch-De-Dominicis theorem, we have

$$\overline{\vec{b}^+_{\vec{k}} \vec{b}^+_{-\vec{k}}} = \overline{\vec{b}_{\vec{k}} \vec{b}_{-\vec{k}}} = 0.$$

In this respect, the equation for the density of *charged sound particles* in the condensate takes the following form:

$$\frac{n_{0,T}}{V} = \frac{n}{V} - \frac{1}{V} \sum_{\vec{k}\neq 0} \frac{L^2_{\vec{k}}}{1 - L^2_{\vec{k}}} - \frac{1}{V} \sum_{\vec{k}\neq 0} \frac{1 + L^2_{\vec{k}}}{1 - L^2_{\vec{k}}} \overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}}. \tag{29}$$

Obviously, at the lambda transition $T = T_\lambda$ the density of *charged sound particles* $\frac{n_{0,T_\lambda}}{V} = 0$. Hence, we note that the mass m and density $\frac{n}{V}$ of *charged sound particles* are expressed via the mass of ions M and density of ions $\frac{N}{V}$ when solving a system of two equations presented in (13) and (20):

$$\frac{n}{V} = \left(\frac{Mc_l}{\hbar} \frac{N}{V}\right)^{\frac{3}{4}} \tag{30}$$

and

$$m = \left(\frac{\hbar}{c_l}\right)^{\frac{3}{4}} \left(\frac{MN}{V}\right)^{\frac{1}{4}}. \tag{31}$$

In conclusion, it should be noted that the given approach opens up a new direction for investigation of BEC of *charged sound particles* in Fermi gases of spin-polarized hydrogen, ^{6}Li and ^{40}K, and in a Bose gas such as ^{87}Rb, because the model of quantum liquid in the uppermost hyperfine state is considered in the same way as superfluid liquid helium. In this letter, we argue for the first time that the superfluid component of Bose or Fermi liquid in the uppermost hyperfine state is determined by means of *charged sound particles* in the condensate. In fact, we argue that the lambda transition point depends on the strain of static magnetic field due to equation (29) and condition for the density of *charged sound particles* $\frac{n_{0,T_\lambda}}{V} = 0$.

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Superfluidity Component of Solid ^{4}He and *Sound Particles* with Spin 1

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We present a new model for solid which is based on such a concept as the fluctuation motion of "solid particles" or "solid points". The fluctuation motion of "solid particles" in solid ^{4}He represents a longitudinal elastic wave which is in turn quantized by neutral longitudinal Bose *sound particles* with spin 1 with the rest mass m. Thus, first we remove a concept of "lattice" for solid by presentation of new model of one as a vibration of *sound particles* by natural frequency Ω_l. In this respect, we first postulate that the superfluid component of a solid ^{4}He is determined by means of *sound particles* with spin 1 in the condensate.

1 Introduction

The quantum solid is remarkable object which reveal macroscopic quantum phenomena, such as superfluidity and Bose-Einstein condensation (BEC) of solid ^{4}He [1] which were reported by many authors [2, 3].

The original theory proposed by Einstein in 1907 was of great historical relevance [4]. In the Einstein model, each atom oscillates relatively to its neighbors in the lattice which execute harmonic motions around fixed positions, the knots of the lattice. He treated the thermal property of the vibration of a lattice of N atoms as a $3N$ harmonic independent oscillator by identical own frequency Ω_0 which was quantized by application of the prescription developed by Plank in connection with the theory of Black Body radiation. The Einstein model could obtain the Dulong and Petit prediction at high temperature but could not reproduce an adequate representation of the the lattice at low temperatures. In 1912, Debye proposed to consider the model of the solid [5], by suggestion that the frequencies of the $3N$ harmonic independent oscillators are not equal as it was suggested by the Einstein model. In addition to his suggestion, the acoustic spectrum of solid may be treated as if the solid represented a homogeneous medium, except that the total number of independent elastic waves is cut off at $3N$, to agree with the number of degrees of freedom of N atoms. In this respect, Debye stated that one longitudinal and two transverse waves are excited in solid. These velocities of sound cannot be observed in a solid at frequencies above the cut-off frequency. Also, he suggested that phonon is a spinless. Thus, the Debye model correctly showed that the heat capacity is proportional to the T^3 law at low temperatures. At high temperatures, he obtained the Dulong-Petit prediction compatible to experimental results.

The other model of solid was presented by the authors of this letter in [6] where the solid was considered as continuum elastic medium consisting of neutral Fermi-atoms, fixed in the knots of lattice. In this case, we predicted that the lattice represents as the Bose-gas of Sound-Particles with finite masses m_l and m_t, corresponding to a longitudinal and a transverse elastic field. On the other hand, the lattice was considered as a new substance of matter consisting of sound particles, which excite the one longitudinal and one transverse elastic waves (this approach is differ from Debye one). These waves act on the Fermi-atoms which are stimulating a vibrations with the natural frequencies Ω_l and Ω_t. In this context, we introduced a new principle of elastic wave-particle duality, which allows us to build the lattice model. The given model leads to the same results as presented by Debye's theory.

However, we consider the model of solid by new way by introducing of such a concept as the fluctuation motion of "solid particles" or "solid points". In this respect, we remove a concept as a lattice of solid or an atoms, fixed in the knots of lattice because we deal with the "solid particle" which exist in any point of the solid. This "solid particle" is a similar to the "fluid particle" on the basis of hydrodynamics [7] (where "fluid particle" is determined as a very small volume V_0, in regard to the volume V of the liquid ($V_0 \ll V$), which consists of a macroscopic number of liquid atoms). The motion of "solid particle" describes the longitudinal elastic wave which in turn represents a Bose gas of neutral *sound particles* with spin 1 with finite mass m. In this letter, we present a new model of solid which describes a vibration of *sound particles* by natural frequency Ω_l. We postulate also that the superfluid component of a solid is determined by means of *sound particles* in the condensate.

2 Analysis

For beginning let us analyze quantization of a quantum liquid (or quantum gas) which consists of N Bose or Fermi atoms with the mass M confined in the volume V. Considering a quantum liquid as a continuum medium, we investigate the fluctuation motion of "fluid particles" on the basis of hydrodynamics (where "fluid particle" is determined as a very small volume V_0, in regard to the volume V of the liquid ($V_0 \ll V$), which consists of a macroscopic number of liquid atoms).

In accordance with the hydrodynamics laws, the mass

density ρ and pressure p for a liquid are presented as

$$\rho = \rho_0 + \rho^{'}$$

and

$$p = p_0 + p^{'},$$

where $\rho_0 = \frac{MN}{V}$ and p_0 are, respectively, the equilibrium mass density and pressure; $\rho^{'}$ and $p^{'}$ are the relative fluctuations of the mass density and pressure.

As is known, the continuity equation has the form:

$$\frac{\partial \rho^{'}}{\partial t} = -\rho_0 \operatorname{div} \vec{v}, \tag{1}$$

which may present as:

$$\rho^{'} = -\rho_0 \operatorname{div} \vec{u}, \tag{2}$$

where $\vec{v} = \frac{\partial \vec{u}}{\partial t}$ is the speed of a fluid particle; $\vec{u} = \vec{u}(\vec{r}, t)$ is the displacement vector of a fluid particle which describes a longitudinal sound wave.

On the other hand, Euler's equation in the first-order-of-smallness approximation takes the reduced form:

$$\frac{\partial \vec{v}}{\partial t} + \frac{\nabla p^{'}}{\rho_0} = 0. \tag{3}$$

Hence, we consider the fluctuation motion of fluid particles as adiabatic, deriving the following equation:

$$p^{'} = \left(\frac{\partial p}{\partial \rho_0}\right)_S \rho^{'} = c_l^2 \rho^{'}, \tag{4}$$

where S is the entropy of liquid; $c_l = \sqrt{\left(\frac{\partial p}{\partial \rho_0}\right)_S}$ is the speed of the longitudinal elastic wave.

As is known, the fluctuation motion of fluid particles represents as a potential one:

$$\operatorname{curl} \vec{v} = \operatorname{curl} \frac{\partial \vec{u}}{\partial t} = 0. \tag{5}$$

Thus, by using the above equation we may get to the wave equation for the vector of displacement $\vec{u} = \vec{u}(\vec{r}, t)$:

$$\nabla^2 \vec{u}(\vec{r}, t) - \frac{1}{c_l^2} \frac{\partial^2 \vec{u}(\vec{r}, t)}{\partial t^2} = 0, \tag{6}$$

which in turn gives a description of the longitudinal sound wave.

Now, we state that the longitudinal elastic wave consists of neutral spinless Bose *sound particles* with the non-zero rest mass m. Then, the displacement vector $u(\vec{r}, t)$ is expressed via a secondary quantization vector of the wave function of spinless Bose *sound particles* directed along the wave vector $\vec{k}$:

$$\vec{u}(\vec{r}, t) = u_l \left(\vec{\phi}(\vec{r}, t) + \vec{\phi}^{+}(\vec{r}, t)\right), \tag{7}$$

where u_l is the normalization constant which is the amplitude of oscillations; $\vec{\phi}(\vec{r}, t)$ is the second quantization vector wave functions for creation and annihilation of one longitudinal *sound particle* with the mass m whose direction $\vec{l}$ is directed towards the wave vector $\vec{k}$:

$$\vec{\phi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} \tag{8}$$

$$\vec{\phi}^{+}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}^{+}_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)} \tag{9}$$

with the condition

$$\int \vec{\phi}^{+}(\vec{r}, t)\, \vec{\phi}(\vec{r}, t)\, dV = n_0 + \sum_{\vec{k} \neq 0} \hat{a}^{+}_{\vec{k}}\, \hat{a}_{\vec{k}} = \hat{n}, \tag{10}$$

where $\vec{a}^{+}_{\vec{k}}$ and $\vec{a}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation for a free *sound particle* with the energy $\frac{\hbar^2 k^2}{2m}$, described by the vector $\vec{k}$ whose direction coincides with the direction $\vec{l}$ of a traveling longitudinal elastic wave; $\hat{n}$ is the operator of the total number of *sound particles*; $\hat{n}_0$ is the total number of *sound particles* at the condensate level with the wave vector $\vec{k} = 0$.

Thus, as is seen, the displacement vector $\vec{u}(\vec{r}, t)$ satisfies wave-equation (6) and in turn takes the form:

$$\vec{u}(\vec{r}, t) = \vec{u_0} + \frac{u_l}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} + \vec{a}^{+}_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)}\right). \tag{11}$$

While investigating a superfluid liquid, Bogoliubov [8] separated the atoms of liquid helium ^{4}He in the condensate from those atoms filling the states above the condensate. In an analogous manner, we may consider the vector operator $\vec{a}_0 = \vec{l}\sqrt{n_0}$ and $\vec{a}^{+}_0 = \vec{l}\sqrt{n_0}$ as c-numbers (where $\vec{l}$ is the unit vector in the direction of propagation of the sound wave) within the approximation of a macroscopic number of *sound particles* in the condensate $n_0 \gg 1$. These assumptions lead to a broken Bose-symmetry law for *sound particles* in the condensate. To extend the concept of a broken Bose-symmetry law for *sound particles* in the condensate, we apply the definition of BEC of *sound particles* in the condensate as was postulated by the Penrose-Onsager for the definition of BEC of helium atoms [9]:

$$\lim_{n_0, n \to \infty} \frac{n_0}{n} = const. \tag{12}$$

On the other hand, we may observe that presence of *sound particles* filling the condensate level with the wave vector $\vec{k} = 0$ leads to the appearance of the constant displacement $\vec{u}_0 = \frac{2u_l \vec{l}\sqrt{n_0}}{\sqrt{V}}$ of the *sound particles*.

To find the normalization constant u_l, we introduce the following condition which allows us to suggest that at absolute zero all *sound particles* fill the condensate level $\vec{k} = 0$.

This reasoning implies that at $n_0 = n$ the constant displacement takes a maximal value $2d = \sqrt{|\vec{u}_0|^2}$ which represents the maximal distance between two neighboring *sound particles*. On the other hand, this distance is determined by the formula $d = \left(\frac{3V}{4\pi n}\right)^{\frac{1}{3}}$, which is in turn substituted into the expression $2d = \sqrt{|\vec{u}_0|^2}$. Then, consequently, we get to the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$.

The condition of conservation of density at each point of the solid stipulates that

$$\rho_0 = \frac{MN}{V} = \frac{mn}{V}, \tag{13}$$

which represents a connection of the mass m and density ρ_0 of *sound particles* with the mass M and density ρ_0 of the liquid helium atoms with mass M.

Now, we consider the Hamiltonian operator $\hat{H}_l$ of a liquid [8]:

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{1}{2}\int\left(\frac{c_l\rho'}{\sqrt{\rho_0}}\right)^2 dV. \tag{14}$$

Substituting ρ' from (2) into (14), we obtain

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{\rho_0}{2}\int(c_l\,\mathrm{div}\;\vec{u})^2\,dV. \tag{15}$$

Using Dirac's approach in [10] for quantization of the electromagnetic field, we have:

$$\frac{\partial\vec{u}(\vec{r},t)}{\partial t} = -\frac{ic_l\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}} k\left(\vec{a}_{\vec{k}}\,e^{-ikc_lt} - \vec{a}^{+}_{-\vec{k}}\,e^{ikc_lt}\right)e^{i\vec{k}\vec{r}} \tag{16}$$

as well as

$$\mathrm{div}\;\vec{u}(\vec{r},t) = \frac{i\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}}\vec{k}\left(\vec{a}_{\vec{k}}\,e^{-ikc_lt} + \vec{a}^{+}_{-\vec{k}}\,e^{ikc_lt}\right)e^{i\vec{k}\vec{r}}. \tag{17}$$

Now, introducing (16) and (17) into (15) and using

$$\frac{1}{V}\int e^{i(\vec{k}_1+\vec{k}_2)\vec{r}} = \delta^3_{\vec{k}_1+\vec{k}_2},$$

we obtain the terms in the right side of the Hamiltonian of the system presented in (15):

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = -\frac{\rho_0c_l^2u_l^2}{2}\sum_{\vec{k}}k^2\left(\vec{a}_{\vec{k}} - \vec{a}^{+}_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} - \vec{a}^{+}_{\vec{k}}\right)$$

and

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = \frac{\rho_0c_l^2u_l^2}{2}\sum_{\vec{k}}k^2\left(\vec{a}_{\vec{k}} + \vec{a}^{+}_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} + \vec{a}^{+}_{\vec{k}}\right).$$

These expressions determine the reduced form of the Hamiltonian operator $\hat{H}_l$ by the form:

$$\hat{H}_l = 2\sum_{\vec{k}}\rho_0u_l^2c_l^2k^2\vec{a}^{+}_{\vec{k}}a_{\vec{k}}, \tag{18}$$

where u_l^2 is defined by the first term in the right side of (18) which represents the kinetic energy of a *sound particle* $\frac{\hbar^2k^2}{2m}$, if we suggest:

$$2\rho_0u_l^2c_l^2k^2 = \frac{\hbar^2k^2}{2m}. \tag{19}$$

Then,

$$u_l^2 = \frac{\hbar^2}{4c_l^2m\rho_0},$$

which allows one to determine the mass m of a *sound particle* using the value of the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$ and (13):

$$m = \frac{\hbar}{c_l}\left(\frac{n}{V}\right)^{\frac{1}{3}}. \tag{20}$$

Thus, the Hamiltonian operator $\hat{H}_l$ describes an ideal Bose gas of a spinless *sound particles*:

$$\hat{H}_l = \sum_{\vec{k}}\frac{\hbar^2k^2}{2m}\vec{a}^{+}_{\vec{k}}a_{\vec{k}}. \tag{21}$$

3 Bose quasiparticles in solid

Now let us analyze quantization of a solid ^{4}He which consists of N atoms with the mass M confined in the volume V. Considering a solid ^{4}He as a continuum medium, we investigate the fluctuation motion of "solid particles" on the basis of hydrodynamics (where "solid particle" is determined as a very small volume V_0, in regard to the volume V of the solid ($V_0 \ll V$), which consists of a macroscopic number of ^{4}He atoms in solid).

To do the transition from quantum liquid to the solid ^{4}He, we introduce a concept as the fluctuation motion of "solid particles" or "solid points". In this respect, we remove such concept as a "lattice" of solid ^{4}He or such concept as an atoms, fixed in the knots of lattice because "solid particles" exist in any point of the solid. The motion of "solid particles" describe an elastic wave consisting of the *sound particles* with spin 1 which in turn are vibrated by the natural frequency Ω_l.

In this respect, we may express the vector displacement of a longitudinal ultrasonic wave $u_l(\vec{r},t)$ via the second quantization vector wave functions of one *sound particle* with spin 1. Then, Eqs. (8) and (9) take the forms:

$$\vec{\phi}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}-kc_lt)} \tag{22}$$

$$\vec{\phi}^{+}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{a}^{+}_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}-kc_lt)} \tag{23}$$

with condition

$$\int\phi^{+}(\vec{r},\sigma)\,\phi(\vec{r},\sigma)\,dV = n_0 + \sum_{\vec{k}\neq0,\sigma}\hat{a}^{+}_{\vec{k},\sigma}\hat{a}_{\vec{k},\sigma} = \hat{n}, \tag{24}$$

where a free sound particles have the mass m and the value of its spin z-component $\sigma = 0; \pm 1$. In this respect, the vector-operators $\vec{a}^{+}_{\vec{k},\sigma}$, $\vec{a}_{\vec{k},\sigma}$ satisfy the Bose commutation relations as:

$$\left[\hat{a}_{\vec{k},\sigma}, \hat{a}^{+}_{\vec{k}',\sigma'}\right] = \delta_{\vec{k},\vec{k}'} \cdot \delta_{\sigma,\sigma'}$$

$$[\hat{a}_{\vec{k},\sigma}, \hat{a}_{\vec{k}',\sigma'}] = 0$$

$$[\hat{a}^{+}_{\vec{k},\sigma}, \hat{a}^{+}_{\vec{k}',\sigma'}] = 0.$$

In this case, the Hamiltonian operator $\hat{H}$ of the solid ^{4}He is represented by the form:

$$\hat{H} = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{1}{2}\int\left(\frac{c_l\rho'}{\sqrt{\rho_0}}\right)^2 dV + \\ + \frac{\rho_0}{2}\int(\Omega_l\vec{u}_l)^2\, dV, \tag{25}$$

where

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = -\frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k},\sigma} k^2\left(\vec{a}_{\vec{k},\sigma} - \vec{a}^{+}_{-\vec{k},\sigma}\right)\left(\vec{a}_{-\vec{k},\sigma} - \vec{a}^{+}_{\vec{k},\sigma}\right),$$

$$\frac{\rho_0}{2}\int(\operatorname{div}\vec{u})^2\, dV = \frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k},\sigma} + \vec{a}^{+}_{-\vec{k},\sigma}\right)\left(\vec{a}_{-\vec{k},\sigma} + \vec{a}^{+}_{\vec{k},\sigma}\right)$$

and

$$\frac{\rho_0}{2}\int(\Omega_l\vec{u}_l)^2\, dV = \frac{\rho_0\Omega^2 u_l^2}{2}\sum_{\vec{k}}\left(\vec{a}_{\vec{k},\sigma} + \vec{a}^{+}_{-\vec{k},\sigma}\right)\left(\vec{a}_{-\vec{k},\sigma} + \vec{a}^{+}_{\vec{k},\sigma}\right).$$

These expressions determine the reduced form of the Hamiltonian operator $\hat{H}$:

$$\hat{H}_l = \sum_{\vec{k}\neq 0,\sigma}\left(\frac{\hbar^2k^2}{2m} + mv^2\right)\vec{a}^{+}_{\vec{k},\sigma}a_{\vec{k},\sigma} + \\ + \frac{mv^2}{2}\sum_{\vec{k}\neq 0,\sigma}\left(\vec{a}^{+}_{-\vec{k},\sigma}\vec{a}^{+}_{\vec{k},\sigma} + \vec{a}_{\vec{k},\sigma}\vec{a}_{-\vec{k},\sigma}\right), \tag{26}$$

where we denote $v = \frac{\hbar\Omega_l}{\sqrt{2}mc_l}$, which in turn is the speed of sound particle in a solid.

For the evolution of the energy level, it is necessary to diagonalize the Hamiltonian $\hat{H}_l$, which can be accomplished by introducing the vector Bose-operators $\vec{b}^{+}_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ [11]:

$$\vec{a}_{\vec{k},\sigma} = \frac{\vec{b}_{\vec{k},\sigma} + L_{\vec{k}}\,\vec{b}^{+}_{-\vec{k},\sigma}}{\sqrt{1 - L^2_{\vec{k}}}}, \tag{27}$$

where $L_{\vec{k}}$ is the unknown real symmetrical function of the wave vector $\vec{k}$.

By substituting (27) into (26), we obtain

$$\hat{H} = \sum_{\vec{k}\neq 0}\varepsilon_{\vec{k}}\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}, \tag{28}$$

where $\vec{b}^{+}_{\vec{k},\sigma}$ and $\vec{b}_{\vec{k},\sigma}$ are the creation and annihilation operators of Bose quasiparticles with spin 1 with the energy:

$$\varepsilon_{\vec{k}} = \left[\left(\frac{\hbar^2k^2}{2m}\right)^2 + \hbar^2k^2v^2\right]^{1/2}. \tag{29}$$

In this context, the real symmetrical function $L_{\vec{k}}$ of the wave vector $\vec{k}$ is found to be

$$L^2_{\vec{k}} = \frac{\frac{\hbar^2k^2}{2m} + mv^2 - \varepsilon_{\vec{k}}}{\frac{\hbar^2k^2}{2m} + mv^2 + \varepsilon_{\vec{k}}}. \tag{30}$$

Thus, the average energy of the system takes the form:

$$\overline{\hat{H}} = \sum_{\vec{k}\neq 0}\varepsilon_{\vec{k}}\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}}, \tag{31}$$

where $\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}}$ is the average number of Bose quasiparticles with spin 1 with the wave vector $\vec{k}$ at the temperature T:

$$\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}} = \frac{1}{e^{\frac{\varepsilon_{\vec{k}}}{kT}} - 1}. \tag{32}$$

Thus, we have found the spectrum of free quasiparticles with spin 1 which is similar to Bogoliubov's one [8]. In fact, the Hamiltonian of system (31) describes an ideal Bose gas consisting of phonons with spin 1 at a small wave number $k \ll \frac{2mv}{\hbar}$ but at $k \gg \frac{2mv}{\hbar}$ the Hamiltonian operator describes an ideal gas of *sound particles*.

4 BEC of *sound particles*

As opposed to London's postulation concerning BEC of atoms [12], we state that *sound particles* in the condensate define the superfluid component of solid ^{4}He. Consequently, statistical equilibrium equation (10) takes the following form:

$$n_{0,T} + \sum_{\vec{k}\neq 0}\overline{\vec{a}^{+}_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}} = n, \tag{33}$$

where $\overline{\vec{a}^{+}_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}}$ is the average number of *sound particles* with the wave vector $\vec{k}$ at the temperature T.

To find the form $\overline{\vec{a}^{+}_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}}$, we use the linear transformation presented in (22):

$$\overline{\vec{a}^{+}_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}} = \frac{1 + L^2_{\vec{p}}}{1 - L^2_{\vec{p}}}\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}} + \\ + \frac{L_{\vec{k}}}{1 - L^2_{\vec{k}}}\left(\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}^{+}_{-\vec{k},\sigma}} + \overline{\vec{b}_{\vec{k},\sigma}\vec{b}_{-\vec{k},\sigma}}\right) + \frac{L^2_{\vec{k}}}{1 - L^2_{\vec{k}}}.$$

According to the Bloch-De-Dominicis theorem, we have

$$\overline{\vec{b}^{+}_{\vec{k},\sigma}\,\vec{b}^{+}_{-\vec{k},\sigma}} = \overline{\vec{b}_{\vec{k},\sigma}\vec{b}_{-\vec{k},\sigma}} = 0.$$

In this respect, the equation for the density of *sound particles* in the condensate takes the following form:

$$\frac{n_{0,T}}{V} = \frac{n}{V} - \frac{1}{V}\sum_{\vec{k}\neq 0,\sigma}\frac{L^2_{\vec{k}}}{1-L^2_{\vec{k}}} - \frac{1}{V}\sum_{\vec{k}\neq 0,\sigma}\frac{1+L^2_{\vec{k}}}{1-L^2_{\vec{k}}}\overline{\vec{b}^{+}_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}}. \quad (34)$$

Obviously, at the lambda transition $T = T_\lambda$ the density of *sound particles* $\frac{n_{0,T_\lambda}}{V} = 0$. Hence, we note that the mass m and density $\frac{n}{V}$ of *sound particles* are expressed via the mass of ions M and density of ions $\frac{N}{V}$ when solving a system of two equations presented in (13) and (20):

$$\frac{n}{V} = \left(\frac{Mc_l}{\hbar}\frac{N}{V}\right)^{\frac{3}{4}} \quad (35)$$

and

$$m = \left(\frac{\hbar}{c_l}\right)^{\frac{3}{4}}\left(\frac{MN}{V}\right)^{\frac{1}{4}}. \quad (36)$$

At $T \to 0$ it follows $\overline{\vec{b}^{+}_{\vec{k},\sigma}\,\vec{b}_{\vec{k},\sigma}} = 0$. Then taking into account the coefficient with number 3 before integral on the right side of equation (34) because it reflects the value of spin z-component $\sigma = 0; \pm 1$, we obtain

$$\frac{n_{0,T}}{n} = 1 - \frac{m^3v^3}{\hbar^3\pi^2\frac{n}{V}}. \quad (37)$$

5 Conclusions

Thus, in this letter, we propose new model for solids which is different from the well-known models of Einstein and Debye because: 1) we suggest that the atoms are the Fermi particles which are absent in the Einstein and Debye models; 2) we remove such concept as lattice of solid by introducing a concept as the fluctuation motion of "solid particles" or "solid points". Thus, we deal with the "solid particle" which exist in any point of the solid; 3) In our model, we argue that the phonons in solid have spin 1 which is different from one presented by Einstein and Debye models; 4) in fact, in this letter, we first postulate that the superfluid component of a solid ^{4}He is determined by means of *sound particles* in the condensate as opposed to London's postulation concerning BEC of atoms [12]. Consequently, such reasoning allows us to consider the model of solid in a new light.

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Fermion-Antifermion Asymmetry

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An event with positive energy transfers this energy photons which carries it on recorders observers. Observers know that this event occurs, not before it happens. But events with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. Since time is irreversible then only the events with positive energy can occur. In single-particle states, events with a fermion have positive energy and occurrences with an antifermion have negative energy. In double-particle states, events with pair of antifermions have negative energy and events with pair of fermions and with fermion-antifermion pair have positive energy.

1 Introduction

Let t, x_1, x_2,x_3 be real numbers, and let $\mathbf{x} := \langle x_1, x_2, x_3\rangle$.

Let $\mathcal{A}$ be some pointlike event.

Let $\varphi(t,\mathbf{x})$ be a 4×1-complex matrix such that

$$\varphi^{\dagger}(t,\mathbf{x})\varphi(t,\mathbf{x}) = \rho(t,\mathbf{x}) \tag{1}$$

where $\rho(t,\mathbf{x})$ is the probability density of $\mathcal{A}$.

Let* $\rho(t,\mathbf{x}) = 0$ if $t > \frac{\pi \mathrm{c}}{\mathrm{h}}$ and/or $|\mathbf{x}| > \frac{\pi \mathrm{c}}{\mathrm{h}}$.

In that case $\varphi(t,\mathbf{x})$ obeys some generalization of the Dirac equation [1]. The Dirac equation for free fermion does have the following form:

$$\left(\frac{1}{\mathrm{c}}\frac{\partial}{\partial t} - \sum_{s=1}^{3}\beta^{[s]}\frac{\partial}{\partial x_s} - \mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}n\gamma^{[0]}\right)\varphi(t,\mathbf{x}) = 0.$$

Here n is a natural number and

$$\beta^{[1]} := \begin{bmatrix} 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & -1 & 0 \end{bmatrix}, \beta^{[2]} := \begin{bmatrix} 0 & -\mathrm{i} & 0 & 0\\ \mathrm{i} & 0 & 0 & 0\\ 0 & 0 & 0 & \mathrm{i}\\ 0 & 0 & -\mathrm{i} & 0 \end{bmatrix},$$

$$\beta^{[3]} := \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}, \gamma^{[0]} := \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

In this case operator $\widehat{H_0}$ is *the free Dirac Hamiltonian* if

$$\widehat{H_0} := \mathrm{c}\left(\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}\frac{\partial}{\partial x_s} + \frac{\mathrm{h}}{\mathrm{c}}n\gamma^{[0]}\right).$$

Let $\mathbf{k}$ be a vector $\langle k_1, k_2, k_3\rangle$ where k_s are integer numbers and let

$$\omega(\mathbf{k}) := \sqrt{k_1^2 + k_2^2 + k_3^2 + n^2}$$

where n is a natural number.

*$\mathrm{c} := 299792458, \mathrm{h} := 6.6260755^{-34}$

Let

$$e_1(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} \omega(\mathbf{k})+n+k_3\\ k_1+\mathrm{i}k_2\\ \omega(\mathbf{k})+n-k_3\\ -k_1-\mathrm{i}k_2 \end{bmatrix},$$

$$e_2(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} k_1-\mathrm{i}k_2\\ \omega(\mathbf{k})+n-k_3\\ -k_1-\mathrm{i}k_2\\ \omega(\mathbf{k})+n+k_3 \end{bmatrix},$$

$$e_3(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} -\omega(\mathbf{k})-n+k_3\\ k_1+\mathrm{i}k_2\\ \omega(\mathbf{k})+n+k_3\\ k_1+\mathrm{i}k_2 \end{bmatrix},$$

$$e_4(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} k_1-\mathrm{i}k_2\\ -\omega(\mathbf{k})-n-k_3\\ k_1-\mathrm{i}k_2\\ \omega(\mathbf{k})+n-k_3 \end{bmatrix}.$$

In that case, functions
$e_1(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$ and
$e_2(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$
are eigenvectors of $\widehat{H_0}$ with eigenvalues $(+\mathrm{h}\omega(\mathbf{k}))$,
and functions
$e_3(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$ and
$e_4(\mathbf{k})(2\mathrm{c}/\mathrm{h})^{3/2}\exp(-\mathrm{i}(\mathrm{h}/\mathrm{c})\mathbf{kx})$
are eigenvectors of $\widehat{H_0}$ with eigenvalues $(-\mathrm{h}\omega(\mathbf{k}))$.

2 Single-Particle States

Let $\mathfrak{H}$ be some unitary space. Let $\widetilde{0}$ be the zero element of $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following conditions:

$$0\widetilde{F} = \widetilde{0}, \widetilde{0} + \widetilde{F} = \widetilde{F}, \widetilde{0}^{\dagger}\widetilde{F} = \widetilde{F}, \widetilde{0}^{\dagger} = \widetilde{0}.$$

Let $\widehat{0}$ be the zero operator on $\mathfrak{H}$. That is any element $\widetilde{F}$ of

$\mathfrak{H}$ obeys to the following condition:

$\widetilde{0F} = 0\widetilde{F}$, and if $\widehat{b}$ is any operator on $\mathfrak{H}$ then

$\widehat{0} + \widehat{b} = \widehat{b} + \widehat{0} = \widehat{b}, \widetilde{0b} = \widetilde{b0} = \widehat{0}.$

Let $\widehat{1}$ be the identy operator on $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following condition:
$\widetilde{1F} = 1\widetilde{F} = \widetilde{F}$, and if $\widehat{b}$ is any operator on $\mathfrak{H}$ then $\widehat{1b} = \widehat{b1} = \widehat{b}$.

Let linear operators $b_{s,\mathbf{k}}$ ($s \in \{1,2,3,4\}$) act on all elements of this space. And let these operators fulfill the following conditions:

$$\left\{b^{\dagger}_{s,\mathbf{k}}, b_{s',\mathbf{k}'}\right\} := b^{\dagger}_{s,\mathbf{k}} b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'} b^{\dagger}_{s,\mathbf{k}} = \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^3 \delta_{\mathbf{k},\mathbf{k}'}\delta_{s,s'}\widehat{1},$$

$$\{b_{s,\mathbf{k}}, b_{s',\mathbf{k}'}\} = b_{s,\mathbf{k}} b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'} b_{s,\mathbf{k}} = \left\{b^{\dagger}_{s,\mathbf{k}}, b^{\dagger}_{s',\mathbf{k}'}\right\} = \widehat{0}.$$

Hence,

$$b_{s,\mathbf{k}} b_{s,\mathbf{k}} = b^{\dagger}_{s,\mathbf{k}} b^{\dagger}_{s,\mathbf{k}} = \widehat{0}.$$

There exists element $\widetilde{F}_0$ of $\mathfrak{H}$ such that $\widetilde{F}^{\dagger}_0 \widetilde{F}_0 = 1$ and for any $b_{s,\mathbf{k}}$: $b_{s,\mathbf{k}}\widetilde{F}_0 = \widetilde{0}$. Hence, $\widetilde{F}^{\dagger}_0 b^{\dagger}_{s,\mathbf{k}} = \widetilde{0}$.

Let

$$\psi_s(\mathbf{x}) := \sum_{\mathbf{k}} \sum_{r=1}^{4} b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}\mathbf{x}\right).$$

Because

$$\sum_{r=1}^{4} e_{r,s}(\mathbf{k}) e_{r,s'}(\mathbf{k}) = \delta_{s,s'}$$

and

$$\sum_{\mathbf{k}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}(\mathbf{x}-\mathbf{x}')\right) = \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \delta(\mathbf{x}-\mathbf{x}')$$

then

$$\begin{aligned}\left\{\psi^{\dagger}_s(\mathbf{x}), \psi_{s'}(\mathbf{x}')\right\} &:= \psi^{\dagger}_s(\mathbf{x})\psi_{s'}(\mathbf{x}') + \psi_{s'}(\mathbf{x}')\psi^{\dagger}_s(\mathbf{x}) \\ &= \delta(\mathbf{x}-\mathbf{x}')\delta_{s,s'}\widehat{1}.\end{aligned}$$

And these operators obey the following conditions:

$$\psi_s(\mathbf{x})\widetilde{F}_0 = \widetilde{0}, \{\psi_s(\mathbf{x}), \psi_{s'}(\mathbf{x}')\} = \left\{\psi^{\dagger}_s(\mathbf{x}), \psi^{\dagger}_{s'}(\mathbf{x}')\right\} = \widehat{0}.$$

Hence,

$$\psi_s(\mathbf{x})\psi_{s'}(\mathbf{x}') = \psi^{\dagger}_s(\mathbf{x})\psi^{\dagger}_{s'}(\mathbf{x}') = \widehat{0}.$$

Let

$$\Psi(t,\mathbf{x}) := \sum_{s=1}^{4} \varphi_s(t,\mathbf{x})\psi^{\dagger}_s(\mathbf{x})\widetilde{F}_0.$$

These functions obey the following condition:

$$\Psi^{\dagger}(t,\mathbf{x}')\Psi(t,\mathbf{x}) = \varphi^{\dagger}(t,\mathbf{x}')\varphi(t,\mathbf{x})\delta(\mathbf{x}-\mathbf{x}').$$

Hence,

$$\int d\mathbf{x}' \cdot \Psi^{\dagger}(t,\mathbf{x}')\Psi(t,\mathbf{x}) = \rho(t,\mathbf{x}).$$

Let a Fourier series of $\varphi_s(t,\mathbf{x})$ has the following form:

$$\varphi_s(t,\mathbf{x}) = \sum_{\mathbf{p}} \sum_{r=1}^{4} c_r(t,\mathbf{p}) e_{r,s}(\mathbf{p}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}\right).$$

In that case:

$$\underline{\Psi}(t,\mathbf{p}) := \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{r=1}^{4} c_r(t,\mathbf{p}) b^{\dagger}_{r,\mathbf{p}}\widetilde{F}_0.$$

If

$$\mathcal{H}_0(\mathbf{x}) := \psi^{\dagger}(\mathbf{x})\widehat{H}_0\psi(\mathbf{x}) \tag{2}$$

then $\mathcal{H}_0(\mathbf{x})$ is called a Hamiltonian $\widehat{H}_0$ density.

Because

$$\widehat{H}_0\varphi(t,\mathbf{x}) = \mathrm{i}\frac{\partial}{\partial t}\varphi(t,\mathbf{x})$$

then

$$\int d\mathbf{x}' \cdot \mathcal{H}_0(\mathbf{x}')\Psi(t,\mathbf{x}) = \mathrm{i}\frac{\partial}{\partial t}\Psi(t,\mathbf{x}). \tag{3}$$

Therefore, if

$$\widehat{\mathbb{H}} := \int d\mathbf{x}' \cdot \mathcal{H}_0(\mathbf{x}')$$

then $\widehat{\mathbb{H}}$ acts similar to the Hamiltonian on space $\mathfrak{H}$.
And if

$$E_{\Psi}\left(\widetilde{F}_0\right) := \sum_{\mathbf{p}} \underline{\Psi}^{\dagger}(t,\mathbf{p})\widehat{\mathbb{H}}\underline{\Psi}(t,\mathbf{p})$$

then $E_{\Psi}\left(\widetilde{F}_0\right)$ is an energy of Ψ on vacuum $\widetilde{F}_0$.

Operator $\widehat{\mathbb{H}}$ obeys the following condition:

$$\widehat{\mathbb{H}} = \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \left(\sum_{r=1}^{2} b^{\dagger}_{r,\mathbf{k}} b_{r,\mathbf{k}} - \sum_{r=3}^{4} b^{\dagger}_{r,\mathbf{k}} b_{r,\mathbf{k}}\right).$$

This operator is not positive defined and in this case

$$E_{\Psi}\left(\widetilde{F}_0\right) = \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{p}} \mathrm{h}\omega(\mathbf{p}) \left(\sum_{r=1}^{2} |c_r(t,\mathbf{p})|^2 - \sum_{r=3}^{4} |c_r(t,\mathbf{p})|^2\right).$$

This problem is usually solved in the following way [2, p. 54]:

Let:

$$\begin{aligned} v_1(\mathbf{k}) &:= \gamma^{[0]} e_3(\mathbf{k}), \\ v_2(\mathbf{k}) &:= \gamma^{[0]} e_4(\mathbf{k}), \\ d_{1,\mathbf{k}} &:= -b^{\dagger}_{3,-\mathbf{k}}, \\ d_{2,\mathbf{k}} &:= -b^{\dagger}_{4,-\mathbf{k}}. \end{aligned}$$

In that case:

$$\begin{aligned} e_3(\mathbf{k}) &= -v_1(-\mathbf{k}), \\ e_4(\mathbf{k}) &= -v_2(-\mathbf{k}), \\ b_{3,\mathbf{k}} &= -d^{\dagger}_{1,-\mathbf{k}}, \\ b_{4,\mathbf{k}} &= -d^{\dagger}_{2,-\mathbf{k}}. \end{aligned}$$

Therefore,

$$\begin{aligned} \psi_s(\mathbf{x}) \;:\; &= \sum_{\mathbf{k}} \sum_{r=1}^{2} \Big(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) + \\ &+ d^{\dagger}_{r,\mathbf{k}} v_{r,s}(\mathbf{k}) \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\Big) \end{aligned}$$

$$\begin{aligned} \widehat{\mathbb{H}} &= \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \sum_{r=1}^{2} \left(b^{\dagger}_{r,\mathbf{k}} b_{r,\mathbf{k}} + d^{\dagger}_{r,\mathbf{k}} d_{r,\mathbf{k}} \right) \\ &\quad -2 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \widehat{1}. \end{aligned}$$

The first term on the right side of this equality is positive defined. This term is taken as the desired Hamiltonian. The second term of this equality is infinity constant. And this infinity is deleted (?!) [2, p. 58]

But in this case $d_{r,\mathbf{k}}\widetilde{F}_0 \neq \widetilde{0}$. In order to satisfy such condition, the vacuum element $\widetilde{F}_0$ must be replaced by the following:

$$\widetilde{F}_0 \to \widetilde{\Phi}_0 := \prod_{\mathbf{k}} \prod_{r=3}^{4} \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 b^{\dagger}_{r,\mathbf{k}} \widetilde{F}_0.$$

But in this case:

$$\psi_s(\mathbf{x})\,\widetilde{\Phi}_0 \neq \widetilde{0}.$$

And condition (3) isn't carried out.

In order to satisfy such condition, operators $\psi_s(\mathbf{x})$ must be replaced by the following:

$$\begin{aligned} &\psi_s(\mathbf{x}) \to \phi_s(\mathbf{x}) := \\ &:= \sum_{\mathbf{k}} \sum_{r=1}^{2} \Big(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) + d_{r,\mathbf{k}} v_r(\mathbf{k}) \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\Big). \end{aligned}$$

Hence,

$$\begin{aligned} \widehat{\mathbb{H}} &= \int d\mathbf{x} \cdot \mathcal{H}(\mathbf{x}) = \int d\mathbf{x} \cdot \phi^{\dagger}(\mathbf{x}) \widehat{H}_0 \phi(\mathbf{x}) = \\ &= \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \sum_{r=1}^{2} \left(b^{\dagger}_{r,\mathbf{k}} b_{r,\mathbf{k}} - d^{\dagger}_{r,\mathbf{k}} d_{r,\mathbf{k}} \right). \end{aligned}$$

And again we get negative energy.

Let's consider the meaning of such energy: An event with positive energy transfers this energy photons which carries it on recorders observers. Observers know that this event occurs, not before it happens. But event with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. This contradicts Theorem 3.4.2 [3]. Therefore, events with negative energy do not occur.

Hence, over vacuum $\widetilde{\Phi}_0$ single fermions can exist, but there are no single antifermions.

3 Two-Particle States

A two-particle state is defined the following field operator [4]:

$$\psi_{s_1,s_2}(\mathbf{x},\mathbf{y}) := \begin{vmatrix} \phi_{s_1}(\mathbf{x}) & \phi_{s_2}(\mathbf{x}) \\ \phi_{s_1}(\mathbf{y}) & \phi_{s_2}(\mathbf{y}) \end{vmatrix}.$$

In that case:

$$\widehat{\mathbb{H}} = 2\mathrm{h}\left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned} \widehat{\mathbb{H}}_a \;:\; &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) - \omega(\mathbf{p})) \sum_{r=1}^{2} \sum_{j=1}^{2} \times \\ &\times \Big\{ v^{\dagger}_j(-\mathbf{k}) v_j(-\mathbf{p}) e^{\dagger}_r(\mathbf{p}) e_r(\mathbf{k}) \times \\ &\times \left(+ b^{\dagger}_{r,\mathbf{p}} d^{\dagger}_{j,-\mathbf{k}} d_{j,-\mathbf{p}} b_{r,\mathbf{k}} \right) + \\ &+ \left(+ d^{\dagger}_{r,-\mathbf{p}} b^{\dagger}_{j,\mathbf{k}} b_{j,\mathbf{k}} d_{r,-\mathbf{p}} \right) + \\ &+ v^{\dagger}_j(-\mathbf{p}) v_j(-\mathbf{k}) e^{\dagger}_r(\mathbf{k}) e_r(\mathbf{p}) \times \\ &\times \left(- b^{\dagger}_{r,\mathbf{k}} d^{\dagger}_{j,-\mathbf{p}} d_{j,-\mathbf{k}} b_{r,\mathbf{p}} \right) + \\ &+ \left(- b^{\dagger}_{r,\mathbf{p}} d^{\dagger}_{j,-\mathbf{k}} d_{j,-\mathbf{k}} b_{r,\mathbf{p}} \right) \Big\} \end{aligned}$$

and

$$\begin{aligned} \widehat{\mathbb{H}}_b \;:\; &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) + \omega(\mathbf{p})) \sum_{r=1}^{2} \sum_{j=1}^{2} \times \\ &\times \Big\{ v^{\dagger}_j(-\mathbf{p}) v_j(-\mathbf{k}) v^{\dagger}_r(-\mathbf{k}) v_r(-\mathbf{p}) \times \\ &\times \left(- d^{\dagger}_{r,-\mathbf{k}} d^{\dagger}_{j,-\mathbf{p}} d_{j,-\mathbf{k}} d_{r,-\mathbf{p}} \right) + \\ &+ \left(- d^{\dagger}_{r,-\mathbf{p}} d^{\dagger}_{j,-\mathbf{k}} d_{j,-\mathbf{k}} d_{r,-\mathbf{p}} \right) \\ &+ e^{\dagger}_r(\mathbf{k}) e_r(\mathbf{p}) e^{\dagger}_j(\mathbf{p}) e_j(\mathbf{k}) \times \\ &\times \left(+ b^{\dagger}_{r,\mathbf{k}} b^{\dagger}_{j,\mathbf{p}} b_{j,\mathbf{k}} b_{r,\mathbf{p}} \right) + \\ &+ \left(+ b^{\dagger}_{r,\mathbf{p}} b^{\dagger}_{j,\mathbf{k}} b_{j,\mathbf{k}} b_{r,\mathbf{p}} \right) \Big\}. \end{aligned}$$

If velocities are small then the following formula is fair.

$$\widehat{\mathbb{H}} = 4\mathrm{h}\left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\widehat{\mathbb{H}}_a \;:=\; \sum_{\mathbf{k}}\sum_{\mathbf{p}} \left(\omega\left(\mathbf{k}\right)-\omega\left(\mathbf{p}\right)\right)\times \\ \times\sum_{r=1}^{2}\sum_{j=1}^{2}\left(d^{\dagger}_{j,-\mathbf{p}}b^{\dagger}_{r,\mathbf{k}}b_{r,\mathbf{k}}d_{j,-\mathbf{p}}-b^{\dagger}_{j,\mathbf{p}}d^{\dagger}_{r,-\mathbf{k}}d_{r,-\mathbf{k}}b_{j,\mathbf{p}}\right)$$

and

$$\widehat{\mathbb{H}}_b \;:=\; \sum_{\mathbf{k}}\sum_{\mathbf{p}} \left(\omega\left(\mathbf{k}\right)+\omega\left(\mathbf{p}\right)\right)\times \\ \times\sum_{j=1}^{2}\sum_{r=1}^{2}\left(b^{\dagger}_{j,\mathbf{p}}b^{\dagger}_{r,\mathbf{k}}b_{r,\mathbf{k}}b_{j,\mathbf{p}}-d^{\dagger}_{j,-\mathbf{p}}d^{\dagger}_{r,-\mathbf{k}}d_{r,-\mathbf{k}}d_{j,-\mathbf{p}}\right).$$

Therefore, in any case events with pairs of fermions and events with fermion-antifermion pairs can occur, but events with pairs of antifermions can not happen.

4 Conclusion

Therefore, an antifermion can exist only with a fermion.

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An Insight into Planck's Units: Explaining the Experimental–Observations of Lack of Quantum Structure of Space-Time

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This letter presents an insight into Planck's natural-units, that they are geometric-mean-values of astronomical-quantities, like total-mass of the universe M_0 and mass corresponding to Hubble's-constant $\left(hH_0/c^2\right)$, providing a theoretical support to the observational findings of Ragazzoni, R., Turatto, M. & Gaessler [Astrophysical Journal,587, L1–L4], Lieu, R. & Hillman, L.W [Astrophysical Journal, 585, L77–L80] and a news item published in Nature [Published on line on 31 March 2003 Nature DOI 10.1038/news030324-13] that there is no observational evidence for the quantum structure of space-time. Physicists have been expecting unification of gravitational and electric forces at Planck's energy; so they wanted to experimentally create a pair of particles whose gravitational-radius is equal to their Compton-wavelength. Whereas this paper shows that in nature there exists a "pair of unequal masses" which satisfies the condition of equality of gravitational and electrostatic potential-energies of the pair. If the universe with its total-mass M_0 and a particle of mass hH_0/c^2 both are electrically charged bodies, then the strengths of electric force and gravitational-force experienced by them will be equal. It is also pointed-out here that P.A.M. Dirac's observation of recurrences of the large-number 10^{40} and its explanation proposed by Tank [Proceedings of Indian National Sci. Acad. A, Vol. 63, No. 6, 469–474 (1997)] in 1997, by Sidharth [arXiv:gen-ph/0509026] in 2005, and by Funkhouser [arXiv:gen-ph/0611115] in 2006, should be viewed as attempts in search of natural system of units; and the recurrences $R_0/r_e = e^2/Gm_e$, $m_p = \left[M_0/m_p\right]^{1/2}$ should be taken more seriously than a mere coincidence, because its explanation by Tank also helped explaining the recurrences of the critical-acceleration of MOND noticed by Sivaram [Astrophys. and Space Sci. 215, (1994), 185–189].

1 Introduction

It has been realized by physicists since long that the conventional system of units, like meter, kilogram and second are arbitrarily chosen units; they do not correspond with any fundamental physical quantities; so we find it difficult to observe any regular pattern. Max Plank proposed a set of natural-units. Physicists have been expecting unification of gravitational and electric forces at the energies where protons attain the masses close to Planck's-mass. Large Hadrons Collider [LHC] was expected to yield some interesting results, because protons were to attain Planck's mass. It was believed that space and time are quantized; Planck-length is the "least-count" for "space" and Planck's unit of "time" is the "least-count" for "time". Whereas this letter shows that Planck's units are statistical-quantities, they are geometric-mean-values of the astronomical-quantities like total-mass of the universe M_0 and mass corresponding to Hubble's constant (hH_0/c^2).

(i) Planck's length L^* is a geometric-mean of: Gravitational-Radius corresponding to total mass of the universe M_0 and Compton-wavelength corresponding to the total-mass M_0 of the universe, i.e.

$$L^* = \left[\left(GM_0/c^2\right)(h/M_0c)\right]^{1/2}.$$

Also, L^* is a geometric-mean of: gravitational-radius of the universe and that of the lightest-particle of mass $\left(hH_0/c^2\right)$. L^* is also a geometric-mean of Compton-wavelengths of M_0 and $\left(hH_0/c^2\right)$.

(ii) Planck's unit of time T^* is a geometric-mean of age-of-the-universe T_0 and the period corresponding the total mass of the universe h/M_0c^2.

(iii) Planck's unit of mass M^* is a geometric-mean of total-mass-of the-universe M_0 and mass-of-the-lightest-particle. So, this letter provides a theoretical explanation for the experimental observations by Ragazzoni et al [1] and Lieu et. al. [2] that there is no evidence for quantum structure of space-time.

(iv) The total mass of the universe M_0 and mass corresponding to Hubble's constant $\left(hH_0/c^2\right)$ form an interesting pair, that: Gravitational-Radius corresponding to total-mass of the universe is equal to Compton-wavelength of the lightest particle, of mass hH_0/c^2.

(v) Gravitational-radius of the lightest particle is equal to the Compton-wavelength of the total-mass of the universe, M_0. Physicists have been trying to generate a pair of particles of equal masses whose gravitational-radius is equal to their Compton-wavelength. But in nature, there exists a pair of unequal masses which satisfies the condition for unification of forces, that their gravitational-potential-energy should be equal to the electrostatic-potential-energy. So this pair is expected to provide some clue to a deeper understanding needed for unification of gravitational and electric forces.

It is also pointed-out here that P. A. M. Dirac's observation of recurrences of the large-number 10^{40} and its explanation proposed by Tank [4] in 1997, by Sidharth [5] in 2005, and by Funkhouser [6] in 2006, should be viewed as attempts in search of natural system of units; and the recurrences $R_0/r_e = e^2/Gm_e$, $m_p = \left[M_0/m_p\right]^{1/2}$ should be taken more seriously than a mere coincidence, because their explanation by Tank also helped explaining the recurrences of the critical-acceleration of MOND noticed by Sivaram [7] and led to further conclusions discussed in the references [8–10].

2 The Derivations

(i) Gravitational-Radius of the universe is equal to Compton-wavelength of the lightest particle, of mass hH_0/c^2:

The gravitational-radius-of-the-universe $R_0 = GM_0/c^2$; Here M_0 is total-mass of the universe. And Compton-wavelength of the lightest-particle of mass hH_0/c^2 ; where H_0 is Hubble's constant, is:

$$\begin{aligned} & h/\left(hH_0/c^2\right)c \\ \text{i.e.} \quad &= c/H_0, \\ \text{i.e.} \quad &= R_0, \\ \text{i.e.} \quad &= GM_0/c^2. \end{aligned}$$

(ii) Gravitational-radius of the lightest particle is equal to Compton-wavelength of the total-mass of the universe, M_0.

$$\begin{aligned} \text{i.e.} \quad &= G\left(hH_0/c^2\right)/c^2, \\ \text{i.e} \quad &= GhH_0/c^4, \\ \text{i.e.} \quad &= GhH_0/GH_0M_0c \end{aligned}$$

(Because $GH_0M_0 = c^3$, based on this author's previous work [4]), i.e.= h/M_0c which is the Compton-wavelength corresponding to the total-mass-of-the-universe.

(iii-a) Planck's length L^* is a geometric-mean of: Gravitational-Radius of the universe and Compton-wavelength corresponding to the total-mass of the universe:

$$\begin{aligned} \text{i.e. } L^* &= \left[\left(GM_0/c^2\right)(h/M_0c)\right]^{1/2}, \\ \text{i.e.} \quad &= \left[hG/c^3\right]^{1/2}. \end{aligned}$$

Similarly, Planck's length is a geometric-mean of gravitational-radius and Compton-wavelengths of every particle of any mass.

(iii-b) Planck's length L^* is also a geometric-mean of: gravitational-radius of the universe and that of the lightest-particle of mass hH_0/c^2:

That is:

$$\begin{aligned} & \left[\left(GM_0/c^2\right)\left(GhH_0/c^4\right)\right]^{1/2}, \\ \text{i.e.} \quad &= \left[G^2M_0hH_0/GM_0H_0c^3\right]^{1/2} \end{aligned}$$

(Because $GH_0M_0 = c^3$, based on this author's previous work [4]),

$$\begin{aligned} \text{i.e.} \quad &= \left[hG/c^3\right]^{1/2}, \\ \text{i.e.} \quad &= L^*. \end{aligned}$$

(iii-c) L^* is also a geometric-mean of Compton-wavelengths of M_0 and $\left(hH_0/c^2\right)$:

That is:

$$\begin{aligned} & \left[(h/M_0c)\left\{h/\left(hH_0/c^2\right)c\right\}\right]^{1/2}, \\ \text{i.e.} \quad &= \left[(h/M_0c)\,(c/H_0)\right]^{1/2}, \\ \text{i.e.} \quad &= \left[(h/M_0c)\,(R_0)\right]^{1/2}, \\ \text{i.e.} \quad &= \left[(h/M_0c)\left(GM_0/c^2\right)\right]^{1/2}, \\ \text{i.e.} \quad &= \left[hG/c^3\right]^{1/2}, \\ \text{i.e.} \quad &= L^*. \end{aligned}$$

The references [1–3] also lead to a conclusion that nothing very special is observed at Planck length; there is no evidence for any quantum structure of space-time. This paper has shown that Planck-length is a statistical-quantity, a geometric-mean-value, not a length of any fundamental-entity.

(iv) Planck's unit of time T^* is a geometric-mean of age-of-the-universe and the period corresponding the total-mass of the universe h/M_0c^2

Age-of-the-universe $T_0 = 1/H_0$.

So the product of the two periods is:

$$\begin{aligned} & (1/H_0)\left(h/M_0c^2\right), \\ \text{i.e.} \quad &= h/H_0M_0c^2, \\ \text{i.e.} \quad &= hG/c^5 \end{aligned}$$

(Because $GH_0M_0 = c^3$, based on this author's previous work [4])

$$\begin{aligned} \text{i.e.} \quad &= T^{*2}, \\ \text{i.e. } T^* &= \left[(T_0)\left(h/M_0c^2\right)\right]^{1/2}. \end{aligned}$$

(v) Planck's unit of mass M^* is a geometric-mean of total-mass-of the-universe M_0 and mass-of-the-lightest--particle :

$$\begin{aligned} \text{i.e.} \quad &= \left[(M_0)\left(hH_0/c^2\right)\right]^{1/2}, \\ \text{i.e.} \quad &= \left[M_0hH_0c/c^3\right]^{1/2}, \\ \text{i.e.} \quad &= \left[M_0hH_0c/GM_0H_0\right]^{1/2} \end{aligned}$$

(Because $GH_0M_0 = c^3$, based on this author's previous work [4]),

$$\text{i.e.} \quad = \quad [hc/G]^{1/2},$$
$$\text{i.e.} \quad = \quad M^*.$$

(vi) P.A.M. Dirac took the classical-radius of the electron e^2/m_ec^2 as a natural unit of length; and found an interesting relation:

$$R_0/r_e = e^2/Gm_em_p = \left[M_0/m_p\right]^{1/2} = 10^{40}.$$

Tank [4] explained the above relation and reached a conclusion that the relation implies: (i) Gravitational potential-energy of the universe is equal to the energy-of-mass of the universe; (ii) Electrostatic potential-energy of the electron is equal to the energy-of-mass of it; and (iii) Strengths of electric-force, strong-force and gravitational-force are proportional to densities of matter within the electron, the pi-meson and the universe respectively. Sidharth [5] and Funkhouser [6] have given a similar explanation for the recurrences of the Large-Number, but they have not drawn any conclusions for further application.

From the above comparison of Planck's natural units and Dirac's natural units we are led to a conclusion that Dirac's choice of natural units leads to interesting new relations. These relations should not be ignored as mere coincidences, because these relations have emerged from right choice of natural-units.

Sivaram [7] noticed the recurrences of the same value of acceleration, equal to the "critical-acceleration" of MOND, at the radial-distance R in the case of the electron, the proton, the nucleus, the globular-clusters, the spiral-galaxies, the galactic-clusters and the universe. Tank [8–10] could explain these recurrences based on equality of potential-energy and energy-of-mass of these systems, the equality which helped him to explain Dirac's large-number-ratios in 1997. Thus, Dirac's attempt to choose natural-units has led to a conclusion, of equality of potential-energy and energy-of-mass of various systems of matter, which helped explaining another set of recurrences noticed by Sivaram, and to draw further conclusions discussed in the references [8–10]

Also, if we measure distances in the units of radius-of-the-universe R_0 and measure masses of bodies in the units of total-mass-of-the-universe M_0 then the gravitational-constant G becomes unity; as follows:

> Gravitational-potential-energy of a system of masses M and m at a distance r is
>
> $$= (M/M_0)\left(mc^2\right)/(r/R_0).$$

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Excited Electronic States of Atoms Described by the Model of Oscillations in a Chain System

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We analyzed the numerical values of half-lifes of excited electronic states of the H, He and Li atom, as well as the Li^+ ion. By means of a fractal scaling model originally published by Müller in this journal, we interprete these half-lifes as proton resonance periods. On the logarithmic scale, the half-lifes were expressed by short continued fractions, where all numerators are Euler's number. From this representation it was concluded that the half-lifes are heavily located in nodes or sub-nodes of the spectrum of proton resonance periods.

1 Introduction

The model of a chain of similar harmonic oscillators was proposed by Müller [1–3] as a phenomenological theory describing physical quantities as proton resonance oscillation modes.

In the most general case, the spectrum of eigenfrequencies of a chain system of many proton harmonic oscillators is given by the continuous fraction equation [2]

$$f = f_p \exp S, \tag{1}$$

where f is any natural oscillation frequency of the chain system, f_p the oscillation frequency of one proton and S the continued fraction corresponding to f. S was suggested to be in the canonical form with all partial numerators equal 1 and the partial denominators are positive or negative integer values

$$S = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3 + \ldots}}}. \tag{2}$$

Particularly interesting properties arise when the numerator equals 2 and all denominators are divisible by 3. Such fractions divide the logarithmic scale in allowed values and empty gaps, i.e. ranges of numbers which cannot be expressed with this type of continued fractions. He showed that these continued fractions generate a self-similar and discrete spectrum of eigenvalues [1], that is also logarithmically invariant. Maximum spectral density areas arise when the free link n_0 and the partial denominators n_i are divisible by 3.

In two previous articles [4, 5] we applied a slightly modified model, where all numerators were substituted by Euler's number. This model was particularly successful describing specific features of the solar system [5].

However, the true physical meaning of the numerator e is not yet clear. It must now be investigated, for which type of data exactly this type of continued fractions can be applied. There might be some data sets, where the numerator is 2, as it was suggested by Müller in a patent [6].

In this article we analyzed a set of very accurately determined half-lifes of excited states of atoms on the logarithmic scale. We show that continued fractions with Euler's number as numerator are adequate to describe these data.

2 Data source and computational details

All atomic spectral data were taken from the web site of the National Institute of Standards and Technology (NIST) [7]. NIST maintains a critical selection of spectral data previously published in regular scientific journals. For the H, He and Li atom, reference was given to a publication by Wiese [8].

Table 1 shows such a data compilation for the Hydrogen atom. We consider here only experimentally observed emission lines (i.e. not Ritz lines), for which the transition probabilities have been determined. We numbered these lines in the order of increasing wavelength and eliminated lines with an already previously listed transition probability. For the Hydrogen atom, this procedure resulted then in a set of 109 lines which have all different transition probabilities. Also, if a transition probability has a numerical error higher than 1% (according to NIST), the corresponding line was ignored.

The transition probability as given by NIST has the unit of frequency $[s^{-1}]$ and is also called the Einstein A coefficient of spontaneous emission. Consider a large number of atoms in an excited state i, decaying to the ground state k (k could also be any lower lying excited state). Equation (3) is then the rate law

$$\frac{\partial N}{\partial t} = -A_{ik} N, \tag{3}$$

which results in

$$N(t) = N_0 \exp(-A_{ik} t), \tag{4}$$

where $N(t)$ is the number of excited atoms at time t, N_0 the number of excited atoms at $t = 0$ and A_{ik} the Einstein A coefficient for the transition $i \to k$. From this exponential law, the half-life $T_{1/2}$ of the transition $i \to k$ can be calculated as

$$T_{1/2} = \frac{\ln(2)}{A_{ik}}. \tag{5}$$

Table 1: Observed emission lines of the Hydrogen atom with corresponding wavelengths and transition probabilities. Obs.: Line no. 18 represents a forbidden transition.

Line no.	Wavelength [Å]	Transition probability [s^{-1}]	Line no.	Wavelength [Å]	Transition probability [s^{-1}]
1	918.125	5.0659×10^4	56	6562.72482	2.2448×10^7
2	919.342	7.8340×10^4	57	6562.77153	2.2449×10^7
3	920.947	1.2631×10^5	58	6562.79	4.4101×10^7
4	923.148	2.1425×10^5	59	6562.85175	6.4651×10^7
5	926.249	3.8694×10^5	60	8392.40	1.5167×10^3
6	930.751	7.5684×10^5	61	8413.32	1.9643×10^3
7	937.801	1.9728×10^7	62	8437.95	2.5804×10^3
8	937.814	1.6440×10^6	63	8467.26	3.4442×10^3
9	949.742	3.4375×10^7	64	8502.49	4.6801×10^3
10	949.742	4.1250×10^6	65	8545.38	6.4901×10^3
11	972.517	1.2785×10^7	66	8598.39	9.2117×10^3
12	972.541	6.8186×10^7	67	8665.02	1.3431×10^4
13	1025.728	1.6725×10^8	68	8750.46	2.0207×10^4
14	1025.728	5.5751×10^7	69	8862.89	3.1558×10^4
15	1215.6699	6.2648×10^8	70	9015.3	5.1558×10^4
16	1215.6699	6.2649×10^8	71	9229.7	8.9050×10^4
17	1215.6701	4.6986×10^8	72	9546.2	1.6506×10^5
18	1215.67312	2.495×10^{-6}	73	10049.8	3.3585×10^5
19	3656.65	9.9657×10^1	74	10938.17	7.7829×10^5
20	3657.25	1.1430×10^2	75	12818.072	2.2008×10^6
21	3658.04	1.3161×10^2	76	15560.46	3.6714×10^3
22	3658.65	1.5216×10^2	77	16411.36	1.6205×10^4
23	3659.41	1.7669×10^2	78	16811.10	2.5565×10^4
24	3660.32	2.0612×10^2	79	17366.885	4.2347×10^4
25	3661.27	2.4162×10^2	80	18179.21	7.4593×10^4
26	3662.22	2.8474×10^2	81	18751.3	8.9860×10^6
27	3663.41	3.3742×10^2	82	21661.178	3.0415×10^5
28	3664.65	4.0224×10^2	83	26258.71	7.7110×10^5
29	3666.08	4.8261×10^2	84	32969.8	6.9078×10^4
30	3667.73	5.8304×10^2	85	37405.76	1.3877×10^5
31	3669.45	7.0963×10^2	86	40522.79	2.6993×10^6
32	3671.32	8.7069×10^2	87	46537.8	3.2528×10^5
33	3673.81	1.0777×10^3	88	51286.5	3.6881×10^4
34	3676.376	1.3467×10^3	89	74599.0	1.0254×10^6
35	3679.370	1.7005×10^3	90	75024.4	1.5609×10^5
36	3682.823	2.1719×10^3	91	81548.4	3.3586×10^3
37	3686.831	2.8093×10^3	92	86644.60	5.0098×10^3
38	3691.551	3.6851×10^3	93	87600.64	3.9049×10^4
39	3697.157	4.9101×10^3	94	93920.3	7.8037×10^3
40	3703.859	6.6583×10^3	95	105035.07	1.2870×10^4
41	3711.978	9.2102×10^3	96	108035.9	2.2679×10^3
42	3721.946	1.3032×10^4	97	113086.81	8.2370×10^4
43	3734.369	1.8927×10^4	98	115395.4	3.3253×10^3
44	3750.151	2.8337×10^4	99	123719.12	4.5608×10^5
45	3770.633	4.3972×10^4	100	123871.53	2.3007×10^4
46	3797.909	7.1225×10^4	101	125870.5	5.0797×10^3
47	3835.397	1.2156×10^5	102	190619.6	2.2720×10^5
48	3889.064	2.2148×10^5	103	278035.0	1.2328×10^5
49	3970.075	4.3889×10^5	104	690717	2.7989×10^4
50	4101.734	9.7320×10^5	105	887610	1.8569×10^4
51	4340.472	2.5304×10^6	106	1118630	1.2709×10^4
52	4861.28694	9.6680×10^6	107	1387500	8.9344×10^3
53	4861.29776	9.6683×10^6	108	1694230	6.4283×10^3
54	4861.35	8.4193×10^6	109	3376000	2.0659×10^3
55	6562.70969	5.3877×10^7			

Finally, the numerical values of continued fractions were always calculated using the the Lenz algorithm as indicated in reference [9].

3 Results and discussion

Half-lifes of exited states of atoms are abundantly available from the NIST web site, however, only for the light atoms such as H, He and Li these data have a very high accuracy. Considering for instance Fe as a heavy element, most of the Einstein A coefficients have uncertainties of 10-18% and are consequently not suitable for a numerical analysis.

Due to results form our previous publications, we suspect that Müller's continued fraction formalism with Euler's number as numerator can still be applied to many data sets, so we set all partial numerators in Müller's continued fractions (given in equation (2)) to Euler's number.

We strictly follow the formalism of previous publications [4–6] and introduce a phase shift p in equation (2). According to [6] the phase shift can only have the values 0 or $\pm\frac{3}{2}$. So we write for the half-lifes of the excited states:

$$\ln \frac{T_{1/2}}{\tau} = p + S, \tag{6}$$

where S is the continued fraction

$$S = n_0 + \cfrac{e}{n_1 + \cfrac{e}{n_2 + \cfrac{e}{n_3 + ...}}} \tag{7}$$

and $\tau = \frac{\lambda_C}{c}$ is the oscillation period of a hypothetical photon with the reduced Compton wavelength of the proton ($\lambda_C = \frac{h}{2\pi mc} = 2.103089086 \times 10^{-16}$ m) and traveling at light speed (numerical value $7.015150081 \times 10^{-25}$ s).

We abbreviate $p + S$ as $[p; n_0 \mid n_1, n_2, n_3, \ldots]$. The free link n_0 and the partial denominators n_i are integers divisible by 3. For convergence reason, we have to include $|e{+}1|$ as allowed partial denominator. This means the free link n_0 is allowed to be $0, \pm3, \pm6, \pm9 \ldots$ and all partial denominators n_i can take the values $e{+}1$, $-e{-}1$, $\pm6, \pm9, \pm12 \ldots$.

For the calculation of the continued fractions we did not consider any standard deviation of the published data. Practically, we developed the continued fraction and determined only 18 partial denominators. Next we calculated repeatedly the data value from the continued fraction, every time considering one more partial denominator. As soon as considering further denominators did not improve the experimental data value significantly (on the linear scale), we stopped considering further denominators and gave the resulting fraction in Table 2. This means we demonstrate how accurately the calculated half-lifes can be expressed through continued fractions. Additionally we also report the numerical error, which is defined as absolute value of the difference between the half-life calculated from the NIST transition probability and the value calculated from the continued fraction representation.

If this numerical error is higher than 1%, we interpret the result as "no continued fraction found", otherwise the continued fraction representation is in satisfying agreement with the experimental data.

As can be seen from Table 2, with one exception, all half-lifes could be expressed in a satisfactory manner by a continued fraction representation. Only one outlier was found,

which underlines the statistical nature of Müller's continued fraction model.

We believe that spectral line number 71 is a true outlier rather than a bad data point, since the Hydrogen spectrum has been thoroughly investigated and is definitely the most easiest one to interpret.

In most cases the numerical errors are several orders of magnitude lower than the data value. This changes when calculating the continued fractions with number 2 as numerator, as it was suggested by Müller in a patent [6]. In this case the number of outliers increases to 12 and the numerical errors of the continued fraction representations are frequently very slightly lower than the 1% limit. So the numerator e is definitely the better choice.

It can be seen that around 25% of the half-lifes could be expressed by two continued fractions, so there is no preferred accumulation of the half-lifes in the neutral zones. The majority of the continued fraction representations terminates with a high partial denominator ($\pm 9, \pm 12, \pm 15 \ldots$). This means there is a general tendency that the half-lifes accumulate in nodes and sub-nodes of the spectrum of the proton resonance periods.

Additionally, in the same manner as here described for the spectral lines of the Hydrogen atom, we analyzed the spectral data of He, Li (neutral atoms) and the Li^+ ion. From the NIST database resulted 142 spectral lines for the He atom, 57 lines for the Li atom and 129 lines for the Li^+ ion.

Again, it was analogously possible to express the half-lifes on the logarithmic scale by continued fraction representations with Euler's number as numerators. Very few outliers were found, 6 in the He data set, only one in the Li data set and 7 in the set of the Li^+ lines (continued fractions not given). Regarding the numerical errors, no significant differences were detected, when comparing with the Hydrogen set.

This result is a contribution to the importance of Euler's number as a possible numerator in the model of oscillations in a chain system. We have now identified the half-lifes of excited states with respect to individual electronic transitions as a further data set where this (still phenomenological) model can be applied. For the half-lifes, apparently it does not matter how many nucleons are in the atom and whether the atom is neutral or charged. It even seems to be that the model applies for both, allowed and forbidden transitions, however, this should be verified with further data; we have here only one forbidden transition in our data set.

4 Conclusions

Numerical investigation of a large data set of 437 half-lifes of electronic transitions from different atoms revealed that Müller's continued fraction model with e as numerator is adequate to express these data on the logarithmic scale. There is a general tendency that half-lifes accumulate in nodes and sub-nodes of the spectrum of proton resonance periods. This accumulation does not seem to be influenced by the atomic charge or the atomic number (chemical element). It can be said that every excited state of an atom (with corresponding transition), has different oscillation properties and goes in resonance with the appropriate proton oscillation. Then, during one proton oscillation period, 50% of the excited atoms become de-excited to a lower-lying state.

This viewpoint has some similarity to the teaching of modern quantum electrodynamics. This theory states that spontaneous emission from atoms is caused by a 50:50 contribution from radiation reaction and vacuum fluctuations [10]. So both models assume an external influence coupled to the atoms, either the proton resonance spectrum or the vacuum fluctuations.

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Table 2: Continued fraction representation of half-lifes of excited states of the Hydrogen atom

Line no.	Half-life [s] Continued fraction representation	Numerical error [s]
1	$1.36826068529 \times 10^{-5}$	
	[0; 45 | -e-1, -e-1, e+1, -6, 6, -e-1, 6]	2.5×10^{-11}
	[1.5; 42 | e+1, -e-1, 24, -6, 9]	1.5×10^{-11}
2	$8.84793439571 \times 10^{-6}$	
	[1.5; 42 | 6, -9, e+1]	3.7×10^{-8}
3	$5.48766669749 \times 10^{-6}$	
	[1.5; 42 | 765]	3.0×10^{-11}
4	$3.23522604695 \times 10^{-6}$	
	[0; 42 | e+1, -e-1, e+1, -9, 12]	4.2×10^{-11}
	[1.5; 42 | -6, e+1, -6, -e-1, -9]	1.6×10^{-11}
5	$1.79135571551 \times 10^{-6}$	
	[0; 42 | 6, e+1, -e-1]	1.7×10^{-8}
6	$9.15843745785 \times 10^{-7}$	
	[0; 42 | -9, -6, 9, 6, -21, 117]	1.7×10^{-19}
7	$3.51351977169 \times 10^{-8}$	
	[0; 39 | -6, e+1, -e-1]	4.7×10^{-10}
	[1.5; 36 | e+1, -e-1, e+1, e+1, -e-1]	4.4×10^{-11}
8	$4.21622372603 \times 10^{-7}$	
	[1.5; 39 | 6, 12, e+1, 6]	6.2×10^{-12}
9	$2.01642816163 \times 10^{-8}$	
	[1.5; 36 | 6, e+1, -6, 6, -e-1, 18]	2.0×10^{-15}
10	$1.68035680136 \times 10^{-7}$	
	[1.5; 39 | -6, 6, e+1, -e-1, e+1]	3.4×10^{-10}
11	$5.42156574548 \times 10^{-8}$	
	[0; 39 | -24, 27, -24, -18]	3.6×10^{-17}
12	$1.01655351621 \times 10^{-8}$	
	[1.5; 36 | -9, -6, -42]	4.2×10^{-14}
13	$4.1443777612 \times 10^{-9}$	
	[0; 36 | 9, -6, -e-1]	4.8×10^{-12}
14	$1.24329102717 \times 10^{-8}$	
	[1.5; 36 | -33]	4.9×10^{-11}
15	$1.106415497 \times 10^{-9}$	
	[1.5; 33 | 6, -6, e+1, e+1, -e-1, 12]	1.1×10^{-15}
16	$1.10639783645 \times 10^{-9}$	
	[1.5; 33 | 6, -6, e+1, e+1, -e-1, 6, 15]	4.3×10^{-17}
17	$1.47522066267 \times 10^{-9}$	
	[0; 36 | -e-1, -39, -e-1, e+1, 9, 6]	4.8×10^{-17}
	[1.5; 33 | e+1, -12, e+1, -9, 12, 12]	8.9×10^{-18}
18	277814.501226	
	[0; 69 | -e-1, 6, -e-1, -141]	0.17
	[1.5; 66 | e+1, 6, -81]	0.97
19	0.00695532858264	
	[0; 51 | -9, e+1, -e-1, e+1]	9.2×10^{-5}
20	0.00606427979493	
	[0; 51 | -6, 6, 15]	5.7×10^{-7}
21	0.00526667563681	
	[0; 51 | -e-1, -e-1, -e-1, 6, 12]	4.2×10^{-8}
	[1.5; 48 | e+1, -e-1, -e-1, 6, -e-1, e+1, -30]	6.8×10^{-10}
22	0.00455538367876	
	[0; 51 | -e-1, 12, e+1, -e-1, 12, 9]	8.9×10^{-10}
	[1.5; 48 | e+1, 90, -e-1, e+1, 9]	7.1×10^{-10}
23	0.00392295648062	
	[0; 51 | -e-1, e+1, -12, -12, 54]	1.2×10^{-10}
24	0.00336283320668	
	[1.5; 48 | 6, 6, 39, -6, -18]	2.1×10^{-11}
25	0.00286874919527	
	[1.5; 48 | 9, e+1, -e-1, e+1, -e-1]	2.7×10^{-5}
26	0.00243431615003	
	[1.5; 48 | 27, e+1, e+1, -18]	3.2×10^{-9}
27	0.00205425635872	
	[1.5; 48 | -39, e+1, -e-1, -e-1, e+1, -33]	2.2×10^{-11}
28	0.00172321793099	
	[1.5; 48 | -12, e+1, -e-1, e+1, -e-1, e+1, 12]	2.3×10^{-9}
29	0.00143624703293	
	[1.5; 48 | -6, -9, e+1, -6, e+1]	3.7×10^{-8}
30	0.00118885013131	
	[0; 48 | e+1, -e-1, -e-1, -e-1, e+1]	1.4×10^{-6}
	[1.5; 48 | -e-1, -e-1, -9, -6, 9]	1.6×10^{-9}
31	0.000976772656962	
	[0; 48 | e+1, 12, -e-1, e+1]	5.4×10^{-7}
32	0.000796089515855	
	[0; 48 | 6, -9, e+1, -e-1]	6.6×10^{-7}
33	0.000643172664526	
	[0; 48 | 9, e+1, -e-1]	4.4×10^{-6}
34	0.000514700512779	
	[0; 48 | 60, e+1, -e-1, -60]	2.0×10^{-11}
35	0.000407613749227	
	[0; 48 | -15, e+1, e+1, -e-1, -9]	1.8×10^{-9}
36	0.000319143229688	
	[0; 48 | -6, -9, -e-1, e+1, -6]	4.3×10^{-9}
37	0.000246733058256	
	[0; 48 | -e-1, -12, -6, 6]	3.3×10^{-9}
38	0.000188094537614	
	[0; 48 | -e-1, e+1, -e-1, -9, 6]	6.2×10^{-9}
	[1.5; 45 | 6, -e-1, e+1, -6, e+1, -351]	6.9×10^{-13}
39	0.000141167630101	
	[1.5; 45 | 12, -e-1, e+1]	8.2×10^{-7}
40	0.000104102726005	
	[1.5; 45 | -51, 9]	4.2×10^{-9}
41	$7.525864591 \times 10^{-5}$	
	[1.5; 45 | -6, -e-1, e+1, -e-1]	9.0×10^{-7}
42	$5.31880893616 \times 10^{-5}$	
	[0; 45 | e+1, -12, -e-1, e+1, -9]	5.3×10^{-10}
	[1.5; 45 | -e-1, -90, e+1, 60]	5.1×10^{-13}
43	$3.66221366598 \times 10^{-5}$	
	[0; 45 | 6, e+1, -15, -e-1, -e-1, 18]	5.8×10^{-13}
44	$2.44608526153 \times 10^{-5}$	
	[0; 45 | -1446]	3.6×10^{-11}
45	$1.57633762522 \times 10^{-5}$	
	[0; 45 | -6, -18, e+1]	3.1×10^{-9}
46	$9.73179614686 \times 10^{-6}$	
	[0; 45 | -e-1, e+1, -12, -e-1, e+1, 9]	1.8×10^{-11}
	[1.5; 42 | e+1, e+1, -e-1, e+1, -e-1, e+1, -6, 6]	2.0×10^{-11}
47	$5.70209921487 \times 10^{-6}$	
	[1.5; 42 | 66, -e-1, e+1, -e-1]	4.6×10^{-10}
48	$3.12961522738 \times 10^{-6}$	
	[0; 42 | e+1, -e-1, 6, -31650]	7.5×10^{-17}
49	$1.57931869161 \times 10^{-6}$	
	[0; 42 | 12, -e-1, e+1, -e-1]	1.8×10^{-8}
50	$7.12235080723 \times 10^{-7}$	
	[0; 42 | -6, e+1, -e-1, 6]	6.3×10^{-10}
	[1.5; 39 | e+1, -e-1, e+1, 9]	4.4×10^{-10}
51	$2.73927908852 \times 10^{-7}$	
	[1.5; 39 | 441, e+1, 12]	1.6×10^{-14}
52	$7.16949917832 \times 10^{-8}$	
	[0; 39 | 15, e+1, -e-1]	3.7×10^{-10}
53	$7.16927671421 \times 10^{-8}$	
	[0; 39 | 15, e+1, -e-1]	3.7×10^{-10}
54	$8.23283622819 \times 10^{-8}$	
	[0; 39 | 9, -48]	5.0×10^{-12}
55	$1.2865363338 \times 10^{-8}$	
	[1.5; 36 | -51, -e-1]	4.8×10^{-12}
56	$3.08779036244 \times 10^{-8}$	
	[0; 39 | -e-1, -9, -30]	8.1×10^{-13}
	[1.5; 36 | e+1, -6, -6, e+1, -9]	4.1×10^{-13}

Line no.	Half-life [s] Continued fraction representation	Numerical error [s]
57	$3.08765281554 \times 10^{-8}$	
	[0; 39 | -e-1, -9, -27, e+1, 30]	5.5×10^{-16}
	[1.5; 36 | e+1, -6, -6, e+1, -6, -69]	2.4×10^{-16}
58	$1.57172667413 \times 10^{-8}$	
	[1.5; 36 | 18, 9]	7.5×10^{-12}
59	$1.07213682783 \times 10^{-8}$	
	[1.5; 36 | -12, 6, e+1, 9]	8.2×10^{-14}
60	0.000457010074873	
	[0; 48 | -36, -e-1, -e-1, 72]	8.3×10^{-12}
61	0.000352872361941	
	[0; 48 | -9, e+1, -6, 66]	6.5×10^{-11}
62	0.000268620051372	
	[0; 48 | -e-1, -e-1, 15, -6, 6]	8.3×10^{-10}
	[1.5; 45 | e+1, -e-1, -9, -6, e+1, 6, -135]	3.4×10^{-14}
63	0.000201250560525	
	[0; 48 | -e-1, e+1, 9, 6]	2.9×10^{-8}
	[1.5; 45 | e+1, e+1, -15, e+1, 6, e+1, -72]	4.0×10^{-14}
64	0.000148105207273	
	[1.5; 45 | 9, 30, -6, e+1, -6, e+1, e+1]	7.5×10^{-13}
65	0.00010680069345	
	[1.5; 45 | -96, -e-1, e+1]	6.3×10^{-9}
66	$7.52463910635 \times 10^{-5}$	
	[1.5; 45 | -6, -e-1, e+1, -e-1]	8.9×10^{-7}
67	$5.16080098697 \times 10^{-5}$	
	[0; 45 | e+1, -39, -e-1, 6, -e-1, -9, -6, 18]	1.6×10^{-15}
	[1.5; 45 | -e-1, 24]	6.5×10^{-8}
68	$3.43023299134 \times 10^{-5}$	
	[0; 45 | 9, -e-1, e+1, -27, -6]	3.1×10^{-11}
69	$2.19642303238 \times 10^{-5}$	
	[0; 45 | -24, -e-1, 6, e+1]	4.6×10^{-10}
70	$1.34440277078 \times 10^{-5}$	
	[0; 45 | -e-1, -e-1, 9, -e-1, e+1]	9.5×10^{-9}
	[1.5; 42 | e+1, -e-1, -15, 12, -e-1, -15]	6.3×10^{-13}
71	$7.78379764806 \times 10^{-6}$	
	[1.5; 42 | 9, -e-1, e+1, -e-1, e+1, -e-1, e+1, -e-1]	1.0×10^{-7}
	no continued fraction found	error 1.3%
72	$4.19936496159 \times 10^{-6}$	
	[1.5; 42 | -9, -e-1, e+1, -e-1, e+1, -e-1]	3.3×10^{-8}
73	$2.06385940319 \times 10^{-6}$	
	[0; 42 | 6, -e-1, 6, 12, 6, 12]	1.3×10^{-13}
	[1.5; 42 | -e-1, e+1, -e-1, 9, e+1, 12]	4.2×10^{-12}
74	$8.90602706652 \times 10^{-7}$	
	[0; 42 | -9, 6, e+1, -e-1]	9.1×10^{-10}
75	$3.14952372119 \times 10^{-7}$	
	[1.5; 39 | 18, e+1, 6, 9, -e-1, -6, 9, -9]	2.6×10^{-17}
76	0.000188796421136	
	[0; 48 | -e-1, e+1, -e-1, -e-1, -e-1, e+1]	7.0×10^{-8}
	[1.5; 45 | 6, -e-1, e+1, -e-1, 6]	6.1×10^{-8}
77	$4.27736612502 \times 10^{-5}$	
	[1.5; 45 | -e-1, e+1, -6, 15]	4.4×10^{-9}
78	$2.71131304737 \times 10^{-5}$	
	[0; 45 | 27, -27, e+1, e+1, 9]	2.0×10^{-13}
79	$1.63682712013 \times 10^{-5}$	
	[0; 45 | -6, -e-1, 138]	3.9×10^{-11}
80	$9.29238910568 \times 10^{-6}$	
	[0; 45 | -e-1, e+1, -e-1, 33]	6.2×10^{-10}
	[1.5; 42 | 6, -e-1, e+1, e+1, -15]	1.3×10^{-10}
81	$7.71363432628 \times 10^{-8}$	
	[0; 39 | 12, -e-1, -e-1, -9]	2.2×10^{-12}
82	$2.27896492047 \times 10^{-6}$	
	[0; 42 | e+1, e+1, e+1, e+1, -e-1, e+1, -6, -15]	1.3×10^{-13}
	[1.5; 42 | -e-1, e+1, e+1, 45]	2.9×10^{-11}

Line no.	Half-life [s] Continued fraction representation	Numerical error [s]
83	$8.9890699074 \times 10^{-7}$	
	[0; 42 | -9, 27]	7.5×10^{-11}
84	$1.00342682266 \times 10^{-5}$	
	[0; 45 | -e-1, e+1, 9, -9]	7.6×10^{-10}
	[1.5; 42 | e+1, e+1, -21]	2.9×10^{-9}
85	$4.9949353647 \times 10^{-6}$	
	[1.5; 42 | -30, -105, 6]	3.2×10^{-13}
86	$2.56787752588 \times 10^{-7}$	
	[1.5; 39 | -48, e+1]	2.5×10^{-10}
87	$2.130924682 \times 10^{-6}$	
	[1.5; 42 | -e-1, e+1, -6, 39, -30]	8.3×10^{-14}
88	$1.87941536444 \times 10^{-5}$	
	[0; 45 | -9, -e-1, e+1, -e-1]	1.4×10^{-7}
89	$6.75977355725 \times 10^{-7}$	
	[0; 42 | -e-1, -e-1, e+1, 6, -6, 12]	8.3×10^{-13}
	[1.5; 39 | e+1, -e-1, -312, 24]	5.5×10^{-15}
90	$4.44068922135 \times 10^{-6}$	
	[1.5; 42 | -12, -e-1, e+1, -e-1]	6.5×10^{-9}
91	0.000206379795319	
	[0; 48 | -e-1, e+1, e+1, -e-1, -e-1, -e-1, e+1, 909]	1.2×10^{-14}
	[1.5; 45 | e+1, e+1, e+1, -e-1, -e-1, -e-1, -12]	5.0×10^{-11}
92	0.000138358253934	
	[1.5; 45 | 12, -12, 111]	1.5×10^{-11}
93	$1.77507024651 \times 10^{-5}$	
	[0; 45 | -9, e+1, e+1]	2.7×10^{-8}
94	$8.88228892141 \times 10^{-5}$	
	[1.5; 45 | -12, -e-1, 9, -e-1, e+1, e+1, -6, -6]	4.5×10^{-13}
95	$5.38575897871 \times 10^{-5}$	
	[0; 45 | e+1, -9, -e-1, e+1, -e-1]	8.3×10^{-8}
	[1.5; 45 | -e-1, -27]	4.5×10^{-8}
96	0.000305633925905	
	[0; 48 | -6, 9, 15, 21, -e-1, -12, -e-1, -18]	1.9×10^{-16}
97	$8.41504407624 \times 10^{-6}$	
	[1.5; 42 | 6, 9, -24, 6, -33, -e-1, 12]	7.3×10^{-17}
98	0.000208446510258	
	[0; 48 | -e-1, 6, -e-1, e+1, -e-1]	8.3×10^{-7}
	[1.5; 45 | e+1, 6, -e-1, 6]	5.7×10^{-8}
99	$1.51979297614 \times 10^{-6}$	
	[0; 42 | 12, 6, e+1]	7.1×10^{-10}
100	$3.01276646481 \times 10^{-5}$	
	[0; 45 | 12, e+1, -e-1, e+1]	9.4×10^{-8}
101	0.000136454353714	
	[1.5; 45 | 12, 6, -e-1, 6, e+1, -6, -12]	1.6×10^{-12}
102	$3.0508238581 \times 10^{-6}$	
	[0; 42 | e+1, -e-1, 27]	7.5×10^{-10}
	[1.5; 42 | -e-1, -e-1, e+1, -6, -9]	8.1×10^{-11}
103	$5.62254364504 \times 10^{-6}$	
	[1.5; 42 | 99, -e-1, e+1]	6.6×10^{-10}
104	$2.476498555 \times 10^{-5}$	
	[0; 45 | 261, -e-1]	7.8×10^{-10}
105	$3.73281911013 \times 10^{-5}$	
	[0; 45 | 6, 6, -30, -6, -6]	2.1×10^{-12}
106	$5.4539678543 \times 10^{-5}$	
	[0; 45 | e+1, -9, 6, -e-1, -30]	4.0×10^{-11}
	[1.5; 45 | -e-1, -18, e+1]	7.3×10^{-8}
107	$7.75818387983 \times 10^{-5}$	
	[1.5; 45 | -9, e+1, -e-1, e+1, -e-1]	6.5×10^{-7}
108	0.000107827447468	
	[1.5; 45 | -147, -6, e+1]	4.3×10^{-10}
109	0.000335518263498	
	[0; 48 | -6, -e-1, e+1, -e-1, e+1]	1.9×10^{-6}

Phenomenological Derivation of the Schrödinger Equation

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The Schrödinger equation is derived classically assuming that particles present local random spatial fluctuations compatible with the presence of the zero-point field. Without specifying the forces arising from this permanent matter-field interaction but exploring its fundamental properties (homogeneity, isotropy and random aspect) to justify the emergence of the continuity equation in one-particle context, these fluctuations are described in terms of the probability density. Specifically, the starting point is the assumption that the local activities, which turn the path followed by the particle totally unpredictable, must be associated with an energy proportional to $\partial P/\partial t$. The polar form of the wave function, which connects the obtained classical equations with the corresponding quantum equation, emerges as a by-product of the approach.

1 Introduction

The evolution of the wave function in single-particle quantum systems is described by the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}, \tag{1}$$

where m is the mass and V is a potential. The complex wave function is generally presented in its polar form

$$\psi = \sqrt{P}\exp(iS/\hbar), \tag{2}$$

where $P=|\psi|^2$ is the probability density, and $S/\hbar$ is a phase. Substituting (2) into (1) results in two equations

$$\frac{\partial P}{\partial t} + \nabla\cdot\left(P\frac{\nabla S}{m}\right) = 0, \tag{3}$$

and

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0, \tag{4}$$

where

$$Q = -\frac{\hbar^2}{4m}\left[\frac{\nabla^2 P}{P} - \frac{1}{2}\frac{(\nabla P)^2}{P^2}\right] \tag{5}$$

is known as quantum potential. At the classical limit ($\hbar \to 0$) Q vanishes and (4) reduces to the Hamilton-Jacobi equation. For this reason, Bohm [1] suggested that S is the classical action function, which relates to the actual velocity, $\mathbf{v} = \nabla S/m$, of the particle. In this way (3) simply expresses the conservation of probability.

This alternative way of writing the Schrödinger equation presents advantages as regards its interpretation in terms of classical variables. However, the problem of ignoring the path followed by the particle persists. And more, we have an obvious increase in complexity: The Schrödinger equation is a single function and quite simple, on the other hand, the equation (4) is somewhat complicated - and still requires the continuity equation to account local activities. And above all, thinking that the quantum revolution, highly non-classical, has its origin in a classical equation with an additional potential is not very easy. In reality, Q is not a traditional potential, but part of the description of the motion, that is, P is playing the role of a dynamical variable at the same footing as S. Thus S and P can be said to codetermine each other. However, in approximate schemes to get information about quantum systems it can be used as a potential [2].

Equation (4) is referred as stochastic Hamilton-Jacobi-Bohm equation. Despite the fact that P is unique for a given quantum system, it is interpreted as a differential equation describing an ensemble of trajectories. This is grounded in the fact that the action S was originally defined as a field variable related with a set of potential trajectories [3].

It is paid much attention to equation (4) and less concern about (3). From a dynamical point of view, the emergence of the continuity equation is the most remarkable result: It highlights the local loss of determinism ($\partial P/\partial t \neq 0$), is valid for one-particle systems (it was obtained in this way), and contains inherently the multiple path aspect of quantum systems [4], exactly how is assigned to equation (4).

Fundamentally, to have $\partial P/\partial t \neq 0$ (change of probability at a given position), and thus to justify the emergence of the equation (3), it is necessary that the particle runs local random spatial fluctuations. Otherwise, there are local preferences, and these combined with the dynamics that emanates from the potential V (deterministic) results in a classical trajectory. Therefore these fluctuations require the presence of external forces with special features. Indeed, these forces exist and are related with the zero-point field (ZPF). They are formally treated in the context of the stochastic electrodynamics [5, 6], and under certain conditions they may be measured [7, 8]. However, their definition is outside the scope of this work; just let's enumerate its indispensable characteristics to justify

the continuity equation in the context of one-particle dynamics.

The above rewriting of the Schrödinger equation starts from propositions valid within the quantum formalism and arrives at seemingly classical equations. What will be done in the present paper is to follow an inverse path. The starting point is the fact that the local changes of the probability density — associated with isotropic random spatial fluctuations impressed by the ZPF — must be related with an energy.

2 The multi path aspect of the motion

Suppose a particle of mass m performing a motion with velocity $\mathbf{v}$. If the associated probability density P is a continuous function of the coordinates and time, then its dynamical evolution along the trajectory is given by

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + \mathbf{v} \cdot \nabla P, \tag{6}$$

where $\partial P/\partial t$ refers to the change of probability at a given position, and the second term accounts for the spatial changes. As P is a probability, then we cannot precise the angle between ∇P and $\mathbf{v}$. Moreover, in principle, ∇P can show an isotropic distribution around each position. Indeed, as P is a conserved quantity, then the change of the probability density inside a given volume Ω (arbitrary), containing the instantaneous position of the particle, must be equal to the probability flux through a surface A surrounding this volume. Formally, we have

$$\frac{\partial}{\partial t}\int_{\Omega} P d\Omega = -\int_{A} P\dot{\mathbf{r}} \cdot d\mathbf{A}, \tag{7}$$

where $\dot{\mathbf{r}}$ is a velocity, and the vector field $P\dot{\mathbf{r}}$ represents all possible probability currents that cross the surface A. Obviously, if the particle is inside this volume, it emerges following one of these possibilities. In accordance with the properties of the ZPF, the field $P\dot{\mathbf{r}}$ must present an isotropic distribution, however, as the velocity of the particle is dictated by the dynamics of the system as a whole, then there are some privileged probability currents (the resulting motion is not a random walk). According to Green's theorem and equation (7), each one of the possible currents obeys

$$\frac{\partial P}{\partial t} + \nabla \cdot (P\dot{\mathbf{r}}) = 0. \tag{8}$$

As this process is repeated at all positions where the particle can be found, linking the successive probability currents, according to which the particle emerges from each volume Ω, is defined a path described by the velocity

$$\mathbf{v} = \frac{\nabla S}{m}, \tag{9}$$

where S is the Hamilton-Jacobi function of one possible path [3, see p. 36]. Therefore equation (8) must be written as (3).

If the local activities are ignored (classical limit), then the function S is defined on a single trajectory. This also can be easily inferred making $\partial P/\partial t = 0$ in equation (7). In this case the probability flux that enters the volume Ω equals the one that emerges from it. This means that the particle has only one possibility (probability current) to leave each successive volume Ω.

If the external field acts on the particle everywhere (homogeneously), without preferred directions (isotropic) and in a totally unpredictable (random) way, that is, like the ZPF, then we will have a local motion compatible with the continuity equation. Therefore, as the particle has several possibilities to leave each position (following one possible current $P\dot{\mathbf{r}}$), this assigns a multi path aspect to the motion. This means that the particle can travel on each one of them indiscriminately; there is no preferred path. Note, not having a preferred path means that all are equally probable. We realize that this fact is consistent with the formulation of quantum mechanics in terms of path integrals, where Feynman and Hibbs [4, see p. 28] begin with the following statement: "Now we can give the quantum-mechanical rule. We must say how much each trajectory contributes to the total amplitude to go from a to b. It is not that just the particular path of extreme action contributes; rather, it is that all the paths contribute. They contribute in equal amounts to the total amplitude, but contribute as different phases. The phase of the contribution from a given path is the action S for that path in units of the quantum of action $\hbar$". Coincidently, this is a description of the evolution operator $\exp(iS/\hbar)$ (unitary), present in (2), which is the core of the path integrals.

3 The main proposition

In a classical system, the particles are actuated by forces in such a way that they move along single predictable trajectories, and this leads to $\partial P/\partial t = 0$ everywhere (the local activities are ignored). By other side, if particles are being actuated by a field, with the characteristics pointed above, local exchange of energy between them occurs in such a way that $\partial P/\partial t \neq 0$. Admitting that this is a fact, let's write an effective stationary action function S_{eff} that, in addition to describing a path through the function S, also takes into account the local activities described in terms of probability density, that is,

$$S_{eff} = S + S_l, \tag{10}$$

where S_l is a local action that depends only on P. Following the same formalism obeyed by the stationary Hamilton's function, the energy and momentum of the particle over a possible path are, respectively, written as

$$H = -\frac{\partial S_{eff}}{\partial t} = -\frac{\partial S}{\partial t} - F\frac{\partial P}{\partial t}, \tag{11}$$

and

$$\mathbf{p} = \nabla S_{eff} = \nabla S + F\nabla P, \tag{12}$$

where $F = \partial S_l/\partial P$ should be a function of P which must comply the dynamics of the system. Specifically, this function must obey the conservation of probability and the local conservation of energy (the particle cannot extract energy from the field indefinitely).

The motion equations of the system can be obtained in the following way: As S and P are taking values on a volume, then the average energy of the multi path system need to be written in the form

$$\bar{H} = \int d^3rPH = \int d^3r\mathcal{H}, \tag{13}$$

where the integral is taken over whole space. Here, $\mathcal{H}$ has the role of Hamiltonian density. With H given by (11) we have

$$\bar{H} = \int d^3rP\left(-\frac{\partial S}{\partial t} - F\frac{\partial P}{\partial t}\right). \tag{14}$$

As $\bar{H}$, written in this way, is a functional of the functions S and P, taking the functional derivatives with respect to these functions, according to the well known rules

$$\frac{\delta\bar{H}}{\delta\xi} = \frac{\partial\mathcal{H}}{\partial\xi} - \frac{\partial}{\partial x_\alpha}\left(\frac{\partial\mathcal{H}}{\partial(\partial\xi/\partial x_\alpha)}\right), \tag{15}$$

where $x_\alpha = x, y, z, t$ and $\xi = S$ or P, we obtain respectively

$$\frac{\delta\bar{H}}{\delta S} = \frac{\partial P}{\partial t} \tag{16}$$

and

$$\frac{\delta\bar{H}}{\delta P} = -\frac{\partial S}{\partial t}. \tag{17}$$

This shows that the proposition (10) preserves the shapes of the canonical equations, where S and P behave as dynamical conjugate variables of the canonically transformed Hamiltonian $\bar{H}$ [1].

Taking into account the momentum (12), the energy (11) can be expressed by

$$H = \frac{|\nabla S + F\nabla P|^2}{2m} + V, \tag{18}$$

then (13) can also be written as

$$\bar{H} = \int d^3rP\left(\frac{|\nabla S + F\nabla P|^2}{2m} + V\right), \tag{19}$$

and, consequently, the canonical equation (16) takes the form

$$\left[\frac{\partial P}{\partial t} + \nabla\cdot\left(P\frac{\nabla S}{m}\right)\right] + (F + PF')\frac{(\nabla P)^2}{m} + PF\frac{\nabla^2 P}{m} = 0, \tag{20}$$

where $F' = \partial F\,/\,\partial P$. The first term, being the continuity equation, is zero, and the trivial solution of the resulting equation gives simultaneously $F = cte/P$ and $F = 0$. However, if this trivial solution is valid, F is not defined in the field of real numbers.

Generalizing the constant to complex numbers, the non zero solution is written as $F = (S_1 + iS_0)/P$, where S_1 and S_0 are real constants (they have dimension of action). Thus, returning this complex shape of F into (19), from (16), results

$$\left[\frac{\partial P}{\partial t} + \nabla\cdot\left(P\frac{\nabla S}{m}\right)\right] + S_1\frac{\nabla^2 P}{m} = 0, \tag{21}$$

which shows that probability conservation is obeyed if F is a pure imaginary ($S_1 = 0$). As this occurs independently of the P^{-1} functionality, then it only justifies the complex aspect of the trivial solution of (20).

Another evidence that F is pure imaginary comes from the fact that the momentum (12) is apparently incompatible with the actual velocity (9); it seems that we should have

$$\mathbf{v} = \frac{\nabla S}{m} + F\frac{\nabla P}{m}. \tag{22}$$

In reality, this behavior is not entirely unexpected, since, as we saw earlier, the actual velocity is the end result of the system dynamics as a whole, that is, S is also dictated by the local activities. Therefore, to reconcile these equations, F shall be such that (9) refers to the real part of (22).

The resulting apparent complex character of the energy (11) and the momentum (12) is only a stage of the calculations. In effect, the canonical equations (16) and (17) can also be obtained even making

$$\int d^3rP\left(-F\frac{\partial P}{\partial t}\right) = 0 \tag{23}$$

in Eq. (14), which makes the average energy (14) real. However, this implies that, on average, the exchange of energy between the particle and the field is zero, meaning that the energy provided by field is promptly returned to it in equal amount. This, besides constituting the desired local energy balance — it can be related with atomic stability [9] — also puts some insight in the complex shape of the mentioned real quantities.

In fact, the local energy balance (23) is satisfied by the trivial solution of (20), expressed by

$$F = \frac{\partial S_l}{\partial P} = \imath\frac{S_o}{P}, \tag{24}$$

as can be easily verified from the normalization of P. So this proven the P^{-1} functionality, which is not achieved only from probability conservation, as pointed above.

Substituting (24) into (19), results in

$$\bar{H} = \int d^3rP\left(\frac{(\nabla S)^2}{2m} + \frac{S_o^2}{2m}\frac{(\nabla P)^2}{P^2} + V\right), \tag{25}$$

which, with the canonical equations (16) and (17), reproduces the equations (3) and (4), respectively, if S_0 is identified with

$\hbar/2$. Therefore, to complete the classical derivation of the Schrödinger equation, the anzatz (2) must also be obtained in a classical context. This is the subject of the next section.

4 Parameterization of the equations

Knowing any one of the solutions (one of the paths) of the motion equations resulting from (25), the energy and momentum at each position, according to the equations (13), (12) and (24), are, respectively, given by

$$H = -\frac{\partial S}{\partial t} - \frac{\imath S_o}{P}\frac{\partial P}{\partial t} \tag{26}$$

and

$$\mathbf{p} = \nabla S + \frac{\imath S_o}{P}\nabla P. \tag{27}$$

Integrating these partial differential equations (minus a possible constant), we obtain the following dimensionless equation

$$\frac{1}{2\imath S_o}\left(\sum_i \int_0^{x_i} p_i dx_i - \int_0^t H dt\right) = \frac{S}{2\imath S_o} + \ln\sqrt{P}, \tag{28}$$

as can be easily verified by following the inverse procedure. The upper limits of the integrals are the coordinates and time of the positions occupied by the particle along a possible path, therefore the left hand side of (28) is a complex function of these parameters, which will be defined in the following way:

$$\ln\psi = \frac{1}{2\imath S_o}\left(\sum_i \int_0^{q_i} p_i dq_i - \int_0^t E dt\right). \tag{29}$$

As both sides of (28) are independent of the path followed by the particle, we can write the following relation between S and P, valid for all paths:

$$\ln\psi = \frac{S}{2\imath S_o} + \ln\sqrt{P}, \tag{30}$$

or

$$\psi = \sqrt{P}\exp\left(\frac{S}{2\imath S_o}\right). \tag{31}$$

This equation with $S_0 = \hbar/2$ is in full agreement with (2). And more, for constant energy and momentum the function defined in (29) is a solution of the Schrödinger equation for a free particle.

Finally, let's re-write the equations obtained in this work in terms of ψ. From (30) and its complex conjugate we obtain the following parametric shapes for S and P:

$$S = \frac{\imath\hbar}{2}\left(\lg\psi - \ln\psi^*\right) \tag{32}$$

and

$$P = \psi^*\psi. \tag{33}$$

Consequently, the equations (25), (26) and (27) can be re-written, respectively, in the forms:

$$\bar{H} = \int d^3 r\left(\frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi + \psi^* V\psi\right), \tag{34}$$

$$\imath\hbar\frac{\partial\psi}{\partial t} = H\psi, \tag{35}$$

and

$$-\imath\hbar\nabla\psi = \mathbf{p}\psi. \tag{36}$$

Applying the divergence operator on both sides of equation (36), allied to fact that $\mathbf{p}$ is coordinate independent (it is independent of the followed path), gives

$$-\imath\hbar\nabla\cdot\nabla\psi = \mathbf{p}\cdot\nabla\psi, \tag{37}$$

and expressing $\mathbf{p}$ in terms of the complex conjugate of (36), we obtain the equality

$$-\psi^*\nabla^2\psi = \nabla\psi^*\cdot\nabla\psi. \tag{38}$$

Therefore the equation (34) can be written in the well known quantum form

$$\bar{H} = \int d^3 r\psi^*\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi. \tag{39}$$

5 Conclusion

The approach shows that the Schrödinger equation and its accessory are necessary and natural equations, parameterized shapes of the complicated — not to say unsolvable — equations resulting from a classical treatment including a special field with homogeneous, isotropic and random characteristics.

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An Analysis of States in the Phase Space: the Anharmonic Oscillator

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The paper introduces a simple quantum model to calculate in a general way allowed frequencies and energy levels of the anharmonic oscillator. The theoretical basis of the approach has been introduced in two early papers aimed to infer the properties of quantum systems exploiting the uncertainty principle only. Although for clarity the anharmonic oscillator is described having in mind the lattice oscillations of atoms/ions, the quantum formalism of the model and approach have general character and can be extended to any oscillating system. The results show that the harmonic energy levels split into a complex system of anharmonic energy levels dependent upon the number of terms of the Hamiltonian that describes the anharmonicity.

1 Introduction

The anharmonic phenomena, well known in physics [1], regard a wide range of properties of practical and theoretical interest; e.g. in acoustics they account for large variations of sound velocity in solids [2], in optics for non-linear interaction of powerful light with lattice vibrations [3]. Moreover are known physical effects that lead to a behavior impossible in harmonic oscillators, like the "foldover effect" [4] and "superharmonic resonance" [5]; both are due to the dependency of the eigenfrequency of nonlinear oscillators on the amplitude and to the non-harmoniticity of the oscillations. In solid state physics, non-linear effects occur when atoms consisting of a positively charged nucleus surrounded by a cloud of electrons are subjected to an electric field; the displacement of nucleus and electrons causes an electric dipole moment, whose interaction with the applied field is linear for small field intensities only [6].

The present paper aims to propose a quantum mechanical approach to tackle the problem of non-harmonic oscillations in a general way, i.e. regardless of the particular issue of specific interest, and in line with the concepts introduced in two papers [7, 8] concerning simple quantum systems, many-electron atoms/ions and diatomic molecules. The basic idea of these papers starts from a critical review of the concepts of positions and momenta of interacting particles in a quantum system, where the dynamical variables are perturbed in a complex way by mutual interactions and change within appropriate ranges of values in agreement with boundary conditions like the minimum total energy.

Consider for instance the hydrogenlike atoms. It is reasonable to regard radial momentum p_ρ and distance ρ between electron and nucleus as variables included within proper ranges of values; so it is certainly possible to write $0 < \rho \leq \Delta\rho$ and $0 < p_\rho \leq \Delta p_\rho$ if $\Delta\rho$ and Δp_ρ have arbitrary sizes, including even the chance of infinite sizes. The only basic hypothesis of the quoted papers was that in general any ranges of conjugate dynamical variables Δx and Δp_x have physical meaning of quantum uncertainty ranges, thus to be regarded according to the basic ideas of quantum statistics; hence

$$\Delta x \Delta p_x = n\hbar, \tag{1.1}$$

with n arbitrary integer.

No hypothesis is necessary about Δx and Δp_x, which are by definition arbitrary, unknown and unpredictable. Eq. (1.1) was the unique assumption in [7, 8] and does so also in the model proposed here. Despite the apparently agnostic character of eq. (1.1), the results inferred in the quoted papers were in all cases completely analogous to that of the usual wave mechanics formalism; in particular it was found that the quantum numbers actually coincide with the numbers of allowed states in the phase space for the concerned systems. Eq. (1.1) only is enough to give the classical Hamiltonian, H_{cl}, the physical meaning of quantum Hamiltonian, H_q; it simply requires considering the ranges of dynamical variables rather than the dynamical variables themselves, which are therefore disregarded since the beginning. For instance, in a one-dimensional problem like that of a mass constrained to oscillate along a fixed direction, it means that hold the positions

$$H_{cl}(x, p_x) \Rightarrow H_q(\Delta x, \Delta p_x) \Rightarrow H_q(\Delta p_x, n). \tag{1.2}$$

The uncertainty is regarded in this way as fundamental principle of nature rather than as mere consequence of commutation rules of quantum operators. The case of the harmonic oscillator, already introduced in [7], has central importance here; its quantum formulation according to eq. (1.1) and positions (1.2) is so short and simple that it is sketched in the next section 2 to make the present paper clearer and self-consistent.

The next section aims also to emphasize how the concepts so far introduced enable the quantum approach. For clarity the anharmonic oscillator is regarded in section 3 having in mind the lattice oscillations of atoms/ions, yet through a very general approach that can be extended to any quantum system. The discussion on the results of the model and the conclusion are reported in sections 4 and 5.

2 The harmonic oscillator

With the positions 1,2, the classical energy equation $E = p^2/2m + k_{har}(x - x_o)^2/2$ of the oscillating mass around the equilibrium position x_o reads $\Delta E = \Delta p^2/2m + k_{har}\Delta x^2/2$, having omitted for simplicity the subscript x; owing to eq. (1.1), $E = E(\Delta p, n)$ is now because of n a random quantity within an energy range ΔE that corresponds to local uncertainty of dynamical variables within Δx and Δp. Both these latter and ΔE are assumed positive by definition. Then, one finds

$$\Delta E = \frac{\Delta p^2}{2m} + \frac{m(n\hbar\omega_{har})^2/2}{\Delta p^2}, \qquad \omega_{har}^2 = \frac{k_{har}}{m}. \tag{2.1}$$

Eq. (2.1) has a minimum as a function of Δp, i.e.

$$\Delta p_{\min} = \sqrt{mn\hbar\omega_{har}}, \qquad \Delta E_{\min} = n\hbar\omega_{har}, \tag{2.2}$$

being now n the number of vibrational states. Although for $n = 0$ there are no vibrational states, the necessity that $\Delta p \neq 0$ compels $\Delta E \neq 0$ and thus $\Delta E_0 = \Delta p_0^2/2m \neq 0$ with $\Delta p_0 = \Delta p_{\min}(n = 0)$. In this particular case, the problem reduces to that of a free particle in the box, i.e. Δp_0 is related to the zero point energy. This requires $\Delta p_0 = \Delta p_{\min}(n = 1)$, because the minimum quantum uncertainty of Δp can be nothing else but that of $\Delta p_{\min}$ for $n = 1$. The numerical correspondence between non-vibrational momentum range, Δp_0, and first vibrational momentum range, $\Delta p_{\min}(n = 1)$, means that at the zero point energy state the mass m is delocalized in a space range, $\Delta x_0 = \Delta x(n = 0)$, equal to that, $\Delta x(n = 1)$, pertinent to the lowest vibrational state. In other words, the oscillation amplitude at the ground energy level is the same as the delocalization range size of the particle with zero point energy only. Hence $\Delta p_0 = \sqrt{m\hbar\omega_{har}}$ defines $E_0 = \Delta p_0^2/2m = \hbar\omega/2$. The minimum of ΔE must be $\Delta E_{\min} = E_{\min} - \hbar\omega_{har}/2$; then, regarding $E_{\min} = E_{har}$ as the harmonic energy level, the known result

$$E_{har} = n\hbar\omega_{har} + \frac{\hbar\omega_{har}}{2} \tag{2.3}$$

is obtained considering uncertainty ranges of eq. (1.1) only, and without any further hypothesis. Note that with $\Delta p = \Delta p_{har}$

$$\frac{\Delta p^2}{2m} = \frac{\omega_{har}^2 mn^2\hbar^2}{2\Delta p^2} = \frac{n\hbar\omega_{har}}{2},$$

in agreement with the virial theorem as $E_{\min}$ is given by the sum of kinetic and potential terms, whereas the zero point term has kinetic character only. Also note in this respect that $\Delta p_{\min}$ and Δp_0 are merely particular range sizes, among all the ones allowed in principle, fulfilling the condition of minimum $E_{\min}$ and E_0.

These results do not contradict the complete arbitrariness of Δp and Δx, since in principle there is no compelling reason to regard the particular ranges of eqs. (2.2) in a different way with respect to all the other ones allowed by eq. (1.1); rather the results merely show the preferential propensity of nature for the states of minimum energy. In effect it is not surprising that the energy calculated with extremal values of dynamical variables in the ranges of eq. (2.1) does not coincide, in general, with the most probable energy. In conclusion, this example highlights that the physical properties of a quantum system can be inferred without solving any wave equation simply replacing the local dynamical variables with the respective quantum uncertainty ranges: the key problem becomes then that of counting correctly case by case the appropriate number of allowed states, as shown in [7,8] for more complex quantum systems.

It appears that, once accepting the eq. (1.1) and the consequent positions 1,2, have actual physical meaning the uncertainty ranges rather than the dynamical variables themselves; these latter are considered here random, unknown and unpredictable within the respective ranges and thus are disregarded since the beginning when formulating the physical problem. Just this is the essence of eq. (2.1). Eventually note that the vibrational quantum number n appears to be here the number of quantum states allowed to the oscillator. Since the present approach gives sensible results for harmonic oscillations, there is no reason to exclude that the same holds for anharmonic oscillations as well. The next paragraph aims to generalize the kind of approach just introduced to the case of anharmonic oscillations.

3 The anharmonic oscillator

The classical Hamiltonian reads now

$$E = p^2/2m + \sum_{i=2}^{N} a_i'(n\hbar)^{-i}(x - x_o)^i, \tag{3.1}$$

being N the arbitrary number of terms of the series including quadratic and anharmonic terms and a_i' proper coefficients assumed known; indeed the values of these coefficients characterize distinctively the specific kind of oscillating system. The signs of a_3' and a_4' are taken here negative [9]; the former expresses the asymmetry of the mutual repulsion between atoms or ions, e.g. in a metallic lattice, the latter describes the softening of the vibration at large amplitudes. The higher order terms allow to describe these effects in a more general way, so their sign and values must agree with the idea that the global consequence of anharmonicity is to lower the potential energy of oscillation; indeed the potential energy reads $a_2'(x - x_o)^2 f(x)$, i.e. it consists of a quadratic term with x-dependent correction factor $f(x) = 1 + \sum_{i=3}^{N}(a_i'/a_2')(x - x_o)^{i-2} < 1$. By analogy with the harmonic case, the coefficient of the quadratic term, anyway related to the force constant k_{an}, is reasonably expected to have still the form $m(n\hbar\omega_{an})^2/2$ with oscillation frequency defined now by $\omega_{an}^2 = k_{an}/m$. Moreover the dependence of this term on ω_{an} suggests that in general $a_i' = a_i'(\omega_{an})$ are to be expected as well.

The following discussion aims to guess this dependence and the relationship between ω_{an} and ω_{har} through the same

approach shown previously; so, as done in section 2, we aim to calculate $\Delta E_{\min}$ and infer next the anharmonic vibrational levels E_{an} and zero point energy E_0, being clearly $\Delta E_{\min} = E_{\min} - E_0$ and $E_{an} = E_{\min}$.

According to the position (1.2) and eq. (1.1), the quantum energy equation corresponding to eq. (3.1) reads

$$\Delta E = \frac{\Delta p^2}{2m} + \sum_{i=2}^{N} \frac{a'_i}{\Delta p^i}. \tag{3.2}$$

This equation, minimized with respect to the range Δp, yields

$$\Delta p_{\min} = m \sum_{i=2}^{N} i a'_i \Delta p_{\min}^{-(i+1)}, \tag{3.3}$$

being

$$\Delta E_{\min} = \Delta E(\Delta p_{\min}), \qquad \Delta p_{\min} = \Delta p_{\min}(\omega_{an}).$$

For assigned coefficients a'_i, the first equation admits in general $N + 2$ solutions $\Delta p_{\min}$, some of which can be however imaginary. Being the momentum uncertainty range Δp real positive by definition, let $I' \leq N + 2$ be the number of positive real roots; so I' possible values of $\Delta p_{\min}$ describe the allowed momentum ranges of the oscillating particle that fulfil the minimum condition. A further limitation to these values is that the series must converge. Disregard also the values of $\Delta p_{\min}$ that with the given a'_i possibly do not fulfil the inequality $\left|(i+1)a'_{i+1}\Delta p_{\min}^{-(i+2)}\right| << \left|ia'_i\Delta p_{\min}^{-(i+1)}\right|$ inferred from eq. (3.2), i.e.

$$\left|a'_{i+1}\right| << \left|a'_i \Delta p_{\min}\right|. \tag{3.4}$$

Then $I \leq I'$ is the number of real roots of physical interest to be considered in the following. Trivial manipulations of eq. (3.2) to eliminate m with the help of eq. (3.3) yield

$$\Delta E = \frac{1}{2}\left(\frac{\Delta p}{\Delta p_{\min}}\right)^2 \sum_{i=2}^{N} \frac{i a'_i}{\Delta p_{\min}^i} + \sum_{i=2}^{N} \frac{a'_i}{\Delta p^i}. \tag{3.5}$$

To extract the allowed physical information from this equation one should minimize with respect to Δp and then proceed as shown in the harmonic case. Actually this minimum condition has been already exploited to infer eq. (3.3) itself, which suggests that eq. (3.5) should not need being minimized once more. To understand this point replace Δp with $\Delta p_{\min}$ in eq. (3.5) and consider first the resulting equation $\Delta E(\Delta p_{\min}) = \sum_{i=2}^{N}(1 + i/2)a'_i\Delta p_{\min}^{-i}$ in the harmonic case; then $N = 2$, i.e. $a'_{i>2} = 0$, yields $3a'_2\Delta p_{\min}^{-2}/2$. By comparison with eq. (2.1) this result takes a more familiar form replacing a'_2 with $a_2\Delta p_{\min}^4/m$ where a_2 is a dimensionless proportionality coefficient linking a'_2 and $\Delta p_{\min}$; in this way one obtains

$$\Delta E(\Delta p_{\min}) = \frac{3a_2\Delta p_{\min}^2}{2m},$$

which has the same form of eqs. (2.2) a proportionality factor apart. As expected, an immediate connection with the harmonic case is possible uniquely on the basis of the condition 3,3 without introducing explicitly neither ω_{har} nor the equations of Δp_{har} and ΔE_{har}. Express thus in general the coefficients a'_i as a function of $\Delta p_{\min}$ as follows

$$a'_i = \frac{\Delta p_{\min}^{i+2}}{m} a_i, \qquad \sum_{i=2}^{N} i a_i = 1, \qquad 1 \leq j \leq I \tag{3.6}$$

where a_i are new constants that fulfil the boundary condition expressed by the second equation, straightforward consequence of eq. (3.3). Note that a'_i are uniquely defined for the specific oscillating system, whereas the appropriate notation of the various a_i should be $a_i^{(j)}$ to emphasize that a set of these coefficients is defined by each solution $\Delta p_{\min}^{(j)}$ of physical interest calculated through eq. (3.3). This would also compel indicating in eq. (3.5) $\Delta E^{(j)}$ and then $\Delta E_{\min}^{(j)}$. To simplify the notations the superscript (j) will be omitted, stressing however once for all that if $N > 2$ then eq. (3.5) actually represents anyone among I admissible equations. Replacing a'_i into the energy equation (3.5), one finds

$$\Delta E = \left(\left(\frac{\Delta p}{\Delta p_m}\right)^2 \frac{1}{2} + \sum_{i=2}^{N} a_i \left(\frac{\Delta p_m}{\Delta p}\right)^i\right) \frac{\Delta p_{\min}^2}{m}.$$

This suggests putting

$$q\frac{\Delta E}{\Delta E_{\min}} = \frac{1}{2}\left(\frac{\Delta p}{\Delta p_{\min}}\right)^2 + \sum_{i=2}^{N} a_i \left(\frac{\Delta p_{\min}}{\Delta p}\right)^i, \tag{3.7}$$

$$\Delta E_{\min} = q\frac{\Delta p_{\min}^2}{m}, \qquad a_2 = \frac{1}{2}\left(1 - \sum_{i=3}^{N} i a_i\right).$$

The proportionality factor q aims to fulfil the reasonable condition $\Delta E = \Delta E_{\min}$ for $\Delta p = \Delta p_{\min}$ and express in a general way the expected link between $\Delta E_{\min}$ and $\Delta p_{\min}^2/m$. Trivial calculations yield

$$q = 1 + \sum_{i=3}^{N}(1 - i/2)a_i. \tag{3.8}$$

Of course q must be intended here as $q^{(j)}$ likewise as $a_i^{(j)}$. Whatever a_i might be, eq. (3.7) does not need being minimized; it simply expresses as a function of $\Delta p/\Delta p_{\min}$ the energy deviation from the harmonic condition for assigned values of the coefficients $a'_{i\geq 2} \neq 0$. Eq. (3.7) and a_2 are uniquely defined in the particular case $a_{i>2} = 0$ only, which corresponds to $q = 1$ as well. Moreover the form of the second equation, analogous to that of eqs. (2.2), suggests that $\Delta p_{\min}$ and $\Delta E_{\min}$ must be also equal or proportional to the respective harmonic quantities Δp_{har} and ΔE_{har}. So putting

in general $\Delta E_{\min} = w^2 \Delta E_{har}$ and $\Delta p_{\min} = w \Delta p_{har}$, with w proportionality factor, one finds

$$\frac{q}{w^2}\frac{\Delta E}{\Delta E_{har}} = \frac{1}{2w^2}\left(\frac{\Delta p}{\Delta p_{har}}\right)^2 + \sum_{i=2}^{N} a_i w^i \left(\frac{\Delta p_{har}}{\Delta p}\right)^i, \qquad (3.9)$$

$$\omega_{an} = w^2 \omega_{har}.$$

Likewise q, also w must be intended in general as $w^{(j)}$. So eqs. (3.9) define I anharmonic frequencies $\omega_{an}^{(j)} \neq \omega_{har}$, here designated shortly ω_{an}, corresponding to the unique harmonic frequency ω_{har}; i.e. the various $\Delta E_{\min}$ describe the splitting of each n-th vibrational energy level $n\hbar\omega_{har}$. The anharmonic potential of eq. (3.9) is expected to depend upon ω_{an} through the dimensionless coefficients a_i, by analogy with the dependence of the harmonic term upon ω_{har}^2. Thus, to complete the task of the present section it is necessary: (i) to define the factor w of eq. (3.9); (ii) to highlight the analytical form of the functions $a_i = a_i(\omega_{an})$; (iii) to express the potential energy of equation (3.9) as a function of ω_{an} through these coefficients.

Rewrite to this purpose the coefficients of eq. (3.2) as shown in following series

$$q\Delta E = \frac{1}{2}\frac{\Delta p^2}{m} + \sum_{i=2}^{N} a_i'' \frac{m^{i/2}(n\hbar\omega_{an})^{i/2+1}}{\Delta p^i}, \qquad (3.10)$$

where the powers of $n\hbar\omega_{an}$ and m have been determined by dimensional consistency of the various terms with both ΔE and Δp^i. Minimizing with respect to Δp and equating to zero, one finds

$$R_E = \frac{1}{2}R_p^2 + \sum_{i=2}^{N} a_i'' R_p^{-i}, \qquad (3.11)$$

where

$$R_E = q\frac{\Delta E}{\Delta E_{har}}, \qquad R_p = \frac{\Delta p}{\Delta p_{har}}, \qquad a_i'' = a_i w^{i+2}.$$

With the coefficients a_i'' and a_i linked by the last position, eq. 3,(11) is identical to eq. (3.9); this consistency supports therefore the positions of both eqs. (3.6) and (3.10). To specify w put first $N = 2$ in eq. (3.9); minimizing $R_p^2/2w^2 + a_2 w^2/R_p^2$ with respect to R_p yields $R_p^4 = 2a_2 w^4$. Since the minimum of R_p can be nothing else but 1 by definition, $w = (2a_2)^{-1/4}$ yields $w = 1$, whereas in this particular case $a_2 = 1/2$. As expected, eq. (3.9) is thus uniquely defined for $a_{i>2} = 0$ only. Note that the coefficient of the quadratic term of eq. (3.10) reads $a_2'' m(n\hbar\omega_{an})^2$; if the result $w = (2a_2)^{-1/4}$ previously obtained for $N = 2$ still holds for any N with a_2 given now by the last eq. (3.7), then $a_2'' = a_2 w^4$ yields $a_2'' = 1/2$ and thus the expected form $m(n\hbar\omega_{an})^2/2$ formerly quoted whatever $a_{i>2}$ might be.

This consideration encourages one to conclude with the help of eq. (3.7)

$$w^2 = (2a_2)^{-1/2} = \left(1 - \sum_{i=3}^{N} i a_i\right)^{-1/2},$$

$$a_i'' = a_i \left(1 - \sum_{i=3}^{N} i a_i\right)^{-i/4-1/2}.$$

Replacing a_i'' in eq. (3.10) one finds

$$\Delta E = \frac{1}{2q}\frac{\Delta p^2}{m} + \\ + \sum_{i=2}^{N} q^{-1} a_i \frac{m^{i/2}(n\hbar\omega_{har})^{i/2+1}}{\Delta p^i}\left(1 - \sum_{i=3}^{N} i a_i\right)^{-\frac{3}{4}(i+2)}. \qquad (3.12)$$

This is the sought generalization of eq. (2.1) when $a'_{i>2} \neq 0$; the positions so far introduced link eq. (3.2) with the harmonic case. Moreover eq. (3.9) yields

$$\omega_{an} = \left(1 - \sum_{i=3}^{N} i a_i\right)^{-1/2} \omega_{har}. \qquad (3.13)$$

With the given choice of w^2, therefore, $a_{i\geq 3} = 0$ yield not only $\omega_{an} = \omega_{har}$ but also $\Delta p_{\min} = \Delta p_{har}$ and $\Delta E_{\min} = \Delta E_{har}$. Hence

$$\Delta E_{\min} = n\hbar\omega_{an} = \left(1 - \sum_{i=3}^{N} i a_i\right)^{-1/2} n\hbar\omega_{har}, \qquad (3.14)$$

$$\Delta p_{\min} = \sqrt{mn\hbar\omega_{an}} = \left(1 - \sum_{i=3}^{N} i a_i\right)^{-1/4} \sqrt{mn\hbar\omega_{har}}.$$

As concerns the zero point energy E_0 hold the considerations of the previous section, i.e. $\Delta E_{\min} = E_{\min} - E_0$; moreover also now for $n = 0$ the minimum of eq. (3.12) reduces to $\Delta p_0^2/2qm$, with $\Delta p_0^2 = \Delta p_{\min}^2 (n = 0)$. As explained before, even in lack of vibrational states $\Delta p_{\min} \neq 0$ compels putting $\Delta p_0 = \Delta p_{\min}^{(0)} (n = 1)$ by virtue of eq. (3.14) so that $E_0 = \left(1 - \sum_{i=3}^{N} i a_i^{(0)}\right)^{-1/2} \hbar\omega_{har}/2q$; since in general are allowed several values of $\Delta p_{\min}$, the notation emphasizes that one must consider here the set of values of $a_i^{(j)}$ corresponding to the smallest among the various $\Delta p_{\min}^{(j)}$.

In conclusion, since the anharmonic energy and momentum must correspond to the respective $\Delta E_{\min}$ and $\Delta p_{\min}$, it is possible to summarize the previous results, with full notation for clarity, as follows with the help of eq. (3.8)

$$E_{an}^{(j)} = \left(1 - \sum_{i=3}^{N} i a_i^{(j)}\right)^{-1/2} n^{(j)}\hbar\omega_{har} + \\ + \frac{1}{2}\left(1 + \sum_{i=3}^{N}(1 - i/2) a_i^{(0)}\right)^{-1} \left(1 - \sum_{i=3}^{N} i a_i^{(0)}\right)^{-1/2} \hbar\omega_{har}, \qquad (3.15)$$

with

$$1 \leq j \leq I,$$

$$\Delta p_{an}^{(j)} = \left(1 - \sum_{i=3}^{N} i a_i^{(j)}\right)^{-1/4} \sqrt{m n^{(j)} \hbar \omega_{har}},$$

$$\omega_{an}^{(j)} = \left(1 - \sum_{i=3}^{N} i a_i^{(j)}\right)^{-1/2} \omega_{har},$$

$$a_i^{(j)} = \frac{m a_i'}{\left(\Delta p_{\min}^{(j)}\right)^{i+2}}.$$

4 Discussion

The strategy of the papers [7, 8] to exploit via eq. (1.1) the classical Hamiltonians of the system of interest was outlined in section 2 and then extended in section 3 to the anharmonic case. The first task of the discussion aims to clarify the classical and quantum ways to regard the harmonic and anharmonic oscillation. The classical potential energy of eq. (3.1), $U_{cl} = U_{cl}(x - x_o)$, concerns a withholding force progressively increasing as a function of $x - x_o$ while the oscillation turns gradually from harmonic into anharmonic behaviour. Moreover if momentum and position of m are both exactly known, U_{cl} can be defined with arbitrary accuracy simply increasing the number of terms of the series.

This description is clearly inadequate for the potential energy, $U_q = U_q(\Delta p, n)$, of the quantum eq. (3.2); in principle the exact elongation of m with respect to the rest position and the corresponding momentum are not jointly specifiable, i.e. the limit $\Delta x \to 0$ could not be described by finite values of $\Delta p_{\min}$. Indeed $\Delta p_{\min} \to \infty$ compels $\Delta p \to \infty$ that yields $\Delta E = \Delta p^2/2m$ regardless of a_i'; this limit corresponds to the classical case of a free particle in a one-dimensional box, of no interest here, rather than to the harmonic limit expected for $\Delta x \to 0$. Eventually the quantum uncertainty compels regarding in a different way also the number of terms of U_{cl} and of U_q: in the former case N is in principle arbitrary, being significant its actual ability to provide a description as detailed as possible of the local state of motion of m, in the latter case does not, being instead significant its actual ability to introduce the allowed physical information into the system.

If for instance the model aims to describe softening and asymmetry effects only, then are justified terms like Δx^i with powers and signs [9] pertinent to these effects only. Solving eq. (3.1) requires exploiting the functional relationship U_{cl} upon Δx through numerical methods, solving eq. (3.2) requires instead a different reasoning because the anharmonic effects inherent the various Δx^i are related to the respective Δp^{-i} through eq. (1.1) only: the previous results show that a general physical principle, the minimum energy, is enough to this purpose. According to the classical eq. (3.1) the harmonicity requires $a'_{i\geq 3}\Delta x^i << a_2'\Delta x^2$ in agreement with the convergence condition (3.4); the quantum eq. (3.2) requires $a'_{i\geq 3}\Delta p^{-i} << a_2'\Delta p^{-2}$, which is still a statement of "small" oscillation amplitudes since $a_i'\Delta p^{-i} \propto a_i'\Delta x^i$. Both definitions are thus equivalent, yet the latter is more interesting because it involves eq. (1.1) and allows further considerations on the classical and quantum concepts of harmonicity. Eq. (3.4) and the first eq. (3.3) yield for $i \geq 3$

$$a'_{i\geq 3}\Delta p^{-i} << a_2'\Delta p^{-2} \Rightarrow a_i \left(\frac{\Delta p_{\min}}{\Delta p}\right)^i << a_2 \left(\frac{\Delta p_{\min}}{\Delta p}\right)^2.$$

Noting that Δp is arbitrary by assumption and that $\Delta p_{\min} \leq \Delta p$ by definition, it turns out that the second inequality can be merely fulfilled by $\Delta p/\Delta p_{\min} >> 1$ regardless of the values of the ratios a_2/a_i and a_2'/a_i'. Since in principle a_i' only are required to fulfil the convergence condition (3.4), whereas the values of a_i are ineffective in this respect because their values are consequently defined in the successive eq. (3.6) only, the conclusion is that small oscillation amplitudes do not require necessarily vanishing $a_{i>2}$. According to eq. (3.13), however, just these latter define w that in turn control ω_{an} and thus the splitting of energy levels. The fact that in general $w \equiv w^{(j)} \neq 1$ even for small oscillations supports the idea that the quantum harmonicity is a particular case, but not a limit case, of the quantum anharmonicity; in other words, an oscillating quantum system does not change gradually from harmonic to anharmonic behaviour.

This conclusion is confirmed also considering the dependence of the constants w on a_i. In eq. (3.6) large values of $\Delta p_{\min}$ entail small a_i and thus w such that the corresponding allowed frequencies ω_{an} are expected to have values similar to ω_{har}; the contrary holds for small values of $\Delta p_{\min}$, to which correspond larger values of w and therefore larger gaps $\omega_{an} - \omega_{har}$.

Hence, when considering the totality of allowed frequencies consistent with the different sizes of all ranges $\Delta p_{\min}$, even small values of a_i' classically compatible with the harmonic condition entail anyway relevant splitting and gap of energy levels with respect to ω_{har} typical of the anharmonicity; otherwise stated, the quantum harmonicity requires $a'_{i\geq 3} = 0$ exactly. The harmonic ground level is a reference energy rather than an attainable limit energy because fails the classical expectation of anharmonic frequencies progressively deviating from ω_{har} along with a_i'; the last eq. (3.7) shows indeed that even the first quadratic coefficient a_2 of potential energy differs from the corresponding harmonic coefficient unless $a_{i\geq 3} = 0$. It is also significant the fact that the unique ω_{har}, classically defined in eq. (2.1) through the force constant k_{har} of Hooke law only, never corresponds to a unique ω_{an} whatever $a'_{i\geq 3} \neq 0$ might be; this latter, although formally introduced in the early eq. (3.3) as $\omega_{an}^2 = k_{an}/m$, has quantum character after being subsequently redefined by eq. (3.9) through the multiplicity of values of w.

It is however worth noting in this respect a further chance to define the oscillation frequency in a mere quantum way through an uncertainty equation having a form seemingly different but conceptually equivalent to eq. (1.1). Introduce the

time range Δt necessary to displace m by Δx with finite average velocity v_x; defining then $\Delta t = \Delta x/v_x$ and $\Delta E = \Delta p v_x$, eq. (1.1) takes a form that introduces new dynamical variables t and E having random, unpredictable and unknown values within the respective uncertainty ranges defined by the same $n\hbar$. Of course Δt and ΔE are completely arbitrary, as they must be, likewise Δx and v_x. Thus, with the constrain of equal n, eq. (1.1) reads equivalently as

$$\Delta E \Delta t = n\hbar, \qquad \Delta t = t - t_o, \tag{4.1}$$

being the constant t_o the arbitrary origin of time coordinates. Eq. (4.1) is not a trivial copy of eq. (1.1): it introduces new information through v_x and shows that during successive time steps Δt the energy ranges ΔE change randomly and unpredictably depending on n. Of course the eq. (1.1) could have been inferred itself in the same way from eq. (4.1), i.e. regarding this latter as the fundamental statement. Relating eqs. (1.1) and (4.1) via the same arbitrary integer n, whatever it might be, means describing the oscillation of m through energy and time uncertainty ranges. This is equivalent to say that the time coordinate is regarded in an analogous way as the space coordinate hitherto concerned.

To show the consequences of this assertion, consider that $1/\Delta t$ has in general physical dimensions of frequency; then eq. (4.1) can be rewritten as $\Delta E_n = n\hbar\omega^{\S}$, being $\omega^{\S}$ a function somehow related to any frequency ω. If in particular $\omega^{\S}$ is specified to be just the previous frequency ω_{har}, whatever the value of this latter might be, eq. (4.1) reads

$$\Delta E_n = n\hbar\omega_{har}. \tag{4.2}$$

The notation emphasizes that the particular case $\omega^{\S} \equiv \omega_{har}$ enables a direct conceptual link with eq. (2.3), i.e. it concerns the harmonicity; having found that n is according to eq. (1.1) the number of vibrational states of the oscillator and $n\hbar\omega_{har}$ their energy levels, then without need of minimizing anything one infers that ΔE_n is again the energy gap between the n-th excited state of the harmonic oscillator and its ground state of zero point energy; the condition of minimum energy and $\Delta p_{\min}$ are now replaced by the specific meaning of Δt.

This conclusion shows that a particular property of the oscillating system is correlated to a particular property of the uncertainty ranges, thus confirming the actual physical meaning of these latter. So E_n falling within ΔE_n are still now random, unpredictable and unknown because of n. While ω_{har} was formerly defined by the formal position 2,1, now eq. (4.1) reveals its actual quantum meaning due to its direct link with the time uncertainty Δt.

This last result is significant for the present discussion: it justifies the different outcomes of the quantum approach with respect to the classical expectation in terms of uncertainty about the dynamical variables of m only; thus, as shown in [7, 8], this result disregards any phenomenological/classical hint to describe the system. In other words, instead of thinking to a withholding spring bound to a mass moving back and forth, the oscillation can be imagined in a more abstract way. It is enough to introduce an arbitrary energy range ΔE_n to which corresponds a respective quantum frequency $1/\Delta t_n$; then the form of eq. (1.1) is suitable to introduce an appropriate potential energy with elongation extent described by a unique quadratic term or by a series of terms, whose coefficients are respectively expressed as a function of ω_{har} or ω_{an} like in eqs. (2.1) or (3.10).

The worth of this conclusion is due to the generality of the resulting concept of oscillation, which skips any information on actual kind of motion of m, particular property of the oscillating mass, specific nature of the withholding force and hypothesis on the allowed range of frequencies. Both time and space uncertainties allow thus to describe an oscillating system in a fully quantum way, without writing and solving its wave equation. The previous results highlight the link of the allowed frequencies to the terms of U_q, see in particular the remarks about eqs. (3.5) and 3,13. A consequence of this point of view is that replacing U_{cl} with U_q compels the existence of several momentum uncertainty ranges $\Delta p_{\min}$ and thus of as many ω_{an} even when one would expect a mere perturbation of the unique ω_{har}: the physical information provided by the quadratic term only is uniquely defined, instead the various values of $\Delta p_{\min}$ and ω_{an} for $N > 2$ in eq. (3.2) reveal according to the last eq. (3.7) multiple anharmonic effects that influence also the quadratic term. The quantum uncertainty is therefore crucial in describing the oscillation.

For instance let us show that, at least for certain frequencies, the anharmonic oscillator appears to be a system intrinsically unstable. Let i be the index of any high order term of the series such that $a'_i/\Delta p^i << a'_2/\Delta p^2$ is true by definition because of the convergence condition; so $a'_i/\Delta p^i$ represents a small contribution to the total energy of oscillation. Let $\delta a'_i/\Delta p^i$ be its value altered by the change of the coefficient a_i because of an external perturbation acting on the oscillator; if for instance an impurity diffuses through the lattice in proximity of the given oscillating atom/ion, the stress field around this impurity or its possible charge field reasonably modify the local repulsion between atoms/ions or the softening effects at large oscillation amplitudes, as a consequence of which the anharmonic coefficients a'_3 and/or a'_4 are expected to change.

Let us exemplify any perturbation like this through a suitable change of some a'_i of the i-th energy terms in eq. (3.2); here however we consider for simplicity one term only to describe the local effect. The proof that some $\Delta p_{\min}$ and resulting $\Delta E_{\min}$ are strongly affected even by a very small change of any $a'_{i>2}$ is easy in the particular case where the series describing the potential energy converges very quickly. Differentiating eq. (3.6) one finds

$$\delta a'_i = a'_i \left((i+2) \frac{\delta \Delta p_{\min}}{\Delta p_{\min}} + \frac{\delta a_i}{a_i} \right).$$

Fix the value of $\delta a_i'$; if the local perturbation of the lattice affects a_i' in such a way that $\delta a_i' >> a_i'$, i.e. it alters significantly a_i, then the quantity in parenthesis is very large. If this happens while holds for $\delta a_i'$ also the condition $\delta a_i'/\Delta p^i << a_2'/\Delta p^2$, still possible because no hypothesis has been made on the strength of the perturbation, then considering that the quadratic term provides the most essential contribution to the total potential energy the result is: even a small perturbation $\delta a_i'/\Delta p^i$ of the whole oscillation energy is able to change significantly both $\Delta p_{\min}$ and a_i that define ω_{an}, see eqs. (3.13) to (3.15). The altered size of the range $\Delta p_{\min}$, actually verified by preliminary numerical simulations carried out with coefficients a_i' arbitrarily chosen to match the aforesaid condition, means in particular that the whole energy of the system admits not only a different ω_{an} allowed to the oscillator but also a larger range of corresponding momenta $p_{\min}$ allowed to m; this does not exclude even the chance of chaotic motion related to a random sequence of values ω_{an} during a weak perturbation transient due to the diffusing impurity.

The reason of such instability rests once again on the different way of regarding the oscillation amplitudes in classical and quantum physics. The former admits the limit $\Delta x \to 0$ regardless of Δp, the latter does not; so the quantum oscillation range of physical interest cannot be arbitrarily small or change arbitrarily without violating the crucial condition of minimum energy. Indeed the oscillation range sizes corresponding to the vibrational levels are quantized themselves

$$\Delta x_{\min} = \sqrt{\frac{n\hbar}{\omega_{an} m}}, \qquad \Delta x_0 = \sqrt{\frac{\hbar}{\omega_{an}^{(0)} m}}.$$

At this point it is worth remembering what has been previously emphasized, i.e. that the sizes of the ranges Δx and Δp are unspecified and indefinable; $\Delta x_{\min}$ and $\Delta p_{\min}$ are merely particular values showing the propensity of nature to fulfil the condition of minimum energy, however without contradicting the assumption that the uncertainty ranges are in principle completely arbitrary. So oscillation ranges that do not fulfil the former condition are certainly possible but unstable because of mere quantum reasons, i.e. they do not correspond to momentum range sizes that minimize the oscillation energy levels.

This conclusion is important because its validity follows uniquely from the assumption of convergence of the potential series only, i.e. it concerns a realistic condition effectively possible for the oscillator rather than an unusual and improbable limit case. Also, this result holds whatever the origin of the anharmonicity might be and confirms the physical diversity of harmonic and anharmonic quantum systems. Note however that the former is actually an ideal abstraction only; what can be expected in practice is a strong or weak anharmonicity, unless some specific physical reason requires just a potential energy with quadratic term only. So the results of the present approach should be regarded as the realistic behaviour of any oscillating system, rather than a sophisticated improvement of the naive harmonic behaviour; now this latter appears thus in general reductive and incomplete, rather than merely approximate. Yet eq. (3.15) shows that the zero point energy is formally analogous in both cases, a numerical difference apart: the only difference between the harmonic and anharmonic cases is that instead of considering the unique $\hbar\omega_{har}/2$ one must select the smallest $\omega_{an}^{(j)}$ to calculate $\hbar\omega_{an}^{(0)}/2$.

Note eventually that easy considerations allow to generalize the concept of perturbed oscillator in the conceptual frame of the present model. So far the present approach aimed to introduce the terms a_3 and a_4 to account for the anharmonicity, so that eqs. (3.2) to (3.15) tacitly assume an isolated oscillating system. Simple considerations however allow to further generalize the physical meaning of eq. (3.2) taking advantage of the fact that the present model works with a number of high order terms in principle arbitrary. In particular coefficients and number of terms could be exploited to describe even an oscillating system perturbed by an external force, for instance due to the interaction with other oscillators; indeed this force can be certainly described as a series development having the form $\sum a_i'' \Delta x^i$ if it is related, in the most general case non-linearly, to the displacement extent of the oscillating mass. Of course i can be even negative if the force vanishes at infinite distance. So, whatever the nature of the perturbation might be, this means that the potential energy of the system changes by an additional amount $-\sum a_i''(1+i)^{-1}\Delta x^{i+1}$ to be summed up with the corresponding terms of eq. (3.1). In any case, however, adding an arbitrary number of such energy terms to those intrinsically characterizing the oscillator does not change in principle the approach so far exposed, except of course the numerical value of the various a_i of eq. (3.9), which are now replaced by the sum $a_i'' + a_i'$ for each i-th power of oscillation elongation. So nothing hinders to regard the energy range ΔE_{an} of this equation as $\Delta E_{an+pert}$ still normalized to that of an isolated harmonic oscillator; it is enough that the coefficients a_i' up to the N-th order are still known, i.e. defined by the particular kind of oscillating system and external perturbation, yet without necessarily assuming any constrain on their signs, now determined by the sum of both effects. Even in the case where the force is described by terms like $\alpha'/\Delta x^i$ one would find an equation like (3.2) containing however terms like $a_k'\Delta p^k$ with $k > 2$. Also in this case, minimizing with respect to Δp would yield an appropriate number of roots $\Delta p_{\min}$ and thus prospective conclusions in principle completely analogous to that previously carried out. In the present case holds therefore the following position

$$\omega_{an+pert} \gtreqless w^2 \omega_{har}.$$

As expected, the previous scheme of vibrational levels is modified the external perturbation that affects w. This last result confirms the very general character of the way to describe any oscillating system simply with the help of the fundamental eq. (1.1).

5 Conclusion

The computational scheme introduced in the present paper is very simple: the most important achievements hitherto exposed do not require numerical calculations, but are consequence of general considerations on basic concepts of quantum mechanics. The general character of the approach, e.g. due to the arbitrary number N of anharmonic terms, and the possibility of extension to the case of a perturbed oscillator, propose the model as a useful tool in a broad variety of physical problems.

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Gravity and the Conservation of Energy

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The Schwarzschild metric apportions the energy equivalence of a mass into a time component, a space component and a gravitational component. This apportionment indicates there is a source of gravitational energy as well as a limit to the magnitude of gravitational energy.

1 Introduction

Albert Einstein asserted that his field equations are in essence a restatement of the conservation of energy and momentum [1, pp. 145–149]. Every solution of the field equations, therefore, must account for all energy in the system described by the solution. How do solutions to the field equations account for gravitational energy?

This paper explains how within Schwarzschild's solution [2] to Einstein's field equations the effects of gravity can be represented as a velocity and as an apportionment of mass-energy equivalence. This allows an accounting for gravitational energy as part of mass-energy equivalence.

The paper first considers a spacetime without gravity, as described by the Minkowski metric. The Minkowski metric can be rewritten as a summation of velocities and as an apportionment of energy equivalence.

The paper then shows the Schwarzschild metric, which adds a spherical non-rotating mass to the spacetime defined by the Minkowski metric, can also be rewritten as a summation of velocities and as an apportionment of energy equivalence. The apportionment of energy equivalence includes a gravitational component. This indicates gravitational energy has a source and a limit to its magnitude.

2 The Minkowski Metric

The Minkowski metric was originally derived based on Hermann Minkowski's fundamental axiom for space-time set out in an address [3] given in September 1908:

> *The substance at any world-point may always, with the appropriate determination of space and time, be looked upon as at rest.*

Minkowski's fundamental axiom for the space-time continuum indicates that for the substance at a world point (e.g., a particle) there exists a local reference frame, with its own local space and time coordinates, in which the substance is at rest with respect to the local space coordinates (but not with respect to the local time coordinate).

For example, assume the local reference frame for a particle has the local space coordinates (ξ, η, ς) and the local time coordinate τ. For the particle, with respect to the local reference frame,

$$\frac{d\xi}{d\tau} = \frac{d\eta}{d\tau} = \frac{d\varsigma}{d\tau} = 0. \tag{1}$$

The Minkowski metric is often expressed using Cartesian reference coordinates (x, y, z, t) and the local time coordinate τ, i.e.,

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2. \tag{2}$$

The Minkowski metric can also be expressed using spherical coordinates, i.e.,

$$c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - (r sin\theta)^2 d\phi^2. \tag{3}$$

3 Selection of a reference frame from which to measure velocity

In order to measure velocity in the Minkowski metric (and the Schwarzschild metric) it is important to select and consistently use a reference frame. In the Minkowski metric there are two reference frames to choose from. The first is the local reference frame defined by local coordinates $(\xi, \eta, \varsigma, \tau)$. The other is the reference frame (referred to herein as the coordinate reference frame) defined by reference coordinates (x, y, z, t).

There is a distinct disadvantage to use of the local reference frame to make measurements: in its own local reference frame an object is always at rest, that is, as indicated by (1) there is no spatial velocity, i.e., no change in the values of the local space coordinates (ξ, η, ς) with respect to passage of time as measured by the time coordinate τ.

In the coordinate reference frame, however, there can be a detectable motion through the space coordinates. This is referred to herein as spatial velocity $(\vec{v}_s)$, which is a vector sum of the motion in three dimensions of space, i.e.,

$$\vec{v}_s = \vec{v}_x + \vec{v}_y + \vec{v}_z, \tag{4}$$

and which has a magnitude v_s where

$$v_s = \left|\vec{v}_s\right| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}, \tag{5}$$

as measured by the coordinate reference frame.

Because of this distinct advantage of making measurements from the coordinate reference frame, this is the reference frame that will be consistently used herein to make measurements.

4 Expressing the Minkowski Metric as a sum of velocities

The Minkowski metric, shown in (2), can be rearranged into the form of a sum of velocities. Since the observer is making measurements from the coordinate reference frame, momentum and energy will need to be measured with respect to changes in the reference time coordinate t. The Minkowski metric is therefore rearranged to show this. Specifically, (2) can be rearranged as

$$c^2dt^2 = c^2d\tau^2 + dx^2 + dy^2 + dz^2, \tag{6}$$

and therefore,

$$c^2 = c^2\left(\frac{d\tau}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2, \tag{7}$$

which can be reduced to

$$c^2 = c^2\left(\frac{d\tau}{dt}\right)^2 + v_s^2. \tag{8}$$

Let a time velocity v_τ be defined as

$$v_\tau = c\frac{d\tau}{dt}, \tag{9}$$

so that v_τ is a measure of the rate of passage of time as measured by the local time coordinate τ with respect to the rate of the passage of time as measured by the reference time coordinate t. This allows (7) to be rewritten as

$$c^2 = v_\tau^2 + v_s^2. \tag{10}$$

Since the time dimension is regarded as being orthogonal to the space dimensions, (10) can be written in the form of a vector sum, i.e.,

$$c = \left|\vec{v}_\tau + \vec{v}_s\right|. \tag{11}$$

Equation (11) is the Minkowski metric written as a sum of velocities. That is, the vector sum of the velocity in the dimensions of time and space is always equal to the speed of light c.

5 Energy equivalence in the Minkowski metric

The Minkowski metric, like all solutions to Einstein's field equations, describes a matterless field [1, p. 143]. In order to see how the Minkowski metric apportions energy equivalence, it is only necessary to place a particle with mass m anywhere in the field. From (11), a momentum of mass m across four dimensions of time and space can be expressed as

$$mc = \left|m\vec{v}_\tau + m\vec{v}_s\right|. \tag{12}$$

Equation (10) can also be rewritten as

$$mc^2 = mv_\tau^2 + mv_s^2. \tag{13}$$

Equation (13) indicates how the Minkowski metric apportions the energy equivalence [4],

$$E = mc^2, \tag{14}$$

of mass m into an energy component E_τ in the time dimension, where

$$E_\tau = mv_\tau^2, \tag{15}$$

and an energy component in the space dimensions, where

$$E_s = mv_s^2, \tag{16}$$

so that

$$E = mc^2 = E_\tau + E_s. \tag{17}$$

6 The Schwarzschild metric

The full Schwarzschild metric for a spherical non-rotating mass M with a Schwarzschild radius R, is typically expressed with the reference coordinates in the form of spherical coordinates, i.e.,

$$c^2d\tau^2 = c^2\left(1-\frac{R}{r}\right)dt^2 - \frac{dr^2}{(1-R/r)} - r^2d\theta^2 - (r\sin\theta)^2d\phi^2. \tag{18}$$

When $M = 0$ and thus $R = 0$, the Schwarzschild metric reduces to the Minkowski metric.

7 Expressing the Schwarzschild Metric as a sum of velocities

In order to express the Schwarzschild metric as a sum of velocities, a gravitational velocity v_g can be defined using the Newtonian definition of gravitational escape velocity, that is

$$v_g = c\sqrt{\frac{R}{r}}. \tag{19}$$

Likewise because in the Schwarzschild metric space is curved a spatial velocity velocity v_{ss} through curved space can be defined as

$$v_{ss} = \sqrt{\frac{1}{1-R/r}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + (r\sin\theta)^2\left(\frac{d\phi}{dt}\right)^2}. \tag{20}$$

The Schwarzschild metric in (18) can now be expressed as a sum of the velocities v_τ, v_g and v_{ss}. That is, (19) can be rearranged as

$$\begin{aligned} c^2dt^2 = c^2d\tau^2 + c^2\frac{R}{r}dt^2 + \frac{dr^2}{(1-R/r)} + \\ + r^2d\theta^2 + (r\sin\theta)^2d\phi^2, \end{aligned} \tag{21}$$

and thus

$$\begin{aligned} c^2 = c^2\left(\frac{d\tau}{dt}\right)^2 + c^2\frac{R}{r} + \frac{1}{1-R/r}\left(\frac{dr}{dt}\right)^2 + \\ + r^2\left(\frac{d\theta}{dt}\right)^2 + (r\sin\theta)^2\left(\frac{d\phi}{dt}\right)^2. \end{aligned} \tag{22}$$

Using the definition of v_{ss} set out in (20), the definition of v_τ set out in (9) and the definition of v_g set out in (19), allows (22) to be simplified to

$$c^2 = v_\tau^2 + v_g^2 + v_{ss}^2. \tag{23}$$

If a gravitational dimension is regarded as being orthogonal to both the dimensions of curved space and the time dimension, (23) can be written in the form of a vector sum, i.e.,

$$c = \left|\vec{v}_\tau + \vec{v}_g + \vec{v}_{ss}\right|. \tag{24}$$

Equation (24) is the Schwarzschild metric written as a sum of velocities. That is, the vector sum of the velocity in the dimensions of time, space and gravity is always equal to the speed of light c.

8 Using the Schwarzschild metric to apportion energy equivalence

In order to see how the Schwarzschild metric apportions energy equivalence, it is only necessary to place a particle with mass m anywhere in the field. From (24), a momentum of mass m across five dimensions of time, space and gravity can be expressed as

$$mc = \left|m\vec{v}_\tau + m\vec{v}_g + m\vec{v}_{ss}\right|. \tag{25}$$

Equation (10) can also be rewritten as

$$mc^2 = mv_\tau^2 + mv_g^2 + mv_{ss}^2. \tag{26}$$

Equation (26) indicates how the Schwarzschild metric apportions the energy equivalence of mass m into an energy component E_τ, an energy component E_{ss} in the space dimensions, and an energy E_g component where

$$E_g = mv_g^2, \tag{27}$$

so that

$$E = mc^2 = E_\tau + E_g + E_{ss}. \tag{28}$$

9 Reciprocity in the apportionment of energy equivalence

In a system of two particles, one particle having a mass m_1 and a Schwarzschild radius of R_1 and the other particle having a mass m_2 and a Schwarzschild radius of R_2, the Schwarzschild metric allows the energy equivalence of each mass to be apportion into, time, space and gravity components. For example, when spatial coordinates (r_1, θ_1, ϕ_1) are measured with respect to m_1 and local time τ_1 is measured at the location of m_2, the energy equivalence of m_2 can be apportioned using the Schwarzchild metric,

$$c^2 d\tau_1^2 = c^2\left(1 - \frac{R_1}{r_1}\right)dt_1^2 - \frac{dr_1^2}{(1 - R_1/r_1)} - r_1^2 d\theta_1^2 - (r_1 \sin\theta_1)^2 d\phi_1^2, \tag{29}$$

into the following apportionment of energy equivalence:

$$m_2 c^2 = m_2 v_{\tau_1}^2 + m_2 v_{g_1}^2 + m_2 v_{ss_1}^2 = E_{\tau_1} + E_{g_1} + E_{s_1}. \tag{30}$$

Likewise, when spatial coordinates (r_2, θ_2, ϕ_2) are measured with respect to m_2 and local time τ_2 is measured at the location of m_1, the energy equivalence of m_1 can be apportioned using the Schwarzschild metric,

$$c^2 d\tau_2^2 = c^2\left(1 - \frac{R_2}{r_2}\right)dt_2^2 - \frac{dr_2^2}{(1 - R_2/r_2)} - r_2^2 d\theta_2^2 - (r_2 \sin\theta_2)^2 d\phi_2^2, \tag{31}$$

into the following apportionment of energy equivalence:

$$m_1 c^2 = m_1 v_{\tau_2}^2 + m_1 v_{g_2}^2 + m_1 v_{ss_2}^2 = E_{\tau_2} + E_{g_2} + E_{s_2}. \tag{32}$$

10 Implications

The Schwarzschild metric apportions the energy equivalence of a mass into a time component, a spatial component and a gravitational component. This suggests that the source of gravitational energy is the energy equivalence of the mass affected by gravity and therefore that the magnitude of gravitational energy cannot exceed the energy equivalence of that mass. As pointed out by Weller [5, 6], this presents a very significant difficulty for those who view gravity as an unlimited source of energy to perform such tasks as forming black holes and creating universes. This also tends to confirm the assertions of Schwarzschild [7] and Einstein [8] that there is indeed a maximum density of matter.

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A Note on the Quantization Mechanism within the Cold Big Bang Cosmology

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In my paper [3], I obtain a Cold Big Bang Cosmology, fitting the cosmological data, with an absolute zero primordial temperature, a natural cutoff for the cosmological data to a vanishingly small entropy at a singular microstate of a comoving domain of the cosmological fluid. This solution resides on a negative pressure solution from the general relativity field equation and on a postulate regarding a Heisenberg indeterminacy mechanism related to the energy fluctuation obtained from the solution of the field equations under the Robertson-Walker comoving elementar line element context in virtue of the adoption of the Cosmological Principle. In this paper, we see the, positive, differential energy fluctuation, purely obtained from the general relativity cosmological solution in [3], leads to the quantum mechanical argument of the postulate in [3], provided this energy fluctuation is quantized, strongly supporting the postulate in [3]. I discuss the postulate in [3], showing the result for the energy fluctuation follows from a discreteness hypothesis.

1 To the Heisenberg Indeterminacy Relation

Recalling the eqn. (53) in [3], purely derived from the general relativity field equations under the cosmological context:

$$\delta E_\rho = \frac{E_0^+}{\sqrt{1-\dot{R}^2/c^2}}\frac{\dot{R}\,\delta\dot{R}}{c^2}, \tag{1}$$

the δE_ρ, given by the eqn. (1), seems to be exclusively valid when $\delta\dot{R}$ is infinitesimal, since this expression is a first order expansion term, where we do tacitly suppose the vanishing of high order terms. But its form will remain valid in a case of finite variation, as derived is this paper, under the same conditions presented in [3]. The eqn. (1), in terms of indeterminacy, says:

- There is an indeterminacy δE_ρ, at a given t, hence at a given $R(t)$ and $\dot{R}(t)$, related to a *small* inteterminacy $\delta\dot{R}(t)$.

A given spherical shell within a t-sliced hypersurface of simultaneity must enclose the following indeterminacy, if the least possible infinitesimal continuous variation given by the field equations in [3], eqn.(1) here, presents discreteness, viz., if the δE_ρ cannot be an infinitesimal in its entire meaning, albeit mantaining its very small value, as a vanishingly small quantity, but reaching a minimum, reaching a discrete quantum of energy fluctuation,

$$\sum_{l=1}^{k}\left(\delta E_\rho\right)_l = \left.\frac{E_0^+\dot{R}/c^2}{\sqrt{1-\dot{R}^2/c^2}}\right|_t \sum_{l=1}^{k}\left(\delta\dot{R}\right)_l. \tag{2}$$

The eqn. (2) is obtained from eqn. (1) by the summation over the simultaneous fluctuations within the spherical shell (since the quantum minimal energy is a spatially localized object, and the t-sliced spherical shell, a $R(t)$-spherical subset of simultaneous cosmological points pertaining to a t-sliced hypersurface of simultaneity, is full of cosmological substratum), where k denotes a partition, k fundamental fluctuating pieces of the simultaneous spacelike spherical shell within a t-sliced hypersurface. This sum gives the entire fluctuation within the shell. Since these pieces are within a hypersurface of simulteneity, they have got the same cosmological instant t. Hence, they have the same $R(t)$ and the same $\dot{R}(t)$ (points within the t-sliced spherical shell cannot have different $R(t)$, since $R(t)$ is a one-to-one function $R(t) : t \mapsto R(t)$, and does not depend on spacelike variables; the t-sliced spherical shell is a set of instantaneous points pertaining to a t-sliced hypersurface of simultaneity such that these points are spatially distributed over an t-instantaneous volume enclosed by a t-instantaneous spherical surface with radius $R(t)$), the reason why the summation index l does not take into account the common factor at the right-hand side of the eqn. (2). From eqn. (57) in [3], we rewrite the eqn. (2):

$$\sum_{l=1}^{k}\left(\delta E_\rho\right)_l = \left.\frac{E_0^+R_0^2}{R^3\sqrt{1-\dot{R}^2/c^2}}\right|_t \sum_{l=1}^{k}\left(\delta R\right)_l. \tag{3}$$

Now, we reach the total instantaneous fluctuations within the spherical shell at the cosmological instant t, a sum of spacelike localized instantaneous fundamental fluctuations within the spherical shell, giving the total instantaneous fuctuation within this shell. Being the instantaneous spherical shell full of cosmological fluid at t, at each fundamental position within the spherical shell we have got a fundamental energy fluctuation with its intrinsical and fundamental quantum $R_0 = \sqrt{2Gh/c^3}$ of indeterminacy [3], an inherent spherically symmetric indeterminacy at each position within the t-sliced spacelike shell.

Hence, the total fluctuation is now quantized:

$$N_t \delta E_\rho = \left. \frac{E_0^+ R_0^2}{R^3 \sqrt{1 - \dot{R}^2/c^2}} \right|_t N_t R_0, \qquad (4)$$

where N_t is the number of instantaneous fundamental domains, the number of fundamental fluctuations within the instantaneous spherical shell contained within a t-sliced hypersurface of simultaneity. Since R_0 is a fundamental quantum of local indeterminacy, the same R_0 is common to all the instantaneous spacelike points within the shell, the same $(\delta R)_l = R_0$ quantum of fluctuation at its respective point within the t-instantaneous spherical shell contained in a t-sliced surface of simultaneity for all the points in this shell, $\forall\ l$.* But N_t is given by:

$$N_t = \frac{R^3}{R_0^3}. \qquad (5)$$

Using the eqn. (5) in the eqn. (4), we obtain:

$$N_t\, \delta E_\rho = \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}}. \qquad (6)$$

The eqn. (6) gives the total positive fluctuation whitin the t-instantaneous spherical shell, the result used in my postulate in [3]. Furthermore, comparing the eqns. (1) and (6), we see the infinitesimal relation given by the eqn. (1) is valid in the finite fluctuation process given by the eqn. (6), provided $\dot{R}\delta\dot{R} \approx c^2$, a result used in the appendix of [3] to obtain its eqn. (56).

The Heisenberg indeterminacy principle reads, for the entire fluctuation at a given t:

$$\left(N_t\, \delta E_\rho\right) \delta t = \frac{E_0^+ \delta t}{\sqrt{1 - \dot{R}^2/c^2}} \geq \frac{h}{4\pi}. \qquad (7)$$

The increasing smearing out indeterminacy over the cosmological fluid, related to the primordial indeterminacy in virtue of the Universe expansion as postulated in [3]:

- The actual energy content of the universe is a consequence of the increasing indeterminacy of the primordial era. Any origin of a comoving reference frame within the cosmological substratum has an inherent indeterminacy. Hence, the indeterminacy of the energy content of the universe may create the impression that the universe has not enough energy, raising illusions as dark energy and dark matter speculations. In other words, since the original source of energy emerges as an indeterminacy, we postulate this indeterminacy continues being the energy content of the observational universe: $\delta E(t) = E^+(t) = E_0^+ / \sqrt{1 - \dot{R}^2/c^2}$,

follows from the increasing N_t, as one infers from the eqns. (5) and (7).

*See [3]. We are in a context of validity of the Cosmological Principle.

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Comments on the Statistical Nature and on the Irreversibility of the Wave Function Collapse

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In a previous preprint, [2], reproduced here within the appendix in its revised version, we were confronted, to reach the validity of the second law of thermodynamics for an unique collapse of an unique quantum object, to the necessity of an ensemble of measures to be accomplished within copies of identical isolated systems. The validity of the second law of thermodynamics within the context of the wave function collapse was sustained by the large number of microstates related to a given collapsed state. Now, we will consider just one pure initial state containing just one initial state of the quantum subsystem, not an ensemble of identically prepared initial quantum subsystems, e.g., just one photon from a very low intensity beam prepared with an equiprobable eigenset containing two elements, an unique observation raising two likelihood outcomes. Again, we will show the statistical interpretation must prevail, albeit the quantum subsystem being a singular, unique, pure state element within its unitary quantum subsystem ensemble set. This feature leads to an inherent probabilistic character, even for a pure one-element quantum subsystem object.

1 A toy: the fair coin eigenset

Let a two-state coin, fifty-fifty, with eigenset $\{\phi_1, \phi_2\}$, be our quantum subsystem. The initial state of this unique subsystem reads:

$$\Psi = \sum_{k=1}^{2} a_k \, \phi_k = \frac{\sqrt{2}}{2} \, \phi_1 + \frac{\sqrt{2}}{2} \, \phi_2 \, , \tag{1}$$

with:

$$a_k = \int_V \phi_k^* \Psi \, dV = \frac{\sqrt{2}}{2} \;\; \forall \;\; k \in \{1, 2\} \, . \tag{2}$$

The eigenstates ϕ_1 and ϕ_2 are different eigenstates.

The unique element [given by eqn. (1)] subsystem plus an unique ideal apparatus subsystem Φ will define an isolated system. The memory state of the subsystem apparatus is initially empty, and the initial state of the system is:

$$\Psi\Phi|_{t=0} = \left(\frac{\sqrt{2}}{2} \, \phi_1 + \frac{\sqrt{2}}{2} \, \phi_2 \right) \Phi_{[\text{void}]} . \tag{3}$$

After a measure operator U acting on $\Psi \otimes \Phi|_{t=0}$, the system propagates forward in time to the $(t = \tau)$-state, the collapsed state for short:

$$\Psi\Phi|_{t=\tau} = \frac{\sqrt{2}}{2} \, \phi_1 \, \phi_{[\phi_1]} + \frac{\sqrt{2}}{2} \, \phi_2 \, \phi_{[\phi_2]} \, . \tag{4}$$

The observer is represented by the Φ apparatus subsystem, being in its own Hilbert state space H_Φ. Since $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$ are different states belonging to H_Φ, these apparatus states are mutually exclusive in H_Φ.

- How many final microstates of the isolated system are there?

The answer depends on which space the apparatus Φ resides. For Φ, the collapsed microstate is a member of H_Φ. The state given by the eqn. (4) cannot be observed in H_Φ, hence cannot be counted from H_Φ by the apparatus subsystem. There are two possible final states for the hypothetical one-element measure that are members of H_Φ, $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$, but both cannot be obtained at the same time. The collapse evolves but just one member of H_Φ subsists as an equilibrium apparatus subsystem state after the collapse. The entropy of a final collapsed state $\Phi_{[\phi_k]}$ in $H_\Phi(\tau)$ is zero, since, under an one-element measure with an unique initial quantum coin state given by eqn. (1), there is just one manner to obtain the $\Phi_{[\phi_k]}$ collapsed state, since the other equally like manner leads to a different collapsed state and should not be considered as being another microstate of the same $\Phi_{[\phi_k]}$. But both the possible collapsed states leads to a same final null entropy. This unique object measure leads to reversible collapse, since the variation of entropy between states is null in any case. We will see this unique object quantum subsystem must be related to a global statistical context.

Choosing an unique coin to accomplish an unique measure, one is establishing there exists just an unique way to obtain the initial coin, to construct the initial coin. But, in fact, there is not. You may make the same coin with another bunch of metallic atoms. We do not take it into account, since a set of identical elements is an unitary set, being irrelevant which element we use to accomplish the measure. But two distinct but identical coins do not necessarily lead to identical outcomes. Hence, if one takes into account the identical man-

ners, including the previous global context within the Universe, from which the system may evolve to the collapse, one does not modify the initial null entropy of the system, since identical coins are identical coins into the input ($t = 0$) but not necessarily identical coins from the output ($t = \tau$). Suppose you may construct the unique coin only from two different ways, $\mathbb{W}_1$ or $\mathbb{W}_2$. Via $\mathbb{W}_1$, there is one possible microstate for each collapsed result as observed by $\Phi_{[\phi_1]}$ *or* $\Phi_{[\phi_2]}$ in the apparatus subsystem reality. In the apparatus reality, the initial number of microstates of the system is also vanished, since the initial number of microstates is 1×1 (in the apparatus world, we do not describe the system via eqn. (3), since this is an object that is not an element of H_Φ. The initial state of the quantum subsystem, our coin, given by the eqn. (1), is unique for $\Phi_{[\text{void}]}$. Initially, there is just one possibility for each subsystem state in the apparatus reality, hence $w_0 = 1 \times 1$ is the initial number of microstates of the [global] system as observed within the apparatus reality. The apparatus dialectics does not handle objects like the ones in the eqns. (3) and (4).). Analogously, via $\mathbb{W}_2$, there is one possible microstate for each collapsed state. But, when $\mathbb{W}_1$ and $\mathbb{W}_2$ are taken into consideration, two possible microstates emerge for each collapsed state, with the same initial null entropy.

When one accomplishes an one-element collapse experiment with various identical initial quantum subsystems (e.g., taking $\mathbb{W}_1$ and $\mathbb{W}_2$ into consideration), the result is one between the possible ones from identical objects (indistinguishable coins). A particular collapse result turns out to be inserted in a global probabilistic context related to the various identical manners by which the Universe may evolve from the past to their states in which there exist identical isolated experiments to be initiated at $t = 0$, in virtue of the entropic evolution of the Universe. The Universe entropically evolves under their various possibilities, and two different manners to construct a same coin are different ways under which the Universe may evolve to a same initial coin state, hence the null entropy, but not necessarily to the same collapsed state. Hence, even an isolated collapse from an unique coin has a global statistical context related to the different manners the Universe might have evolved, and an unique coin exhibits its global statistical bias. Since the Universe is large, a given initial subsystem, our two-state coin initial quantum subsystem, has a miriad of possible histories up to $t = 0$, say N_1, but with none of these manners giving a different object, all giving the same Ψ at $t = 0$. Analogously, one has, as $\Phi_{[\text{void}]}$ possible initial states, a bag with N_2 identical elements, all given by $\Phi_{[\text{void}]}$. When you isolate the system, you obtain an isolated bag with $N_1 \times N_2$ identical elements given by the eqn. (3). The number of microstates related to this bag is $w_0 = 1$. The number of microstates related to $\Phi_{[\phi_k]}$ is not $w_f = 1$ anymore, but [2]:

$$\lim_{N_1 N_2 \to \infty} \sum_{l=1}^{N_1 N_2} \xi_l^p = \lim_{N_1 N_2 \to \infty} \left[\frac{N_1 N_2}{2} + f(N_1 N_2) \right] > 1, \quad (5)$$

being $N_1 \times N_2$ the number of final histories of collapse, where the histories are, now, being instantaneously counted at τ, within the Universe entropic evolution.

Taking into account the the different manners by which an initial subsystem may be obtained does not change the probability of a given collapsed state, conversely, defines it via a natural frequentist interpretation within a global context. The probability associated to a given collapsed state when an unique experiment is accomplished with an unique one-element initial state is the one associated to the frequentist interpretation taking into account the various manners to construct the initial state. Since the Universe may provide infinitely many manners to construct an isolated system, when one takes an exemplar into account, the probabilistical character is inherent to individual processes, since a particular result resides within a global statistical context related to different states of the Universe that leads to the same initial isolated system. Even a single photon within a low intensity beam may be constructed by different manners. A single photon does not know this, obviously, but it behaves under a global statistical context related to the different manners by which the Universe may evolve to that in which a beam of a single photon is within an isolated system with an apparatus.

There are not two final microstates, $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$, for the collapsed apparatus, and one should not say the entropy variation is $\Delta S = k \ln 2 - k \ln 1$, since different microstates are physically distinguishable a posteriori, carrying different measurable physical properties, encapsulated within the difference between the eigenvectors $\Phi_{[\phi_1]}$ and $\Phi_{[\phi_2]}$. In fact, an unique one-element collapse is a reversible process for quantum initial subsystems with just an unique element. But it is very difficult to observe, since the Universe entropically evolves among a miriad of possibilities leading to identical initial quantum subsystems, inserting an individual measure within the Universe's entropy evolution statistical context, being the number of final collapsed microstates of a given collapsed state greater than 1, leading to an irreversible collapse even with a single photon beam as initial quantum subsystem, e.g., since this single photon within the beam turns out to be in a context of a very large number of available microstates for each possible collapsed state, a context in which the final entropy of a given collapsed state is greater than the initial null entropy.

2 Appendix: comments on the entropy of the wave function collapse

2.1 The Boltzmann formula: a source of misconception for a reckless vision

At a first glance [1], one may think the wave function collapse violates the second law of thermodynamics, since a quantum system prepared as a superposition of eigenstates of a given operator suddenly undergoes to a more restrictive state. But this is not the case, in virtue of the fact that a superposition

and a eigenstate are states on equal footing. The use of the Boltzmann formula:

$$S = k \ln w, \tag{6}$$

for the entropy S of a thermodynamically closed system leads, at a first glance, to the impression that the entropy should have a greater value before the collapse, under an erroneous assumption that the initial number, w_0, of microstates, w, should be greater than the final number of microstates, w_f, in virtue of the needed quantity of eigenstates, $w_0 > 1$, used to construct the wave function before the collapse, in contrast to the apparent $w_f = 1$ after the collapse, where $k = 1.38 \times 10^{-23}\, JK^{-1}$ is the Boltzmann constant. We will see that the converse occurs. Furthermore, one should, firstly, define the thermodynamically closed system as consisting of two subsystems: the quantum object subsystem plus the classical apparatus subsystem.

2.2 A simple solution for this apparent paradox

Consider a quantum subsystem Ψ [4]: prepared as a superposition of the n eigenstates $\{\phi_k\}$, with $1 \leq k \leq n$, of a given operator Φ with finite non-degenerated spectrum:

$$\Psi = a_1 \phi_1 + \cdots + a_n \phi_n = \sum_{k=1}^{n} a_k \phi_k, \tag{7}$$

where:

$$a_k = \int_V \phi_k^* \Psi dV, \tag{8}$$

is the inner product the Hilbert state space is equipped with. The * denotes the complex conjugation and dV the elementar volume of the physical space V of a given representation.

Up to the measure, before the interaction between a classical apparatus subsystem, designed to obtain observable eingenvalues of the operator Φ, and a quantum subsystem Ψ given by eqn. (7), there exists just one microstate of the global system consisted by apparatus subsystem plus quantum subsystem, since these two subsystems are not initially correlated and the initial microstate of the quantum subsystem Ψ is just the unique state Ψ as well the initial microstate of the classical apparatus subsystem is the unique one in which it has no registered eigenvalue.

Hence, in virtue of the initial independence of the subsystems, the initial microstate of the global thermodynamically closed system has multiplicity $w_0 = 1 \times 1 = 1$, being the initial entropy of the global system given by:

$$S_0 = k \ln 1 = 0, \tag{9}$$

in virtue of the eqn. (6).

One may argue the initial state of the classical apparatus subsystem has got a multiplicity greater than 1, since this subsystem seems to have internal modes compatible with an empty memory. We emphasize this is not the case, since the state of the memory defines the apparatus state, being this state an empty one in spite of any apparatus internal modes before an accomplished measure*. The same comment is valid for the quantum subsystem, since the state of this subsystem is Ψ, previously defined by the superposition of a Φ operator eigenstates, $\{\phi_k\}$, being the object Ψ an unique one. These objects, by definition, are initially constrained to these defined states, and one does not need to take into account the different manners by which these subsystems should equally evolve to their respective initial states.

Once a measure is accomplished, there will exist n possible eigenvalues to be registered within the memory of the classical apparatus subsystem, viz., since there are n different final situations for the global system, where n is the number of non-degenerated eigenvectors of the Φ operator. A reckless short-term analysis would lead to the conclusion that the final number of microstates of the global system, w_f, should be $w_f = n$, since it seems to be the number of ways by which a final collapsed state is reached. But such a conclusion is wrong, since the final state is not simply a collapsed one with a label on it. Differently from a case in which a pair of unbiased dice is thrown, where a particular result of a throw of dice is not physically different from any other result, except for the labels on them, a given collapsed state encapsulates physical content. Each collapsed state is a different final state with its characteristic multiplicity, and one should not enroll the possible collapsed states within a same bag with $w_f = n$ possible collapsed elements. Comparing with the throw of dice case, if you erased the dice numbers, their labels, you could not infer the difference between the results, but the physical content within the collapsed wave function result would lead one to infer the difference between different results, between different outcomes of collapse of Ψ.

- Different physical characteristics imply different outcomes for the wave function collapse and define evolutions from the initial global system to new states of the global system, instead of different configurations for a same final state.

In the throw of dice example, the different outcomes are different configurations of a same final state. If the collapsed

*The irrelevance of the apparatus internal modes compatible with a given apparatus memory state asserts the hypothesis of an unbiased apparatus subsystem. Any result to be measured by the apparatus subsystem must have the same number of equally like apparatus microstates. If some result was related to a different number of apparatus compatible microstates, the results with the maximal number of apparatus compatible microstates would be biased. The collapse should not be caused by apparata biases. In virtue of this hypothesis, one may neglect the apparatus internal modes compatible with a particular apparatus memory state, since the same number of internal modes is common to all the memory states, and the variation of entropy cancels out the same common number (say w_a): $\Delta S = S_f - S_0 = k \ln\left(w_f \times w_a\right) - k \ln(1 \times 1 \times w_a) = k \ln w_f - k \ln(1 \times 1)$, where w_f is the number of microstates of a given final state of the global isolated system in which the apparatus has registered the respective collapsed state, considering the apparatus memory state as its unique degree of freedom.

wave function was a state with n different possible configurations for this same collapsed state, the final number of microstates would be $w_f = n$, but this is not the case.

For the collapsed states, the multiplicities of the possible final results are not necessarily the same, since they depend on the outcome probabilities of their respective eigenvalues. Let p be the label of the eigenvalue with the least reliable ($\neq 0$)* outcome probability. The outcome probability of a given eigenvalue is given by Max Born's rule, from which the least probability, of the p-labeled eigenvalue, is simply given by $a_p^* a_p$, where [see eqn. (8)]:

$$a_p^* a_p = \left| \int_V \phi_p^* \Psi dV \right|^2 \neq 0. \tag{10}$$

Applying a frequentist interpretation for the probability, the least multiplicity of microstates is† $N a_p^* a_p$, where N is the quantity of *state-balls* within an *a posteriori interpreted quantum-subsystem-urn* (we are emphasizing that the interaction with the classical apparatus subsystem permits a classical‡, under the frequentist sense, a posteriori, interpretation of probabilities, since any quantum effects of probabilistic superposition of amplitudes cease after the collapse, permitting a frequentist interpretation via Born's rule). Such a frequentist interpretation requires $N \to \infty$, i.e., infinitely many measures to be accomplished on identical quantum subsystems by the classical apparatus subsystem, but we will back to this point later.

The least final entropy of the global system, related to the outcome probability of the p-labeled eigenvalue, reads:

$$S_f = k \ln \left(N a_p^* a_p \right). \tag{11}$$

From the eqns. (9) and (11), the least possible entropy variation turns out to be:

$$\Delta S = S_f - S_0 = k \ln \left(N a_p^* a_p \right). \tag{12}$$

From the eqn. (12), we infer that the second law of thermodynamics holds *iff*:

$$N a_p^* a_p \geq 1 \Rightarrow a_p^* a_p \geq \frac{1}{N}, \tag{13}$$

since $N > 0$. Now, we will prove the following theorem:

Theorem: *The second law of thermodynamics holds for the wave function collapse under a frequentist interpretation via Max Born's rule and, once accomplished the collapse, the collapse is an irreversible phenomenon.*

Proof: Suppose the converse, i.e., that the second law of thermodynamics does not hold for the wave function collapse under a frequentist interpretation via Max Born's rule. In virtue of eqn. (12), one has:

$$\Delta S = S_f - S_0 = k \ln \left(N a_p^* a_p \right) < 0 \Rightarrow N a_p^* a_p < 1. \tag{14}$$

Since§ $a_p \neq 0$, $N \geq 1/(a_p^* a_p)$ violates the condition stated by the eqn. (14). But $N \to \infty$, in virtue of the frequentist interpretation, hence $N > 1/(a_p^* a_p)$, and the eqn. (14) is an absurd. We conclude the second law of thermodynamics holds within the terms of this theorem. The proof the collapse is an irreversible phenomenon follows as a corollary of this theorem. In fact:

$$N > 1/(a_p^* a_p) \Rightarrow N a_p^* a_p > 1 \therefore$$

$$\Delta S = k \ln \left(N a_p^* a_p \right) > 0, \tag{15}$$

and the collapse of the wave function is an irreversible phenomenon, being $\Delta S > 0$ the entropy variation of the thermodynamically closed system: quantum subsystem plus classical apparatus subsystem. ■

The law of large numbers states the probability of an event p, P_p, is given by the limit:

$$\lim_{N \to \infty} \frac{\sum_{l=1}^N \xi_l^p}{N} = P_p \,, \tag{16}$$

where ξ_l^p assumes the value 1 when the event p occurs, or zero otherwise. If $a_p^* a_p \equiv P_p \neq 0$, the limit must obey:

$$\lim_{N \to \infty} \frac{\sum_{l=1}^N \xi_l^p}{N} = \frac{\lim_{N \to \infty} \sum_{l=1}^N \xi_l^p}{\lim_{N \to \infty} N} \neq 0. \tag{17}$$

From eqn. (17), we conclude $\lim_{N \to \infty} \sum_{l=1}^N \xi_l^p$ cannot be finite, since N grows without limit. Hence:

$$\lim_{N \to \infty} \sum_{l=1}^N \xi_l^p > 1. \tag{18}$$

Particularly, the eqn. (18) gives the number of microstates of the p-labelled eigenstate, proving the above theorem. Rigorously, one should substitute:

$$N \to N + \frac{f(N)}{a_p^* a_p}, \tag{19}$$

*If $a_p = 0$, the respective eigenstate ϕ_p, within the superposition representing Ψ [see eqn. (7)], turns out to be an impossible collapsed state. Such consideration would be totally void, since the final microstate associated to it would never occur, being $\Delta S = k \ln 0 - k \ln 1 = -\infty$ [see eqns. (6) and (9)] a violation of the second law of thermodynamics, in accordance with the impossibility of a final microstate with $a_p = 0$.

†See the discussion leading to the eqns. (19) and (20), regarding the meaning of N.

‡Here, the classical designation resides within the counting process after the collapse. We are not saying the final collapsed state leads to a classical interpretation of the quantum object, we are emphasizing that the dialectics after the collapse to interpret frequency of a given collapsed state is the classical one via Born's rule. One does not count quantum waves, but the discrete signals of a collapsed object. Surely, alluding, e.g., to the double-slit canonical example, the diffraction pattern on the screen has not a discrete counterpart, but the points on the screen, when the intensity of the source is reduced, have and may be counted.

§Remember the reliability defining the p-labeled eigenstate, see eqn. (10) again and its inherent paragraph.

within the above theorem proof, with:

$$\lim_{N\to\infty} \frac{f(N)}{N} = 0. \tag{20}$$

Such choice leads to:

$$\sum_{l=1}^{N} \xi_l^p = N a_p^* a_p = \left(N + \frac{f(N)}{a_p^* a_p}\right) a_p^* a_p = \\ = N\left(a_p^* a_p + \frac{f(N)}{N}\right) \quad \therefore \tag{21}$$

$$\frac{\sum_{l=1}^{N} \xi_l^p}{N} = a_p^* a_p + \frac{f(N)}{N}. \tag{22}$$

Taking the limit $N \to \infty$ in eqn. (22), we recover the law of large numbers. Taking the limit $N \to \infty$ in eqn. (21), one obtains in virtue of the eqn. (18):

$$\lim_{N\to\infty} \sum_{l=1}^{N} \xi_l^p = \lim_{N\to\infty} \left(N + \frac{f(N)}{a_p^* a_p}\right) a_p^* a_p > 1\,, \tag{23}$$

therefore

$$\lim_{N\to\infty} \left(N + \frac{f(N)}{a_p^* a_p}\right) > \frac{1}{a_p^* a_p}\,. \tag{24}$$

Eqn. (24) is the argument used to prove the theorem, as one infers from the eqn. (19).

3 Conclusion

Finally, we conclude the reversible collapse of the wave function is an extremely rare statistical phenomenon. Once a collapse is reached, it is irreversible since there are a miriad of indistinguishable but distinct outcomes that may be equally reached, leading to a large number of Universe microstates with this same collapsed result. Hence, if one seeks to overcome the collapse: there exist fundamental issues to bypass.

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Our Mathematical Universe: I. How the Monster Group Dictates All of Physics

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A 4th family b' quark would confirm that our physical Universe *is* mathematical and is discrete at the Planck scale. I explain how the Fischer-Greiss Monster Group dictates the Standard Model of leptons and quarks in discrete 4-D internal symmetry space and, combined with discrete 4-D spacetime, *uniquely* produces the finite group Weyl E_8 x Weyl E_8 = "Weyl" SO(9,1). The Monster's j-invariant function determines mass ratios of the particles in finite binary rotational subgroups of the Standard Model gauge group, dictates Möbius transformations that lead to the conservation laws, and connects interactions to triality, the Leech lattice, and Golay-24 information coding.

1 Introduction

The ultimate idea that our physical Universe *is* mathematical at the fundamental scale has been conjectured for many centuries. In the past, our marginal understanding of the origin of the physical rules of the Universe has been peppered with huge gaps, but today our increased understanding of fundamental particles promises to eliminate most of those gaps to enable us to determine with reasonable certainty whether this conjecture is true or false.

My principal goal is to show that if a 4th quark family exists, the physical rules of the Universe follow directly from mathematical properties dictated by the Fischer-Greiss Monster Group via the Monster's j-invariant function and the Möbius transformation in discrete spacetime, with everything related to the Golay-24 information code for the Leech lattice.

In a series of articles and conference talks beginning in 1992 [1–3] I have been predicting that a 4th quark family with a b' quark at about 80 GeV and a t' quark at about 2600 GeV will be produced at the colliders. Its detection will support these proposals:

1. The Standard Model (SM) of leptons and quarks provides an excellent approximation to the actual *discrete* symmetry groups of these fundamental particles and requires little modification for extension to the Planck scale.
2. There are 3 lepton families and 4 quark families, each family of two states defined by a different finite binary rotational subgroup of the $SU(2)_L$ x $U(1)_Y$ part of the SM gauge group.
3. The leptons are 3-D polyhedral entities, and the quarks are 4-D polytope entities which combine into 3-D colorless hadrons, color being a 4-D property with exact symmetry derived from 4-D rotations.
4. Lepton and quark approximate mass values are determined by the j-invariant function of elliptic modular functions, being related to the above subgroups and Möbius transformations in both discrete lattice spaces and continuous spaces.
5. Both 4-D spacetime and 4-D internal symmetry space are discrete at the Planck scale, and both spaces can be telescoped upwards mathematically by icosians to 8-D spaces that *uniquely* combine into 10-D discrete spacetime with discrete Weyl E_8 x Weyl E_8 symmetry (not the E_8 x E_8 Lie group of superstrings/M-theory).
6. All the above is related to the Fischer-Greiss Monster Group which herein I argue actually dictates all the rules of physics, except perhaps the entropy law.
7. Consequently, our physical Universe *is* mathematical with only one set of rules and physical constants, which eliminates any multiverse with different values.
8. We live in the only possible Universe, the one with 4-D discrete spacetime dictated by the Monster Group and its relation to information coding and the Leech lattice.

My discrete geometrical approach briefly outlined above fits within the realm of the SM, so its past successes should still apply. One simply must "discretize" the SM lagrangian. Even Noether's theorem works in discrete spaces [4] to connect conservation laws to symmetries, the conserved quantity being continuous but periodic.

2 Brief orientation for discreteness

A few years ago a comprehensive review [5] summarized many of the historical mathematical and physical arguments for considering the Universe to be mathematical. Included were the three hypotheses: (1) the External Reality Hypothesis (ERH) — there exists an external physical reality completely independent of us humans; (2) the Mathematical Universe Hypothesis (MUH) — our external physical reality is a mathematical structure; and (3) the Computable Universe Hypothesis (CUH) — the mathematical structure that is our external physical reality is defined by computable functions. Recall that a computable function must be specifiable by a finite number of bits. The mathematical details are in that article.

The ERH is relatively easy to accept, for the universe certainly existed long before we humans came on the scene. The MUH is the conjecture for which I hope the data from the

colliders will help us decide. One assumption here is that Gödel's Incompleteness Theorem is not an impediment, i.e., there is no limit to being able to determine the ultimate source of all the rules of Nature and what these rules actually are.

The most interesting statements [5] regarding challenges to the CUH are "... virtually all historically successful theories of physics violate the CUH ..." and "The main source of CUH violation comes from incorporating the continuum, usually in the form of real or complex numbers, which cannot even comprise the input to a finite computation since they generically require infinitely many bits to specify." To me, therein lies the problem: *continuous spaces*.

In particle physics, we consider two spaces: (1) a continuous spacetime for particle movement such as translations, rotations and Lorentz transformations, and (2) a continuous internal symmetry space at each spacetime point for the local gauge interactions of the Standard Model. In both spaces we have successfully used continuous functions for our descriptions of the behavior of Nature.

My proposed solution to this problem is to consider both spaces to be discrete spaces "hidden" underneath the continuous approximation, as if we do not yet have enough resolution to detect this discreteness. All our successful physics theories are then excellent *effective* theories containing continuous fields and continuous wave function amplitudes in this approximate world.

We will not be entering a strange new world by considering a discrete approach, for we use difference equations, lattice models, and discrete computations to approximate continuum physics all the time in numerical calculations, and the results are quite reliable and amazingly accurate. Therefore, I suggest that a fundamental discreteness at the Plank scale of about 10^{-35} meters is not unreasonable [3].

The possibility that the Monster Group, whose influence looms over all of mathematics, could dictate all of physics was put forth in several of my previous papers and conference talks over the last two decades, but other physicists have conjectured a similar proposal. What the others have not realized is the direct connection in a *discrete* internal symmetry space from the Monster to the lepton and quark states via the j-invariant of elliptic modular functions. In this article, I provide additional essential arguments to establish the hegemony of the Monster Group and I arrive at the conclusions that spacetime is discrete and our Universe *is* mathematical.

3 The Monster and the j-invariant

The very large discrete symmetry group called the Monster group M is a finite simple group because it has only two normal subgroups, the trivial one-dimensional group and the whole group itself. Finite simple groups can be used as building blocks in that any other type of finite group can be constructed from them. The list of all finite simple groups is: (i) the cyclic groups C_p, with p prime, (ii) the alternating groups A_n, n > 4, (iii) 16 infinite families of Lie groups, and (iv) 26 sporadic groups. The smallest sporadic is the Mathieu Group M_{11} of order 7920 discovered in 1861, while the largest sporadic is the Monster M constructed in 1980 with order of about 8 x 10^{53}. The Monster has 194 different irreducible representations, with the smallest irreducible matrix representations of M being in space dimensions 1, 196883, 21296876, and 842609326.

As I explain in the next section, the most direct connection of M with the SM of leptons and quarks is via the j-invariant of elliptic modular functions

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + \dots \quad (1)$$

where $q = e^{2i\pi\tau}$ and τ is a ratio for a 2-D lattice that we will define in a later section. I.e., this 2-D lattice approach in our discrete spaces leads directly to the symmetry groups for the lepton and quark families and for the Lorentz transformations in spacetime.

As has been determined by mathematicians, the coefficients of the powers of q are simple linear combinations of dimensions of irreducible representations of the identity operation of M, a correlation known as "Monstrous Moonshine". E.g., 196884 = 1 + 196883, and 21493760 = 21296876 + 196883 + 1, etc. More mathematical and historical information about the Monster can be learned from the online papers and books by T. Gannon [6].

4 Binary rotation groups and the j-invariant

Here I review the connection between the j-invariant and the discrete symmetry groups for the leptons and quarks. I have proposed [1–3] that the lepton and quark flavors, being electroweak eigenstates, correspond to orthogonal states in specific discrete symmetry groups called finite binary rotational groups. These seven subgroups of the SM local gauge group act in the $\mathbb{R}^3$ and $\mathbb{R}^4$ real subspaces of the 2-D unitary space $\mathbb{C}^2$ for $SU(2)_L$ x $U(1)_Y$. In fact, I am using discrete $\mathbb{R}^3$ and $\mathbb{R}^4$.

The lepton families correspond to the 3-D finite binary rotational groups called the binary tetrahedral group 2T, the binary octahedral group 2O, and the binary icosahedral group 2I, also labelled as [3, 3, 2], [4, 3, 2], and [5, 3, 2], respectively, in Table 1. These are groups of discrete symmetry rotations and reflections. Binary here refers to the double cover of the SO(3) rotation group by Spin(3) = SU(2), so these groups are finite subgroups of SU(2) and $SU(2)_L$ x $U(1)_Y$.

Having exhausted the group possibilities in $\mathbb{R}^3$, one moves up one real spatial dimension to $\mathbb{R}^4$ in order to define the quark families, which then correspond to the finite binary rotation groups [3, 3, 3], [4, 3, 3], [3, 4, 3], and [5, 3, 3] of the regular 4-D convex polytopes. One may not need the number of quark families to match the number of lepton families for anomaly cancellation because this geometrical approach defines leptons and quarks as 3-D and 4-D entities, respectively. I.e., the interactions are not among point particles.

Leptons						Quarks					
group	order	family	N	Pred. Mass (MeV)	Emp. Mass (MeV)	**group**	order	family	N	Pred. Mass (GeV)	Emp. Mass (GeV)
					.						.
					.	**[3, 3, 3]**	120	$d^{-1/3}$	1/4	0.011	0.007
					.			$u^{+2/3}$		0.38	0.004
[3, 3, 2]	24	e^-	1	[1]	0.511	**[4, 3, 3]**	384	$s^{-1/3}$	1	0.046	0.2
		ν_e		0?	0.0?			$c^{+2/3}$		[1.5]	1.5
[4, 3, 2]	48	μ^-	108	108	103.5	**[3, 4, 3]**	1152	$b^{-1/3}$	108	[5]	5.0
		ν_μ		0?	0.0?			$t^{+2/3}$		160	171.4
[5, 3, 2]	120	τ^-	1728	1728	1771.0	**[5, 3, 3]**	14400	$b'^{-1/3}$	1728	~ 80	?.?
		ν_τ		0?	0.0?			$t'^{+2/3}$		~ 2600	?.?

Table 1: Lepton and quark families for the binary rotational groups [a, b, c], their j-invariant proportionality constant N, and the predicted mass values for the quarks based upon group-to-group N ratios with the charm quark mass [1.5 GeV] and bottom quark mass [5 GeV] as reference masses for ratios of the "up-like" and "down-like" quark states, respectively. These are the "bare" mass predictions. Drawings with these symmetries are online [3].

Each lepton group represents the *binary* rotational symmetries of familiar 3-D regular polyhedrons, the tetrahedron, the octahedron, and the icosahedron. In terms of two complex variables z_1 and z_2, there are three algebraic equations for each regular polyhedron that remain invariant under the operations of its binary group, corresponding to the complex equations for the vertices, the face centers, and the edge centers. Call these three equations W_1, W_2 and W_3, respectively. F. Klein, in a famous 1884 book [7], reported that these three equations are not independent because they form a mathematical syzygy. He showed that two independent equations W_1 and W_2, say, have a ratio proportional to the j-invariant

$$j(\tau) = \frac{W_1}{NW_2} \tag{2}$$

where N is a specific integer, being 1, 108, and 1728, for the three groups, 2T, 2O, and 2I, respectively. Certain integrals, including a mass integral, for the particle states would involve these N values as important factors.

The four binary rotational groups for the quarks are handled [8] by projecting their physical 4-D polytopes onto the 2-D unitary plane $\mathbb{C}^2$ and realizing that their symmetries lead to the same invariant algebraic equations as for the leptons, with the addition of one other symmetry group syzygy for [3, 3, 3]. The corresponding N values are thus 1/4, 1, 108, 1728.

These N values suggest the pairings of the lepton families to quark families as shown horizontally in Table 1. Notice that these family pairings are different from the traditional ad hoc pairings that are normally suggested for the SM because here there exist fundamental geometrical connections.

5 Particle mass values

The influence of the j-invariant of the Monster continues. In spaces where the j-invariant applies, all rational functions (ratio of two polynomials) are proportional to the j-invariant and invariant under all fractional linear transformations (also called Möbius transformations). For physics purposes, mass of a fundamental particle is proportional to the j-invariant because mass is an invariant under Möbius transformations. Conservation laws in physics can be related to Möbius transformations in both discrete and continuous spaces.

At this stage there is no absolute mass scale, so I must use mass ratios only, selecting a different reference mass value for the "up" states and for the "down" states. For the lepton mass values, we have the N ratios 1:108:1728. Table 1 shows the predicted and the actual values. The patterns of ratios match roughly and they were the clue to considering these binary rotational groups.

Note that without using the reference empirical masses for the ratios, the two predicted states in each family would be degenerate with the same mass. One should form two new orthogonal linear superposition states from these original degenerate states. These states would have different "bare mass" values and would be sensitive to the "vacuum" environment.

For the electroweak interactions, a zero-order approximation to the quark CKM mixing matrix and the lepton PMNS mixing matrix follows directly from the characteristic equations of the 3-D and 4-D symmetries projected to the unitary plane $\mathbb{C}^2$, producing unitary eigenvectors and eigenvalues λ_j = exp[iϵ_j]. The two angles (ϵ_1, ϵ_2) are (π, π) for [3,3,2], (2π/3,

$4\pi/3$) for [4,3,2], ($2\pi/5$, $8\pi/5$) for [5,3,2], ($2\pi/5$, $8\pi/5$) for [3,3,3], ($\pi/3$, π) for [4,3,3], ($\pi/6$, $7\pi/6$) for [3,4,3], and ($\pi/15$, $19\pi/15$) for [5,3,3].

One can define a 3 x 3 unitary matrix [9] and substitute angle difference values for the lepton mixing matrix PMNS and a 3 x 3 quark mixing matrix CKM3, producing

$$PMNS = \begin{pmatrix} 0.5 & 0.866 & \epsilon \\ -0.579 & 0.335 & 0.743 \\ 0.643 & -0.372 & 0.669 \end{pmatrix} \quad (3)$$

$$CKM3 = \begin{pmatrix} 0.978 & 0.208 & \epsilon \\ -0.180 & 0.847 & 0.5 \\ 0.104 & -0.489 & 0.866 \end{pmatrix} \quad (4)$$

with ϵ small. Several of the off-diagonal values in VCKM3 would require higher order corrections in order to better agree with empirically determined values.

A 4 x 4 unitary mixing matrix for our four quark families that brings in c_{34} and s_{34} in the 3rd and 4th rows leads to

$$VCKM4 = \begin{pmatrix} 0.978 & 0.208 & \epsilon_1 & \epsilon_2 \\ -0.180 & 0.847 & 0.5 & \epsilon_3 \\ 0.099 & -0.465 & 0.842 & 0.309 \\ -0.032 & 0.151 & -0.268 & 0.951 \end{pmatrix} \quad (5)$$

with all the ϵ values small. Adjustments can be made by considering higher order corrections.

One should not ignore the fact that a degrees-of-freedom argument would make neutrinos that are zero mass exactly. My two lepton states in each family have 4 d.o.f. total, which can partition into the massive electron state with 3 d.o.f., leaving just 1 d.o.f. for the neutrino state. Thus, the neutrino is massless and can have one helicity state only. Alternately, if both lepton states per family share the 4 d.o.f. equally with 2 d.o.f. each, then these would be two massless states, i.e., possibly two sterile neutrino states. Nature appears to have chosen the unequal split, but sterile neutrinos are still a possibility. As to the quarks, the two 4-D quark family states have a total of 6 d.o.f. to split 3-3, guaranteeing the existence of the two massive quark states per family we measure.

The discovery of the b' quark, probably by the FCNC decay b' $\rightarrow$ b + γ, is the acid test of this geometrical approach toward understanding the SM. There is already some hint in the Fermilab data for this decay but the signal/noise ratio is not good enough. The 4th quark family has recently been in vogue because the baryonic particle-antiparticle asymmetry in the Universe (BAU) can then be explained by CP violation with a new value for the Jarlskog invariant that is about 10^{13} times larger [10] than for only 3 quark families. As far as I know, the b' quark remains a viable possibility.

6 Discrete internal symmetry space

In this geometrical approach, the internal symmetry space is discrete $\mathbb{C}^2$ at the Planck scale. Therefore we must consider the mathematical properties of a 2-D hexagonal lattice (or of a 2-D rectangular lattice) of mathematical nodes either with two real axes $\mathbb{R}^2$, or two complex axes $\mathbb{C}^2$, or two quaternion axes $\mathbb{H}^2$, etc. All its nodes can be represented by integer linear combinations of two complex numbers that we label ω_1 and ω_2 forming a right-handed basis (ω_1, ω_2). We can change these two numbers without changing the lattice by letting

$$\begin{aligned} \omega_1' &= a\omega_1 + b\omega_2 \\ \omega_2' &= c\omega_1 + d\omega_2 \end{aligned} \quad (6)$$

where a, b, c, and d, are integer elements of a 2 x 2 matrix with determinant 1. Such matrices form a symmetry group called the "modular group" SL(2, Z) which is related to elliptic curves. Actually, all that matters is the ratio $\tau = \omega_1/\omega_2$ which defines the τ for the j-invariant in Eq. 1. Since

$$f(\tau) = f\left(\frac{a\tau + b}{c\tau + d}\right), \quad (7)$$

all modular functions $f(\tau)$ on the lattice depend only upon its shape. The j-invariant is such a function, and all other SL(2, Z)-invariant functions are rational functions of j(τ).

Eq. 7 defines the fractional linear transformations, i.e., the Möbius transformations, which are based upon the transformations $\tau \rightarrow 1 + \tau$ and $\tau \rightarrow -1/\tau$ for translations, rotations, etc. In the limit when the node spacing approaches zero, the continuous approximation appears and the Möbius transformations include the continuous symmetry transformations.

7 Geometry of the boson interactions

The 12 bosons of the SM, 8 gluons and 4 EW bosons, operate on the fermion states in a continuous internal symmetry space. For a continuous space one can map the complex plane $\mathbb{C} = \mathbb{R}^2$ and unitary plane $\mathbb{C}^2 = \mathbb{R}^4$ to the 2-D Riemann sphere. Its 2-D surface has no demarcations, thus allowing any small or large rotation. Consequently, the symmetry group for the SM interactions is the continuous gauge group of operations.

In my geometrical approach this internal symmetry space is discrete, so only specific finite rotation groups can produce these boson operations. However, when the internal symmetry space is discrete and particle symmetries are defined by the specific finite binary rotation groups for leptons and quarks, the Riemann surface is tessellated, i.e., composed of identical equilateral triangles, their number uniquely determined by the binary rotation group. Then the number of rotational operations becomes severely restricted and each boson operator must respect the integrity of the symmetry group for the lepton or quark families participating in the interaction.

Geometry provides the important clue. We desire a small group in our discrete space for defining these interactions (i.e., producing the appropriate rotations by the bosons), and we find the binary icosahedral group 2I or [5, 3, 2]. However, there will be some missed operations on the symmetry for the binary octahedral group 2O. But if we take 2I twice, i.e., including its "reciprocal" [5, 3, 2], then we get it all.

In order to appreciate this geometry, quaternion algebra simplifies the game. Recall that the SU(2) matrix representation and the unit quaternion **q** are related by

$$\mathbf{q} = w\mathbf{1} + x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Longleftrightarrow \begin{pmatrix} w + iz & x + iy \\ -x + iy & w - iz \end{pmatrix} \qquad (8)$$

where the **i**, **j**, and **k** are unit imaginaries, the coefficients w, x, y, and z are real, and $w^2+x^2+y^2+z^2 = 1$. We can represent the two orthogonal lepton or quark states in each family by two orthogonal unit quaternions in $\mathbb{C}^2$.

There is also a conjugate plane $\mathbb{C}'^2$ for the antiparticles and its Riemann sphere. The conjugate quaternion is **q'** =w**1** - x**i** - y**j** - z**k**. What we discuss for the particle states works for the antiparticle states, too. Having a conjugate space is very special. Clifford algebra and Bott periodicity dictate that only $\mathbb{R}^4$, $\mathbb{R}^8$, and other real spaces $\mathbb{R}^n$ with dimensions divisible by four have two equivalent conjugate spaces. This specific mathematical property dictates a world with both particle states and their antiparticle states for these dimensions only.

One more mathematical fact. The group $U(1)_Y$ for weak hypercharge Y in $SU(2)_L$ x $U(1)_Y$ has the important role of reducing the symmetry between the two spaces, normal and conjugate, in $\mathbb{R}^4 = \mathbb{C}^2$ from being simply equivalent to their being *gauge* equivalent. The physics consequence is that particles and antiparticles have the same positive mass but all other properties can be opposite sign. Alternately, we can use the 2-element inversion group C_i to accomplish the same distinction as well as to determine the intrinsic parity of the particle states, odd for particles and even for antiparticles.

Furthermore, the use of quaternions for the electroweak operations tells us that the L in $SU(2)_L$, which means left-handed chirality only for the weak interaction, is really dictated by quaternion properties, so that the left-handed physics restriction for the weak interaction in $\mathbb{C}^2$ follows. That is, in the normal unitary plane all unit quaternions have left-handed screw transformations that mix the two orthogonal states and right-handed screw transformations that do not. Put another way, the quaternions transform the two orthogonal flavor states as left-handed doublets and right-handed singlets. For example, in the first lepton family, they are (ν_{eL}, e_L) and (ν_{eR}) and (e_R). In the conjugate unitary plane for antiparticles, the quaternion transformations have the opposite handedness.

Now back to rotating the Riemann sphere. In the simplest electroweak (EW) interactions of a boson with an incoming fermion, the fermion state either remains the same (via γ or Z^0) or changes from the initial state to an orthogonal state (via $W^{\pm}$). As examples, the γ may be the identity and the Z^0 may produce a 4π rotation, while the $W^{\pm}$ operates between different states. The 120 operations of the binary icosahedral group 2I are represented by 120 unit quaternions, and 2I contains almost all the rotation operations needed for the 7 fermion family groups. However, several symmetry operations of 2O would be absent. One needs to add the "reciprocal" binary icosahedral group to include all the operations of 2O, making a grand total of 240 operations. (n.b. One could also consider just the generators to realize the same result.)

Here comes an interesting and unexpected mathematical consequence. The first set of 120 quaternions can be expressed as 120 special unit quaternions known as *icosians* which telescope 4-D discrete-space quaternions up to being 8-D discrete-space octonions to locate points that form a special lattice in $\mathbb{R}^8$ called D_8. The second set of 120 quaternions does the same, forming another D_8 lattice in $\mathbb{R}^8$ by filling the holes in the first D_8 lattice.

The icosians are special unit quaternions q_i which have the mathematical form

$$q_i = (e_1 + e_2\sqrt{5}) + (e_3 + e_4\sqrt{5})i + \\ +(e_5 + e_6\sqrt{5})j + (e_7 + e_8\sqrt{5})k \qquad (9)$$

where the eight e_j are special rational numbers. The important mathematical fact here is that in each pair, such as (e_3 + $e_4\sqrt{5}$), exactly one of the e_j is nonzero. Therefore, even though the icosians are telescoping us up to an 8-D space, their primary importance is that they represent 4-D operations in $\mathbb{R}^4$ even though we can now define identical quaternion operations via octonions in the much larger $\mathbb{R}^8$ space also.

Together, these two D_8 lattices of 120 icosians each combine to form the 240 octonions that define the famous E_8 lattice in $\mathbb{R}^8$. The symmetry group for this E_8 lattice is not the Lie group E_8 but the discrete group Weyl E_8.

Therefore, the operations of the SM occur in discrete 4-D internal symmetry space, but they operate also in the discrete 8-D space because these icosians span both spaces simultaneously.

8 Quark color, gluons, and hadron states

Now I must back up to show that the gluon interactions can occur in $\mathbb{R}^4$ for $SU(3)_C$ even though one normally expects the larger space $\mathbb{C}^3$. Because 4-D rotations are simultaneous rotations in two orthogonal planes, each of the three quark color charges Red, Green, and Blue, can be assigned to the three possible rotation plane pairs [wx, yz], [xy, zw], and [yw, xz], respectively. Actually, because these three 4-D rotation pairs are equivalent and we could have made the color assignments in any order, we learn the mathematical reason for color being an *exact* physical symmetry.

Contained within the above specific icosians are the gluon operations on the color states, but one can use a specific 4 x 4 rotation block matrix R to define the transition from one color state in the 4-D space to another. There are 8 orthogonal gluon matrices in agreement with the 8 gluons of the $SU(3)_C$ gauge group of the SM.

Hadrons are colorless quark combinations, so they occur when the combined resultant 4 x 4 matrices produce no net 4-D rotation, i.e., are the identity matrix. One can show that this

colorless state exists for three combinations of quark states only: (1) the quark-antiquark pair with color and anticolor, (2) three quarks, or (3) three antiquarks, with the appropriate linear combinations of colors or anticolors.

The mathematics itself distinguishes quarks (and baryon number) from leptons: the quarks are 4-D entities and the leptons are 3-D entities, with only the 4-D entities capable of the color interaction because color is an exact symmetry in $\mathbb{R}^4$. Quark *confinement* results because isolated quarks are 4-D entities which cannot exist in a 3-D space, so one can never have an isolated single quark in our 3-D spatial world.

The colorless hadron states, being those special mathematical combinations of quark 4-D entities, are now actually 3-D entities like the lepton states are. That is, the colorless combinations of quarks are 3-D composite particle states because their geometrical intersections define 3-D geometric entities.

Therefore, in my geometrical version of the SM, we have 3-D lepton states, 3-D hadron states, 3-D electroweak boson states, but 4-D quark states and 4-D gluon states. The 4-D quark and gluon states are confined, i.e., they cannot exist as separate entities in our 3-D spatial world, but the 3-D lepton, 3-D hadron, and 3-D electroweak boson states can move through 4-D discrete spacetime with its 3 spatial dimensions.

9 Geometry of discrete 4-D spacetime

Our 3-D particles move in discrete 4-D spacetime. We know that continuous 4-D spacetime has symmetries related to its continuous Lorentz group SO(3,1). For a discrete 4-D spacetime and its Lorentz transformations we need to determine a finite subgroup of SO(3,1) for its discrete symmetry.

A clever mathematical approach to 4-D spacetime was introduced by R. Penrose [11] long ago, who showed how to utilize his "heavenly sphere" to account for Lorentz transformations, etc. This "heavenly sphere" is actually 4-D spacetime (t, x, y, z) mapped onto the Riemann sphere. Consider being in the center of the "heavenly sphere" so that light rays from stars overhead pass through unique points on the unit celestial sphere surrounding you. A Lorentz boost is a conformal transformation of the star locations: the constellations will look distorted because the apparent lengths of the lines connecting the stars will change but the angles between these connecting lines will remain the same.

In our discrete 4-D spacetime we need to tesselate this Riemann surface into identical equilateral triangles and then perform the symmetry transformations of the sphere. But we have already achieved this tesselation earlier with the binary rotation groups when we considered the discrete internal symmetry space mapped to the Riemann sphere, so we know the result. Using the isomorphism SO(3,1) = PSL(2,C), we see [2] that the group mathematics connects the conformal transformations just described to the Möbius group via

$$\text{SO}(3,1) = \text{Möbius group} = \text{PSL}(2,\mathbb{C}), \qquad (10)$$

with the discrete Lorentz transformations of the tessellated Riemann sphere already contained in SO(3,1). Thus, we have a unit quaternion group PSL(2,C) (equivalently, an SU(2) matrix or spinor) representation of the Lorentz transformation.

Therefore, we are back to our discrete symmetries of the binary polyhedral groups because they are finite modular subgroups of the Möbius group PSL(2,C). Therefore, the 240 special quaternions called icosians are now required for discrete Lorentz boosts and discrete rotations in the discrete 4-D spacetime. We obtain a second E_8 lattice in $\mathbb{R}^8$ with symmetry group Weyl E_8.

10 Unification of spacetime and the Standard Model

We can now unite the discrete internal symmetry space operations with the discrete spacetime operations [2]. The direct product of our two Weyl E_8 groups results in a subgroup of the continuous group PSL(2,$\mathbb{O}$), where $\mathbb{O}$ represents all the unit octonions. For the continuous case, PSL(2,$\mathbb{C}$) has become PSL(2,$\mathbb{O}$) = SO(9,1), the Lorentz group in 10-D spacetime. That is, the final combined spacetime is *bigger* than I expected, being isomorphic to a 10-D spacetime instead of an 8-D spacetime.

Applying this result to our discrete case, the combined finite subgroup

$$\text{finite PSL}(2,\mathbb{O}) = \text{finite SO}(9,1), \qquad (11)$$

the finite Lorentz group in discrete 10-D spacetime. The same result, expressed in terms of the direct product of the Weyl E_8 groups is

$$\text{Weyl } E_8 \text{ x Weyl } E_8 = \text{"Weyl"} \text{ SO}(9,1), \qquad (12)$$

a finite subgroup of SO(9,1).

Therefore, the big surprise is that the combination of a 4-D discrete spacetime with a 4-D discrete internal symmetry space creates a *unique* connection to 10-D discrete spacetime, not to an 8-D discrete spacetime. Unlike the situation with continuous spaces, we do not have a 6-D "curled up" internal symmetry space with about 10^{500} possibilities.

The mathematics has dictated a beautiful result: there seems to be *only one way* for our Universe to exist when spacetime is discrete.

11 A physical particle model

Even though the mathematics telescopes us up from $\mathbb{R}^4$ to $\mathbb{R}^8$, we still need a physical model of particles in the discrete 4-D spacetime defining our Universe. The leptons, hadrons, and the electroweak bosons are non-point-like 3-D entities that appear to be point-like particles at our normal size resolution of about 10^{11} times larger than the Planck scale.

Peering in at the Planck scale, however, I expect the discrete 4-D internal symmetry space at each spacetime point to conjoin into the discrete 4-D spacetime. In order to do

so, each particle must *emerge* by "gathering in" nodes of the lattice to make its 3-D or 4-D entity with its correct symmetry. For example, if the particle is an electron, we expect the symmetry of the node collection will be tetrahedral to agree with its [3, 3, 2] symmetry. If the lattice of nodes was originally uniformly spaced in this region of discrete space, then the existence of the electron has distorted this lattice with a decreasing distortion amount for increasing distance from the electron's center.

Note that this geometrical approach assumes that the lattice nodes themselves do not have any *measurable* physical properties. Consequently, we have arrived finally at the end of the hierarchy of physical particles within particles. At this point in the geometrical approach we simply must accept this gathering-of-nodes process because the mathematics dictates this process via graph theory and Kuratowski's theorem.

Kuratowski's theorem is important here because it states that a graph is planar if and only if it does not contain a Kuratowski subgraph K_5 or $K_{3,3}$. For example, if an n-dimensional graph (a lattice of nodes) in a spatial dimension higher that 2-D does not contain a Kuratowski K_5 subgraph, also known as the complete graph of five vertices, then this n-D graph reduces to 2-D.

But the first quark family's binary rotational group [3, 3, 3] symmetry is the rotationally symmetric version of the Kuratowski subgraph K_5. Therefore, at least one quark state of the first quark family is stable as it moves through the lattice, while all other quark families have states that will decay down to [3, 3, 3] quark states. Indeed, the physics agrees with this mathematical prediction.

At the DISCRETE'08 conference in December, 2008 where I tried to present this geometrical approach in my allotted 20 minutes (!), C. Jarlskog asked me an interesting question: Why can't the universe have only quarks and gluons? I.e., a QCD world seems complete by itself. Why complicate the material world with leptons and the electroweak interaction? To which I immediately answered: Kuratowski's Theorem in mathematics does not allow such a world, but I was not encouraged to elaborate with any of the details.

Here is the rest of my argument. If quarks are 4-D entities, most quark states decay because they do not have the structure of K_5 (or $K_{3,3}$), so the initial structure will re-form into two or more new particles. In a universe with only quarks and gluons, a problem arises because gluons change only the color state for a particular quark but cannot change one quark flavor into another. In order to obey Kuratowski's theorem, Nature had no choice but to bring in more particles, notably the leptons and the electroweak interaction bosons. Voilà!

The immortality of the electron with group [3,3,2] seems to depend upon its close geometrical relation to the regular K_5 symmetry group [3, 3, 3]. Of course, the electron could annihilate with its antiparticle (and so can a quark).

At this point one might be concerned about the emergence of fermion particles from the "vacuum" state. In order to account for all the particles in the known Universe, the equivalent of about one new hydrogen atom per cubic meter per 10 million years is required. This process can occur because fermions are represented by spinors, and spinors originate from zero-length vectors. That is, according to E. Cartan, one zero-length vector splits into a spinor and conjugate spinor mathematically. The spinor is the fermion such as an electron and the conjugate spinor is the anti-fermion positron, for example. If their total energy remains zero by adding up all energy forms, then this creation process is viable.

As the electron or any 3-D particle moves through the lattice, I would expect that the particle's lattice distortion effect moves with it, with its previous distortions relaxing back toward being a regular lattice while the oncoming positions become more distorted. Mathematically, the Möbius transformations guarantee the integrity of this movement. That is, for our lattice, the transformation $\tau \rightarrow 1+\tau$ ensures that the movement process is identical everywhere in the lattice. The second Möbius transformation $\tau \rightarrow -1/\tau$ when combined with the one above allows rotations and other linear transformations to occur in the lattice.

This lattice distortion by a particle in 4-D discrete spacetime is the "warping of spacetime" associated with the gravitational interaction proposed by A. Einstein in the general theory of relativity. In this way, gravitation appears to be different from the other fundamental interactions which appear to be more localized.

More details of this particle model, such as the geometry of the gravitational interaction, the origin of the rules of quantum mechanics, the origin of time, and the information coding of the fundamental particles, will be discussed thoroughly in the second paper of this series.

12 Triality, the Leech lattice, and information coding

We know that particle EW interactions can be described in lowest order by the Feynman diagram (Fig. 1) involving three particles with three lines meeting at a point. There can be two fermions interacting with one of the electroweak ($W^{\pm}$, Z^0, or γ) or color (8 gluons) bosons. There can be three gluons interacting. More complicated diagrams can be drawn but they will all be made from combinations of this generic one.

This lowest order Feynman diagram with two fermions and one boson is a mathematical *triality* diagram with the fermions representing spinors and the boson representing a vector Jordan algebra entity. Triality is a relationship between three vector spaces over a field F that are all isomorphic to each other. Thus, the common vector space is isomorphic to $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, or $\mathbb{O}$, i.e., involving spinors in dimensions 1, 2, 4, and 8, respectively [12].

In our 4-D discrete spacetime the fermion states can be represented by quaternions. In fact, Clifford algebra tells us that there will be two quaternion representations in $\mathbb{R}^4$ called the right-handed spinor representation S_4^+ and the left-handed

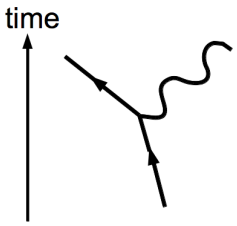

Fig. 1: The incoming fermion emits or absorbs a boson and a fermion exits. E.g., an electron emits a photon and continues in a different direction. This diagram represents triality among two spinors (electron in and electron out) and a vector boson.

spinor representation S_4^-. In general, for even dimensional spacetimes, i.e., even n values, the two spinor representations have dimension $2^{n/2-1}$, but the vector representation has dimension n. For example, in n = 4 space, the boson vector representation is a 4 x 4 real matrix and the fermion spinor representation is a 2 x 2 complex matrix or, equivalently, also a 4 x 4 real matrix. I.e., the fermions and bosons are the same dimension.

We know that the icosians telescope us up to discrete 8-D. With n = 8, the spinor representations are again the same size as the vector representation, both represented by 8 x 8 real matrices. Even so, they are not equivalent representations. However, one can permute the vector, left-handed spinor, and right-handed spinor representations into each other [12]. In 4-D, for example, there is a parity operator that can do this change of a left-handed spinor into a right-handed spinor and vice-versa.

For the generic Feynman diagram, one can think about the two fermions and the one boson as being three E_8 lattices which come together momentarily to form a 24-D lattice called the Leech lattice. The Monster Group again plays its governing role through the j-invariant function. The numerator of $j(\tau)$, being $1 + 720\ q + 146512\ q^2 + \ldots$, is the generating function for the lattice vectors in this product of three copies of the E_8 lattice. And for conformal field theories, the j-invariant is the partition function for the Monster Group [13].

Another very important mathematical connection takes us to information coding theory. One could say that each particle in the triality diagram brings in its 8-bit Hamming code word to temporarily form the 24-bit binary Golay code word or, equivalently, the 12-bit ternary Golay code word, related to the Leech lattice. The 8-bit Hamming code has 72 distinct code words in 9 different but overlapping sets [14], the exact number required for the fundamental particles of the SM: 6 leptons plus 8 x 3 = 24 quarks sums to 30 fermion states; when doubled for anti-particles, makes 60 particle states; then add the 12 bosons to get 72. The 24-bit Golay code word encodes 12 data bits defining up to $2^{12} = 4096$ different items, easily covering the possible interaction triples of the SM.

These code words support the hegemony of the Monster Group because the allowed SM interactions of the leptons and quarks can be related to information theory in 24 dimensions. The second article includes details of the Turyn construction for these Golay-Leech lattice code words and their relationship to quantum information theory and the Monster Group.

13 Conclusion

In this brief article I have outlined specific connections between the mathematics of the Monster Group and fundamental physics particles and rules. These connections support the three hypotheses ERH, MUH, and CUH, so I conclude that the Universe *is* mathematical and that we live in the only possible one. I await the empirical confirmation by the discovery of the 4th quark family, particularly the b' quark at about 80 GeV. Hopefully, the wait will not be long.

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Spin, Isospin and Strong Interaction Dynamics

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The structure of spin and isospin is analyzed. Although both spin and isospin are related to the same SU(2) group, they represent different dynamical effects. The Wigner-Racah algebra is used for providing a description of bound states of several Dirac particles in general and of the proton state in particular. Isospin states of the four $\Delta(1232)$ baryons are discussed. The work explains the small contribution of quarks spin to the overall proton spin (the proton spin crisis). It is also proved that the addition of QCD's color is not required for a construction of an antisymmetric state for the $\Delta^{++}(1232)$ baryon.

1 Introduction

The isospin notion has been conceived by W. Heisenberg in 1932 [1, see p. 106]. It aims to construct a mathematical basis that represents the proton-neutron similarity with respect to the strong nuclear force. Both spin and isospin have the same SU(2) group structure. Thus, like spin multiplets of a quantum state, one combines corresponding states of nuclear isobars in an isospin multiplet. For example, the ground state of the ^{14}C, ^{14}O and the $J^{\pi} = 0^{+}$ excited state of ^{14}N are members of an isospin triplet. Obviously, one must remember that isospin is a useful *approximation* that neglects proton-neutron differences that are related to their mass and their electromagnetic interactions.

Later developments have shown that the proton-neutron similarity stems from the similarity between the *u, d* quarks. It follows that the usefulness of isospin symmetry extends to particle physics. For example, the three pions are members of an isospin triplet. Due to historical development, isospin notation takes different form in nuclear and particle physics. Here T and I denote isospin in nuclear and particle physics, respectively. In this work the symbol T is used, mainly because of the following reason. In the case of spin, the symbols J and j denote total and single particle angular momentum operators, respectively. Similarly, the symbols T and t denote the corresponding isospin operators. Thus, due to the same underlying SU(2) group, isospin relations can be readily borrowed from their corresponding spin counterparts. The operators T and t are used in the discussion presented in this work.

This work examines states of electrons and quarks. These particles have spin-1/2 and experimental data are consistent with their elementary pointlike property. Evidently, a theoretical analysis of an elementary pointlike particle is a much simpler task than that of a composite particle. The discussion begins with an examination of relevant properties of electronic states of atoms. The mathematical structure of the SU(2) group is used later for a similar analysis of isospin states.

Two important conclusions are derived from this analysis. First, it is well known that quarks' spin carry only a small fraction of the entire proton's spin [2]. This experimental evidence, which is called the second EMC effect and also the proton spin crisis, is shown here to be an obvious result of the multi-configuration structure of states of more than one Dirac particle. Another result is that the anti-symmetric state of the $\Delta^{++}(1232)$ baryon is well understood and there is no need to introduce a new degree of freedom for its explanation. It means that the historical starting point of the QCD construction has no theoretical basis. (Below, the symbol Δ refers to this isospin quartet of baryons.)

Generally, in order to simplify notation, the specific value of normalization factor is omitted from the expressions. The second and the third sections analyze spin and isospin, respectively. The fourth section provides an explanation for the proton spin crisis. The fifth section explains the antisymmetric structure of the Δ^{++} baryon (without using color). The last section contains concluding remarks.

2 Spin States

A comprehensive discussion of angular momentum can be found in textbooks [3]. In this short work some elements of this theory are mentioned together with a brief explanation. This is done for the purpose of arriving rapidly at the main conclusions. A relativistic notation is used and for this reason the jj coupling [3] takes place.

Let us begin with a discussion of spin and spatial angular momentum. These quantities are dimensionless and this property indicates that they *may* be coupled. Now, the magnetic field depends on space and time. Moreover, the theory must be consistent with the experimental fact where both spatial angular momentum and spin of an electron have the same kind of magnetic field. Thus, it is *required* to construct a relativistically consistent coupling of these quantities. This is the theoretical basis for the well known usage of spin and spatial angular momentum coupling in the analysis of electronic states of atoms.

A motionless free electron is the simplest case and the spin-up electron state is [4, see p. 10]

$$\psi(x^\mu) = Ce^{-imt}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \tag{1}$$

where m denotes the electron's mass.

A second example is the state of an electron bound to a hypothetical pointlike very massive positive charge. Here the electron is bound to a spherically symmetric charge Ze. The general form of a j^π hydrogen atom wave function is [5, see pp. 926–927]

$$\psi(r\theta\phi) = \begin{pmatrix} F\mathcal{Y}_{jlm} \\ G\mathcal{Y}_{jl'm}\end{pmatrix}, \tag{2}$$

where $\mathcal{Y}_{jlm}$ denotes the ordinary Y_{lm} coupled with a spin-1/2 to j, $j = l \pm 1/2$, $l' = l \pm 1$, F, G are radial functions and the parity is $(-1)^l$.

By the general laws of electrodynamics, the state must be an eigenfunction of angular momentum and parity. Furthermore, here we have a problem of *one* electron (the source at the origin is treated as an inert object) and indeed, its wave function (2) is an eigenfunction of both angular momentum and parity [5, see p. 927].

The next problem is a set of n-electrons bound to an attractive positive charge at the origin. (This is a kind of an ideal atom where the source's volume and spin are ignored.) Obviously, the general laws of electrodynamics hold and the system is represented by an eigenfunction of the total angular momentum and parity J^π. Here a single electron is affected by a spherically symmetric attractive field *and* by the repulsive fields of the other electrons. Hence, a single electron does not move in a spherically symmetric field and it *cannot* be represented by a well defined single particle angular momentum and parity.

The general procedure used for solving this problem is to expand the overall state as a sum of configurations. In every configuration, the electrons' single particle angular momentum and parity are well defined. These angular momenta are coupled to the overall angular momentum J and the product of the single particle parity is the parity of the entire system. The role of configurations has already been recognized in the early decades of quantum physics [6]. An application of the first generation of electronic computers has provided a numerical proof of the vital role of finding the correct configuration interaction required for a description of even the simplest case of the ground state of the two electron He atom [7]. The result has proved that several configurations are required for a good description of this state and no configuration dominates the others. This issue plays a very important role in the interpretation of the state of the proton and of the Δ^{++}.

For example, let us write down the 0^+ ground state He_g of the Helium atom as a sum of configurations:

$$\begin{aligned}\psi(\mathrm{He}_g) \;=\; & f_0(r_1)f_0(r_2)_{\frac{1}{2}^+\frac{1}{2}^+} + f_1(r_1)f_1(r_2)_{\frac{1}{2}^-\frac{1}{2}^-} + \\ & f_2(r_1)f_2(r_2)_{\frac{3}{2}^-\frac{3}{2}^-} + f_3(r_1)f_3(r_2)_{\frac{3}{2}^+\frac{3}{2}^+} + \\ & f_4(r_1)f_4(r_2)_{\frac{5}{2}^+\frac{5}{2}^+} + \ldots\end{aligned} \tag{3}$$

Here and below, the radial functions $f_i(r)$, $g_i(r)$ and $h_i(r)$ denote the two-component Dirac radial wave function (multiplied be the corresponding coefficients). In order to couple to $J = 0$ the two single particle j states must be equal and in order to make an even total parity both must have the same parity. These requirements make a severe restriction on acceptable configurations needed for a description of the 0^+ ground state of the He atom.

Higher two-electron total angular momentum allows the usage of a larger number of acceptable configurations. For example, the $J^\pi = 1^-$ state of the He atom can be written as follows:

$$\begin{aligned}\psi(\mathrm{He}_{1^-}) \;=\; & g_0(r_1)h_0(r_2)_{\frac{1}{2}^+\frac{1}{2}^-} + g_1(r_1)h_1(r_2)_{\frac{1}{2}^+\frac{3}{2}^-} + \\ & g_2(r_1)h_2(r_2)_{\frac{1}{2}^-\frac{3}{2}^+} + g_3(r_1)h_3(r_2)_{\frac{3}{2}^-\frac{3}{2}^+} + \\ & g_4(r_1)h_4(r_2)_{\frac{3}{2}^-\frac{5}{2}^+} + g_5(r_1)h_5(r_2)_{\frac{3}{2}^+\frac{5}{2}^-} + \\ & g_6(r_1)h_6(r_2)_{\frac{5}{2}^+\frac{5}{2}^-}\ldots\end{aligned} \tag{4}$$

Using the same rules one can apply simple combinatorial calculations and find a larger number of acceptable configurations for a three or more electron atom. The main conclusion of this section is that, unlike a quite common belief, there are only *three* restrictions on configurations required for a good description of a J^π state of more than one Dirac particles:

1. Each configuration must have the total angular momentum J.
2. Each configuration must have the total parity π.
3. Following the Pauli exclusion principle, each configuration should not contain two or more identical single particle quantum states of the same Dirac particle.

These restrictions indicate that a state can be written as a sum of many configurations, each of which has a well defined single particles angular momentum and parity of its Dirac particles.

The mathematical basis of this procedure is as follows. Take the Hilbert sub-space made of configurations that satisfy the three requirements mentioned above and calculate the Hamiltonian matrix. A diagonalization of this Hamiltonian yields eigenvalues and eigenstates. These eigenvalues and eigenstates are related to a set of physical states that have the given J^π. As pointed out above, calculations show that for a quite good approximation to a quantum state one needs a not very small number of configurations and that no configuration has a dominant weight. These conclusions will be used later in this work.

3 Isopin States

Spin and isospin are based on the same mathematical group called SU(2). Its three generators are denoted j_x, j_y, j_z. An equivalent basis is [1, see pp. 357–363]

$$j_+ = j_x + ij_y, \quad j_- = j_x - ij_y, \quad j_z. \tag{5}$$

All the j operators mentioned above commute with the total j^2 operator. For this reason, if one of them operates on a member of a $(2J + 1)$ multiplet of an SU(2) irreducible representation then the result belongs to this multiplet. The two $j_\pm$ operators are of a particular importance. Thus, let $\psi_{J,M}$ denote a member of such a multiplet and one finds

$$J_z J_- \psi_{J,M} = (M - 1) J_- \psi_{J,M}. \tag{6}$$

This relation means that J_- casts $\psi_{J,M}$ into $\psi_{J,M-1}$

$$J_- \psi_{J,M} = \sqrt{J(J + 1) - M(M - 1)}\, \psi_{J,M-1}, \tag{7}$$

where the appropriate coefficient is written explicitly. Analogous relations hold for the J_+ operator.

Let us turn to isospin. The required operators are simply obtained by taking the mathematical structure of spin and replacing the total spin operator J and the single particle spin operator j by the corresponding isospin operators T, t. (Here, like in the spin case, M, m denote the eigenvalue of T_z, t_z, respectively.) The issue to be examined is the structure of the isospin multiplet of the four baryons:

$$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}. \tag{8}$$

These $\Delta(1232)$ baryons have the lowest energy of the family of the Δ baryons [8]. The Δ^{++} baryon has three u quarks and $\psi_\Delta(uuu)$ denotes its state. Therefore, its isospin state is $T = 3/2$, $M = 3/2$ and the isospin component of the wave function is symmetric with respect to an exchange of any pair of quark.

Let us examine the operation of T_- on Δ^{++}.

$$\begin{aligned} T_- \psi_\Delta(uuu) &= (t_{1-} + t_{2-} + t_{3-}) \psi_\Delta(uuu) \\ &= \psi_\Delta(duu) + \psi_\Delta(udu) + \psi_\Delta(uud), \end{aligned} \tag{9}$$

where t_{i-} operates on the ith quark. This is the way how one obtains a yet unnormalized expression for the Δ^+ baryon from that of Δ^{++}. A successive application of T_- yields expressions for every member of the isospin quartet (8).

Now, the Δ^{++} state is symmetric with respect to its quark constituents and the same symmetry holds for the isospin operator $T_- = t_{1-} + t_{2-} + t_{3-}$. Hence, also the Δ^+ is symmetric with respect to its *uud* quark states. This argument proves that isospin space of *every* member of the baryonic quartet (8) is symmetric. The same result can be obtained from a different argument. The u, d quarks are fermions and their overall state must be antisymmetric with respect to an interchange of any pair of quarks. Now, the isospin operators used above do not affect other coordinates of quarks. It means that for every members of the isospin quartet (8), the entire symmetry of the other coordinates remain antisymmetric and the isospin coordinate is symmetric.

The data confirms the similarity between members of an isospin multiplet. Thus, for example, the mass difference between the Δ^0 and Δ^{++} baryons is less than 3 MeV [8], whereas the mass difference between the Δ multiplet and the nucleons is about 300 MeV. This evidence shows the goodness of the isospin notion, where strong interactions dominate the state of members of an isospin multiplet and the effect of all other interactions can be regarded as a small perturbation.

4 The Proton Spin Crisis

The proton's $J^\pi = 1/2^+$ state is determined by three valence *uud* quarks. The non-negligible probability of the existence of an additional quark-antiquark pair [1, see p. 282] indicates that it is a highly relativistic system. The discussion of section 2 holds for the spin-1/2 point-like quarks and the expansion in configurations is a useful approach. Here the three single particle j^π represent the *uud* quarks, in that order. Evidently, each configuration must satisfy the three requirement written few lines below (4). However, the Pauli exclusion principle of restriction 3 does not hold for the d quark. Thus, in analogy to (3) and (4) one expands the proton's wave function as a sum of terms of specific configurations. A truncated expression for this expansion is shown below:

$$\begin{aligned} \psi(uud) = {} & f_0(r_1) f_0(r_2) h_0(r_3) {}^{\frac{1}{2}^+}_{\frac{1}{2}^+}(0)^{\frac{1}{2}^+} + \\ & f_1(r_1) f_1(r_2) h_1(r_3) {}^{\frac{1}{2}^-}_{\frac{1}{2}^-}(0)^{\frac{1}{2}^+} + \\ & f_2(r_1) g_2(r_2) h_2(r_3) {}^{\frac{1}{2}^+}_{\frac{1}{2}^+}(1)^{\frac{1}{2}^+} + \\ & f_3(r_1) g_3(r_2) h_3(r_3) {}^{\frac{1}{2}^-}_{\frac{1}{2}^-}(1)^{\frac{1}{2}^+} + \\ & f_4(r_1) g_4(r_2) h_4(r_3) {}^{\frac{1}{2}^+}_{\frac{1}{2}^-}(0)^{\frac{1}{2}^-} + \\ & f_5(r_1) g_5(r_2) h_5(r_3) {}^{\frac{1}{2}^+}_{\frac{1}{2}^-}(1)^{\frac{1}{2}^-} + \\ & f_6(r_1) g_6(r_2) h_6(r_3) {}^{\frac{1}{2}^+}_{\frac{3}{2}^+}(1)^{\frac{1}{2}^+} + \\ & f_7(r_1) g_7(r_2) h_7(r_3) {}^{\frac{1}{2}^-}_{\frac{3}{2}^+}(1)^{\frac{1}{2}^-} + \\ & f_8(r_1) g_8(r_2) h_8(r_3) {}^{\frac{1}{2}^+}_{\frac{1}{2}^+}(1)^{\frac{3}{2}^+} + \\ & f_9(r_1) g_9(r_2) h_9(r_3) {}^{\frac{1}{2}^-}_{\frac{1}{2}^-}(1)^{\frac{3}{2}^+} + \\ & f_a(r_1) g_a(r_2) h_a(r_3) {}^{\frac{1}{2}^-}_{\frac{3}{2}^-}(1)^{\frac{1}{2}^+} + \\ & f_b(r_1) g_b(r_2) h_b(r_3) {}^{\frac{1}{2}^+}_{\frac{3}{2}^-}(1)^{\frac{1}{2}^-} + \\ & f_c(r_1) g_c(r_2) h_c(r_3) {}^{\frac{1}{2}^+}_{\frac{1}{2}^-}(1)^{\frac{3}{2}^-} + \ldots \end{aligned} \tag{10}$$

The symbols $_{0\ldots9,a,b,c}$ are used for enumerating the terms of (10). Here, like in (3) and (4), $f_i(r)$, $g_i(r)$ and $h_i(r)$ denote the Dirac two-component radial wave function of the *uud* quarks, respectively (multiplied be the corresponding coefficients). In each term, the number in parentheses indicates how the two angular momenta of the *uu* quarks are coupled. Below, J_{uu} denotes the value of this quantity.

The following remarks explain the form of these terms. An important issue is the coupling of the two *uu* quark that

abide by the Pauli exclusion principle. For this reason, J_{uu} is given explicitly in each term. Another restriction stems from the rule of angular momentum addition. Thus, for every term, the following relation must hold in order to yield a total spin-1/2 for the proton: $J_{uu} = j_d \pm 1/2$. These rules explain the specific structure of each term of (10) which is described below.

In terms 0,1 the two spin-1/2 are coupled antisymmetrically to $J_{uu} = 0$ and the two radial function are the same. In terms 2,3 these spins are coupled symmetrically to $J_{uu} = 1$ and antisymmetry is obtained from the two orthogonal radial functions. In terms 4,5 the different orbitals of the *uu* quarks enable antisymmetrization. Thus, the two spin-1/2 functions are coupled to $J_{uu} = 0$ and $J_{uu} = 1$, respectively. The radial functions are not the same because of the different orbitals. In terms 6,7 the spins are coupled to $J_{uu} = 1$. In terms 8,9 we have a symmetric angular momentum coupling $J_{uu} = 1$ and the antisymmetry is obtained from the orthogonality of the radial function $f_i(r)$, $g_i(r)$. Terms *a,b* are analogous to terms 6,7, respectively. In term *c* the different *uu* orbitals enable antisymmetrization and they are coupled to $J_{uu} = 1$.

A comparison of the expansion of the He atom ground state (3) and that of the proton (10) shows the following points:

1. If the expansion is truncated after the same value of a single particle angular momentum then the number of terms in the proton's expansion is significantly larger.
2. This conclusion is strengthened by the fact that the proton has a non-negligible probability of an additional quark-antiquark pair. Evidently, an inclusion of this pair increases the number of acceptable configurations.
3. Calculations show that the number of configurations required for the ground state spin-0 of the two electron He atom is not very small and that there is no single configuration that dominates the state [7]. Now the proton is a spin-1/2 relativistic particle made of three valence quarks. Therefore, it is very reasonable to assume that its wave function takes a multiconfiguration form.

Using angular momentum algebra, one realizes that in most cases an individual quark does not take the proton's spin direction. This is seen on two levels. First, the upper and the lower parts of the quark single particle function have $l = j \pm 1/2$. Furthermore, the relativistic quark state indicates that the coefficients of the upper and the lower part of the Dirac four component function take a similar size. Hence, for the case where $j = l - 1/2$, the Clebsch-Gordan coefficients [3] used for coupling the spatial angular momentum and the spin indicate that the spin of either the upper or the lower Dirac spinor has no definite direction and that the coefficient of the spin down is not smaller than that of the spin up [3, see p. 519].

Let us turn to the coupling of the quark spins. The 3-quark terms can be divided into two sets having $j_{uu} = 0$ and $j_{uu} > 0$, respectively. For $j_{uu} = 0$ one finds that the single particle $j_d = 1/2$ and this spin is partially parallel to the proton's spin. For cases where $j_{uu} > 0$, the proton's quark spins are coupled in a form where they take both up and down direction so that they practically cancel each other. The additional quark-antiquark pair increases spin direction mixture. It can be concluded that the quark spin contribute a not very large portion of the proton spin and the rest comes from the quark spatial motion. This conclusion is supported by experiment [9].

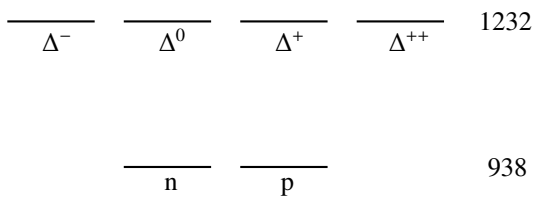

Fig. 1: Energy levels of the nucleon and the Δ isospin multiplets (MeV).

5 The State of the Δ^{++} Baryon

In textbooks it is argued that without QCD, the state of the Δ^{++} baryon demonstrates a fiasco of the Fermi-Dirac statistics [10, see p. 5]. The argument is based on the claim that the Δ^{++} takes the lowest energy state of the Δ baryons [11] and therefore, its spatial wave function consists of three single particle symmetric s-waves of each of its three *uuu* quarks. Now the $J^\pi = 3/2^+$ state of the Δ baryons shows that also their spin is symmetric. It means that the Δ^{++} is regarded to have space, spin and isospin symmetric components of its wave function. As stated above, textbooks claim that this outcome contradicts the Fermi-Dirac statistics. However, using the physical issues discussed in this work and the energy level diagram (see Fig. 1) of the nucleon and the Δ baryons, it is proved that this textbook argument is incorrect.

- As explained in section 3, all members of an isospin multiplet have the same symmetry. Hence, if there is a problem with the Fermi-Dirac statistics of the Δ^{++} then the same problem exists with Δ^+ and Δ^0. It follows that if the above mentioned textbook argument is correct then it is certainly incomplete.
- *The data described in fig. 1 shows that* Δ^+ *is an excited state of the proton.* Hence, its larger mass is completely understood. Thus, there is no problem with the Fermi-Dirac statistics of the Δ^+ baryon. Analogous relations hold for the neutron and the Δ^0 baryons. Using the identical statistical state of the four Δ baryons (8), one realizes that there is no problem with the Fermi-Dirac statistics of the Δ^{++} and the Δ^- baryons.
- The multi-configuration structure of a bound system of

Dirac particles is known for about 50 years [7]. In particular, the multi-configurations structure of all baryons (like in (10)) proves that, contrary to the above mentioned textbook argument [10, see p. 5], the single particle spatial wave functions of the three *u* quarks of the Δ^{++} baryon *are not a pure s-wave*.

6 Conclusions

This work uses the Wigner-Racah mathematical structure and proves two very important points. It explains the small contribution of quark's spin to the overall proton spin. Therefore, it eliminates the basis for the proton spin crisis. It also proves that everything is OK with the Fermi-Dirac statistics of the Δ^{++} baryon. It follows that there is no need to introduce the QCD's color degree of freedom in order to build an antisymmetric wave function for this baryon.

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11. Today more than 10 different Δ baryonic multiplets are identified [8].

Einstein's Planetary Equation: An Analytical Solution

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Einstein's planetary equation can be solved by the method of successive approximations.This yields two linearly independent solutions. An analytical solution is presented for this equation. This solution produces eight linearly independent mathematical solutions, two of which are given approximately by the well-known method of successive approximations.

1 Introduction

Einstein's planetary equation is given [1] by

$$\frac{d^2u}{d\phi^2} + u - \frac{k}{l^2} = \frac{3k}{c^2}u^2 \tag{1}$$

where ϕ and u are the instantaneous angular and reciprocal radial displacements of the planet in the fixed plane of motion, with the Sun as origin, l is the constant angular momentum per unit mass [2] and

$$k = GM \tag{2}$$

where M is the rest mass of the Sun, G is the universal gravitational constant and c is the speed of light in vacuum. The method of successive approximations yields the solution of equation (1) [1] as:

$$r(\phi) = \frac{1}{u(\phi)} = \frac{\left(1-\epsilon_0^2\right)a_0}{1+\epsilon_0\cos\left[\left(1-\frac{3k^2}{c^2l^2}\right)\phi+\alpha\right]} \tag{3}$$

where ϵ_0 is the eccentricity, a_0 the semi-major axis and α is the epoch. The second solution of equation (1) obtained from the method of successive approximations is the solution (3) with sine instead of cosine. The effect revealed by these two approximate solutions is an anomalous precession of the planetary orbit in which the perihelion advances by an angle per revolution Δ given [1] by

$$\Delta = \frac{6\pi k^2}{c^2l^2}. \tag{4}$$

In this article, Einstein's planetary equation (1) is solved analytically.

2 Analytical Solution

Suppose the analytical solution of equation (1) is in the form of a Taylor or Laurent series given as

$$u(\phi) = \sum_{n=0}^{\infty} A_n\, exp\left\{ni\left(\omega\phi + \phi_0\right)\right\} \tag{5}$$

where A_n, ω and ϕ_0 are constants. Then, substituting (5) into (1), applying the linear independence of the exponential functions and equating corresponding coefficients on both sides yields the following system of equations:

$$\frac{3k}{c^2}A_0^2 - A_0 + \frac{k}{l^2} = 0 \tag{6}$$

$$\omega^2 = 1 - \frac{6k}{c^2}A_0 \tag{7}$$

$$A_1 = arbitrary\ constant \tag{8}$$

$$A_2 = \frac{3k}{c^2}\left(1 - 2^2\omega^2 - \frac{6k}{c^2}A_0\right)^{-1}A_1^2 \tag{9}$$

$$\begin{aligned} A_3 \;=\; & \frac{18k^2}{c^4}\left[\left(1 - 2^2\omega^2 - \frac{6k}{c^2}A_0\right)\right. \\ & \left.\left(1 - 3^2\omega^2 - \frac{6k}{c^2}A_0\right)\right]^{-1} A_1^3 \end{aligned} \tag{10}$$

and so on. Equation (6) is a binomial in A_0 and has two possible roots given by

$$A_{0^-} = \frac{c^2}{6k}\left[1 - \left(1 - \frac{12k^2}{c^2l^2}\right)^{1/2}\right] \tag{11}$$

and

$$A_{0^+} = \frac{c^2}{6k}\left[1 + \left(1 - \frac{12k^2}{c^2l^2}\right)^{1/2}\right] \tag{12}$$

It follows from substituting (11) into (7) that they are two possible values of the parameter ω given as:

$$\omega_1 = \left\{1 - \left[1 - \left(1 - \frac{12k^2}{c^2l^2}\right)^{1/2}\right]\right\}^{1/2} \tag{13}$$

and

$$\omega_2 = -\left\{1 - \left[1 - \left(1 - \frac{12k^2}{c^2l^2}\right)^{1/2}\right]\right\}^{1/2} \tag{14}$$

Similarly, by substituting (12) into (7) other two possible values of the parameter are obtained as:

$$\omega_3 = \left\{1 - \left[1 + \left(1 - \frac{12k^2}{c^2l^2}\right)^{1/2}\right]\right\}^{1/2} \tag{15}$$

and

$$\omega_4 = -\left\{1 - \left[1 + \left(1 - \frac{12k^2}{c^2 l^2}\right)^{1/2}\right]\right\}^{1/2}. \quad (16)$$

It follows from equation (9) that A_2 has eight possible values given by

$$A_{2^1} = \frac{3k}{c^2}\left(1 - 2^2\omega_1^2 - \frac{6k}{c^2}A_{0^+}\right)^{-1} A_1^2 \quad (17)$$

$$A_{2^2} = \frac{3k}{c^2}\left(1 - 2^2\omega_1^2 - \frac{6k}{c^2}A_{0^-}\right)^{-1} A_1^2 \quad (18)$$

$$A_{2^3} = \frac{3k}{c^2}\left(1 - 2^2\omega_2^2 - \frac{6k}{c^2}A_{0^+}\right)^{-1} A_1^2 \quad (19)$$

$$A_{2^4} = \frac{3k}{c^2}\left(1 - 2^2\omega_2^2 - \frac{6k}{c^2}A_{0^-}\right)^{-1} A_1^2 \quad (20)$$

$$A_{2^5} = \frac{3k}{c^2}\left(1 - 2^2\omega_3^2 - \frac{6k}{c^2}A_{0^+}\right)^{-1} A_1^2 \quad (21)$$

$$A_{2^6} = \frac{3k}{c^2}\left(1 - 2^2\omega_3^2 - \frac{6k}{c^2}A_{0^-}\right)^{-1} A_1^2 \quad (22)$$

$$A_{2^7} = \frac{3k}{c^2}\left(1 - 2^2\omega_4^2 - \frac{6k}{c^2}A_{0^+}\right)^{-1} A_1^2 \quad (23)$$

$$A_{2^8} = \frac{3k}{c^2}\left(1 - 2^2\omega_4^2 - \frac{6k}{c^2}A_{0^-}\right)^{-1} A_1^2 \quad (24)$$

Similarly, it follows from (10) that A_3 has eight possible values. The above sequence may be continued to derive the eight possible corresponding values for each of the constants A_4, A_5, ... in terms of the arbitrary constant A_1 . This sequence implies eight mathematically possible analytical solutions of Einstein's planetary equation of the form:

$$\begin{aligned} u(\phi) &= A_0 + A_1 exp\left[i\left(\omega\phi + \phi_0\right)\right] + \\ &\quad f_2(A_1) exp\left[2i\left(\omega\phi + \phi_0\right)\right] + ... \\ &\quad f_n exp\left[ni\left(\omega\phi + \phi_0\right)\right] + ... \end{aligned} \quad (25)$$

where ϕ_0 and A_1 are arbitrary.

Now, consider the first exact analytical solution corresponding to equations (12) and (14). In this case, it follows from (9) that

$$A_2 = f_2(A_1) = -\frac{k}{c^2}\left(1 - \frac{6k}{c^2}A_{0^-}\right)^{-1} A_1^2 \quad (26)$$

and

$$A_3 = f_3(A_1) \quad (27)$$

and in general

$$A_n = f_n(A_1),\ n = 4,\ 5,\ ... \quad (28)$$

In this case, the exact analytical solution of Einstein's planetary equation is a complex function of ϕ which may be written in Cartesian form as

$$u(\phi) = x(\phi) + iy(\phi) \quad (29)$$

where

$$\begin{aligned} x(\phi) &= A_{0^-} + A_1\cos\left(\omega_1\phi + \phi_0\right) + \\ &\quad f_2(A_1)\cos 2\left[\left(\omega_1\phi + \phi_0\right)\right] + \ldots \end{aligned} \quad (30)$$

and

$$\begin{aligned} y(\phi) &= A_{0^-} + A_1\sin\left(\omega_1\phi + \phi_0\right) + \\ &\quad f_2(A_1)\sin 2\left[\left(\omega_1\phi + \phi_0\right)\right] + \ldots \end{aligned} \quad (31)$$

Therefore it may be expressed in Euler form as

$$u(\phi) = R(\phi)e^{i\Phi(\phi)} \quad (32)$$

where R is the magnitude given by

$$R(\phi) = \left\{x^2(\phi) + y^2(\phi)\right\}^{\frac{1}{2}} \quad (33)$$

and Φ is the argument given by

$$\Phi(\phi) = \tan^{-1}\left\{\frac{y(\phi)}{x(\phi)}\right\}. \quad (34)$$

Hence by definition the instantaneous radial coordinate of the planet from the Sun, r , is given by

$$r(\phi) = R^{-1}(\phi)\,\ell^{-i\Phi(\phi)}. \quad (35)$$

3 Physical Interpretation of First Analytical Solution

The instantaneous complex radial displacement r of the planet from the Sun is given in terms of the angular displacement Φ as

$$r(\phi) = R^{-1}(\phi)\,\ell^{-i\Phi(\phi)}. \quad (36)$$

Therefore the magnitude of the instantaneous complex radial displacement of the planet from the Sun can be considered to be the real physically measurable instantaneous radial displacement, r_p. Thus,

$$r_p(\phi) = R^{-1}(\phi) = \left\{x^2(\phi) + y^2(\phi)\right\}^{-\frac{1}{2}}. \quad (37)$$

It may be noted from (9) and (10) that for $n > 1$ $f_n(A_1)$ is of order at most c^{-2n}. Therefore as a first approximation let us neglect all terms in $f_n(A_1)$ for $n > 1$. Then it follows from (37) and (31)–(32) that

$$r_p(\phi) = \frac{A}{1 + \varepsilon_1\cos\left(\omega_1\phi + \phi_0\right)} \quad (38)$$

where

$$A = \frac{1}{A_{0^-}}\left(1 + \frac{A_1^2}{A_{0^-}^2}\right)^{-\frac{1}{2}} \quad (39)$$

and

$$\varepsilon_1 = \frac{A_1}{A_{0-}}\left(1 + \frac{A_1^2}{A_{0-}^2}\right)^{-1}. \tag{40}$$

Consequently, the orbit is a precesing conic section with eccentricity and hence semi-major axis given by

$$a = \frac{A}{1 - \varepsilon_1^2} \tag{41}$$

and perihelion displacement angle Δ given by

$$\Delta = 2\pi\left(\omega_1^{-1} - 1\right). \tag{42}$$

It follows from (42) and (14) that the perihelion displacement angle from this analytical method is given explicitly as

$$\Delta = \frac{6\pi k^2}{c^2 l^2} + \frac{54\pi k^4}{c^4 l^4}. \tag{43}$$

This is an advance precisely as obtained from the method of successive approximations. The leading term in (43) is identically the same as the leading term of the corresponding advance from the method of successive approximations [1]. Moreso, this analytical method reveals the exact corrections of all orders of c^{-2} to the leading term in (44).

It also follows from (40) and (12) that the orbital eccentricity ε_1 from this analytical method is given explicitly as

$$\begin{aligned} \varepsilon_1 \;=\; & \frac{l^2 A_1}{k}\left(1 + \frac{3k^2}{c^2 l^2} + \ldots\right)^{-1} \\ & \left[1 + \frac{l^4 A_1^2}{k^2}\left(1 + \frac{3k^2}{c^2 l^2} + \ldots\right)^{-2}\right]^{-1}. \end{aligned} \tag{44}$$

Thus, an experimental measurement of the orbital eccentricity ε_1 in equation (45) is sufficient to determine the parameter A_1 that occurs in the exact analytical solution. It also follows from this result that the analytical method in this article reveals post-Newtonian corrections of all order of c^{-2} to the planetary orbital eccentricity which have not been derived from the method of successive approximations.

It also follows from equations (41) and (14) that the orbital semi-major axis from this analytical method is given explicitly as

$$\begin{aligned} a \;=\; & \frac{l^2}{\left(1 - \varepsilon_1^2\right)k}\left(1 + \frac{3k^2}{c^2 l^2} + \ldots\right)^{-1} \\ & \left[1 + \frac{l^4 A_1^2}{k^2}\left(1 + \frac{3k^2}{c^2 l^2} + \ldots\right)^{-2}\right]^{-1}. \end{aligned} \tag{45}$$

Thus, this analytical method reveals post-Newtonian corrections of all orders of c^{-2} to planetary semi-major axis, which have not been derived from the method of successive approximations.

4 Conclusion

This article uncovers an analytical solution to Einstein's planetary equation. The first analytical solution to the order of c^{-2}, reveals post-Newtonian corrections to the orbital eccentricity and semi-major axis of a planet. Moreover, up to the second iterate there is no such correction from the method of successive approximations. Consequently, these unknown corrections to orbital eccentricity revealed by the analytical approach in this article are opened up for experimental investigation.

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Building Galactic Density Profiles

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The principal objective of this study is to provide a method to build galactic density profiles. The models developed in this study were tested against the zCosmos deep field galactic survey. The herein study suggests that light travel distances need to be converted into Euclidean distances in order to derive the galactic density profile of the survey which is the evolution of galactic density over time. In addition, the present study indicates an Ω_m of 0.19.

1 Introduction

The main purpose of the herein study is to provide a method to build galactic density profiles which requires the conversion of light travel distances (LTD) to Euclidean distances. The LTD is the distance traversed by a photon between the time it is emitted and the time it reaches the observer. In astronomical units, the Euclidean distance is defined as the equivalent distance that would be traversed by a photon between the time it is emitted and the time it reaches the observer if there were no expansion of the Universe.

The zCosmos deep field was used to derive the galatic density profile based on a sampling method, and to compute an estimate of the mean mass density of the Universe.

2 Mathematical development and methods

Galactic density profiles have been derived from the normalization of the galactic counts between redshift buckets by dividing by the corresponding sample volume. For the scenario with additive LTD, the LTDs were directly fed into the sampling volume formula eq. (2). For the scenario with a model of the motion of the photon in an expanding space, the Euclidean distances were fed into the sampling volume formula.

2.1 Method to build galactic density profiles

2.1.1 Normalisation of galactic counts

Let us consider an observer positioned at the center of a sphere of radius r and looking at a cone of sky in the z direction. The observer is counting galaxies within this cone, and measures the redshift for each object. A histogram of the galactic counts versus redshifts is obtained by counting the set of objects contained within each redshift bucket. This histogram is required to be normalised in order to obtain the density profile. Below is derived the expression of the sampling volume of the buckets, function of r_0 the lower radius of the sampling bucket, and Δr the radius width of the bucket. The sampling volume in spherical coordinates is described by the following integral:

$$V_{r_0 o,\Delta r} = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\theta_0} \sin\theta \, d\theta \, d\varphi \int_{r_0}^{r_o+\Delta r} r^2 dr. \quad (1)$$

By solving integral (1), the sampling volume for a spherical sampling ($\theta_0 = \pi$) is expressed as following:

$$V_{r_0,\Delta r} = \frac{4\pi}{3}\left((r_0+\Delta r)^3 - r_0^3\right), \quad (2)$$

where $V_{r_0,\Delta r}$ is the sampling volume for a given bucket, r_0 the lower radius of the bucket, and Δr the radius width of the bucket.

In order to use eq. (2), the galactic counts need to be converted into spherical values, by multiplying the counts by the sphere to survey solid angle ratio (η). Given the zCosmos survey spectroscopic area of 0.075 square degrees which is the solid angle, this ratio is the following:

$$\eta = \frac{4\pi\,(180/\pi)^2}{0.075} = 550'038. \quad (3)$$

The reported survey coverage area of the zCosmos-deep field is 1 deg2, [8]. However, what is required is the solid angle which is measured by the area of the survey projected in the plan described by the right ascension in degrees and $180/\pi * \sin(declination)$. Note that the sine of declination term is due to the Jacobian for spherical coordinates. The spectroscopic area obtained with this procedure is 0.075 deg2 (surface coverage in figure 1).

2.1.2 Conversion of redshifts to LTDs

Two approaches are available for converting the redshifts from observed galaxies into LTDs, one based on cosmological redshifts and the other one on dopplerian redshifts. First, let us introduce the method based on cosmological redshifts from the calculator of Wright [16] which uses a Lambda-CDM cosmology. The followings are generally assumed for this model: a flat Universe, with parameters: Ω_M = 0.27, Ω_{vac} = 0.73 and H_o = 71 [km s^{-1} Mpc^{-1}].

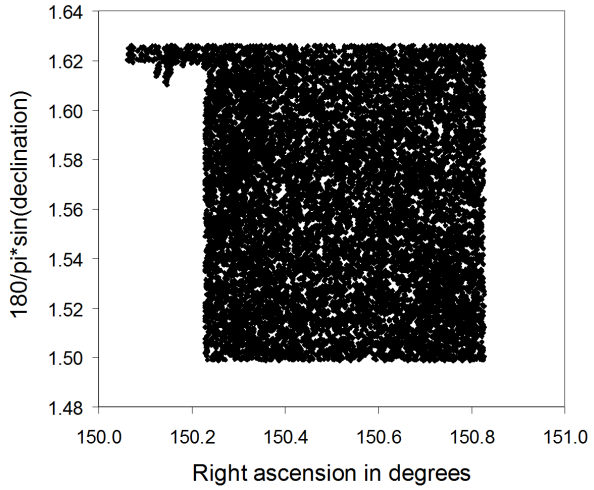

Fig. 1: Procedure to compute the spectroscopic area for the zCosmos survey as defined by the solid angle.

In the dopplerian redshift method, the relationship between redshifts and recession velocities is the following:

$$1 + z = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}. \quad (4)$$

From this equation, one may compute the recession velocity for a given redshift. Then the distance is computed as following:

$$distance = \frac{v}{H_o}. \quad (5)$$

From subsequent calculations an Ω_M of 0.19 was obtained which was used to derive the galactic density profile. Both methods give comparable distances with differences less than 5 % for redshifts up to 5.2 using Ω_M = 0.19. The difference between dopplerian and cosmological redshifts is discussed by Bedran [2]. Historically, the first solution to compute distances from cosmological redshifts was obtained by Mattig [9] which is based on Friedmann equations of general relativity. Mattig equation with q_o = 0.5 also provides distances close to what is obtained using dopplerian redshifts; however, Mattig had to assume that conservation of mass is applicable to the Universe in his derivations which is a big bang cosmology. On the other hand, dopplerian redshifts do not require any assumption on the cosmology, and present the advantage that they also explain blueshifts that are being observed such as for Andromeda.

2.1.3 Sources of data

The zCosmos galactic survey Data Release DR1 was used [8].

2.2 Propagation of light in an expanding space

The main hypothesis for the development of a model for the propagation of light in an expanding space, is that the speed of light is frame-independent. Considering redshifts, this means that the relative movement of a light source does not change the speed of light emitted; however, it does add or subtract energy to the photon. In a dopplerian world, this change in energy level changes the frequency of the source of light, and not the speed. However, as space between the photon and the observer expands, this expansion is added to the overall distance the photon has to travel in order to reach the observer - in over words the speed of light is frame-independent with respect to the local space. This implies that there exists a distance for which the recession speed between the observer and the photon equals the speed of light, which is the Hubble sphere, and that recession speed can exceed than the speed of light for large distances. The frame-independent hypothesis for the speed of light has been established in the past with the experiment of Michelson-Morley [10]. Based on observations of double stars [14, 4] it was shown that the velocity of propagation of light does not depend on the velocity of motion of the body emitting the light.

As a consequence of the above, LTDs are not anymore additive, meaning that if we have three points aligned in space, the distance between the two extremes is not anymore equal to the sum of the two sub-segments as measured in LTDs.

Based on the above hypothesis, the Euclidean distance between the photon and the observer is described by the following differential equation:

$$\frac{dy}{dt} = -c + H_o \cdot c \cdot T, \quad (6)$$

where y is the Euclidean distance between the photon and the observer, T the LTD between the observer and the photon, c the celerity of light, and H_o the Hubble constant.

2.3 Conversion of light travel distances to Euclidean distances

Let us consider a photon initially situated at a Euclidean distance y_o from the observer and moving at celerity c in the direction of the observer. Let us say T is the initial LTD between the photon and the observer, and define the Hubble constant function of LTDs.

The differential equation describing the motion of the photon in the LTD framework is described by eq. (6). By taking a reference point in time in the past, and T_b be today time from this reference point, we get $T = T_b - t$. Hence, $dt = -dT$. Therefore, eq. (6) becomes:

$$\frac{dy}{dT} = c - Ho \cdot c \cdot T, \quad (7)$$

with boundary conditions $y(T) = y_o$ and $y(0) = 0$.

By integration from 0 to T, the following relationship relating Euclidean distances y to light travel distances T is obtained:

$$y = c \cdot T - \frac{c \cdot H_o \cdot T^2}{2}. \quad (8)$$

The corresponding horizon computed by setting $\frac{dy}{dT} = 0$ is $T_h = \frac{1}{H_o}$ which is the Hubble sphere.

2.4 The Hubble constant was determined with respect to LTDs

In general the literature refers to the Hubble constant measured with respect to LTDs. A common way to obtain the Hubble constant is based on standard candles with supernovae and cepheids [13, 1] and the Tully-Fisher relation [5]. Both the standard candle and Tully-Fisher method rely on the distance modulus. As shown below the distance modulus gives a measure of LTDs and not Euclidean distances.

Let us recall the derivation of the distance modulus. The magnitude as defined by [12] is:

$$m = -2.5 \log F + K, \quad (9)$$

where m is the magnitude, F the brightness or flux and K a constant. The absolute magnitude is defined as the apparent magnitude measured at 10 parsecs from the source.

Planck's law for the energy of the photon leads to a redshift correction to the distance modulus

$$E = \frac{h \cdot c}{\lambda}, \quad (10)$$

where E is the energy of the photon, h the Planck's constant, and λ the light wavelength.

The ratio of observed to emitted energy flux is derived from eq. (10), leading to

$$\frac{E_{obs}}{E_{emit}} = \frac{\lambda_{emit}}{\lambda_{obs}} = \frac{1}{1+z}. \quad (11)$$

From geometrical considerations, the projected surface of the source of light on the receptor diminishes with a relationship proportional to the inverse of square distance from the source of light; hence, the following relationship is obtained for the brightness or flux:

$$F_{obs} \propto \frac{L_{emit}}{d^2} \cdot \frac{E_{obs}}{E_{emit}}, \quad (12)$$

where L_{emit} is the emitted luminosity and d the distance to the source of light.

Combining eq. (9), (11) and (12), we obtain:

$$m = -2.5 \log \left(\frac{L_{emit}}{d^2 \cdot (1+z)} \right) + K. \quad (13)$$

And, because z is close to zero at 10 Parsec:

$$M = -2.5 \log \left(\frac{L_{emit}}{100} \right) + K, \quad (14)$$

where M is the absolute magnitude.

Hence, the distance modulus, eq. (13) minus (14) is:

$$m - M = -5 + 5 \log d + 2.5 \log(1+z), \quad (15)$$

with d in parsec and log means the logarithm to base 10.

The expansion of the Universe adds up to the Euclidean distance, and therefore the apparent magnitude of the source of light is fainter than if no expansion was present.

2.5 Evolution of the galactic density assuming no new galaxy formation

Assuming cosmological redshifts we have:

$$1 + z = \frac{a_o}{a_1}, \quad (16)$$

where a_o and a_1 are respectively the present scale factor and the scale factor at z.

From the conservation of mass the density is proportional to the inverse of the cubic scale factor:

$$\rho \propto \frac{1}{a^3}. \quad (17)$$

Therefore, the model for the evolution of the density with respect to the present density is the following:

$$\rho_t = \rho_o \cdot (1+z)^3, \quad (18)$$

where ρ_t is the density in the past at redshift z and ρ_o is the present density.

3 Results

3.1 A flat density profile using Euclidean distances

Galactic density profiles have been derived for the two antagonistic scenarios respectively assuming that LTDs are additive, and with the propagation of light in an expanding space (figure 2). Note that the galactic density profiles obtained with cosmological redshifts and dopplerian redshifts are very similar. The highest redshift galaxies observed for the survey (z = 5.2) are very close to the Hubble sphere (which are at 13.65 Glyr) as calculated from cosmological redshifts with Ω_m=0.19.

The theoretical evolution of the galactic density with respect to the present density assuming no new formation of galaxies (figure 3) was computed assuming cosmological redshifts with eq. (18). Note that the first point in the galactic density profile is not representative of the average density as the sample volume is very small; hence, the measure represents the density in the neighbouring galactic cluster of the Milky Way (figure 2 and 3).

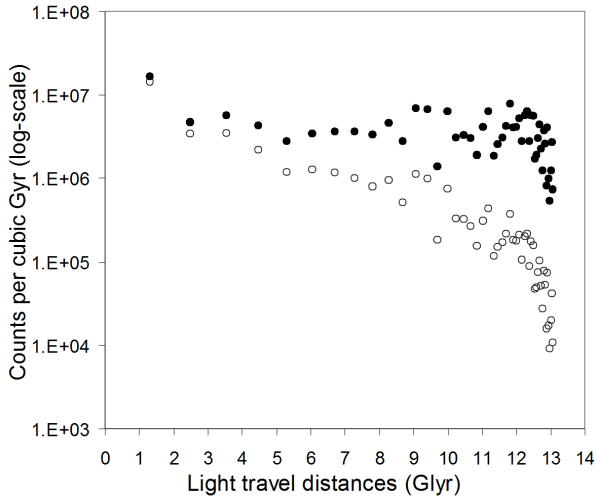

Fig. 2: Galactic density profile derived from the equivalent spherical sampling, where Glyr are billion light years from today. LTDs are obtained from redshift conversion with dopplerian redshifts. The blank dots indicate densities based on LTDs. The solid dots indicate densities obtained with Euclidean distances on the basis of dopplerian redshifts.

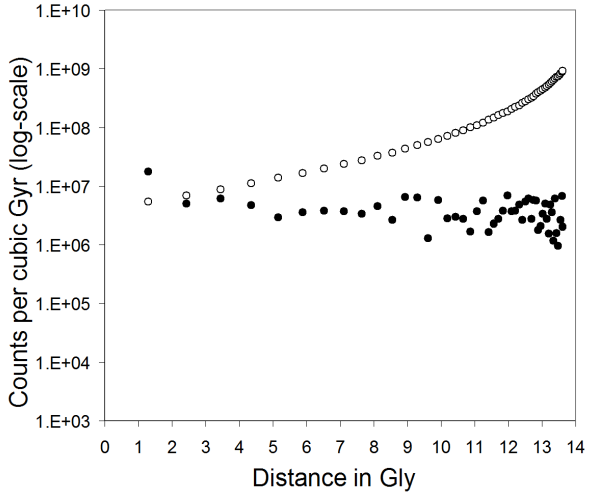

Fig. 3: Galactic density profile derived from the equivalent spherical sampling, where Gly are billion light years from today. LTDs are obtained from redshift conversion with cosmological redshifts (omega matter of 0.19).The solid dots indicate densities obtained with Euclidean distances on the basis of cosmological redshifts. The blank dots indicate the theoretical evolution of galaxies assuming that the survey is incomplete (with no new galaxy formation).

3.2 Estimation of Ω matter from galactic counts

The average galactic mass estimated from light deflection [15] is $1.7 \times 10^{11}\ M_{\odot}$. The Universe mean density is obtained by multiplying this figure with the average galactic count per cubic Glyr. Using dopplerian redshits the galactic count density is 4.6×10^{6} counts per cubic Glyr, leading to a mean Universe density of $1.84 \times 10^{-30}\ g/cm^3$. Using a Hubble constant of 71 $km/s/Mpc$ and recent estimates of the gravitational constant of $6.67 \times 10^{-8}\ cm^3/g/sec^2$ [11], the critical density is estimated at $9.47 \times 10^{-30}\ g/cm^3$ (from $\rho_c = \frac{3H^2}{8\pi G}$). Therefore, the corresponding Ω_m equals to 0.19. Note that smaller values of the Hubble constant would lead to a higher Ω_m.

3.3 Estimation of the number of galaxies in the visible Universe

Another challenge is to estimate the number of galaxies in the visible Universe. Using the galactic density in the nearby Universe from figure 2 expressed per cubic Glyr LTD, and the volume of the sphere of radius 14 Gly LTD, the number of galaxies in the visible Universe is estimated at 175 billion. Gott et al. [6] estimated a number of galaxies in the visible Universe at about 170 billion based on the Sloan Digital Sky Survey luminosity function data using the Press-Schechter theory. Both figures are consistent with each other; however, the author believes that these figures need to be reviewed to account only for the Euclidean radius when computing the volume of the visible Universe. As the galactic density profile is flat, it is expected that the estimated number of galaxies in the visible Universe is internally consistant with the bulk amount of galaxies observed in the survey converted to spherical values, i.e. multiplying the number of galaxies in the survey (10046 galaxies) by the sphere to survey solid angle ratio, which leads to 5.5 billion galaxies (see Table 1).

4 Discussion

A new approach is proposed in the present study to derive the galactic density profile which is based on the conversion of light travel distances to Euclidean distances. The method has been tested by computing the galactic density profiles based on the data from the zCosmos deep field survey.

In the scenario using LTDs with the sampling method, the galactic count per cubic Glyr grows according to a steep slope (figure 2), without accounting for the effect of the expansion which should add up to this growth. There is no explanation for such result - this scenario appears to be unrealistic. The scenario using Euclidean distances, shows a flat profile for the galactic counts per cubic Gyr (figure 3). However, there is still a gap between the computed galactic density profile and the theoretical evolution of galactic densities assuming no new galaxy formation. Leaving aside model bias, this gap may be interpreted as if galaxies grow in number over time. Another hypothesis is that the galactic survey is incomplete meaning that faint galaxies are left asside from the zCosmos survey at large distances, which would account for the missing galaxies causing the gap in figure 3. The theoretical density obtained by conservation of mass is too large by a factor

Table 1: Estimation of the number of galaxies in the visible Universe (radius 14 Glyr) using LTD distances and Euclidean distances.

	Radius of the visible Universe	Galactic density	Estimated number of galaxies
Using LTDs	14 Glyr	1.52×10^7 counts per cubic Glyr	175 billion
Using Euclidean distances with dopplerian redshifts	6.90 Glyr	4.60×10^6 counts per cubic Glyr	6.3 billion
Galaxy count of the survey converted to spherical values			5.5 billion

of order 200 at redshift 5.2. This discrepancy is unrealistically to large. Clearly more detailed work needs to be carried out to investigate this gap.

By applying conservation of mass, as we approach the singularity of the big bang, the Universe would have been so dense that it is difficult to explain how gravity did not prevent the early Universe from collapsing. A possibility is that the Hubble constant was much higher in the past leading to a higher critical density - cosmic inflation would still be necessary to overcome this issue. From the present study, the galactic density appears to be constant over time, which would corroborate the steady state cosmology of [3, 7]. The other condition being that the Hubble constant remains unchanged over time.

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An Analysis of States in the Phase Space: Uncertainty, Entropy and Diffusion

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The paper aims to show the physical link between Fick's laws and entropy increase in an isolated diffusion system, initially inhomogeneous and out of the thermodynamic equilibrium, within which transport of matter is allowed to occur. Both the concentration gradient law and the entropic terms characterizing the diffusion process are inferred from the uncertainty equations of statistical quantum mechanics. The approach is very general and holds for diffusion systems in solid, liquid and gas phases.

1 Introduction

Diffusion concerns the transport of matter activated by thermal motion of atoms and molecules. Theoretical and experimental reviews on the mechanisms of mass transfer in solid, liquid and gas phases are widely reported in literature, e.g. [1, 2]. The importance of diffusion is well recognized in the kinetics of microstructural changes, nucleation of new phases, phase transformations, homogeneization and recrystallization of alloys and so on [3]; for instance electric conduction includes phenomena closely related to the transport mechanisms of ions and electrons.The theoretical background of the diffusion is based on an intuitive hypothesis: the driving energy that governs the mass transfer is related to the concentration gradient of molecules or atoms or ions in a diffusion medium, which can be simply the vacuum or a gas/liquid/solid phase. Such an assumption is so simple and reasonable to skip a more profound consideration just about the physical meaning of its general character. It is sensible to expect that this generality, and that of the related concentration gradient driving force itself, should be in fact consequence of some general principle of nature. This consideration recalls in effect the second law of thermodynamics, as concerns in particular the probabilistic character of the entropy. Consider an arbitrary number of particles "a" diffusing within a medium "b"; whatever the former might be, e.g. ions, atoms, molecules and so on, in the following they will be shortly referred to as particles, whereas the system formed by "a" and "b" will be referred to as diffusion system. One expects that after a proper time range, the system attains the most probable configuration, i.e. a uniform distribution of "a" into "b" regardless of the particular initial configuration assumed in general in a non-equilibrium state. So a net mass flow was necessarily occurring before reaching this limit situation, after which it is no longer allowed to occur. The entropy seems to be the thermodynamic concept most closely related to describe the transient and final configurations. This means that: (i) the dimensionless entropy formula $-\sum_i w_i \log(w_i)$, where the index i numbers the thermodynamic states allowed to the diffusing particles, should be involved since the beginning into the concentration gradient formulation of any diffusion problem; (ii) this formula should reduce to the simpler Boltzmann form $-\log(w_{eq})$ when the equilibrium configuration is effectively attained; (iii) the mass flow $\mathbf{J}$ is by consequence different from zero only during the time step (i), whereas it reduces to zero at the asymptotic time step (ii). Our knowledge on the diffusion process is thus based on a phenomenological hypothesis, the concentration gradient law, and on a general principle of nature, the entropy. It would be significant to regard both concepts as a natural consequence of a unique and more general principle of nature, without the need of phenomenological assumptions. Of course a general approach to this problem cannot leave out the quantum aspect of any problem inherent the dynamics of particles on microscopic scale. Justifying from the quantum point of view the concentration gradient driven diffusion law would provide a sound physical basis to the general problem of mass transport, whereas the continuity equation, if applicable, would also appear itself as a corollary identified by well-defined physical requirements about the diffusion system. On the one side it is certainly significant to demonstrate by means of a unique general principle the quantum origin of the macroscopic equations describing how the configuration of the diffusion system evolves as a function of time because of the mass transfer. On the other side this task seems further noteworthy if carried out within the same theoretical frame that allows describing the quantum properties of matter. The purpose of the present paper is to investigate the quantum basis hidden into the gradient law, i.e. to demonstrate that the uncertainty is the basic quantum principle leading to the first Fick law as a corollary. Moreover the theoretical model proposed here also confirms through a simple and straightforward approach that the entropy of the diffusion system is the other key concept underlying the mechanisms of mass transport.

2 Classical background

For simplicity, let us regard the diffusion system as an isolated thermodynamic system formed by an isotropic body of matter and introduce the mass flow as follows:

$$\mathbf{J} = c\mathbf{v}, \tag{2,1}$$

where c is the concentration or more in general the activity of the diffusing particle and $\mathbf{v}$ its displacement velocity. Eq.

2,1 is simply a definition. A further equation appears necessary to introduce a physical hypothesis about the thermodynamic force **F** that triggers the flow. Expressing this hypothesis through the following equation, known as first Fick law

$$\mathbf{J} = -D\nabla c, \qquad c = c(x,t) \tag{2,2}$$

and combining these equations, one finds indeed

$$\mathbf{v} = -\frac{D}{k_B T}\nabla[k_B T \log(c/c_0)], \qquad c_0 = c_0(t), \tag{2,3}$$

where c_0 is an arbitrary reference concentration not dependent upon x but possibly dependent on time. The definition of mobility β of the diffusing particle

$$\mathbf{v} = \beta\mathbf{F} \tag{2,4}$$

entails therefore as a consequence at constant T

$$D = \beta k_B T, \qquad \mathbf{F} = -\nabla[k_B T \log(c/c_0)]. \tag{2,5}$$

One finds therefore through the definition of mobility both the sought force, which reasonably results equal to the gradient of the potential energy $\mu = k_B T \log(c/c_0)$, and the well known Einstein equation linking β to D. The form of **F** provides a partial answer to the aforesaid point (iii): if c is equal everywhere in the diffusion system, then it does not longer depend upon x; so, defining c_0 equal or proportional to this uniform limit value of c, one finds $\mathbf{F} = 0$ and thus $\mathbf{v} = 0$ everywhere. This shows that **F** accounts for the net mass flow in the diffusion system until $c \to c_0$. These preliminary considerations highlight that the diffusion law can be effectively related to a thermodynamic function, the chemical potential, that describes the driving force allowing the transport of matter. Exploit now again the basic definition eq. 2,1 to evidence how arbitrary changes of both c and **v** affect **J**. Consider then

$$\delta\mathbf{J} = \mathbf{v}\delta c + \delta\mathbf{J}', \qquad \delta\mathbf{J}' = c\delta\mathbf{v} \tag{2,6}$$

in the time range δt during which $\delta\mathbf{J}$ is allowed to occur. Note that δc can be due: (i) to the change δm of m within the reference volume V defining c or (ii) to the change δV of V for fixed m or (iii) to both reasons. In any case, defining the space range $\delta x = v_x\delta t$ where the particles are allowed to diffuse along the x-direction during δt, the x-component of eq. 2,6 reads $\delta J_x/\delta x = \delta c/\delta t + c\delta v_x/\delta x$. So, for infinitesimal changes dc and $d\mathbf{v}$ of the process parameters and of the dynamical variables dt and dx, the last equation reads $\nabla\cdot\mathbf{J} = \partial c/\partial t + c\nabla\cdot\mathbf{v}$, i.e. in general

$$\nabla\cdot\mathbf{J} = \frac{\partial(c+C)}{\partial t}, \qquad C = \int_{t_o}^{t} c'\nabla\cdot\mathbf{v}'dt', \qquad C = C(x,t) \tag{2,7}$$

with the integral calculated between the fixed time t_o, e.g. the beginning of the diffusion process, and the current time t. If holds the condition $\nabla\cdot\mathbf{v} = 0$, then $\nabla\cdot\mathbf{J} = \partial c/\partial t$ describes a particular diffusion process where the rate of concentration change is equal to the gradient of related mass flow, which necessarily means lack of sinks or sources of matter within the volume element where is defined c. Since $c\nabla\cdot\mathbf{v}$ results because of the term $\delta\mathbf{J}'$ additional to $\delta\mathbf{J}$, it appears that the well known second Fick equation is a particular case of eq. 2,6 for $\delta\mathbf{J}' = 0$. Actually $\delta\mathbf{J}' \neq 0$ is due not only to a possible chemical reaction that involves the diffusing particle and modifies the local concentration of the diffusion system but, more in general, also to any local force field that attracts or repels the diffusing particles and perturbs their motion. Note indeed that $\delta\mathbf{J}' = c\mathbf{a}\delta t = \mathbf{F}'_V\delta t$ yields

$$\frac{\delta\mathbf{J}'}{\delta t} = \frac{\mathbf{F}'}{V} = \mathbf{F}'_V,$$

being in general $\mathbf{F} \neq \mathbf{F}'$. The force per unit volume $\mathbf{F}'_V$ that controls the perturbation term $\delta\mathbf{J}'$, appearing in eq. 2,6 as a perturbation of **J** is particularly interesting for charged particles diffusing in an ionic medium where polarized impurities are active. Note indeed that $\mathbf{v}\cdot\mathbf{J}$ has physical dimensions of energy per unit volume; then $\mathbf{v}\cdot\delta\mathbf{J}' = (m\delta v^2/2)V^{-1}$, i.e. the effect of $\mathbf{F}'_V$ is that of perturbing the kinetic energy of the particle in the interaction volume V. It is usually acknowledged that the time enters into the diffusion equation thanks to the continuity condition that leads to the second Fick law. Yet the mere definition of eq. 2,1 entails an interesting conclusion: regardless of the aforesaid effects related to δm that possibly alter the plain diffusion process, the time evolution of the system is actually consequence of the concentration gradient law; although the Fick hypothesis does not contain explicit reference to the time, this latter enters indeed into the problem through **v**. The present considerations show therefore that the ancillary condition of continuity is not necessary to infer the second Fick law; rather, simply taking into account the finite range δt required to justify $\delta\mathbf{J}$, as nothing changes instantaneously in nature, the continuity condition appears to be itself a corollary of the definition of mass flow and not an additional boundary condition. Otherwise stated, even from a merely classical point of view the time coordinate appears a necessary ingredient together with the space displacement to account for the mass transfer in any diffusion problem; consequently the position $\nabla\cdot\mathbf{v} = 0$ does not represent a supplementary hypothesis "ad hoc" but simply a possible chance allowed for $\delta\mathbf{J}$. This conceptual basis, to be further implemented by quantum considerations reasons in the next section, is characterized by three physical features summarized as follows: (i) the definition of mass flow, eq. 2,1; (ii) the gradient concentration law; (iii) the necessity of introducing diffusion driven displacement $\delta\mathbf{r}$ and time range δt linked by $\delta\mathbf{r} = \mathbf{v}\delta t$, which also introduces the energy range $\delta\varepsilon = (\mathbf{v}\cdot\delta\mathbf{J})V$ corresponding to $\mathbf{F}\cdot\delta\mathbf{r}$ within the reference volume V defining c.

3 Preliminary quantum considerations

This section introduces the basic ideas to describe the diffusion system according to the uncertainty relationships

$$\Delta x \Delta p_x = n\hbar = \Delta t \Delta \varepsilon, \tag{3,1}$$

where n is an arbitrary number of quantum states allowed to any particle moving in the space range Δx with conjugate momentum falling in the momentum range Δp_x; the ranges are taken positive by definition. As already shown in [4], the second equality is obtained from the first one defining formally $\Delta t = \Delta x / v_x$ and $\Delta \varepsilon = \Delta p_x v_x$ linked by the same n; v_x is the velocity with which the particle travels within Δx. No hypothesis is required about the ranges that quantify the concepts of space and time uncertainty. Their sizes, in principle arbitrary, can vary from zero to infinity; moreover nothing is known about their analytical form, e.g. any local functional relationship like $p_x = p_x(x)$ within Δx is physically meaningless because both p_x and x are assumed random, unknown and unpredictable. Yet, despite such an agnostic point of view, relevant features of the ranges are apparent. First, v_x must be upper bounded. Consider a free particle in finite sized Δx and Δp_x with n finite as well: if $v_x \to \infty$ then $\Delta t \to 0$ would require $\Delta \varepsilon \to \infty$, which in turn would allow in principle an infinite energy ε; but this is impossible once having merged both uncertainties via a unique n, as $\varepsilon \to \infty$ is inconsistent with any p_x falling within the finite range Δp_x and thus necessarily finite itself. Hence the simple fact of having regarded together space and time uncertainties, i.e. admitting that both dynamical variable concur to describe any physical system, requires $v_x \leq v_x^{\max}$; eqs. 3,1 entail as a corollary the well acknowledged existence of an upper limit for the propagation rate of any signal. Moreover put $\Delta x = x - x_o$ and consider that the coordinate x_o, whatever it might be, is defined in an appropriate reference system that defines position and size of Δx and v_x as well; yet, being x_o indeterminate and indeterminable, the present approach based on Δx only does not specify in fact any particular reference system. The same holds for course also for the other ranges of eqs. 3,1, in particular for the time frame. Also, in lack of constrains or hypotheses the reference system could be in principle Cartesian or curvilinear or inertial or non-inertial or anything else. This means that any physical problem discarding "a priori" the local dynamical variables and exploiting eqs. 3,1 only, i.e. replacing

$$x \to \Delta x, \qquad p_x \to \Delta p_x, \qquad t \to \Delta t \tag{3,2}$$

holds by definition in any space-time reference system R. Hence eqs. 3,1 entail that all reference systems are indistinguishable and thus equivalent in describing the properties of quantum particles. If so, it eventually follows that the upper value allowed to v_x, whatever it might be, must be invariant in any R. Indeed v_x is defined by its own reference system; being the former arbitrary, the latter is arbitrary as well. Consider instead a well specified value of v_x, e.g. just its maximum value $v_x^{\max}$; this latter must be uniquely defined in R and in any other R' otherwise R and R' could be identified depending on their own $v_x^{\max}$, e.g. because of a greater velocity allowed in either of them, thus contradicting their indistinguishability. It appears therefore that equivalence of all reference systems and invariance of $v_x^{\max}$ are strictly linked. The time coordinate, previously introduced to account for the finite rate with which occurs the mass flow change $\delta \mathbf{J}$, still appears here as a consequence of the finite velocity v_x with which any particle moves within Δx and entails a finite time range to change the configuration of the diffusion system. Yet now Δt takes a more general physical meaning, as it appears from the previous considerations and it will be shown in the next sections. The uncertainty inherent eqs. 3,1 requires innately a time range for particles delocalized in Δx, i.e.: any physical process characterized by an energy spread $\Delta \varepsilon$ requires a time range Δt during which is to be expected a momentum change falling within Δp_x too. Previous papers [5, 6] have shown that this way of regarding eqs. 3,1 is enough to calculate the energy levels of hydrogenlike and many electron atoms/ions and diatomic molecules without solving any wave equation; then is attracting the idea that even the diffusion model can be formulated in terms of particles randomly spreading within their own delocalization space ranges conceptually arbitrary, unknown and unknowable themselves. As in the quoted papers, the statistical formulation of the quantum uncertainty is the only assumption necessary also in the context of the present problem. Suppose of having N particles in N_V elementary volumes Δx^3 of diffusion medium at a fixed time of the diffusion process. Regardless of the equilibrium or non-equilibrium situation at the given time, let

$$W_{cl} = \binom{N}{N_V}, \quad N = N(t), \quad N_V = N_V(t), \quad V = \Delta x^3 \tag{3,3}$$

be the number of ways to distribute N classical particles in N_V available sites of the diffusion medium. From a quantum point of view the combinatorial calculus still holds in principle also in the case of identical particles, as it is done in the Fermi-Dirac and Bose-Einstein statistics; one must simply replace W_{cl} with the pertinent expressions of numbers of states taking into account the indistinguishability of identical particles. Note in this respect the characteristic way of working of eqs. 3,1: once accepting the replacements 3,2, the physical interest about the system moves from the constituent particles to their phase space. On the one side just this feature of eqs. 3,1 entails the corollary of quantum indistinguishability of identical particles when considering uniquely ranges of dynamical variables where *any* particle could be found, rather than the actual dynamical variables of the particle itself; indeed this latter is never specified "a priori". On the other side this explains the general worth of the eqs. 3,1 regardless of the specific system concerned: the present model

holds in principle for diffusion processes in solid or liquid or gas phase, since no hypothesis is formulated about N and N_V of W_{FD} or W_{BE}. Further information on the process, e.g. the role of lattice defects on the effectiveness of mass transport, are to be introduced "a posteriori" through specific values of the coefficient D only, see eq. 2,2, whose quantum root will be indeed highlighted in the next section. It is important however that regardless of the kind of diffusion system, the computation of the number of allowed states accessible to the particles requires calculating the ways of distributing N objects into N_V volume elements of sizes $\Delta x^3_{1\leq i\leq N_V}$; this is possible even in the present approach because the combinatorial computation of allowed states does not require knowing where exactly are located these volumes in the diffusion system, which indeed would be prevented by eqs. 3,1. Just this computation yields the corresponding entropy of the diffusion system. At the very beginning of the diffusion process one can imagine an isolated ordered system S_0 where all particles are confined in some arbitrary volume of the system; as the particles are allowed to walkover randomly to occupy a greater volume, the number of allowed thermodynamic states progressively increases as a function of time. For $t \to \infty$ the system reaches an asymptotic state S_∞ to which corresponds a net mass flow $\mathbf{J} = 0$. The driving force of the diffusion process is thus certainly correlated to the tendency of the system towards its state of thermodynamic equilibrium and maximum entropy. Thus eqs. 3,3 simply tell that in non-equilibrium conditions the system $S(t)$ at the time t is such that $S_0 \leq S(t) < S_\infty$, until the distribution of particles corresponds to the maximum number of quantum states inherent $S_\infty \neq 0$; correspondingly $\mathbf{J} \neq 0$ describes net mass flow in the system tending the maximum entropy, until when $\mathbf{J} \to 0$. The next section aims to show that this intuitive picture of diffusion process will be inferred together with the concentration gradient law through eqs. 3,1 only, without need of any phenomenological hint.

4 Diffusion quantum model

By definition the uncertainty ranges of eqs. 3,1 include any position and momentum of the particles during the diffusion process, despite both dynamical variables are expected to change as a function of time by effect of an appropriate driving force $\mathbf{F}$. In principle one could think Δx and Δp_x large enough to include any possible change of x and p_x from the initial stage of the diffusion process to the final state of thermodynamic equilibrium; indeed the eqs. 3,1 admit possible interactions of these particles with the surrounding medium along the diffusion path $\delta\Delta x = v_x\delta t$ from $\delta t = 0$ to $\delta t \to \infty$, e.g. by elastic and anelastic collisions, through an appropriate size of the energy range $\Delta\varepsilon$. Owing to the complete arbitrariness of the ranges, however, this approach although sensible does not appear far reaching to get relevant information about the process. Yet it is also possible, and more heuristic, to require that Δx and Δp_x are allowed to change themselves as a function of time without contradicting their arbitrariness and without requiring any information on the local values x and p_x; in effect eqs. 3,1 can be differentiated with respect to t and x whatever the current time and space coordinates of particles might be. Consider thus $\delta\Delta x$ and $\delta\Delta p_x$, rather than δx and δp_x, regardless of whether the displacement of matter from two different points of the diffusing medium occurs with or without net mass flow; $\delta\Delta x$ describes the change of delocalization range to which is related the assumed change of momentum $\delta\Delta p_x$ by effect of $\mathbf{F}$. The force is here easily justified by eqs. 3,1 themselves, regardless of other specific motivations: $\Delta\dot{x}$ defining $\delta\Delta x = \Delta\dot{x}\delta t$ requires $\Delta\dot{p}_x$, which therefore affects the range of values allowed to any p_x; in turn the change of p_x, allowed to occur and thus in fact occurring, entails $F_x = m\partial v_x/\partial t$. Since it is possible to write $\delta\Delta p_x = (\partial\Delta p_x/\partial t)\delta t$, then

$$\frac{\partial\Delta p_x}{\partial t} = -n\hbar\Delta x^{-2}v_x = F_x = m\frac{\partial v_x}{\partial t}, \qquad v_x = \frac{\partial\Delta x}{\partial t}. \quad (4,1)$$

Note that here v_x is not the diffusion velocity of the particle but the rate with which changes Δx, so F_x is defined in the phase space of the particle. Yet this information is enough as concerns the diffusion problem: by effect of F_x the particle is allowed to move faster, being however still delocalized within the larger range $\Delta x' = \Delta x + \delta\Delta x$. This is why the momentum of the particle is allowed to change along with $\delta\Delta x$. The notation of velocity is unique to emphasize that v_x of eq. 4,1 and v_x of the particle defining eqs. 3,1 are both arbitrary and thus assumed coincident. On the one side this representation is consistent with well known ideas of the diffusion process, e.g. particle jumps through different sites in a crystal lattice or particle collisions randomly occurring in gas phase; on the other side it suggests that the local concentration change is described by a constant amount of mass m allowed to move slower or faster in a decreasing or increasing phase space delocalization range depending on the sign of the velocity component v_x. In this way the force component F_x introduced via the deformation of the momentum range is conceptually consistent with that of eq. 2,5: to the momentum change rate that defines the classical force corresponds now, from the point of view of eqs. 3,1, the existence of a force field $\Delta\dot{p}_x$ necessary to account for any possible $\dot{p}_x$ during the diffusion process. Let us differentiate now eqs. 3,1 with respect to x to link the change of size of the delocalization range $\delta\Delta x$ and that of the momentum range $\delta\Delta p_x$ when the particle displaces by δx; this yields

$$\frac{\partial\Delta p_x}{\partial x} = -n\hbar\Delta x^{-2}\frac{\partial\Delta x}{\partial x}. \quad (4,2)$$

Eqs. 4,1 and 4,2 describe the dynamics of the diffusing particle as a function of time in agreement with eqs. 3,1. The classical eqs. 2,6 and 2,7 have introduced $\mathbf{v}$ as macroscopic average velocity describing the net mass flow due to

the displacement rate of the particle; now the quantum approach shows how the uncertainty compels regarding a random mass flow in the phase space of the particle: the deterministic force of eq. 2,5, exactly defined at any point of the diffusion system, is now replaced by the random force of eq. 4,1 controlled by arbitrary values of n and Δx. Let us show now that this agnostic point of view, far from being elusive of the problem, is actually source of relevant physical information. The fact that the diffusion is allowed in a given volume $V = (n\hbar)^3 \Delta p_x^{-3}$ suggests exploiting an approach conceptually identical but formally different from that introduced in section 2. If the motion of the particle is random, the orientation of its momentum $\mathbf{p}$ is defined in general within a sphere of radius $|\Delta\mathbf{p}|$ whose volume is thus $\propto \Delta p_x^3$ once taking $\Delta p_x \equiv |\Delta\mathbf{p}|$; since the medium is isotropic and the uncertainty ranges are arbitrary and unknown, there is no necessity to introduce explicitly separate ranges Δp_x, Δp_y and Δp_z. So, instead of starting from $\partial \Delta p_x / \partial x$, it is more convenient considering $a''' \Delta p_x^2 \partial \Delta p_x / \partial x$, where a''' is a proper proportionality factor; indeed $\hbar^{-3} \Delta p_x^2 d\Delta p_x$ is proportional to the number of particles whose momentum was initially included in a sphere of radius Δp_x and takes after the time range δt values falling in the section of sphere between Δp_x and $\Delta p_x + d\Delta p_x$. So introducing the quantity $a'' \partial \Delta p_x^3 / \partial x$ means considering a volume element in the momentum space of the particle, which yields in turn with the help of the eq. 3,1 $a' \partial \Delta x^{-3} / \partial x$; here a'' and a' are trivial numerical factors. In conclusion, although starting from a 1D equation, we have introduced a volume element $V = \Delta x^3$ that represents an elementary volume of the diffusion medium where is located a given amount of diffusing mass m corresponding to the concentration c. This defines the equation

$$-\frac{a'}{V^2}\frac{\partial V}{\partial x} = \frac{a'}{V}\frac{\partial \log(V_o/V)}{\partial x}, \qquad (4,3)$$
$$V = V(x,t), \qquad V_o = V_o(t),$$

where the arbitrary constant V_o is a reference volume by definition not dependent on x but possibly dependent on t. Consider first the left hand side of this identity, which reads

$$-\frac{a'}{V^2}\frac{\partial V}{\partial x} = -\frac{a' m}{V^2}\frac{\partial c^{-1}}{\partial x} = \frac{a' m}{c^2 V^2}\frac{\partial c}{\partial x} = \frac{a'}{m}\frac{\partial c}{\partial x},$$
$$c = \frac{m}{V}, \qquad c = c(x,t),$$

where c has here the same physical meaning introduced in the early eq. 2,1, although the equation concerns now the phase space rather than a selected volume of matter. This result regards m as a constant with respect to x, i.e. c depends on x through the volume Δx^3 around m only. This point of view, extended to various volumes Δx_i^3 in which the diffusion medium can be ideally divided, entails that the deformation extents $(\Delta x_i + \delta \Delta x_i)^3$ change as a function of x in order that the respective δc_i represent by consequence these changes; this holds when a total amount of matter $\sum_i m_i$ is simply redistributed along x, thus changing the reference volumes that physically define the respective c_i only, or when $\sum_i m_i$ is subjected to change itself because of sinks or sources of matter in the diffusion medium; this is why the time has been explicitly introduced in eqs. 3,3. The right hand side of the first eq. 4,3 depends certainly upon time through V_o; the same holds therefore for the left hand side, i.e. $a' = a'(t)$. Moreover a' depends in general on x as well; indeed it accounts for how $\partial \Delta x^{-3} / \partial x$ changes in general as a function of x, so $a' = a'(x,t)$. Eventually a' must be consistent with the idea of a mass m crossing the momentum space surface proportional to Δp_x^2 during the time range δt, i.e. the physical dimensions of a' must be $\mathrm{mp^2t} = \mathrm{ml^2t^{-1}}$ like that of $\hbar$; this point will be better emphasized in section 5. Specifying thus purposely the proportionality factor a' in order that also the right hand side of eq. 4,3 depends on c, one finds

$$J_x = -D\frac{\partial c}{\partial x}, \qquad a' = -Dm, \qquad D = D(x,t). \qquad (4,4)$$

The physical dimensions of D are therefore $\mathrm{l^2t^{-1}}$. This result represents the first task of the present paper: to infer the concentration gradient law governing any diffusion process as a consequence of the fundamental eq. 3,1, thus showing the quantum origin of the first Fick law. To proceed further, consider now the right hand side of eq. 4,3 rewritten with the help of the second eq. 4,4 as

$$J_x = -Dc_o f \frac{\partial \log(f)}{\partial x}, \qquad f = \frac{c}{c_o}, \qquad c_o = \frac{m}{V_o}, \qquad c_o = c_o(t).$$

The first expression calculated in an arbitrary point $x = x_a$ defines $f = f_a$ through the local concentration c_a and reads, with obvious meaning of symbols,

$$J_a = -D_a c_o f_a \left.\frac{\partial \log(f)}{\partial x}\right|_{f_a} = -D_a \left.\frac{\partial c}{\partial x}\right|_{x=x_a}, \qquad (4,5)$$
$$f_a = \frac{c_a}{c_o}, \qquad D_a = D(x_a,t).$$

Let us expand in series the function $\log(f)$ around x_a

$$\log(f) = \log(f_a) +$$
$$+ \left.\frac{\partial \log(f)}{\partial x}\right|_{f_a}(x - x_a) + \frac{1}{2}\left.\frac{\partial^2 \log(f)}{\partial x^2}\right|_{f_a}(x - x_a)^2 + \ldots$$

and calculate this expression in another point x_b, arbitrary as well; this yields

$$\left.\frac{\partial \log(f)}{\partial x}\right|_{f_a} = \frac{\log(f_b) - \log(f_a)}{x_b - x_a} - \frac{1}{2}\left.\frac{\partial^2 \log(f)}{\partial x^2}\right|_{f_a}(x_b - x_a) - \ldots,$$
$$f_b = \frac{c_b}{c_o}.$$

Replacing in eq. 4,5 and putting $J_o = -D_a c_o/(x_b - x_a)$ one finds

$$\frac{J_a}{J_o} = -f_a \log(f_a) + \\ + \left(f_a \log(f_b) - \frac{f_a (x_b - x_a)^2}{2} \left. \frac{\partial^2 \log(f)}{\partial x^2} \right|_{f_a} + \cdots \right). \tag{4,7}$$

Rewrite now c_o not yet defined as $c_o = (c_b - c_a)/\gamma$, being γ a dimensionless proportionality factor; this position entails

$$J_o = -\frac{D_a}{\gamma} \frac{c_b - c_a}{x_b - x_a}, \tag{4,8}$$

$$f_a = \gamma \frac{c_a}{c_b - c_a}, \qquad f_b = \gamma \frac{c_b}{c_b - c_a}, \qquad \gamma = \gamma(t).$$

The last position agrees with the dependence of c_o upon time through V_o. In this way J_o agrees conceptually with J_a and thus with the definition of concentration gradient driven mass flow yet with a different diffusion coefficient $D_o = \gamma^{-1} D_a$; it reduces indeed to the usual differential form $J_o = -D_o \partial c/\partial x$ in the limit $x_b \to x_a$ that necessarily entails $c_b \to c_a$. One would expect that in this limit $J_o \to J_a$, which should require $\gamma \to 1$; however the fact that in general $\gamma \neq 1$, as it is shown below, suggests that J_o is physically consistent with but numerically different from J_a. Before concerning this point, note that the second and third eqs. 4,8 require $f_b = \gamma + f_a$; so eq. 4,7 reads

$$\frac{J_a}{J_o} = -f_a \log(f_a) + \left(f_a \log(f_a + \gamma) - d_{ab}^2 \left. \frac{\partial^2 \log(f)}{\partial x^2} \right|_{f_a} \right),$$

$$d_{ab}^2 = f_a \frac{(x_b - x_a)^2}{2}, \tag{4,9}$$

having neglected for simplicity the higher order terms of series development of $\log(f)$. The time function γ is therefore a parameter controlling the evolution of the ratio J_a/J_o, which results to be also a function of $x_a - x_b$ and $c_a - c_b$ via f_a. To explain this result, let x_b be the coordinate of a particle at the beginning of the diffusion process and x_a that of the particle at a later time, while c_b and c_a are the respective concentrations. In general $f_a \neq f_b$ for $x_a \neq x_b$ since $c_a \neq c_b$. Consider however in this respect the particular limit condition $c_b \to c_a$ to be expected in two relevant cases: (i) at the very beginning of the diffusion process, when the particle has traveled an infinitesimal path so that x_a is very close to its initial position x_b; (ii) at the end of the diffusion process, when the particle has traveled a finite path with x_a arbitrarily far from x_b but the concentration is uniform throughout the diffusion system. In both cases it is convenient to define $\gamma \to 0$ in order that the undetermined form $\gamma/(c_b - c_a) \to 0/0$ does not necessarily cause divergent values of f_a and f_b. If $c_b \to c_a$ simply because $x_b \to x_a$, case (i), elementary manipulations of eq. 4,9 show that both sides tend to γ provided that $\gamma/f_a \to 0$; in effect this is verified because by definition $\gamma/f_a = (c_b - c_a)/c_a$, see eq. 4,8. The result is thus

$$\lim_{\substack{c_b \to c_a \\ x_b \to x_a}} \frac{J_a}{J_o} = \gamma, \qquad t \to 0, \qquad \gamma \to 0. \tag{4,10}$$

This simply means that at $t = 0$ there is no net flow of matter as $J_a = 0$. This is reasonable, because after a very short path the particle has high probability to return to its initial position. The second chance for $c_b \to c_a$ even though $x_a \neq x_b$ yields, putting again $\gamma \to 0$,

$$\lim_{\substack{c_b \to c_a \\ x_b \neq x_a}} \frac{J_a}{J_o^{eq}} = \frac{D_a}{J_o^{eq}} \left. \frac{\partial c}{\partial x} \right|_{x = x_a} = \gamma - d_{ab}^2 \left. \frac{\partial^2 \log(f)}{\partial x^2} \right|_{f^{eq}}, \tag{4,11}$$

$$t \to \infty, \qquad \gamma \to 0.$$

Note that γ can fulfill both conditions if its form is, for instance, like $t/(t^2 + t_o)$. Also note that in fact the behavior of γ can be consistent with any $c_b - c_a$, i.e. whatever this limit might be depending on the kind of diffusion system; being γ defined here by its limit condition only, one could hypothesize any stronger/weaker time dependence, e.g. like $t^k/(t^{k+1} + t_o)$, with k ensuring a finite value of $\gamma(c_b - c_a)^{-1}$ no matter how rapidly $c_a \to c_b$ case by case. Put therefore by definition

$$\lim_{c_a \to c_b} \gamma/(c_b - c_a) = \gamma_{ab}, \qquad \gamma_{ab} \neq 0. \tag{4,12}$$

The left hand side of eq. 4,11 has now the form

$$(x_b - x_a)\gamma(c_b - c_a)^{-1} (\partial c/\partial x)_{x = x_a}.$$

The right hand side vanishes for $\gamma \to 0$ if $c_a = c_b = const$ everywhere in the diffusion system because f is now a constant defined by the limit $f_a \to f^{eq}$, whence the notation J_o^{eq}. Hence $x_a \neq x_b$ and γ such that γ_{ab} remains finite require $\partial c/\partial x$ vanishing at x_a. As expected, the situation of uniform concentration entails on microscopic scale the asymptotic condition of thermodynamic equilibrium without net mass transfer. Hence the maximum chance of displacement is expected at times intermediate between 0 and infinity. If c_a is the same everywhere because x_a is arbitrary, then actually neither side of eq. 4,11 depends on x; so must hold also on a macroscopic statistical scale the conclusion that a uniform distribution of particles in the diffusion system makes the ratio J_a/J_o^{eq} of eq. 4,9 inconsistent with a net flow of particles. In fact this requires verifying that also the sum of all terms of eq. 4,9 over the indexes a and b fulfills the condition

$$\sum_{b,a} \lim_{\substack{c_b \to c_a \\ x_b \neq x_a}} \frac{J_a}{J_o^{eq}} = 0, \qquad t = \infty, \tag{4,13}$$

whereas in general, since f_a never diverges,

$$\sum_{b,a} \frac{J_a}{J_o} \neq 0, \qquad t > 0. \tag{4,14}$$

Actually the sums are extended to all paths of particles from the respective starting points x_b to their end points x_a, which also means summing over all elementary volumes $V_a = \Delta x_a^3$ and $V_b = \Delta x_b^3$ of the diffusing medium in which the particles are found with corresponding concentrations c_a and c_b; since both coordinates are arbitrary, this picture represents in fact any path between any points in the diffusion system. Before demonstrating eq. 4,13, note that the sum has conceptual meaning because in fact it does not require computing anything; it is introduced in principle because neither x_a nor x_b are known but are merely referred to their own V_a and V_b only, wherever their position in the diffusion system might be. Also note that the ratio J_a/J_o entails two harmonized but different definitions of mass flow: at numerator appears a local term, characterized by a concentration difference between two coordinates infinitely close each other, at denominator a macroscopic term characterized by coordinates arbitrarily apart. The flow described by J_a is thus a net flow of matter only controlled by D_a, since by definition an effective concentration gradient corresponds to it. The fact that the sum of ratios is finite in eq. 4,14 and equal to 0 in eq. 4,13 suggests that J_o must concern a macroscopic diffusion term controlled by $D_o = D_a\gamma^{-1}$, describing total displacement of matter that consists in principle of both vanishing and non-vanishing net mass flows because $J_o \neq 0$ even though $J_a = 0$; both flows are in fact allowed to occur in a macroscopic volume of diffusion system, so that neither of them can be excluded. Hence the ratio J_a/J_o in eq. 2,3 represents a sort of "displacement efficiency" corresponding to the thermodynamic force F_x of eq. 4,1, i.e. the chance that the random motion of particles produces an effective flow of matter between two arbitrary volumes within the diffusion system. Eq. 4,13 is then easily justified noting that J_o^{eq} changes sign by exchanging x_a and x_b if $c_a = c_b$, whereas J_a does not for the simple reason that its definition has nothing to do with x_b. In effect just the presence of a concentration gradient makes the environment around the coordinates x_a and x_b physically different; if the coordinates belong to different volumes V_a and V_b that define the respective non-equilibrium concentrations, the displacement of a particle between two points out of the equilibrium is distinguishable from that obtained keeping fixed c_a and c_b with reversed path. Instead the sums $\sum_{a,b}$ and $\sum_{b,a}$ at the equilibrium must be in principle identical, because a uniform distribution of particles within the diffusion system makes indistinguishable starting points and end points; if the diffusion system is perfectly homogeneous, then all volumes $V_i = \Delta x_i^3$ where $c \neq 0$ are identical. This is consequence of having defined c as due to a unique value of m into different volumes of phase space that define V_a and V_b of the diffusing medium. Thus the only chance for a sum to coincide with its own value of opposite sign is that the sum is null. Eq. 4,13 is in fact possible from a mathematical point of view because

$$\partial^2 \log(f)/\partial x^2 = -f^{-2}(\partial f/\partial x)^2 + f^{-1}\partial^2 f/\partial x^2, \qquad (4,15)$$

i.e. the former addend is certainly negative whereas the second can take in principle both signs; hence in principle the sum of terms at right hand side of eq. 4,11 can vanish for an appropriate value of $f_a = f_b = f^{eq}$. Let us return now to eq. 4,9 and note with the help of eq. 4,8 that for $f_a = 0$, i.e. $c_a = 0$, the ratio J_a/J_o is identically null in agreement with its probabilistic meaning. Then, since each coordinate x_a belongs to its own volume V_a that defines c_a, summing over all the possible indexes a means summing over states really accessible to the particles; empty volumes V_a with $c_a = 0$ do not contribute to the sum. It is clear therefore that each f_a represents a possible state allowed for the diffusion system: the values f_a, $f_{a'}$, $f_{a''}$, ... in various points labeled by a, a', a'', ... quantify the ways of distributing the total mass M into various elementary volumes reached by the diffusing species during the diffusion process. Summing both sides of eq. 4,9 over the indexes a and b as done before, means therefore estimating the total probability of mass transport within the diffusion system; then let us introduce, even without carrying out any explicit calculation,

$$\sum_{a,b}\frac{J_a}{J_o} = -\sum_{a,b} f_a \log(f_a) + \\ + \sum_{a,b}\left(f_a \log(f_a+\gamma) - d_{ab}^2 \frac{\partial^2 \log(f)}{\partial x^2}\bigg|_{f_a}\right). \qquad (4,16)$$

Summing over all probabilities of diffusion paths, one finds the resulting configuration change of the diffusion system at any time. A few remarks are enough to guess what to expect from this equation. At $t \to 0$ one finds a sum of terms $f_a \log(1+\gamma/f_a)$, which for $\gamma \to 0$ tend to γ, plus terms that contain the factor d_{ab}; since in this limit $x_a - x_b \to 0$, neither of them contributes to the sum. At $t > 0$ both addends contribute to the sum. At the equilibrium asymptotic time where again $\gamma = 0$ the sum vanishes according to eq. 4,15 because $f_a \to f^{eq}$ everywhere; this result agrees with the statistical limit $\sum_{a,b} J_a/J_o = 0$ previously inferred, which actually is the macroscopic result revealed by the experience. The first addend at right hand side is clearly an entropic term, whereas f_a defined in eq. 4,5 must have the probabilistic significance of thermodynamic state related to the current configuration of the diffusion system. In effect it is possible to define the limit value f^{eq} such that $\sum_{a,b}(f^{eq}) = 1$ whatever the number of terms of the sum might be; indeed according to eq. 4,12 the finite limit γ_{ab} for $c_b \to c_a$ and $\gamma \to 0$ has been defined finite but not specified; the value of γ_{ab} can be therefore taken as that fulfilling the required property of f^{eq}. If so the first sum of eq. 4,16 is such that when the system evolves towards the equilibrium then

$$-\sum_{a,b} f_a \log(f_a) \to -\sum_{a,b} \log(f^{eq}).$$

The possibility of relating f_a to the thermodynamic prob-

ability of states allowed to the diffusing particles defines the physical meaning of the time parameter γ: depending on the value of this latter the totality of possible values of x_a and x_b, whatever they might be, corresponds to a possible arrangement of diffusing particles at the current time starting from an arbitrary initial configuration in the diffusing medium. According to eq. 4,10 it appears that $\gamma = 0$ at $t = 0$ defines the initial configuration. So, through the totality of possible paths from any x_b to any x_a, the parameter $\gamma > 0$ provides an indication of the order→disorder evolution of the configuration of the diffusion system as a function of time. Rewrite now eq. 4,16 as follows

$$\sum_{a,b} \frac{J_a}{J_o} = \frac{S_t}{k_B} - \frac{S_o}{k_B}, \tag{4,17}$$

where

$$\frac{S_t}{k_B} = -\sum_{a,b} f_a \log(f_a), \tag{4,18}$$

$$\frac{S_o}{k_B} = -\sum_{a,b} \left(f_a \log(f_a + \gamma) - d_{ab}^2 \frac{\partial^2 \log(f)}{\partial x^2}\bigg|_{f_a} \right).$$

The ratio J_a/J_o has been previously identified as the local chance of net mass flow between two arbitrary points of the diffusion system; the sum at left hand side is therefore the flow efficiency throughout the whole diffusion system, i.e. $\Pi_{netflow} = \sum_{a,b} J_a/J_o$. It is possible therefore to introduce the total chance of mass transfer, Π_{tr}, with and without net mass flow such that of course $\Pi_{tr} = \Pi_{netflow} + \Pi_{nonetflow}$ with obvious notation. This kind of definition is suggested by the possibility of normalizing Π_{tr} to 1. Hence comparing with eqs. 4,17 and 4,18 one infers

$$\Pi_{tr} = \frac{S_t}{k_B}, \qquad \Pi_{nonetflow} = \frac{S_o}{k_B}.$$

Of course S_t, the most general statistical definition of entropy, is also the most general way to describe the configuration of N diffusing particles in the N_V volumes available in the diffusion system, regardless of whether or not the configuration entails a net displacement of matter; instead S_o, which does not refer to net transfer of atoms, counts simply the number of ways to arrange any prefixed distribution of particles and thus the thermodynamic probability of any configuration. Hence the entropic terms concern two different kinds of diffusion mechanisms allowed to occur as a function of time. In effect the possibility that $x_b \to x_a$ is not excluded in the present model even at times t_1, t_2,..; it would be enough to define γ for instance as $t(t-t_1)(t-t_2)/(t+t_o)^4$ in agreement with the previous considerations at $t \to 0$ and $t \to \infty$ and at any time where $x_b \to x_a$ entails $c_b \to c_a$ too. Further considerations are possible about the results hitherto obtained.

5 Discussion

The eqs. 3,1 only have been exploited to highlight the link between concentration gradient law and entropy of diffusion system through elementary considerations. Both concepts have been extracted through elementary algebraic manipulations of the left and right hand sides of the unique eq. 4,3. No hypotheses "ad hoc" have been introduced about the physical features of the diffusion system and its driving mechanisms, leading for instance to Markovian jumps or not, interstitial or defect activated jumps, collisions in gas phase and so on. This is due to the general worth of eqs. 3,1 regardless of the specific system concerned: the present conclusions hold in principle for diffusion processes in solid or liquid or gas phase.

Regarding the statistical formulation of the uncertainty as fundamental principle of nature, the diffusion particles result randomly delocalized within elementary volumes $V = \Delta x^3$ into which can be ideally subdivided the whole system, whose size is however inessential to infer the entropic terms $-f \log f$; these volumes control the concentrations c, which in turn define the thermodynamic states allowed to the diffusing particles in relation to their occupation probability. No assumption was made about the coordinates of the points x_a and x_b falling within the respective elementary volumes, whose number, size and position indeed have been never specified in section 4. In fact such a kind of local information is irrelevant to calculate the entropy; it is enough to compute how N particles can be distributed in N_V volume elements, regardless of how many and where these latter might actually be in the diffusion medium. For this reason the model describes the time evolution of the whole system even without knowing in detail how is progressively modified the configuration of particles and volumes as a function of time. Actually eqs. 4,17 admit also empty elementary volumes that however do not contribute to the total entropy of the system, in fact determined by the distribution of particles only. So S_o in eq. 4,17 corresponds in general to the ways of distributing particles into available microstates described by Δx^3, possibly taking into account the indistinguishability of identical particles, through a dynamical pattern of particles exchanging their occupation volumes even without net mass flow. In effect, also this kind of information does not require a detailed knowledge on the local motion of particles. Nothing is known about this motion within their own Δx^3, because it would require some sort of local information about x and p_x. Being impossible to establish if within this arbitrary volume the motion is for instance Markovian or not, one must admit that both chances are in fact allowed; this also justifies why the diffusing species is involved in mass transfer process with and without net displacement of particles.

This conclusion does not conflict with the fact that J_x entails explicitly an effective concentration gradient; eq. 4,4 is simply the differential formulation of a physical law related

to the driving force that triggers the displacement, see eqs. 4,1 and 2,3 and 2,5 as well. The quantum approach behind this step accounts for the physical basis of eq. 2,2, whereas the definition 2,1 has now the rank of a corollary of eq. 4,4 rather than a mere definition: now the physical dimensions of eq. 2,1 are required by quantum motivations, rather than being suggested by a reasonable assumption. Indeed the available information about the diffusion system is inferred in the typical way of quantum mechanics, i.e. without requiring an exact local knowledge about position and momentum of the particles, as follows:

(i) from a macroscopic point of view, through J_o of eq. 4,8 and the entropic terms of eq. 4,17;

(ii) through the probabilistic meaning of the ratio J_a/J_o, which indeed represents the probability of effective mass transport as concerns the chances of Marcovian or non-Marcovian displacements.

Non-trivial consequence of these constrains about our degree of information is the heuristic achievement resulting from the quantum approach with respect to that provided by the classical physics where, from the point of view of the continuity equation, the general character of both Fick's laws is merely due to the lack of sinks/sources perturbing the diffusion process. This fact appeared already in the classical section 2 when it was found that in general $\mathbf{F} \neq \mathbf{F}'$, i.e. the driving force controlling the mass transport is in principle different from that due to local perturbations; the former was uniquely inferred from general hypotheses, eqs. 2,1 and 2,2, the latter remained instead unspecified and does so still now. This is not incompleteness of the present model, but rather the statement that the local perturbations must be purposely specified case by case depending on the physical features of the diffusion system. The worth of any theoretical approach depends on its ability to be generalized beyond the specific problem for which it was formerly conceived. In the case of diffusion the generalization is evident: several important physical laws are expressed through the gradient of a well defined function.

One example is the Fourier equation, $\mathbf{J}_Q = K\nabla T$, where K is the heat conductivity and $\mathbf{J}_Q$ the heat flow; also the Ohm law, $\mathbf{I} = R^{-1}\nabla V$, exhibits a similar form involving the electrical resistance R and the electric potential V to describe the displacement of charges per unit time. Although a common gradient law describes in the former case the transport of heat and in the latter that of electrons, both equations involve forms of kinetic energy, respectively due to the oscillation frequency of atoms/ions/molecules within the heat diffusion thermodynamic system and to the velocity of electrons propagating within a conductor. The entropic aspects in these systems are clear. In the former case they were already evidenced by the crucial Boltzmann intuition, although in lack of any quantum reference; it is not surprising that indeed the statistical definition of entropy inferred here goes back to the early times when the thermodynamics was essentially the science of heat exchanges. The entropy difference in the absence and presence of an electric field is also evident in the latter case: without electric field the motion of the electrons is random, in the $\mathbf{k}$ space it is represented by a sphere; the presence of the field instead orients the motion of the electrons along a preferential direction. The applied field triggers thus a more ordered motion of electrons, which suggests in turn a loss of total entropy. The analogy with the case discussed in section 4 is clear, although the respective entropy changes have opposite sign. This is not surprising: in an isolated system the entropy always increases, in a system interacting with an external field this is not necessarily true. In all cases however the gradient-like laws, mass diffusion, heat diffusion and Ohm law, are similarly consistent with entropic terms describing the actual numbers of accessible states during the displacement of matter or energy. Another consequence of the generality of the present model concerns the driving force of the diffusion process. In section 2, eq. 2,5 was inferred from eqs. 2,1 and 2,2, the only equations available. Of course the same can be done identically here, though on a more profound quantum basis. Yet the approach carried out in section 4 allowed inferring eq. 4,1, which introduces the concept of force directly as a consequence of eqs. 3,1 and deserves thus further considerations.

First of all, the quantum nature of the mass flow can be evidenced replacing v_x of eq. 4,1 into the x-component of eq 2,1, which yields thanks to eq. 2,5

$$J_x = \frac{k_B T}{n\hbar}\Delta x^2 c \frac{\partial \log(c/c_o)}{\partial x}. \qquad (5,1)$$

So, simply identifying F_x of eq. 2,5 with that of eq. 4,1 appear again terms of J_x having the form $c\partial\log(c)/\partial x$, which can be handled in a completely analogous way as in section 4 to infer entropic terms like $c\log(c/c_o)$ of eq. 4,5. Moreover $J_x \to 0$ for $n \to \infty$ agrees with eq. 4,16; an increase of entropy due to the increase of states accessible to the diffusion system corresponds to the reaching of asymptotic equilibrium where the net mass flow vanishes. As expected, the result obtained via the time coordinate defining v_x agrees with that previously obtained through the space coordinate only. Yet it is worth remarking that the combined information of the first eq. 4,18 plus eq. 5,1 regards this time behavior of any isolated diffusion system as a spontaneous evolution process: indeed $t \to \infty$ requires $J_x \to 0$ that in turn requires a maximum number of allowed states $n \to \infty$. Two fundamental statements of thermodynamics appear here as corollaries of eqs. 3,1: the statistical formula of entropy and the entropy increase in an isolated system.

Let us exploit eq. 5,1 noting that $k_B T/n\hbar$ has physical dimensions of time. So compute this equation at the time τ where the total diffusion spread lies within an average value of Δx^2 computed starting from $\Delta x^2 \to 0$ at $t = 0$ up to the value $\Delta x^2 = \Delta x_\tau^2$ at the time τ; this means assigning to Δx^2 the particular mean value $\overline{\Delta x^2} = \Delta x_\tau^2/2$ averaged between

zero and Δx_τ^2. Comparing with eq. 4,4, one finds immediately the known Einstein's one-dimensional result

$$D = \frac{1}{2}\frac{\Delta x_\tau^2}{\tau}.$$

6 Heuristic aspects of the quantum uncertainty

The present section, based on wide-ranging considerations about v_x, extends somewhat the preliminary remarks introduced in section 3 and has prospective worth. The aim is to emphasize that F_x of eq. 4,1 has actually a physical meaning much more general and contains much more information than the mere eq. 2,5. The byproduct of eqs. 3,1 proposed here is so short, straightforward and relevant to deserve being sketched although, strictly speaking, beyond the mere purposes of the present model; accordingly, however, the results hitherto inferred appear as a particular kind of selected physical information extracted from a broader context able to link topics apparently dissimilar.

Key tools of the following considerations are the replacements 3,2 that compel changing the way to formulate any physical property P from the usual form $P(x, p_x, t)$ to $P(\Delta x, \Delta p_x, \Delta t)$ and thus to $P(\Delta x, n, \Delta\varepsilon)$. In effect the paper [4] has shown that the number n of states coincides with the quantum number appearing in the eigenvalues of the harmonic oscillator, while the papers [5, 6] show that this is true in general; e.g. the number l of states calculated for the angular momentum coincides with the orbital quantum number. The first remark concerns the two ways of expressing F_x in eqs. 4,1:

(i) F_x follows from the definition of momentum itself, $\Delta\dot{p}_x = m\dot{v}_x$, and involves directly the mass m, previously introduced with mere reference to the concentration of diffusion particles and now regarded in general as the mass of any particle accelerated in Δx;

(ii) $F_x = -n\hbar\Delta x^{-2}v_x$ does not involve directly any mass but the deformation rate, $\Delta\dot{x} = v_x$, of Δx only.

Why in (ii) the mere time deformation of Δx in the phase space surrogates the presence of an accelerated mass? The answer rests on the same considerations already introduced in section 4: if a growing/shrinking range is accessible to a particle, then this latter can move faster/slower while being still therein delocalized; the fact that the particle can accelerate/decelerate simply reaffirms once more that nothing in known about how any dynamical variables change within the respective delocalization ranges.

However, in lack of constraining hypotheses, there is no reason to exclude that this idea holds regardless of whether the range sizes are stationary or not. Otherwise stated: slow motion in a short range or faster motion in a larger range are two indistinguishable chances, both allowed to occur for a particle by the lack of local information inherent the eqs. 3,1 and in fact both occurring. This rationalizes why just the uncertainty of x, p_x, ε and t links the deformation rate of time dependent range sizes of the phase space to the acceleration of any particle, possible and thus actual. The size and position of any range require a reference system to be defined in principle, although never quantifiable.

Consider for instance $\Delta x = x_t - x_o$ and $\Delta p_x = p_t - p_o$: the coordinate x_o, whatever it might be, is defined with respect to the origin O of an arbitrary reference system R, while the same also holds for the momentum p_o of the range Δp_x conjugate to Δx. So a free particle is described in R by its own Δx and Δp_x; indeed eqs. 4,1 have been inferred in R keeping constant x_o and regarding x_t as a time function. Yet, if neither of these boundaries is specifiable, one could also think x_t fixed and x_o time function. The difference is apparent: the displacement of x_o means that now Δx deforms while contextually moving in R, as O displaces at rate $-\partial x_o/\partial t$ with respect to x_t. Thus it is possible to introduce another reference system R_o solidal with x_o such that a particle accelerated in R is at rest in R_o, which moves with the same acceleration in R. Clearly still acts on the particle a force that justifies the acceleration of R_o in R, although however the particle is in fact at rest in R_o.

The conclusion of this reasoning is well known: a particle at rest in an accelerated reference frame is subjected to a force F_x indistinguishable from that due to the presence of mass. Of course with large sized Δx one can speak about average force F_x, whereas in a small sized range F_x takes a value better and better defined. This statement is nothing else but the equivalence principle, here inferred as a corollary of eqs. 3,1. After having introduced in eqs. 4,1 $F_x = m\dot{v}_x$, can be inferred also the link between F_x and Newton's law after these preliminary remarks? Of course let us start again from eqs. 4,1 with v_x and $\dot{v}_x$ defined in any R.

First of all, the fact that the mass in eq. 4,1 is unique and that the equivalence principle has been obtained elaborating independently both sides of $m\dot{v}_x = F_x = -n\hbar\Delta x^{-2}v_x$ shows the identity of inertial and gravitational mass. Moreover just the fact the unique mass m must somehow appear also in the second equality compels putting $v_x = \zeta' m$ via an appropriate dimensional factor ζ'; hence $F_x = -n\hbar\zeta' m\Delta x^{-2}$ with the acceleration no longer appearing explicitly in this expression, which rather has the form of an interaction force $F_x^{m,\zeta'}$ between m and another entity that can be nothing else but ζ'.

This result suggests a more interesting form of F_x putting $\zeta' = n\sum_k \zeta_k m'^k$, being ζ_k coefficients of the power series development of ζ' and m' a further arbitrary mass that interacts with m. The series truncated at the first order only yields approximately $\zeta' \simeq n\zeta m'$, with ζ unique proportionality factor; here n is inessential and does not play any role because, being m' arbitrary, $m'n$ is another value arbitrary as well. In this way one finds $F_x^{m,m'} \approx -\hbar\zeta(m/\Delta x)(m'/\Delta x)$ at the first order of approximation, i.e. an attractive force is originated between the linear densities $m/\Delta x$ and $m'/\Delta x$ of masses by definition delocalized within Δx. This sensible result appears better under-

standable thinking to particle waves that propagate through Δx rather than to point particles moving randomly within Δx.

Moreover the proportionality factor ζ can be regarded as a constant since the arbitrary masses m and m' account for the arbitrariness of v_x. With the notation $\zeta = G/\hbar$ one recognizes the approximate Newton law; the classical distance $x_{m,m'}$ between local coordinates exactly known of particles is replaced by any random distance falling within the uncertainty range including them.

Obviously Δx^{-2} shows that the functional dependence of $F_x^{m,m'}$ on all possible distances between the masses is like $x_{m'm}^{-2}$. This confirms that effectively the diffusion particles are acted by the force F_x, whose physical meaning can be extended even to the gravitational interaction. Note however that actually both signs are allowed for the velocity component v_x along x, which correspond to the signs of $\partial\Delta x/\partial t$ depending on whether Δx shrinks or expands as a function of time.

In agreement with the idea of phase space-time deformation in the presence of mass, one would expect thus $v_x = \pm\zeta' m$, i.e. even a negative value of m. This conclusion emphasizes nothing else but the existence of antimatter. After this instance about how eqs. 3,1 can be purposely exploited, let us proceed with another example short enough to be mentioned here, i.e. the Coulomb law. It is not a chance that even this latter has a form similar to that of the Newton law, with the charges playing the role of the masses. To emphasize the reason of this similarity, let us introduce in eq. 4,1 the fine structure constant $\alpha = e^2/\hbar c$. Eliminating $\hbar$ eq. 4,1 reads $F_x = e'e/\Delta x^2 = m_e a_x$, where now m_e is the electron mass and $e' = nv_x(c\alpha)^{-1}e$. This latter reads more expressively $e' = \pm n\,|v_x|\,(c\alpha)^{-1}e$. Again, the charges interact through their linear densities $e/\Delta x$ and $e'/\Delta x$ for the reasons previously explained. Also the electron charges appear therefore because of the phase space-time deformation in the presence of the mass m_e. Once more is crucial the characteristic value of v_x of charged particles; for instance $v_x = 0$ would describe a neutral particle, whereas it also appears that a massless particle would be chargeless as well. A boundary condition of the problem is that for an appropriate value n^* of the integer n one must find $e' = e$, as nothing hinders indeed just such a possibility. So $e' = \pm(n/n^*)e$; e.g. for a couple of electrons one must take $n = n^*$ i.e. $|v_x| = c\alpha$, whose value seems therefore to be a combined constant of nature. It is reasonable to assume $n^* = 3$ since actually one should consider v_x, v_y and v_z for the respective components replacing the early F_x, for simplicity the only one hitherto considered, whereas the number n of states should be counted as $n = n_x + n_y + n_z$. Take the ground values $n_x = n_y = n_z = 1$ and consider the three chances $v_x \neq 0, v_y \neq 0, v_z \neq 0$ and $v_x \neq 0, v_y \neq 0$ and $v_x \neq 0$ only. This means considering the charges of particles resulting from $n = 1, 2, 3$ with $n^* = 3$. As inferred before, $n = n^*$ holds for protons and electrons. Yet, in addition to $e' = \pm e$, possible values of e' result respectively to be $e' = \pm e/3$ or $e' = \pm(2/3)e$ as well, i.e. particles with fractional charges should also exist in nature. But, being n arbitrary, what about hypothetical charges described by $n > n^*$?

A full discussion on this question is clearly far beyond the purpose of the present paper; further work is in progress on this specific topic. As concerns the results hitherto introduced, it is enough to conclude that the formal analogy between the Fourier law and the mass/charge transport laws is due to their common quantum basis, discussed here with reference to the entropic aspects too, that goes back to the intimate quantum nature of the entropy and Newton and Coulomb forces themselves.

7 Conclusion

The quantum origin of the diffusion law has been described with the help of eqs. 3,1 only. The assumption of uncertainty that allowed to calculate the energy levels of many-electron atoms and diatomic molecules, enables even the basic law controlling the transport of matter to be inferred in a very simple way. It is also remarkable that elementary considerations on eqs. 3,1 open the way to further results much more general than the specific task to which they were initially addressed in the present diffusion model. This emphasizes the heuristic character of eqs. 3,1: the uncertainty, regarded itself as a fundamental principle of nature rather than as mere corollary of commutation rules of operators, appears a key tool to infer a conceptual background unifying seemingly different physical phenomena. As concerns the present model, the level of comprehension provided by the approach based on the phase space-time uncertainty opens the way to more specific considerations on the possible mechanisms of transport in solid, liquid and gas phases.

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Dynamical Space: Supermassive Galactic Black Holes and Cosmic Filaments

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The unfolding revolution in observational astrophysics and cosmology has lead to numerous puzzles: "supermassive" galactic central black holes, galactic "dark matter" halos, relationships between these black hole "effective" masses and star dispersion speeds in galactic bulges, flat spiral galaxy rotation curves, cosmic filaments, and the need for "dark matter" and "dark energy" in fitting the Friedmann universe expansion equation to the supernovae and CMB data. Herein is reported the discovery of a dynamical theory for space which explains all these puzzles in terms of 3 constants; G, α - which experimental data reveals to be the fine structure constant $\alpha \approx 1/137$, and δ which is a small scale distance, perhaps a Planck length. It is suggested that the dynamics for space arises as a derivative expansion of a deeper quantum foam phenomenon. This discovery amounts to the emergence of a unification of space, gravity and the quantum.

1 Dynamical Space

The many mysteries of cosmology, such as supermassive galactic black holes, cosmic filaments, "dark matter" galactic haloes, flat spiral-galaxy rotation curves, "dark energy" effects in expansion of the universe, and various unexplained correlations between galactic black hole masses and star velocities, all suggest that we have an incomplete account of space and gravity. We report herein the discovery of such a theory and its successful testing against the above phenomena, and as well against laboratory and geophysical gravity experiments. If space is, at a deep level, a quantum system, with dynamics and structure, then we expect a derivative expansion would give a classical/long-wavelength account. In the absence of that quantum theory we construct, phenomenologically, such an account in terms of a velocity field [1]. In the case of zero vorticity we obtain

$$\nabla\cdot\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) + \frac{\alpha}{8}\left((trD)^2 - tr(D^2)\right) + \\ + \frac{\delta^2}{8}\nabla^2\left((trD)^2 - tr(D^2)\right) + ... = -4\pi G\rho$$

$$\nabla\times\mathbf{v} = \mathbf{0}, \quad D_{ij} = \frac{\partial v_i}{\partial x_j} \tag{1}$$

where the major development reported herein is the discovery of the significance of the new $\delta-$term, with δ having the dimensions of a length, and presumably is the length scale of quantum foam processes. This term is shown to be critical in explaining the galactic black hole and cosmic filament phenomena. This δ is probably a Planck-like length, and points to the existence of fundamental quantum processes. If $\delta = 0$ (1) cannot explain these phenomena: δ must be non-zero, no matter how small, and its value cannot be determined from any data, so far. G is Newton's constant, which now appears to describe the dissipative flow of quantum foam into matter, and α is a dimensionless self-coupling constant, that experiment reveals to be the fine structure constant, demonstrating again that space is fundamentally a quantum process. We briefly outline the derivation of (1). Relative to the non-physical classical embedding space, with coordinates $\mathbf{r}$, and which an observer also uses to define the velocity field, the Euler constituent acceleration of the quantum foam is

$$\mathbf{a} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} \tag{2}$$

and so, when $\alpha = 0$ and $\delta = 0$, (1) relates this acceleration to the density of matter ρ, and which will lead to Newton's account of gravity. The matter acceleration is found by determining the trajectory of a quantum matter wavepacket. This is most easily done using Fermat's maximum proper-travel time τ:

$$\tau = \int dt\sqrt{1 - \frac{\mathbf{v}_R^2(\mathbf{r}_0(t), t)}{c^2}} \tag{3}$$

where $\mathbf{v}_R(\mathbf{r}_o(t), t) = \mathbf{v}_o(t) - \mathbf{v}(\mathbf{r}_o(t), t)$, is the velocity of the wave packet, at position $\mathbf{r}_0(t)$, wrt the local space. This ensures that quantum waves propagating along neighbouring paths are in phase, and so interfere constructively. This maximisation gives the quantum matter geodesic equation for $\mathbf{r}_0(t)$

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + (\nabla\times\mathbf{v})\times\mathbf{v}_R \\ - \frac{\mathbf{v}_R}{1 - \dfrac{\mathbf{v}_R^2}{c^2}}\frac{1}{2}\frac{d}{dt}\left(\frac{\mathbf{v}_R^2}{c^2}\right) + ... \tag{4}$$

with $\mathbf{g} \equiv d\mathbf{v}_o/dt$. The 1st term in $\mathbf{g}$ is the Euler space acceleration $\mathbf{a}$, the 2nd term explains the Lense-Thirring effect, when the vorticity is non-zero, and the last term explains the precession of orbits. In the limit of zero vorticity and neglecting

relativistic effects (1) and (4) give

$$\nabla \cdot \mathbf{g} = -4\pi G\rho - 4\pi G\rho_{DM}, \quad \nabla \times \mathbf{g} = \mathbf{0} \tag{5}$$

where

$$\rho_{DM} \equiv \frac{\alpha}{32\pi G}\left((trD)^2 - tr(D^2)\right) + \frac{\delta^2}{32\pi G}\nabla^2\left((trD)^2 - tr(D^2)\right) + ... \tag{6}$$

This is Newtonian gravity, but with the extra dynamical terms which has been used to define an effective "dark matter" density. This ρ_{DM} is not a real matter density, of any form, but is the matter density needed within Newtonian gravity to explain dynamical effects caused by the α and δ-terms in (1). It is purely a space/quantum-foam self-interaction effect. Eqn.(3) for the elapsed proper time maybe written in differential form as

$$d\tau^2 = dt^2 - \frac{1}{c^2}(d\mathbf{r}(t) - \mathbf{v}(\mathbf{r}(t), t)dt)^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \tag{7}$$

which introduces a curved spacetime metric $g_{\mu\nu}$ for which the geodesics are the quantum matter trajectories when freely propagating through the quantum foam. When $\alpha = 0$ and $\delta = 0$, and when ρ describes a sphere of matter of mass M, (1) has, external to the sphere, a static solution $\mathbf{v}(\mathbf{r}) = -\sqrt{2GM/r}\hat{\mathbf{r}}$, which results in Newton's matter gravitational acceleration $\mathbf{g}(\mathbf{r}) = -GM/r^2\hat{\mathbf{r}}$. Substituting this $\mathbf{v}(\mathbf{r})$ expression in (7), and making the change of time coordinate

$$t \to t' = t - \frac{2}{c}\sqrt{\frac{2GMr}{c^2}} + \frac{4GM}{c^3}\tanh^{-1}\sqrt{\frac{2GM}{c^2 r}}, \tag{8}$$

(7) becomes the standard Schwarzschild metric, and which is the usual explanation for the galactic black hole phenomenon, see [3–5], namely a very small radius but very massive concentration of matter. To the contrary we show here that the observed galactic black holes are solutions of (1), even when there is no matter present, $\rho = 0$. These solutions are quantum foam solitons.

The above $\mathbf{v}(\mathbf{r}) = -\sqrt{2GM/r}\hat{\mathbf{r}}$ solution also explains why the $\alpha-$ and $\delta-$terms in (1) have gone unnoticed, namely that for these solutions $(trD)^2 - tr(D^2) = 0$. It is for this reason that the $\alpha-$ and $\delta-$terms are now included, namely that Newton's inverse square law for gravity is preserved for solar system situations, and from which Newton determined his theory from Kepler's analysis of Brahe's planetary data. The key point is that the solar system is too special to have revealed the full complexity of the phenomenon of gravity.

However just inside a planet the $\alpha-$term becomes detectable, and it results in the earth's matter acceleration g being slightly larger than that predicted by Newtonian gravity, and we obtain from (1)

$$\Delta g = g_{NG}(d) - g(d) = -2\pi\alpha G\rho(R)d + O(\alpha^2), \quad d > 0 \tag{9}$$

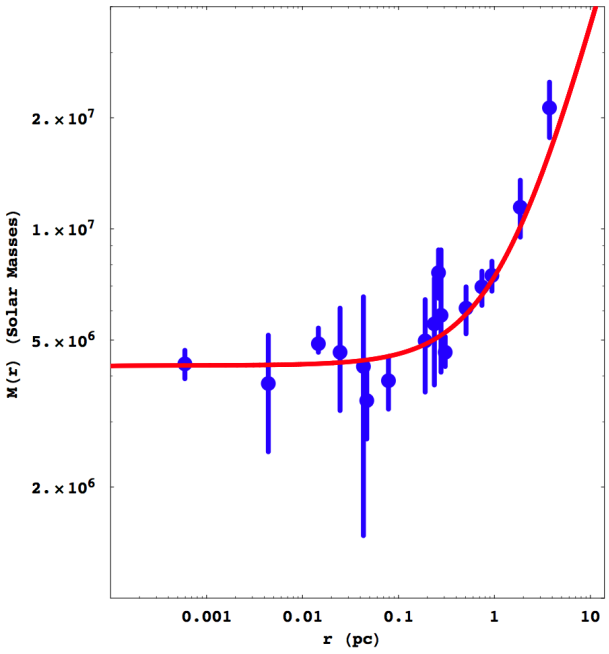

Fig. 1: The $M(r)$ data for the Milky Way SgrA* black hole, showing the flat regime, that mimics a point-like mass, and the rising form beyond $r_s = 1.33$pc, as predicted by (12), but where M_0 and r_s parametrise a quantum foam soliton, and involves no actual matter. The left-most data point is from the orbit of star S2 - using the Ghez *et al.* [3] value $M_0 = 4.5 \pm 0.4 \times 10^6$ solar masses. The other data is from Camenzind [5], but which requires these remaining data points to be scaled up by a factor of 2, presumably arising from a scaling down used to bring this data into agreement with a smaller initial value for M_0.

down a bore hole at depth d. This involves only α as the δ-term is insignificant near the surface. The Greenland Ice Shelf bore hole data [6] and Nevada bore hole data [7], both give $\alpha \approx 1/137$ to within observational errors, even though the ice and rock densities $\rho(R)$ differ by more than a factor of 2 [2]. So this result for α is robust, and shows that α is the fine structure constant $\alpha = e^2\hbar/c$, with α probably the more fundamental constant, and now showing up in the quantum foam account for gravity. As well laboratory measurements of G, modified Cavendish experiments, have always shown anomalous and inconsistent results [10, 11], revealing a systematic effect not in Newtonian gravity. Indeed the Long 1976 laboratory experiment to measure G, reported the anomaly to have magnitude $\delta_L = 0.0037 \pm 0.0007$ [8] (this δ_L is not related to δ in (1)), which equals $0.5/(136 \pm 26)$, showing that α can be measured in laboratory gravity experiments, of the type pioneered by Long.

1.1 Black Holes and Filaments as Quantum Foam Solitons

For the special case of a spherically symmetric flow, and in the absence of matter $\rho = 0$, we set $\mathbf{v}(\mathbf{r}, t) = \hat{\mathbf{r}}v(r)$. Then (1) has exact static two-parameter, v_0 and $\kappa \geq 1$, solutions

$$v(r)^2 = v_0^2(\kappa - 1)\frac{\delta}{r}\left(1 - {}_1F_1\left[-\frac{1}{2} + \frac{\alpha}{4}, -\frac{1}{2}, -\frac{r^2}{\delta^2}\right]\right) + \\ + v_0^2\kappa\left(\frac{4-2\alpha}{3}\right)\frac{r^2}{\delta^2}\frac{\Gamma(\frac{2-\alpha}{4})}{\Gamma(-\frac{\alpha}{4})}\,{}_1F_1\left[1 + \frac{\alpha}{4}, \frac{5}{2}, -\frac{r^2}{\delta^2}\right], \tag{10}$$

where ${}_1F_1\,[a, b, w]$ is the confluent hypergeometric function. Here v_0 is a speed that sets the overall scale, and κ is a structural parameter for the black hole, and sets the relative significance of the two terms in (11) and (12), and which is determined by the history of the black hole: in-falling matter increases κ, and values of both v_0 and κ are affected by surrounding matter if $\rho \neq 0$. In the limit $r \gg \delta$

$$v(r)^2 \approx A\frac{\delta}{r} + B\left(\frac{\delta}{r}\right)^{\alpha/2}. \tag{11}$$

However $v(r) \to 0$ as $r \to 0$ when $\delta \neq 0$, and so the δ-term dynamics self-regulates the interior structure of the black hole, which has a characteristic radius of $O(\delta)$. Inside this radius the in-flow speed goes to zero, and so there is no singularity. Hence there is a naturally occurring UV cutoff mechanism. Eqn. (**??**) gives an asymptotic form for $g(r)$, which is parametrised by an "effective mass" $M(r)$ within radius r: $g(r) = GM(r)/r^2$. In terms of observable $M(r)$ (11) gives a two-parameter description

$$M(r) = M_0 + M_0\left(\frac{r}{r_s}\right)^{1-\alpha/2} \tag{12}$$

r_s is the distance where $M(r_s) = 2M_0$. $M(r)$ from the Milky Way SgrA* black hole [3–5] is shown in Fig.1, and the best fit gives $r_s = 1.33$pc. This remarkable data comes from observations of orbits of stars close to SgrA*, and in particular the star S2, which has an elliptical orbit with a period of 15.2±0.11 years, and is the left-most data point in Fig.1. This dynamical space solution exhibits an effective point-like mass acceleration for $r < r_s$, where $M(r)$ is essentially constant, and for $r > r_s$ an increasing $M(r)$. At the outer-most data point the presence of stars within the galactic core begin to become apparent, with $M(r)$ becoming larger than the form predicted in (12). Note that if $\delta = 0$, then the flat feature in $M(r)$ is absent, while if $\alpha = 0$ the rise in $M(r)$ is absent, and the flat feature continues outwards. Intriguingly then the role of the $\delta-$term dynamics is critical to the effective point-like mass description of the inner region of the black hole, even though there is no actual matter present. It is this region of $M(r)$ that explains the inner star elliptical orbits - with $\delta = 0$ the $\alpha-$term produces a "weak" black hole, but with $g(r) \sim 1/r^{1+\alpha/2}$, which does not produce the observed star orbits. Eqn. (12) is in terms of observables. If we best-fit the data using an $M(r)$ directly from (10), by varying v_0, κ and δ, we find that there is no unique value of δ - v_0 and κ rescale to compensate for a deceasing δ, in the regime outside of the inner core to the black hole, but δ cannot be set to zero. This is evidence of the existence of a finite, but very small, structure to space, suggestive of a Planck-like fundamental length.

This black hole also explains the so-called "dark matter" halo. Asymptotically $\rho_{DM}(r)$ is related to the matter-less $M(r)$ via

$$M(r) = \int_0^r 4\pi r^2 \rho_{DM}(r) dr \tag{13}$$

giving

$$\rho_{DM}(r) = \frac{(1 - \alpha/2)M_0}{4\pi r_s^{1-\alpha/2} r^{2+\alpha/2}} \tag{14}$$

which decreases like $r^{-\gamma}$ with $\gamma = 2 + \alpha/2$. The value of the exponent γ has been determined by gravitational lensing for numerous elliptical galaxies in the Sloan Lens ACS Survey [12], and all give the generic result that $\gamma = 2$. Higher precision data may even permit the value of α to be determined. So the space dynamics completely determines ρ_{DM} in terms of observables M_0 and r_s.

Unlike the point-mass parametrisation of black holes, the above shows that the quantum foam black hole is an extended entity, dominating the galaxy from the inner regions, to beyond the central bulge, and even beyond the spiral arms. Indeed the $\rho_{DM}(r)$ in (14) predicts flat rotation curves, with orbital speed given by

$$v_{orb}^2(r) = GM_0\left(\frac{r_s}{r}\right)^{\alpha/2}\frac{1}{r_s} \tag{15}$$

but to which must be added the contribution form the matter density. For the Milky Way, we get the black hole contribution is $v_{orb} = 117$km/s at the location of the solar system, $r = 8$kpc, and determined by M_0 and r_s. That the black hole is an extended structure explains various observed correlations, such as that in [9] which reported a correlation between M_0 and the stellar speed dispersion in the bulge.

Eqn. (1), but only when $\delta \neq 0$, also has exact filament solutions

$$v(r)^2 = v_0^2\frac{r^2}{\delta^2}\,{}_1F_1\left[1 + \frac{\alpha}{8}, 2, -\frac{r^2}{2\delta^2}\right] \tag{16}$$

where r is the distance perpendicular to the axis of the filament, and $v(r)$ is the in-flow in that direction. In the limit $r \gg \delta$

$$v(r)^2 \sim 1/r^{\alpha/4} \;\text{ giving }\; g(r) \sim 1/r^{1+\alpha/4} \tag{17}$$

producing a long range gravitational attraction. Such cosmic filaments have been detected using weak gravitational lensing combined with statistical tomographic techniques. Again $v(r) \to 0$ as $r \to 0$ when $\delta \neq 0$, and so the δ-term dynamics self-regulates the interior structure of the filament, which has a characteristic radius of $O(\delta)$.

1.2 Expanding Universe

The dynamical 3-space theory (1) has a time dependent expanding universe solution, in the absence of matter, of the Hubble form $v(r,t) = H(t)r$ with $H(t) = 1/(1 + \alpha/2)t$, giving a scale factor $a(t) = (t/t_0)^{4/(4+\alpha)}$, predicting essentially a uniform expansion rate. This results in a parameter-free fit to the supernova redshift-magnitude data. In contrast the Friedmann model for the universe has a static solution - no expansion, unless there is matter/energy present. However to best fit the supernova data fictitious "dark matter" and "dark energy" must be introduced, resulting in the ΛCDM model. The amounts $\Omega_\Lambda = 0.73$ and $\Omega_{DM} + \Omega_M = 0.27$ are easily determined by best fitting the ΛCDM model to the above uniformly expanding result, without reference to the observational supernova data. But then the ΛCDM has a spurious exponential expansion which becomes more pronounced in the future.

2 Conclusions

The notion that space is a quantum foam system suggests a long-wavelength classical derivative-expansion description, and inspired by observed properties of space and gravity, such an effective field theory has been determined. This goes beyond the Newtonian modeling in terms of an acceleration field description - essentially the quantum foam is accelerating, but at a deeper level the acceleration is the Euler constitutive acceleration in terms of a velocity field. This velocity field has been detected experimentally, with the latest being from spacecraft earth-flyby Doppler shift data [13]. The dynamics of space now accounts for data from laboratory experiments through galactic black holes and filaments, to the expansion of the universe. We note that there is now no known phenomenon requiring "dark energy" or "dark matter". The black hole and cosmic filament phenomena require the existence of both α - the fine structure constant, and δ which is presumably a quantum foam characteristic Planck-like length scale. Gravity is now explainable as a emergent phenomenon of quantum foam dynamics, but only if we use as well a quantum wave description of matter - gravitational attraction is a quantum matter wave refraction effect, and also causes EM wave refraction. Hence the evidence is that we are seeing the unification of space, gravity and the quantum, pointing to a revolution in physics, and in our understanding of reality.

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A Generalized Displacement Problem in Elasticity

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By solving a special coupling boundary value problem for vector Helmholtz equations it is shown how the displacement boundary value problem in elasticity can be solved. It is shown that the generalized displacement problem possesses at most one solution.

1 Statement of the Problem

By D_i we denote a bounded domain in $\boldsymbol{R}^3$ with boundary S belonging to the class C^2, and by D_e the unbounded domain $D_e := \boldsymbol{R}^3 \setminus \overline{D_i}$. We assume that the normal vector n on S is directed into the exterior domain D_e. The physical meaning is that D_i is a fixed elastic solid with no volume forces present and D_e represents a homogeneous isotropic linear solid which is characterized by the density $\rho = 1$ (this is no loss of generality) and the Lamé parameters λ and μ. We consider time-harmonic elastic waves with circular frequency ω and it will be assumed that all Lamé constants and the frequency are positive. We assume that the elastic medium D_e is in welded contact to the rigid inclusion D_i, which means that we consider displacement boundary conditions.

To formulate the elasticity problems we introduce the following function spaces. By $C^{0,\alpha}(S)$ and $C_T^{0,\alpha}(S)$ we denote the spaces of Hölder continuous functions and Hölder continuous tangential fields ($0 < \alpha < 1$), respectively. The space $C_D^{0,\alpha}(S)$ denotes the subspace of Hölder continuous tangential fields possessing Hölder continuous surface divergence in the sense of the limit integral definition given by Müller [1]. Defining the differential operator $\Delta^* := \Delta + \frac{\lambda+\mu}{\mu}\operatorname{grad}\operatorname{div}$, where Δ is the Laplace operator and λ and μ are the Lamé elastic constants with $\mu > 0$ and $\lambda + 2\mu > 0$. For a positive frequency ω the wavenumbers κ_p and κ_s are defined by $\kappa_p := \omega/\sqrt{\lambda + 2\mu}$ and $\kappa_s := \omega/\sqrt{\mu}$. Now, the time-harmonic exterior displacement problem in elasticity can be formulated as

Problem D: Find a vector field $u \in C^2(D_e) \cap C(\overline{D_e})$ satisfying the time-harmonic elasticity equation

$$\Delta^* u + \kappa_s^2 u = 0, \text{ in } D_e, \tag{1}$$

the welded contact boundary conditions

$$u = f, \text{ on } S, \tag{2}$$

and the Sommerfeld radiation condition

$$(x, \operatorname{grad} u_j(x)) - i\kappa_j u_j(x) = o(\frac{1}{|x|}), \text{ for } |x| \to \infty,\ j = s, p, \tag{3}$$

uniformly for all directions $x := x/|x|$, where

$$u_p := \frac{-1}{\kappa_p^2}\operatorname{grad}\operatorname{div} u \text{ and } u_s := \frac{1}{\kappa_p^2}\operatorname{grad}\operatorname{div} u + u. \tag{4}$$

Here $f \in C^{0,\alpha}(S)$ is a given vector field.

By (a, b) and $[a, b]$ we denote the scalar product and vector product of the vectors a and b, respectively. The vector fields u_s and u_p are known as the rotational and irrotational parts of u, respectively. The rotational part corresponds to a dilatational or compressional wave and the irrotational part corresponds to a shearing wave. The wave numbers κ_s and κ_p are known as the slownesses of the rotational and irrotational waves, respectively.

That Problem D possesses at most one solution has already been discussed by Kupradze [2] and Ahner [3]. The existence of a solution has been shown by Hähner and Hsiao [4].

For any domain $D \subset \boldsymbol{R}^3$ with boundary ∂D we introduce the linear space of vector fields $u : D \to \boldsymbol{R}^3$ by

$$F(D) := \{u \mid u \in C^2(D) \cap C(\overline{D}),\ \operatorname{curl} u, \operatorname{div} u \in C(\overline{D})\ \}.$$

From the integral representation theorem for solutions of the time-harmonic elasticity equation, known as the Betti formulas [2], we see that the displacement field is analytic. Therefore, by using (4) u can be split into $u = u_p + u_s$. Differentiating both, u_p and u_s, we see that u_p is *curl*-free and that u_s is *divergence*-free. Furthermore, u_j is a solution of the vector Helmholtz equation $\Delta u_j + \kappa_j u_j = 0$, in D_e, for $j = s, p$.

This motivates us to study the following slightly more general coupling

Problem HD: Find two vector fields $u_s, u_p \in F(D_e)$ satisfying the vector Helmholtz equations

$$\left.\begin{array}{lll} \Delta u_s + \kappa_s^2 u_s = 0, & \text{in } D_e, & \kappa_s \neq 0,\ \Im(\kappa_s) \geq 0, \\ \Delta u_p + \kappa_p^2 u_p = 0, & \text{in } D_e, & \kappa_p \neq 0,\ \Im(\kappa_p) \geq 0, \end{array}\right\} \tag{5}$$

the coupling boundary conditions

$$\left.\begin{array}{lllll} [n, u_s] & + & [n, u_p] & = & c, \\ \operatorname{div} u_s & & & = & \gamma, \\ & & [[\operatorname{curl} u_p, n], n] & = & d, \\ (n, u_s) & + & (n, u_p) & = & \delta, \text{ on } S, \end{array}\right\} \tag{6}$$

and the radiation conditions

$$[\operatorname{curl} u_j, \hat{x}] + \hat{x} \operatorname{div} u_j - i\kappa_j u_j = o(1/|x|), \text{ for } |x| \to \infty, \tag{7}$$

and $j = s, p$, uniformly for all directions $\hat{x} := x/|x|$. Here $c \in C_D^{0,\alpha}(S)$ and $d \in C_T^{0,\alpha}(S)$ are given tangential fields and $\gamma, \delta \in C^{0,\alpha}(S)$ are given functions.

2 Uniqueness

By Problem HDS we denote the special case of Problem HD, with

$$\kappa_p^2 = \frac{\omega^2}{\lambda + 2\mu} \text{ and } \kappa_s^2 = \frac{\omega^2}{\mu}, \tag{8}$$

and the right-hand sides

$$\gamma = 0 \text{ and } d = 0.$$

Now we have the following equivalence

Theorem 3.1: 1) Let u be a solution of Problem D corresponding to the boundary data $f := n\delta - [n, c]$. Then

$$u_p := \frac{-1}{\kappa_p^2}\text{grad div}\, u \text{ and } u_s := \frac{1}{\kappa_p^2}\text{grad div}\, u + u,$$

is a solution of Problem HDS.

2) Let u_p, u_s be a solution of Problem HDS corresponding to the boundary data $c := [n, f]$, $\gamma = 0$, $d = 0$ and $\delta := (n, f)$. Then $u := u_p + u_s$ is a solution of Problem D.

Proof: We will show only part 2). Let u_p, u_s be a solution of Problem HDS corresponding to the boundary data $c := [n, f]$, $\gamma = 0$, $d = 0$ and $\delta := (n, f)$. Representing u_s via the representation theorem for solutions of the vector Helmholtz equation [6] it can be seen that $\text{div}\, u_s$ is a solution of the scalar Helmholtz equation $\Delta\text{div}\, u_s + \kappa_s^2\text{div}\, u_s = 0$ in D_e satisfying the homogeneous Dirichlet boundary condition $\text{div}\, u = 0$ and the Sommerfeld radiation condition. From the uniqueness theorem for the exterior Dirichlet problem [5, 6] we obtain $\text{div}\, u_s = 0$ in D_e.

Using the integral representation theorem for solutions of the vector Helmholtz equation [6] it can be seen that $\text{curl}\, u_p$ solves the vector Helmholtz equation $\Delta\text{curl}\, u_p + \kappa_p^2\text{curl}\, u_p = 0$ in D_e, fulfills the homogeneous electric boundary condition $[[\text{curl}\, u_p, n], n] = 0$ and $\text{div curl}\, u_p = 0$, on S, and the radiation condition (7). From the uniqueness theorem for the exterior electric boundary value problem [6] we obtain $\text{curl}\, u_p = 0$ in D_e.

That $u := u_p + u_s$ is a solution of $\Delta^* u + \kappa_s^2 u = 0$ in D_e, follows by straightforward calculations. Since the cartesian components of every solution of the vector Helmholtz equation satisfying the radiation condition (7) also satisfy the radiation condition of Sommerfeld [6, see Corollary 4.14], we obtain that u fulfills the radiation condition (3).

That u fulfills the boundary conditions (2) is easily seen by

$$\begin{aligned} u &= u_s + u_p = n(n, u_s + u_p) - [n, [n, u_s + u_p]] \\ &= n(n, f) - [n, [n, f]] = f, \text{ on } S. \end{aligned}$$

From the uniqueness theorem for Problem D we obtain the following uniqueness

Theorem 3.2: Problem HD possesses at most one solution if for κ_p and κ_s the condition (8) holds.

Proof: Let u_p, u_s be a solution of the homogeneous Problem HD. As in the proof of Theorem 3.1 we can see that $u := u_s + u_p$ is a solution of Problem D but now to the homogeneous boundary condition. Therefore, by the uniqueness theorem for the exterior displacement problem we derive $u = 0$ in D_e.

Now we have $u_s = -u_p$ in D_e and there holds $\text{div}\, u_p = 0$ and $\text{curl}\, u_s = 0$ in D_e. From this we conclude

$$-\kappa_j^2 u_j = \Delta u_j = \text{grad div}\, u_j - \text{curl curl}\, u_j = 0,$$

and therefore $u_j = 0$ in D_e, for $j = s, p$. This means that Problem HD possesses at most one solution.

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On the Neutrino Opera in the CNGS Beam

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In this brief paper, we solve the relativistic kinematics related to the intersection between a relativistic beam of particles (neutrinos, e.g.) and consecutive detectors. The gravitational effects are neglected, but the effect of the Earth rotation is taken into consideration under a simple approach in which we consider two instantaneous inertial reference frames in relation to the fixed stars: an instantaneous inertial frame of reference having got the instantaneous velocity of rotation (about the Earth axis of rotation) of the Cern at one side, the lab system of reference in which the beam propagates, and another instantaneous inertial system of reference having got the instantaneous velocity of rotation of the detectors at Gran Sasso at the other side, this latter being the system of reference of the detectors. Einstein's relativity theory provides a velocity of intersection between the beam and the detectors greater than the velocity of light in the empty space as derived in this paper, in virtue of the Earth rotation. We provide a simple calculation for the discrepancy between a correct measure for the experiment and a measure arising due to the effect derived in this paper.

1 Definitions and Assumptions

Consider a position vector for CERN in relation to the center of the Earth, vector $\vec{C}$, and a position vector for the Gran Sasso receptors in relation to the center of the earth, vector $\vec{G}$. Consider the angular velocity vector of the Earth along its axis of rotation, vector $\vec{\omega}$. The velocity of rotation of $\vec{C}$ in relation to Earth's axis is given by $\vec{v}_C = \vec{\omega} \times \vec{C}$. Analogously, the velocity of rotation of $\vec{G}$ in relation to Earth's axis is given by $\vec{v}_G = \vec{\omega} \times \vec{G}$. Consider a baseline $\mathcal{L}_{CG}$ connecting $\vec{C}$ and $\vec{G}$ along the vector $\vec{G} - \vec{C}$; CERN's and Gran Sasso's latitudes $(\updownarrow_S^N)$, λ_C and λ_G, respectively, and CERN's and Gran Sasso's longitudes $(\leftarrow WE \rightarrow)$, α_C and α_G, respectively.

Since the effect related to the velocity of the neutrinos depends on its own velocity at the completion of the calculation and on the rotation of the Earth, viz., such effect does not depend on the specific values of the lateral velocity (to be defined below) of the receptors, as we will see, we may consider some geometric assumptions to simplify the geometry related to the baseline path $\vec{G} - \vec{C}$ along $\mathcal{L}_{CG}$ through the Earth.

Firstly, we will consider $\vec{C}$ and $\vec{G}$ having got the same latitude λ^*:

$$\lambda_C = \lambda_G = \lambda. \tag{1}$$

These latitudes would be important if the effect to be derived here was related to specific values of latitude, its fluctuations, systematic and/or statistical errors related to it etc., related to the six standard deviations that characterizes the claim related to the experiment. But that is not the case. Now, consider the plane $\mathbf{\Pi}$, orthogonal to $\vec{\omega}$, that cross the Earth through the hypothetically common latitude containing the points $\vec{C}$ and $\vec{G}$. Trace two lines pertaining to $\mathbf{\Pi}$: the line $\mathcal{L}_{CA}$, from the point $\vec{C}$ to Earth's rotation axis, and the line $\mathcal{L}_{GA}$, from the point $\vec{G}$ to Earth's rotation axis. $\mathcal{L}_{CA}$ and $\mathcal{L}_{GA}$ cross the rotation axis at the point $\vec{A}$. Also, trace the mediatriz line $\mathcal{L}_{MA}$, from the point $\vec{A}$ to the point $\vec{M} = (1/2)\left(\vec{G} + \vec{C}\right)$, equally dividing $\mathcal{L}_{CG}$. The angle between $\mathcal{L}_{CA}$ and $\mathcal{L}_{MA}$ equals the angle between $\mathcal{L}_{MA}$ and $\mathcal{L}_{GA}$, being this angle given by:

$$\alpha = \frac{1}{2}\left(\alpha_G - \alpha_C\right). \tag{2}$$

Upon the previous remarks regarding the geometric simplifications, the same remarks hold for the radius of the Earth, i.e., we will consider the Earth as a sphere. Thus, the following relation holds:

$$\left|\vec{C}\right| = \left|\vec{G}\right| = R_E = \frac{R}{\cos\lambda}, \tag{3}$$

where R_E is the radius of the Earth, its averaged value $R_E = 6.37 \times 10^6 m$, and $R = \left|\vec{C} - \vec{A}\right| = \left|\vec{G} - \vec{A}\right|$.

2 Defining two Instantaneous Inertial Reference Frames

The relativistic kinematics will run in the plane $\mathbf{\Pi}$ previously defined. The line $\mathcal{L}_{CG}$ will define an axis: Ox, with the origin O at the point $\vec{C}$, being the unitary vector of the axis Ox, $\hat{e}_x$, given by:

$$\hat{e}_x = \frac{\vec{G} - \vec{C}}{\left|\vec{G} - \vec{C}\right|}. \tag{4}$$

Now, define the Oz axis contained in the $\mathbf{\Pi}$ plane such that its unitary vector, $\hat{e}_z$, is given by:

$$\hat{e}_z = -\hat{e}_x \times \frac{\vec{\omega}}{\left|\vec{\omega}\right|}. \tag{5}$$

*The latitudes of CERN and Gran Sasso are, respectively: $46^{\text{deg}}14^{\text{min}}3^{\text{sec}}$(N) and $42^{\text{deg}}28^{\text{min}}12^{\text{sec}}$(N). The longitudes of CERN and Gran Sasso are, respectively: $6^{\text{deg}}3^{\text{min}}19^{\text{sec}}$(E) and $13^{\text{deg}}33^{\text{min}}0^{\text{sec}}$(E).

The axis Oy is trivially obtained with its unitary vector being given by:

$$\hat{e}_y = \hat{e}_z \times \hat{e}_x. \quad (6)$$

Now, we define the system at the Gran Sasso's detectors, $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$, such that its origin $\tilde{O}$ is at the point $\vec{G}$, being the unitary vector of the axis $\tilde{O}\tilde{x}$, the same $\hat{e}_x \equiv \hat{e}_{\tilde{x}}$. The axis $\tilde{O}\tilde{z}$ is parallel to Oz, with the same unitary vector $\hat{e}_z \equiv \hat{e}_{\tilde{z}}$, with analogous reasoning to obtain the axis $\tilde{O}\tilde{y}$ and its unitary vector $\hat{e}_{\tilde{y}} \equiv \hat{e}_y$. In other words, $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ is the parallel pure translation of $Oxyz$ from $\vec{C}$ (CERN) to $\vec{G}$ (Gran Sasso).

To define the two instantaneous inertial reference frames to accomplish, simply, the effect of the Earth rotation, we, firstly, write down the rotation velocities of the points $\vec{C}$ and $\vec{G}$ about Earth's rotation axis, i.e., we write down the rotation velocities of (CERN) and (Gran Sasso) about Earth's axis. For CERN, the rotation velocity $\vec{v}_C$ reads:

$$\vec{v}_C = \vec{\omega} \times \vec{C} = \omega R \hat{e}_\phi = \omega R\,(\cos\alpha\,\hat{e}_x - \sin\alpha\,\hat{e}_z)\,, \quad (7)$$

where the auxiliar unitary vector has been the azimutal ϕ-versor of the spherical coordinates, the spherical coordinates with origin at the center of the Earth with its equatorial dextrogyre plane Ξ such that $\vec{\omega} \cdot \vec{\xi} = \vec{0}\ \forall\ \vec{\xi} \in \Xi$. For Gran Sasso, the rotation velocity $\vec{v}_G$ reads:

$$\vec{v}_G = \vec{\omega} \times \vec{G} = \omega R \hat{e}_{\tilde{\phi}} = \omega R\,(\cos\alpha \hat{e}_{\tilde{x}} + \sin\alpha\,\hat{e}_{\tilde{z}})\,, \quad (8)$$

where $\hat{e}_{\tilde{\phi}}$ is the azimutal ϕ-versor previously defined, but now at Gran Sasso.

We see via the eqs. (7) and (8) that both the frames of reference, $Oxyz$ and $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$, are instantaneously under a null relative translation through the common axis $Ox \equiv \tilde{O}\tilde{x}$, and under a reverse translation through their respective parallel axes $Oz \parallel \tilde{O}\tilde{z}$. We will *inertially* consider this quite instantaneous* effect of the reverse translation (Newton's first law will hold, we will instantaneously neglect the gravitational field through the neutrino travel to Gran Sasso, as well as the weak characteristic for neutrino interactions with matter) via the following approach:

- We will consider a system of reference $O_C x_C y_C z_C$ that exactly coincides with $Oxyz$ at the instant $t_C = 0$, but with the following constant velocity of translation in relation to the fixed stars: $\vec{v}_\epsilon = \omega R \cos\alpha\,\hat{e}_{x_C} - \omega R \sin\alpha\,\hat{e}_{z_C} = \omega R \cos\alpha\,\hat{e}_x - \omega R \sin\alpha\,\hat{e}_z = \vec{v}_C$, such that the neutrino travel will be in this inertial referential. The subscript ϵ is to asseverate this referential is being considered for the neutrino travel during the entire process (emission→detection), but with $\epsilon \approx 0$ in the sense given in the previous footnote [$\epsilon \equiv \delta t_v / T << 1$]. Considered this, we will drop the subscript C in $O_C x_C y_C z_C$, for the sake of economy of notation, and rename it simply as $Oxyz$, although this latter is not the original one;
- We will consider a system of reference $O_G x_G y_G z_G$ that exactly coincides with $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ at the instant $t_G = t_C = 0^\dagger$, but with the following velocity of translation in relation to the fixed stars: $\vec{u}_\epsilon = \omega R \cos\alpha\,\hat{e}_{x_G} + \omega R \sin\alpha\,\hat{e}_{z_G} = \omega R \cos\alpha\,\hat{e}_{\tilde{x}} + \omega R \sin\alpha\,\hat{e}_{\tilde{z}} = \vec{v}_G$. Considered this, we will drop the subscript G in $O_G x_G y_G z_G$, for the sake of economy of notation, and rename it simply as $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$, although this latter is not the original one;
- We will consider a system of reference travelling with the beam of neutrinos, but this will be explained in the next section.

3 From CERN to the Flux through the Gran Sasso Detectors

From now on, we model the lattice (strips, emulsion, cintilators etc) distribution through the Grand Sassos' detectors from the perspective of an $Oxyz^\ddagger$ observer with the following characteristics:

- The average proper (no Lorentz contraction in $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$) displacement of detectors along $\tilde{O}\tilde{x}$ is $d_{\tilde{O}\tilde{x}}$;
- The average proper (no Lorentz contraction in $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$) displacement of detectors along $-\tilde{O}\tilde{y}$ is $d_{\tilde{O}\tilde{y}}$;
- The average proper (no Lorentz contraction in $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$) displacement of detectors along $-\tilde{O}\tilde{z}$ is $d_{\tilde{O}\tilde{z}}$;
- The detectors in $Oxyz$ will be abstracted to a tridimensional $d_{0x} \times d_{0y} \times d_{0z}$ othogonally spaced lattice falling upward [see the eqs. (7) and (8)] at the velocity $\vec{v}_G - \vec{v}_C = 2\omega R \sin\alpha\,\hat{e}_z$, being the basis vectors of these sites given by $\{\vec{d}_{0x} = d_{0x}\hat{e}_x,\ \vec{d}_{0y} = -d_{0y}\hat{e}_y,\ \vec{d}_{0z} = -d_{0z}\hat{e}_z\}$, where $\{\hat{e}_x,\ \hat{e}_y,\ \hat{e}_z\}$ is the canonical spacelike 3D euclidian orthonormal basis of $Oxyz$.
- We will neglect relativistic (Einstein's) effects related to the movement of the lattice of detectors, the movement of $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ in $Oxyz$, as previously stated, but such effects will become important in the referential of the neutrino beam (to be defined below).

Now, we define the neutrino frame of reference $O'x'y'z'$ in the canonical configuration with the frame of reference $Oxyz$, i.e., coincident origins at $t = t' = 0$ keeping the spacelike parallelism of the axes $x \equiv x'$, $y \equiv y'$ and $z \equiv z'$ and

*The time spent by a neutrino beam to accomplish the race from $\vec{C}$ to $\vec{G}$, δt_v, obey $\delta t_v / T << 1$, where T is the period of Earths's rotation about its axis, thus quite instantaneous in relation to the Earth daily kinematics.

†The relativistic effects between the systems of reference at CERN and at Gran Sasso related to time synchronization is being neglected due to the order of magnitude related to the velocities due to the Earth rotation and due to the magnitude of the gravitational field as previously stated. Furthermore, we are undressing these effects between these systems at $\vec{C}$ and $\vec{G}$ to asseverate the relevant relativistic effects that will lead to the neutrino velocity will raise in virtue of relativistic motion in relation to the detectors in Gran Sasso, as we will see.

‡From now on, we are working with the inertial frames (in relation to the fixed stars) defined above, viz., from now on: $Oxyzt$ means $O_C x_C y_C z_C t_C$ (see the two final paragraphs of the previous section); $\tilde{O}\tilde{x}\tilde{y}\tilde{z}\tilde{t}$ means $O_G x_G y_G z_G$ (see the two final paragraphs of the previous section).

boosted with velocity $v_\nu\hat{e}_x$ in relation to the $Oxyz$ frame. Consider a neutrino beam entering the block of detectors in Gran Sasso in the $Oxyz$ frame of reference. The beam passes a lattice of detectors stated above, being these detectors raining upward with velocity $\vec{v}_G - \vec{v}_C = 2\omega R\sin\alpha\hat{e}_z$ through the beam in the $Oxyz$ frame of reference. A horizontal neutrino beam, along Ox, may contact a horizontal [vertical means along Oz, upward means in the $\hat{e}_z$ direction, and horizontal means parallel to the xy plane] lattice of detectors parallelly raining upward in virtue of the Earth rotation as discussed before (raining upward with velocity $\vec{v}_G - \vec{v}_C = 2\omega R\sin\alpha\hat{e}_z$). Once an interaction occurs between the horizontal beam and consecutively located detectors in this horizontal lattice, this interaction is simultaneous in the $Oxyz$ (rigorously $Oxyzt$, but the context is clear here) world, implying non-simultaneity for these raindrops of detectors in the $O'x'y'z'$ world. The distribution of these raindrops of detectors must have, *instantaneously* at t' in $0'x'y'z'$ world, the following characteristics:

- The displacement between two consecutive raindrops of detectors correlated to the respective simultaneous ones in $Oxyz$, these latter displaced by the proper distance $x_{i+1} - x_i = d_{0x}$ along Ox and belonging to the falling upward xy plane of detectors in $Oxyz$, is given by:

$$x'_{i+1}(t') - x'_i(t') = \gamma^{-1}(x_{i+1} - x_i) = \gamma^{-1}d_{0x}, \tag{9}$$

being $\gamma = 1/\sqrt{1 - v_\nu^2/c^2}$, c the speed of light in the empty space, v_ν the speed of the neutrino, whose velocity is along the $\hat{e}_x$ direction in the $Oxyz$ world (the beam of neutrinos is at rest in its referential $O'x'y'z'$, as previously seem).

- The displacement between two consecutive raindrops of detectors correlated to the respective simultaneous ones in $Oxyz$, these latter displaced by the proper distance $z_i - z_{i+1} = 0$ along Oz and belonging to the falling upward xy plane of raining detectors in $Oxyz$, is given by:

$$z'_{i+i}(t') - z'_i(t') = 2\frac{v_\nu d_{0x}}{c^2}\omega R\sin\alpha. \tag{10}$$

- The vertical distance between consecutive (consecutive but inclined in the $O'z'y'z'$ world; the parallel to xy planes of detectors parallelly raining upward in $Oxyz$ become inclined in $O'x'y'z'$) raining planes of detectors Π'_i and Π'_{i+1}, $\forall\, i$, remains the same d_{0z} distance, the distance between consecutive parallelly raining planes of detectors. The raining upward planes turn out to be inclined in relation to the $x'y'$ plane of the neutrino world $O'x'y'z'$ by the angle:

$$\theta = \pi - \arctan\left(2\frac{\gamma v_\nu}{c^2}\omega R\sin\alpha\right). \tag{11}$$

Indeed, let's derive these facts. Firstly, instantaneously at t in $Oxyz$, two consecutive raindrops* Ox along, are time delayed in $O'x'y'z'\times\{t'\}$ world by the amount:

$$t'_{i+1} - t'_i = \gamma\left(t - \frac{v_\nu}{c^2}x_{i+1}\right) - \gamma\left(t - \frac{v_\nu}{c^2}x_i\right) = -\gamma\frac{v_\nu}{c^2}(x_{i+1} - x_i) = -\gamma\frac{v_\nu}{c^2}d_{0x}, \tag{12}$$

in virtue of the Lorentz transformations $(x,t)\to(x',t')$. Here, we see a detection that occurs at the position x_{i+1} pertaining to the horizontal lattice of detectors in Gran Sasso, at the plane $\tilde{x}\tilde{y}$ within the block of detectors in Gran Sasso, hence more internal, (remember $\tilde{x}\tilde{y}\parallel xy$), *must occur earlier* than the detection at the position x_i in the frame of reference of the beam of neutrinos, and the i-raindrop is late in relation to the $(i+1)$-raindrop. Hence, backwarding the t'_i clocks down to the the t'_{i+1} instant (backwarding the movie, maybe better: backwarding the neutrino's opera), i.e., comparing the non-simultaneous events in the beam of neutrinos frame, the event $i+1$ ocurring when the $i+1$-raindrop crosses the beam of neutrinos and the event i when the i-raindrop crosses the beam of neutrinos (remember these events are simultaneous in $Oxyz$) previously to infer the instantaneous (at t'_{i+1}) position of the i-raindrop when the $i+1$ raindrop crosses the beam of neutrinos at the instant $t'_{i+1} < t'_i$ in the $O'x'y'z't'$ frame, the i-raindrop must move the amounts (backwarding the movie from the instant t'_i at which the i-raindrop crosses the beam of neutrinos in the $O'x'y'z't'$ world to the non-simultaneous instant $t'_{i+1} < t'_i$ at which the $i+1$-raindrop crosses the beam of neutrinos in the $O'x'y'z't'$ world): $\delta z'$ downward and $\delta x'$ to the right, being these amounts given by:

$$\delta z' = \left(\frac{2\omega R\sin\alpha}{\gamma}\right)\times\left(-\gamma\frac{v_\nu}{c^2}d_{0x}\right) = -\frac{2\omega d_{0x}v_\nu R\sin\alpha}{c^2};$$

$$\delta x' = (-v_\nu)\times\left(-\gamma\frac{v_\nu}{c^2}d_{0x}\right) = \frac{v_\nu^2\gamma d_{0x}}{c^2}, \tag{13}$$

since $(-v_\nu\hat{e}_{x'} + (2\omega R\sin\alpha/\gamma)\hat{e}_{z'})$ is the velocity of raindrops in $0'x'y'z'$, obtained from the Lorentz transformations $L(\vec{u})$ for the 3-velocities of the Gran Sasso lattice block of sensors, the raining raindrops lattice of sensors, from the $Oxyz$ to the beam of neutrinos frame $O'x'y'z'$:

$$(0,0,2\omega R\sin\alpha)|_{Oxyz}\xrightarrow{L(\vec{u})}(-v_\nu,0,2\omega R\sin\alpha/\gamma)|_{O'x'y'z'}\,. \tag{14}$$

But, at t, the i-raindrop and the $(i+1)$-raindrop have got the same z coordinate, since they are in a xy plane, and, since the $z\to z'$ Lorentz map is identity, these raindrops must have the same z' coordinate at their respective transformed instants

*From now on, we will call raindrops the detectors in the lattice of detectors within the block of detectors at Gran Sasso. Thus, raindrops ≡ detectors within the lattice of detectors defined at the beginning of this section; 1 raindrop ≡ 1 detector within the lattice of detectors within the block of detectors at Gran Sasso.

(of course, since at *each* of these transformed instants, different instants in $O'x'y'z'$ in virtue of the non-simultaneity in this frame, the z' coordinate will read the same, since these reaindrops will cross the beam and the beam has the same coordinate z' in its own frame of reference, viz., the beam is parallel to $O'x'$). Hence, backwarding t'_i clocks down to the the t'_{i+1} instant, one concludes that the $\delta z'$ in the eq. (13) is the instantaneous, at same t', height shift between consecutive raindrops that simultaneously cross the beam of neutrinos in $Oxyz$. The $x \to x'$ Lorentz map is not identity, implying one must calculate the $x'_{i+1} - x'_i$ shift at the $Oxyz$ instantaneous t:

$$x'_{i+1}(t) - x'_i(t) = \gamma(x_{i+1} - v_\nu t) - \gamma(x_i - v_\nu t) = \gamma(x_{i+1} - x_i)$$

$$= \gamma d_{0x}. \tag{15}$$

This shift is related to different instants, t'_i, t'_{i+1}, in the beam of neutrinos frame. Thus, backwarding t'_i clocks down to the the t'_{i+1} instant (backwarding the movie to observe the earlier t'_{i+1} instantaneous), this amount given by the eq. (15) is reduced by the amount $\delta x'$ given by eq. (13):

$$x'_{i+1}(t') - x'_i(t') = \gamma d_{0x} - \gamma d_{0x}\frac{v_\nu^2}{c^2} = \gamma d_{0x}\left(1 - \frac{v_\nu^2}{c^2}\right)$$

$$= \gamma^{-1} d_{0x}. \tag{16}$$

The first eq. (13) gives the eq. (10), since eq. (13) gives the z' position of the i-raindrop at the previous instant t'_{i+1} before the i-raindrop crosses the beam of neutrinos in the $O'x'y'z'$, therefore:

$$z'_i\left(t'_{i+1}\right) = z'_\nu - \frac{2\omega d_{0x} v_\nu R\sin\alpha}{c^2}, \tag{17}$$

where z'_ν is *a* constant z' coordinate of the beam of neutrinos in its own frame; and, since the z' position of the $(i+1)$-raindrop at the t'_{i+1} instant is z'_ν (due to the very fact the $(i+1)$-raindrop crosses the beam at the instant t'_{i+1} in the $O'x'y'z'$ world), one has $z'_{i+1}\left(t'_{i+1}\right) = z'_\nu$, from which, with the eq. (17), one has got:

$$z'_{i+1}\left(t'_{i+1}\right) - z'_i\left(t'_{i+1}\right) = 2\frac{v_\nu d_{0x}}{c^2}\omega R\sin\alpha, \tag{18}$$

reaching the eq. (10). The non-instantaneous displacement (non-instantaneous in $O'x'y'z'$) given by eq. (15) is the distance between two sucessive non-instantaneous interactions with the beam, raindrops marks assigned upon the beam in $O'x'y'z'$. This fact is easy to understand, as these instantaneously assigned marks (instantaneous in $Oxyz$) would become splayed in $O'x'y'z'$, since the beam turns out to be contracted in $Oxyz$ due to Lorentz contraction. Also, one shall infer that eq. (16) gives the t' instantaneous displacement of falling upward raindrops along $O'x'$. The reason why the distance between consecutive raindrops marks γd_{0x} are bigger than the contracted distance $\gamma^{-1}d_{0x}$ of the two consecutive falling raindrops is explained by the non-simultaneity between these raindrops when touching the proper beam in the $0'x'y'z'$ world, straightforwardly seem by the inclination (the horizontal planes of raindrops in $Oxyz$ inclines in $O'x'y'z'$) between the raindrop plane containing these two consecutives raindrops in $O'x'y'z'$ and the proper plane $\Pi'_\nu \parallel x'y'$ containing the neutrinos beam in $O'x'y'z'$; i.e., when the first sensor raindrop crosses the beam, assigning the first interaction, the second travels an amount $\delta x'$ to the left given by the second eq. (13) before crossing the beam, assigning the second interaction. A xy instantaneous falling upward plane of sensors within the block of sensors at Gran Sasso containing raindrops in $Oxyz$ world becomes an inclined instantaneous falling upward plane in $O'x'y'z'$ world, being the inclination, eq. (11), easily derived from eqs. (16) and (18):

$$\tan(\pi - \theta) = \frac{\delta z'(t')}{x'_{i+1}(t') - x'_i(t')} = 2\frac{\gamma v_\nu}{c^2}\omega R\sin\alpha, \tag{19}$$

giving the eq. (11).

4 Faster than Light Effects in Gran Sasso

To understand the effect, first, consider two sensors, say i-raindrop and $(i+1)$-raindrop. If these sensors are constructed to tag the instants, t_{i+1} and t_i, at which two events are registered at their exact locations and a team of physicists obtains the time variation interval by $t_{i+1} - t_i$, being $x_{i+1} - x_i$ the distance between these sensors, one would have:

$$\frac{\delta x}{\delta t} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} = \infty, \tag{20}$$

for simultaneous events ($t_i = t_{i+1}$), if one expects a signal is travelling between the sensors. Furthermore, if one expects a privileged direction along which the signal should travel from the i-raindrop (first) to the $(i+1)$ raindrop (later), if the $(i+1)$-raindrop registered a signal before the i-raindrop, violating the expected sequential direction of detections, one would say the signal would have been registered from the future to the past direction. In the previous section the instantaneous events in the $Oxyz$ became non-instantaneous in the beam frame of reference, and the internal register within the Gran Sasso block along the direction $Ox \equiv \tilde{O}\tilde{x}$, at the position x_{i+1} registered the interaction with the beam at the same instant the internal register at the position x_i registered, since these events were hypothetically simultaneous in $Oxyz$, in virtue of the Earth rotation. From the point of view of the neutrino beam, these registers occurred in the order: x'_{i+i} before, x_i later, due to the inclination of the raindrops planes in virtue of the Earth rotation. We are forced to conclude the rotation of the Earth may provide a kinematics of intersection between beams and sequential sensors that may led to the conclusion the sensors are registering time intervals related to quasi-simultaneous events that are cintilated by different particles at different positions almost at the same time, leading to an errouneous conclusion that the signal would have travelled between the sensors generating the time tag data. E.g.,

suppose two ideal clocks, perfect ones, gedanken ones, that register the instants: t_i at which a beam of neutrinos enters the block of raindrop sensors in Gran Sasso and t_o at which this beam of neutrinos emerges from the block. Let d_B be the lenght travelled through the block. One team of physicists will measure the velocity of the beam by $d_B/(t_o - t_i)$ with no use of data from the sensors within the block. Another team will perform the calculation from the data obtained from a sequence of sensors (raindrops) located Ox along. This second team may obtain registers at diferent positions x_{i+1} and x_i related to the lateral intersection between these sensors and the beam entirely into the block of sensors but with the beam travel not entirely accomplished through the block. The data of this second team would be mistaken, since the registers at the different locations x_{i+1} and x_i would not have been made by the same neutrino, implying the clocks at x_{i+1} and x_i would be registering two quasi-simultaneous events *not related to a same neutrino*, concluding erroneously that the time variation between these events was so small that the particle that generated these events would be travessing with a velocity greater than c.

Einstein's theory of relativity does not avoid velocities greater than the light in the empty space, but avoids *an unique* particle propagating with velocity greater than the velocity of light in the empty space. To infer that a velocity greater than c may arise from the discussion through this brief article, consider the velocity two *different* raindrops interact with the beam of neutrinos in the beam $O'x'y'z'$ frame of reference. These events are non-simultaneous in the beam frame as previously discussed, but the beam crosses two sucessive interactions with a propagation that is faster than c, since the distance between two sucessive interactions along the beam in the beam frame of reference is given by the eq. (15), γd_{0x}, being the time spent given by the eq. (12), $(\gamma v_\nu d_{0x})/c^2$. Thus, the 2-propagation $\mathcal{V}'_{(i+1)\to(i)}$ (the number 2 to denote two bodies are related to a single propagation velocity):

$$\mathcal{V}'_{(i+1)\to(i)} = \frac{c}{v_\nu}c \Rightarrow \mathcal{V}'_{(i+1)\to(i)} > c. \tag{21}$$

As asseverated this is not a propagation of a single particle, but a ratio between the covered distance along the beam in the beam frame and the time interval spent to interact, non-simultaneously, with two sequential but distinct sensors (raindrops x_{i+1} and x_i). Of course, if $v_\nu \to 0$, these distinct interactions will tend to become simultaneous, leading to the result discussed at the beginning of this section (eq. 20). It follows that is not difficult to conclude that the time elapsed between two distinct sensors must be related to just an unique particle if one is intended to use their time tags for velocity computations.

5 The Consequence of the Effect

A simple calculation provides the discrepancy obtained by the set of CNGS detections intended to obtain the correct velocity of the neutrino particle announced few days ago. Let v_c be the correct value for the neutrino's velocity, δS_1, the distance between the CERN and the point at which the neutrino enters the block of detectors at Gran Sasso, δS_2 the lenght of the block of detectors to be internally covered by the neutrino at Gran Sasso, δt_1 the elapsed time spent by the neutrino to cover δS_1, δt_2 the elapsed time spent by the neutrino to cover δS_2, δt_f a fake elapsed time due to the effect previously discussed, and v_f a fake velocity that would arise from an erroneous measure for the elapsed time through δS_2. Hence:

$$v_c = \frac{\delta S_1 + \delta S_2}{\delta t_1 + \delta t_2}, \tag{22}$$

$$v_f = \frac{\delta S_1 + \delta S_2}{\delta t_1 + \delta t_f}. \tag{23}$$

A simple calculation gives:

$$\frac{v_f}{v_c} = 1 + \frac{\delta t_2 - \delta t_f}{\delta t_1 + \delta t_f} \Rightarrow \frac{v_f - v_c}{v_c} = \frac{\delta t_2 - \delta t_f}{\delta t_1 + \delta t_f}. \tag{24}$$

One should not write $v_f = \delta S_2/\delta t_f$, since, as previously discussed, δt_f is not related to a propagation of a particle, but to the time elapsed between two detectors in the same xy plane in the $Oxyz$ frame of reference. If a sequence of cintilations within the block of detectors are generated by different neutrinos due to the effect previously discussed, and this sequence is interpreted as a path traced by a single neutrino, the measure of the distance covered within the block of detectors at Gran Sasso would encapsulate an error for each estimated path having got the effect encrusted within it. This distance, an erroneous one, is δS_2, although this distance may be defined as a correct one for purposes of comparison with a case in which (hypothetically) a neutrino travelled this distance with the correct velocity v_c. Obviously, the δt_f would not be related to the time spent to cover this distance, since, in a case in which the effect, as previously explained, was generated due to simultaneous time tagging at two different sensors (raindrops) due to two different neutrinos in the $Oxyz$ reference frame, one would have $\delta t_f = 0$ for a continuous laterally traced path in virtue of the Earth rotation, from which $\delta S_2 \neq 0$. δS_2 would arise, under the effect discussed in this paper, from a path misinterpretation. But, once one defines δS_2 as the distance to be covered in a comparison case with a neutrino used to cover it with the correct velocity v_c:

$$\delta t_2 = \frac{\delta S_2}{v_c}, \tag{25}$$

turns out to be the correct elapsed time. From the eq. (24), one reaches:

$$v_f\left(\delta t_1 + \delta t_f\right) - v_c\delta t_1 = v_c\delta t_2. \tag{26}$$

If the computation is done taken into consideration the erroneous *elapsed* time δt_f, being this $\delta t_f = 0$ in a case of

simultaneity due to the effect previously discussed, the eq. (26) reads:

$$v_f \delta t_1 - v_c \delta t_1 = v_c \delta t_2, \tag{27}$$

and the effect turns out to increase the fake velocity in relation to the correct velocity in a manner in which the neutrino with the fake velocity would be winning the race by an amount of distance given by $\delta S_2 = v_c \delta t_2$, accomplished the path δS_1, from the eq. (27). Hence, eq. (24) turns out to read:

$$\frac{v_f - v_c}{v_c} = \frac{\delta t_2}{\delta t_1} = \frac{\delta S_2}{v_c} \frac{v_c}{\delta S_1} = \frac{\delta S_2}{\delta S_1}. \tag{28}$$

With the values [1]:

$$\frac{v_f - v_c}{v_c} = \left[2.48 \pm 0.28\ (stat.) \pm 0.30\ (sys.)\right] \times 10^{-5}, \tag{29}$$

and:

$$\delta S_1 = 733 \times 10^3 m, \tag{30}$$

we reach for the discrepancy between the covered distances:

$$\delta S_2 = \left[18.2 \pm 2.05\ (stat.) \pm 2.20\ (sys.)\right] m. \tag{31}$$

6 Conclusion

We conclude the relativistic effect discussed here in virtue of the Earth rotation may lead to a misinterpretation of the elapsed time within the block of detectors at Gran Sasso.

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Open Letter by the Editor-in-Chief: Declaration of Academic Freedom (Scientific Human Rights)
The Portuguese Translation[*]

Declaração de Liberdade Acadêmica

(Direitos Humanos Científicos)

Artigo 1: Preâmbulo

O início do século XXI reflete, mais do que em qualquer outra época na história da Humanidade, a profundidade e a significância do papel da Ciência e da tecnologia nas relações humanas.

A natureza poderosamente pervasiva da Ciência e da tecnologia modernas tem levado a uma percepção corriqueira de que descobertas-chave somente podem ser feitas, ou principalmente, por intermédio de grandes grupos de pesquisa sob tutela corporativa ou governamental com acesso à instrumentação de alto custo e a vasto suporte pessoal.

Todavia, a percepção corriqueira é mítica, esconde a verdadeira natureza de como as descobertas científicas são realmente feitas. Enormes e caros projetos tecnológicos, complexos ou não, não são senão o resultado da aplicação das profundas percepções de pequenos grupos de pesquisadores dedicados ou cientistas solitários, frequentemente trabalhando isolados. Um cientista trabalhando sozinho está e estará, tanto agora quanto no futuro, assim como ocorrera no passado, apto a fazer uma descoberta que pode influenciar substancialmente o destino da humanidade e mudar a face de todo o planeta sobre o qual nós tão insignificantemente residimos.

Descobertas revolucionárias são geralmente feitas por indivíduos trabalhando em posições subordinadas dentro de agências governamentais, em instituições de ensino e pesquisa, ou em empresas comerciais. Consequentemente, o pesquisador, com frequencia, está vinculado ou limitado por diretores de instituições e corporações que, trabalhando em uma prioridade diferente, visam controlar e aplicar perquisa e descoberta científicas para benefício pessoal, organizacional, ou engrandecimento pessoal.

O registro histórico de descobertas científicas está repleto de instâncias de supressão e ridicularização por parte do poder estabelecido, já há muito se tendo revelado e reivindicado pela inexorável marcha de necessidade prática e iluminação intelectual. Também assim se tem corrompido e sujado o registro histórico por plágio e deliberada perversão de fatos, perpetrados pelos inescrupulosos, motivados por inveja e avareza. Assim também o é hoje em dia.

O objetivo desta declaração é manter e incentivar a doutrina fundamental de que a pesquisa científica deve estar livre de latentes e abertas influências repressivas advindas de diretivas burocráticas, políticas, religiosas e pecuniárias, e de que a criação científica é um direito humano, não menos do que outros de tais direitos e árduas esperanças que se propuserem em tratados e leis internacionais.

Todos os cientistas que a apoiam devem ser fiéis a esta Declaração, como uma indicação de solidariedade para com a comunidade científica internacional interessada, e para conceder o Direito dos cidadãos do mundo à livre criação científica de acordo com suas habilidades e disposição individuais, para o avanço científico, por sua extrema habilidade como cidadãos decentes em um mundo indecente, [para] o benefício da Humanidade.

Artigo 2: Quem é um cientista

Um cientista é qualquer pessoa que faz Ciência. Qualquer pessoa que colabora com um cientista no desenvolvimento e proposição de idéias e dados em pesquisa ou aplicação é também um cientista. A posse de uma qualificação formal não é um pré-requisito para que uma pessoa seja um cientista.

Artigo 3: Onde a Ciência é produzida

A pesquisa científica pode ser desenvolvida em qualquer lugar, por exemplo, em um lugar de trabalho, durante um curso formal de educação, durante um programa acadêmico patrocinado, em grupos, ou, de modo independente, por indivíduos em suas casas.

Artigo 4: Liberdade de escolha do tema de pesquisa

Muitos cientistas que trabalham por graus de pesquisa mais avançados ou em outros programas de pesquisa em instituições acadêmicas tais como universidades e centros de estudos avançados são privados de trabalhar em um tema de pesquisa de sua própria escolha por acadêmicos seniores e/ou funcionários administrativos, não por falta de instrumentos de apoio, mas, em vez disso, por causa de hierarquia acadêmica e/ou pelo que outros funcionários não aprovam a linha de pensamento em virtude de seu potencial conflito com dogma estabelecido, teorias favorecidas, ou financiamento de projetos outros que possam ser desacreditados pela pesquisa proposta. A autoridade da maioria ortodoxa é muito frequentemente invocada para percalçar um projeto de pesquisa tal que a autoridade e seus pressupostos não sejam incomodados. Essa prática comum é uma obstrução deliberada ao livre pensamento científico, não sendo científica ao extremo, e criminosa. Ela não pode ser tolerada.

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Um cientista trabalhando para qualquer instituição acadêmica, autoridade ou agência, deve ser completamente livre para escolher um tema de pesquisa, limitado apenas pela disponibilidade de recursos materiais e aptidões intelectuais aptas a serem oferecidas pela instituição educacional, agência ou autoridade. Se um cientista procede com a pesquisa sendo membro de um grupo colaborativo, os diretores de pesquisa e os líderes de equipe devem estar limitados ao contexto de colsultores e conselheiros em relação ao que fora escolhido por um cientista do grupo como sendo um tema de pesquisa relevante.

Artigo 5: Liberdade de escolha de métodos de pesquisa

Frequentemente, tem-se o caso de se exercer pressão sobre um cientista por parte de funcionários administrativos ou acadêmicos seniores em relação a um programa de pesquisa conduzido dentro de um ambiente acadêmico, de se forçar um cientista a adotar métodos de pesquisa alheios aos que o cientista escolheu, por nenhuma razão outra senão preferência pessoal, viés, política institucional, mando editorial, ou autoridade coletiva. Essa prática, a qual está muito espalhada, é uma negação deliberada de liberdade de pensamento e não pode ser permitida.

Um cientista não comercial ou acadêmico tem o direito de desenvolver um tema de pesquisa em qualquer caminho razoável e por quaisquer meios que considere ser os mais efetivos. A decisão final sobre como a pesquisa será conduzida deve ser feita pelo próprio cientista.

Caso um cientista não comercial ou acadêmico trabalhe como membro de uma equipe colaborativa não comercial ou acadêmica de cientistas, os líderes de projeto e diretores de pesquisa deverão ter apenas direitos de conselheiros e consultores, não devendo, todavia, influenciar, mitigar ou limitar os métodos de pesquisa ou tema de pesquisa de um cientista dentro do grupo.

Artigo 6: Liberdade de participação e colaboração em pesquisa

Há um elemento significativo de rivalidade institucional na prática de Ciência moderna, concomitante com elementos de inveja pessoal e preservação de reputação a todo custo, independente das realidades científicas. Isso tem, com frequencia, levado os cientistas a não convidar colegas competentes de instituições rivais ou outros sem afiliação acadêmica. Essa prática também é uma obstrução deliberada ao progresso da Ciência.

Caso um cientista não comercial ou acadêmico requeira a colaboração de outrem e este estiver de acordo em oferecê-la, aquele cientista terá a liberdade de convidar este outro para que lhe preste essa ou qualquer outra ajuda, supondo-se que a assistência requerida esteja sob um contexto pressuposto de pesquisa associada. Se tal colaboração estiver fora desse escopo pressuposto, o cientista ainda terá a liberdade de escolher o outro para sua discrição, livre de qualquer interferência de quem quer que seja.

Artigo 7: Liberdade de discordar em discussão científica

Em consequencia de ciúmes furtivos e interesse adquirido, a Ciência moderna repudia discussões abertas e premeditadamente bane aqueles cientistas que questionam as visões ortodoxas. Muito frequentemente, cientistas de habilidade extraordinária, que apontam deficiências em teorias vigentes ou em interpretação de dados, são rotulados de crackpots (excêntricos tolos), de modo às suas interpretações serem convenientemente ignoradas. Eles são particular e privadamente ridicularizados e são barrados dos congressos científicos, seminários e colóquios de modo às suas idéias não encontrarem audiência. Falsificação deliberada de dados e má representação de teoria são ferramentas frequentes e atuais dos inescrupulosos na supressão de fatos, tanto técnicos quanto históricos. Comitês internacionais de meliantes científicos têm sido formados e esses conselhos abrigam e dirigem convenções internacionais nas quais apenas os seus acólitos são permitidos a apresentar artigos, independente da qualidade de conteúdo. Esses conselhos angariam grades somas de dinheiro público para financiar seus projetos patrocinados, por intermédio de engano e mentira. Qualquer objeção às suas propostas, ainda que objetadas sobre bases científicas, é silenciada por quaisquer meios disponíveis, de modo que o dinheiro continue fluindo para dentro de suas contas de projeto, e que os garanta empregos bem pagos. Cientistas que se opõem a essa praxe têm sido exonerados a pedido daqueles; outros têm sido impedidos de ocupar posições acadêmicas por uma rede de cúmplices corruptos. Em outras situações, alguns têm sido expulsos de suas candidaturas a programas de educação superior tal como doutoramento, por expressar idéias que minam uma teoria da moda, a despeito do arraigo temporal que uma teoria ortodoxa todavia possa ter. O fato fundamental de que nenhuma teoria científica é definitiva e inviolável, estando portanto aberta à discussão e à reexaminação, é completamente ignorado. O fato de que um fenômeno pode ter um número de explicações plausíveis é também ignorado, e maliciosamente põem em descrédito qualquer explicação que não esteja de acordo com a opinião ortodoxa, recorrendo, sem arguir exceções, ao uso de argumentos não científicos para justificar suas opiniões tendenciosas.

Todos os cientistas devem ser livres para discutir sua pesquisa e a pesquisa de outrem sem medo de que sejam pública ou privadamente ridicularizados sem fundamento, de que sejam acusados, depreciados, impugnados ou postos em descrédito de qualquer outra forma por alegações não consubstanciadas. Nenhum cientista deve ser posto em uma posição pela qual seu sustento ou reputação estejam sob risco em consequencia de expressão de uma opinião científica. A liberdade de expressão científica deve ser suprema. O uso de autoridade na refutação de um argumento científico não é científico e não deve ser usada para amordaçar, suprimir, intimidar, ostraci-

zar, ou, por qualquer forma coercitiva, barrar um cientista. A supressão deliberada de fatos científicos ou argumentos, seja por ato ou omissão, e a manipulação deliberada de dados para sustentar um argumento ou para por em descrédito uma visão oposta é fraude científica, perfazendo um crime científico. Princípios de evidência devem guiar toda discussão científica, seja tal evidência física, teórica ou também uma combinação.

Artigo 8: Liberdade para publicar resultados científicos

Uma censura deplorável de artigos científicos tem agora se tornado a prática padrão dos conselhos editoriais dos maiores jornais e arquivos eletrônicos, e de seus bandos de alegados árbitros especialistas. Os árbitros são em sua maior parte protegidos pelo anonimato de modo que um autor não tem como verificar suas alegadas especialidades. Artigos são atual e rotineiramente rejeitados caso o autor não concorde com ou contradiga uma teoria preferida ou a corrente ortodoxa principal. Muitos artigos são agora automaticamente rejeitados em virtude do aparecimento na lista de artigos de autor de um cientista em particular que não tenha encontrado favor entre os editores, entre os árbitros, ou entre outros censores especialistas, sem qualquer consideração que seja feita ao conteúdo do artigo. Existe uma listagem negra de cientistas dissidentes e esta lista é comunicada aos e entre os participantes de conselhos editoriais. Tudo isso contribui para o aumento da não isenção, da tendenciosidade, e para a punível supressão de livre pensamento, devendo ser condenado pela comunidade científica internacional.

Todos os cientistas devem ter o direito de apresentar seus resultados de pesquisa científica, no todo ou em parte, em conferências científicas relevantes, de publicar os mesmos em jornais científicos impressos, em arquivos eletrônicos, e em qualquer outro meio. Nenhum cientista deve ter seus artigos ou relatórios rejeitados quando submetidos à publicação em jornais científicos, em arquivos eletrônicos, ou em outro meio, simplesmente por que seu trabalho questiona a opinião majoritária corrente, por que conflita com as visões de um conselho editorial, pelo que mine as bases de outra corrente ou projetos de pesquisa planejados por outros cientistas, por estar em conflito com qualquer dogma político, credo religioso, ou opinião pessoal de outrem; e nenhum cientista deve ser inserto em listagem negra ou por outra forma censurado, impedido, por quem quer que seja, de publicar. Nenhum cientista deve bloquear, modificar, ou de outra forma interferir na publicação de um trabalho de cientista sob promessa de presentes ou qualquer forma de suborno.

Artigo 9: Coautoria de artigos científicos

É um segredo alardeado em círculos científicos, que muitos coautores de artigos de pesquisa têm, de fato, pouca ou nenhuma relação com as pesquisas ali relatadas. Muitos supervisores de estudantes graduados, por exemplo, não são contrários à colocação de seus nomes em artigos escritos por aquelas pessoas que apenas nominalmente trabalham sob suas supervisões. Em muitos de tais casos, a pessoa que de fato escreve o artigo tem um intelecto superior ao do supervisor nominal. Em outras situações, novamente pelos propósitos de notoriedade, reputação, dinheiro, prestígio, e os similares, as pessoas não participantes são incluídas no artigo como coautoras. Os autores de fato de tais artigos podem apenas objetar sob risco de serem subsequentemente penalizados de alguma forma, ou mesmo de serem expulsos de suas candidaturas a graduações de pesquisa mais elevadas ou de suas equipes de pesquisa, conforme o caso. Muitas têm de fato sido expulsas sob tais circunstâncias. Essa prática horrorosa não pode ser tolerada. Apenas aquelas pessoas responsáveis pela pesquisa deveriam ter autoria oficialmente reconhecida.

Nenhum cientista deve convidar outra pessoa para ser incluída e nenhum cientista deve permitir que seu nome seja incluso como coautor de um artigo científico caso não tenha contribuído significativamente para a pesquisa que se relata no artigo. Nenhum cientista deve permitir que ele mesmo ou ela mesma seja coagido por qualquer representante de uma instituição acadêmica, corporação, agência governamental, ou qualquer outra pessoa, a incluir seu nome como coautor em relação à pesquisa para qual não tenha significativamente contribuído, e nenhum cientista deve permitir que seu nome seja usado como coautor à guisa de presentes ou quaisquer subornos. Nenhuma pessoa deve induzir ou tentar induzir um cientista, por qualquer forma, a um caminho que permita que o nome do cientista seja incluído como coautor de um artigo científico relacionado a assuntos para os quais não tenha significativamente contribuído.

Artigo 10: Independência de afiliação

Muitos cientistas estão agora empregados sob contratos de curto período. Com o término do contrato de emprego, também termina a afiliação acadêmica. É frequente a política de conselhos editoriais em que pessoas sem uma afiliação acadêmica ou comercial não publiquem. Na ausência de afiliação, muitos recursos não estão disponíveis ao cientista, e as oportunidades de apresentar palestras e artigos em conferências são reduzidas. Essa é uma prática viciosa que deve ser parada. Ciência não reconhece filiação.

Nenhum cientista deve ser impedido de apresentar artigos em conferências, colóquios ou seminários, de publicar em qualquer meio, de acessar bibliotecas acadêmicas ou publicações científicas, de participar de encontros científicos, ou de dar conferências, por necessitar de afiliação a uma instituição acadêmica, instituto científico, laboratório governamental ou comercial, ou de qualquer outra organização.

Artigo 11: Acesso aberto à informação científica

Muitos livros especializados sobre assuntos científicos e muitos jornais científicos rendem pouco ou nenhum lucro, de modo que editores comerciais não estão dispostos a publicá-

los sem uma contribuição em dinheiro de instituições acadêmicas, agências governamentais, fundações filantrópicas, e correlatos. Sob tais circunstâncias, editores comerciais deveriam permitir acesso livre a versões eletrônicas das publicações, e esforçar-se por manter o custo dos materiais impressos num mínimo.

Todos os cientistas deverão se esforçar para assegurar que seus artigos de pesquisa estejam disponíveis à comunidade científica internacional de modo gratuito, ou alternativemante, se tal não puder ser evitado, a um mínimo custo. Todos os cientistas deverão tomar medidas ativas para fazer com que seus livros técnicos estejam disponíveis ao custo mais baixo possível de modo a poder estar a informação disponível à mais ampla comunidade científica internacional.

Artigo 12: Responsabilidade ética de cientistas

A História testifica que descobertas científicas são usadas tanto para fins bénéficos quanto malévolos, para o benefício de alguns e para a destruição de outros. Dado que o progresso da Ciência e da tecnologia não pode parar, meios para que se contivesse a aplicação malévola deveriam ser estabelecidos. Apenas um governo democraticamente eleito, laico, com liberdade racial e não tendencioso, pode salvaguardar a civilização. Apenas governos, tribunais e comitês democraticamente eleitos podem salvaguardar o direito de livre criação científica. Hoje em dia, vários estados não democráticos e regimes totalitários conduzem pesquisa ativa em física nuclear, química, virologia, engenharia genética etc, com propósito de produzir armas nucleares, químicas e biológicas. Nenhum cientista deveria ter interesse em colaborar com estados não democráticos ou regimes totalitários. Qualquer cientista coagido a trabalhar no desenvolvimento de armas para tais estados deveria encontrar meios de diminuir o progresso de programas de pesquisa e de reduzir a produção científica de forma que a civilização e a democracia possam finalmente prevalecer.

Todos os cientistas adquirem uma responsabilidade moral por suas criações científicas e descobertas. Nenhum cientista deve voluntariamente se engajar no desenho ou construção de armas de qualquer tipo, para o que quer que seja, para estados não democráticos ou regimes totalitários ou permitir que suas habilidades científicas e conhecimentos sejam aplicados no desenvolvimento do que quer que seja prejudicial à Humanidade. Um cientista deve viver pelo dito de que todo governo não democrático e toda violação de direitos humanos são crime.

22 de novembro de 2005 / Traduzido em 17 de maio de 2011

Open Letter by the Editor-in-Chief: Declaration of Academic Freedom (Scientific Human Rights)
The Arabic Translation*

إعلان الحرية الأكاديمية
(الحقوق العلمية للأنسانية)

المادة ١ : مقدمة

إن بداية القرن الحادى و العشرون بعد الميلاد تعكس أكثر من أي وقت مضى في تاريخ البشرية عمق وأهمية دور العلم والتكنولوجيا للبشرية. ونظرا لطبيعة الأنتشار بقوة المميزة للعلم والتكنولوجيا الحديثين فأنه يمكن إحراز اكتشافات ثورية أخرى سواء كانت بمشاركة او بشكل حصرى من قبل الحكومة أو المجموعات البحثية المؤسسية الممولة مما يؤهلها للوصول إلى أجهزة باهظة الثمن بشكل كبير وجحافل من الباحثين و الأفراد.

إن الإدراك و الأتفاق المشترك للعلم هو امر من المستحيل ويناقض الطبيعة الحقيقية لكيفية صنع الاكتشافات العلمية. ذلك لأن المشاريع التكنولوجية الكبيرة والمكلفة – مهما كانت معقدة – ليست سوى نتيجة لتطبيق الأفكار العلمية العميقة لمجموعات صغيرة من الباحثين أو العلماء المنفردين الذين يعملون – في كثير من الأحيان – في عزلة. إن عالم واحد – سواء في الحاضر والمستقبل – وحده هو تماما كما في الماضي قادرة اكتشاف ما يمكن أن يؤثر بشكل كبير على مصير الإنسانية وتغيير وجه هذا الكوكب الذي نقطنه.

تتم عادة الاكتشافات الرائدة من قبل أفراد يعملون في مناصب ثانوية داخل الوكالات الحكومية أومؤسسات البحث العلمى أوالتعليمي أو المؤسسات التجارية. وبالتالي فإن الباحث في كثير من الأحيان مقيد أو يتم قمعه من قبل مديري المؤسسات والشركات الذين يعملون لأهداف مختلفة، ساعون إلى السيطرة وتطبيق الاكتشافات العلمية والبحوث من أجل الربح الشخصي أو المؤسسي أو حتى الترقي الشخصي.

إن السجل التاريخي لاكتشاف العلمي مليء بحالات القمع والسخرية من قبل المؤسسات و تنكشف هذه الحالات و تبرر بعدها بسنوات بسبب المسير الغير قابل للتغيير للعلم و الضرورة العملية والتنوير الفكري. كذلك حالات الإشقاء وتلطيخ السمعة و الانتحال والتحريف المتعمد و التي يرتكبها عديمي الضمير- بدافع الحسد والجشع- وهكذا هو الحال حتى الان.

الهدف من هذا الإعلان هو دعم وتعزيز مبادئ البحث العلمي الأساسية التي يجب أن تكون خالية من التأثيرات القمعية العلنية والمستترة و من التوجيهات والبيروقراطية السياسية والمالية (إن عدم الأعتقاد فى توجيهات الأديان هو خروج عن اخلاق العلم وعدم أتباع لحق الأنسان في الاعتقاد بالله و قد يقصد هنا التوجيهات الزائدة عن منهج الدين، المترجم) والمالية، وأن صنع العلم هو حق من حقوق الإنسان لا تقل عن تلك الحقوق و الأمال الأخرى كما نصت عليه المواثيق الدولية والقانون الدولي.

يجب دعوة جميع العلماء المناصرين بالألتزم بهذا الإعلان باعتباره مؤشرا للتضامن مع المجتمع العلمي الدولي المعني وتعاطفا مع حقوق المواطنة ذلك من أجل صنع إنجازات علمية دون قيود وفقا لمهاراتهم و تصوراتهم الفردية - من أجل النهوض بالعلم – وذلك لأقصى قدرتهم كمواطنين شرفاء فى تلك البيئة الغير ملائمة. تلك المنفعة هى العائد على البشرية بعدما امضت العلوم والتكنولوجيا فترة طويلة جدا في مطال الاضطهاد.

المادة ٢ : من هو العالم

العالم هو أي شخص يتناول العلم و أي شخص يتعاون مع أحد العلماء في تطوير والإقتراح الأفكار والبيانات في مجال البحث العلمي أو تطبيقه هو أيضا عالما. و أن اكتساب مؤهل رسمي ليس شرطا مسبقا للإنسان لكى يكون عالما.

المادة ٣ : ما الذى ينتج العلم

ويمكن إجراء البحث العلمي في أي مكان على الإطلاق، على سبيل المثال في مكان العمل أو خلال مقرر دراسى أو خلال برنامج اكاديمي مدعوم وذلك خلال مجموعات أو كأفراد في اطار إجراءات مستقلة.

المادة ٤ : حرية اختيار اطار البحث

مُنع العديد من العلماء الذين يعملون للحصول على درجة أعلى في برامج البحث العلمى أو غيرها من برامج البحوث في المؤسسات الأكاديمية مثل الجامعات والكليات من العمل على موضوع بحث من اختيارهم وذلك من قبل المشرفين الأكاديميين أو المسؤولين الإداريين، وليس ذلك لعدم وجود امكانيات فنية لكن - و بدلا من ذلك - بسبب التسلسل الهرمي الأكاديمي و/ أو غيره من الأمور الرسمية و ذلك ببساطة لعدم أتفاقه و العقيدة السائدة أو النظريات المفضلة أو مسار التمويل للمشاريع الأخرى التي قد تفقدت مصداقيتها نتيجة البحوث المقترحة. وكثيرا ما ثارت السلطة الغالبة لافشال مشروع بحثي بحيث لا يزعج السلطة والميزانيات. هذه الممارسة الشائعة هي عرقلة متعمدة للفكر العلمي الحر و هي غير علمية في نهايتها وتعد جنائية و بالتالي لا يمكن التغاضي عنها.

أن اي عالم يعمل في أي مؤسسة أكاديمية أو هيئة أو وكالة يكون حرا تماما في اختيار موضوع البحث، مرتبطا فقط بالدعم المادي والمهارات الفكرية القادرة على أكتسابها من المؤسسات التعليمية أو الهيئات أو الوكالات. إن قام أحد العلماء بإجراء البحوث بوصفه عضوا في مجموعة تعاونية فإنه يقتصر على مديريه أو قادة الفريق البحثى القيام بالأدوار الاستشارية والتشاور في ما يتعلق باختيار موضوع البحوث ذات الصلة.

المادة ٥ : حرية اختيار طرق البحث

يتكرر كثيرا أن يتم التسبب بإجبار العالم من قبل أفراد إداريين أو كبار الأكاديميين في ما يتعلق ببرنامج البحوث التي أجريت في البيئة الأكاديمية على أتباع أساليب بحثية مختلفة عن تلك التي اختارها العالم

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إن من حق العالم الغير هادف للربح أو الغير أكاديمي تطوير موضوع البحث بأي وسيلة و بأي سبب معقول يعتبره أكثر فعالية. القرار النهائي بشأن الكيفية التي سيتم بها إجراء البحوث سيقوم بها العالم وحده.
إذا كان عالما غير هادف للربح أو غير أكاديمي يعمل كعضو في فريق تعاوني من العلماء غير هادف للربح أو غير أكاديمي فإن قادة ومدراء المشاريع البحثية لديهم حقوق النصح و الأرشاد فقط ويجب ألا يؤثروا خلاف ذلك بتخفيف أو تقييد طرق البحث أو البحوث المعنية لعالم داخل المجموعة.

المادة ٦ : حرية المشاركة والتعاون في مجال البحوث

هناك عنصر للتنافس بين المؤسسات في ممارسة العلم الحديث - بالتزامن مع عناصر من الحسد الشخصي والحفاظ على السمعة مهما كان الثمن - بغض النظر عن الحقائق العلمية. وقد أدى ذلك في كثير من الأحيان إلى منع العلماء من التعاون بين الزملاء المختصصين في المؤسسات المتنافسة أو غيرهم من الذين دون أي انتماءات أكاديمية. هذه الممارسة هي أيضا عرقلة متعمدة لتقدم العلمي.
إذا طلب عالما غير هادف للربح مساعدة من شخص أخر بعد موافقته فإن للعالم الحرية في دعوة هذا الشخص لتقديم أي مساعدة بشرط تضمين المساعدة في ميزانية البحوث المرتبطة بها. إذا كانت المساعدة مستقلة عن أي اعتبارات مالية فإن للعالم وحده الحرية في الانخراط في طلب المساعدة من الشخص و في تقديره بشكل خالي من أي تدخل على الإطلاق من قبل أي شخص مهما كان.

المادة ٧ : حرية الاختلاف في المناقشة العلمية

بسبب الغيرة الدفينة وتشعب الأهتمامات المكتسب يتجنب العاملون في العلم الحديث المناقشة مفتوحة عمدا ويتجنب أولئك العلماء الذين يشككون من وجهات النظر المتعصبة. في كثير من الأحيان يوصف العلماء ذوي القدرة البارزة - الذين يشيرون إلى أوجه القصور في النظرية الحالية أو الى تفسير بياناتها بالمجازيب حتى أنه يحدث تجاهل لوجهات نظرهم بشكل عفوي ويكونون - سرا وعلانية - وبصورة منهجية ممنوعون من حضور المناقشات العلمية والحلقات الدراسية والندوات بحيث اصبحت أفكارهم لا تجد طريقة للاعلان عن نفسها. إن التزييف المتعمد للبيانات والسخرية من نظرية ما هي الآن أدوات معتادة من عديمي الضمير في طمس الحقائق، سواء الفنية أوالتاريخية. وقد تم تشكيل لجان دولية مِن من يوصفون بالعلماء واستضافة هذه اللجان ومباشرتها للمقابلات الدولية التي يسمح بها فقط لمساعديهم بتقديم أوراق بحثية بغض النظر عن جودة المحتوى. هذه اللجان استقطبت مبالغ كبيرة من المال من الأموال العامة لرعاية تمويل مشاريعها من خلال اللجوء إلى الخداع والكذب. وأي اعتراض على مقترحاتهم على أسس علمية يتم منعه بأي وسيلة تحت تصرفهم، لذلك يمكن ان يستمر تدفق الاموال الى حسابات مشايعهم التي تضمن لهم وظائف جيدة الأجر. وقد تم عزل العلماء المعارضين لهم بأمر منهم و منع الآخرين من الحصول على التعيينات الأكاديمية من خلال شبكة من المتواطئين. في حالات أخرى تم طرد البعض من الترشيح في برامج الدرجة الأعلى مثل الدكتوراه للتعبير هم عن الأفكار التي تقوض "نظرية المألوف". مع ذلك هذه النظرية موجودة منذ فترة طويلة و قد تكون تقليدا دينيا عمره الاف السنين. الحقيقة الأساسية بأنه لا توجد نظرية علمية محددة وغير قابلة للانتهاك - وبالتالي فهي مفتوحة للمناقشة وإعادة النظر – تم تجاهلها تماما. كأن نتجاهل حقيقة أن أي ظاهرة قد يكون لها عدد من التفسيرات المعقولة - وهذا لا يتفق مع الرأي التقليدي-و تستخدم و دون تردد الحجج الغير علمية لتبرير آراء متحيزة.
جميع العلماء يجب أن يكونوا احرارا في مناقشة أبحاثهم وبحوث الآخرين دون خوف من السخرية العامة و خاصة تلك التي لا تملك أساسا ماديا أو محض اتهام أو استخفاف أو طعنا فيه أو خلاف ذلك مما لا أساس له من ادعاءات فقدت مصداقيتها. و لا يجب وضع اي عالم في موقف يعرض اسلوب معيشتة أو سمعته للخطر بسبب التعبير عن الرأي العلمي. حرية التعبير عن الرأي يجب أن تكون الأولى بالتقدير. استخدام السلطة في دحض حجة علمية ليست علما ويجب ألا تستخدم لإسكات أوقمع أوترهيب أو نبذ أو غير ذلك من إكراه أو تقييد للعالم. القمع المتعمد للحقائق العلمية أو الحجج إما بفعل أو الامتناع عن فعل والجبر المتعمد للبيانات لدعم حجة أو لتشويه سمعة الرأي المخالف والغش العلمي الأمور التي تصل إلى حد الجريمة العلمية. ومبادئ الأستدلال يجب أن تحرك المناقشة العلمية و يكون الدليل المادي أو النظري أو مزيج منها هما الأساس.

المادة ٨ : حرية نشر النتائج العلمية

الآن أصبحت الرقابة ضعيفة على الأوراق العلمية و من الممارسات العامة المعمول بها في هيئات تحرير عدد من المجلات الرئيسية ومكتبات الحفظ الإلكترونية وعلى شرائح من محكمي المجلات المزعومين. إن المحكم هو القسم الاكبر المحمي بهدف عدم الكشف عن الهويته حتى أن العالم الذي يتقدم للنشر لا يمكن التحقق من خبراته المفترضة. والآن و بشكل روتيني يتم رفض البحوث إذا لم يتفق المؤلف أو تعارض مع النظرية المفضلة لدى المحكم أوالعقيدة السائدة. و في الوقت الحالي يتم رفض العديد من الأوراق البحثية تلقائيا بشكل ظاهري بتأثير ظهور العالم في قائمة مؤلفات عالم اخر ليس مقبولا لدى احد من المحررين أوالمحكمين أو حتى المرقبين و الخبراء الأخرين دون أي اعتبار على الإطلاق لمحتويات الورقة البحثية. هناك قوائم سوداء للعلماء المعارضين وتبلغ هذه القائمة بين هيئات التحرير للمجلات المتعاونة. كل هذه التراكم من الانحياز والقمع للتفكير الحر وجب أدانتها من قبل المجتمع العلمي الدولي.
جميع العلماء لديهم الحق في تقديم نتائج بحوثهم العلمية، كليا أو جزئيا، في المؤتمرات العلمية ذات الصلة، ونشرها في المجلات العلمية المطبوعة والالكترونية أو بأية وسيلة أخرى. لا يجوز رفض الأوراق العلمية او التقارير لأي عالم عند تقديمها للنشر في المجلات العلمية المطبوعة والإلكترونية أو/ وسائل الإعلام الأخرى وذلك– و بكل بساطة – بسبب نقاط بحوثهم المعنية أو تعارضها مع وجهات النظر لهيئة التحرير أو تقويضها للأسس الحالية أو الأسس المخطط لها لمشاريع البحوث من قبل العلماء الآخرين أو أن تتعارض مع أي عقيدة سياسية أو ملية (مع الأخذ في الاعتبار العامل الاخلاقي للبحث الاكاديمي، المترجم) أو رأي شخصي آخر. ولا يجب وضع أي عالم في القوائم السوداء أو لومه على خلاف ذلك ومنعه من النشر من قبل أي شخص أي كان. لا يجوز لأي عالم حجب أو تعديل أو التداخل في نشر بحث بتقديم الوعود أو أي مظهر اخر من مظاهر الرشوة من أي نوع.

المادة ٩ : المشاركة في تأليف المنشورات العلمية
من الصعب كتمان أنه في كثير من الأروقة العلمية تكون مشاركة المؤلفين فى الأوراق البحثية في الواقع ضئيلة أو لا علاقة لهم بتاتا بالبحث الذى ذكروا فيه. فالعديد من المشرفين على طلاب الدراسات العليا- على سبيل المثال - لا يتورعوا عن وضع أسمائهم على أوراق مكتوبة من قبل أولئك الأشخاص الذين وضعوا اسميا تحت إشرافهم. في حالات كثيرة من هذه يكون للشخص الذي يكتب الورقة في الواقع سبق التفكير على المشرف الاسمي و في حالات أخرى – و لأغراض الشهرة والسمعة والمال والمكانة وغيره مما سبق ذكره- لا يشاركون نهائيا فى الورقة أو البحث الذى تضمن اسمائهم. يمكن للمؤلفين الفعلي لهذه الأوراق الأعتراض مما يعرضهم لخطر العقاب لاحقا بطريقة ما أو حتى أستبعاده من التقدم للترشيح لدرجة علمية أو ادارية أعلى فى الفريق البحثي، كما هو الحال. وقد تم بالفعل طرد الكثير في مثل هذه الظروف. لا يمكن السكوت عن هذه الممارسة المرعبة. يجب فقط على الأشخاص المسؤولين عن البحث أعتماد مصداقية المشاركة فيه.
لا يجوز لأي عالم أن يدعو شخص آخر إلى إدراج أسمه ولا يجوز تضمين اسماء باعتبارهم مؤلفون مشاركون في ورقة علمية اذا لم يسهموا إسهاما كبيرا في الدراسة التى نشرت. لا يجوز لأي عالم أو عالمة السماح بإكراه نفسه او نفسها - من قبل ممثل أي مؤسسة أكاديمية أو تعاونية أو وكالة حكومية أو أي شخص آخر- بأن تدرج أسمائهم كمؤلف مشترك للبحوث التي لم يساهم فيها بشكل كبير ولا يجوز لأي عالم السماح بأستخدام اسمهم كمؤلف مشارك في مقابل أي هدايا أو أى رشاوى أخرى. لا يجوز لأي شخص التحريض أو محاولة التحريض لإقناع العالم بأى طريقة كانت للسماح بتضمين اسم باحث ومؤلف كمشارك في ورقة علمية تتعلق بالمواضيع لم يسهم فيها الاخير بشكل ملحوظ.

المادة ١٠ : الاستقلال عن الانتماء
يعمل الآن كثير من العلماء في إطار عقود قصيرة الأجل و مع انتهاء عقد العمل ينتهي الانتماء الأكاديمي لهذا العالم. و غالبا ما تكون سياسة هيئات التحرير أن لا تنشر بحوثا للأشخاص بدون انتماء أكاديمي أو تجاري. في غياب الانتماء فأن كثيرة من الموارد ليست متاحة للعالم ويتم تقليص فرصه لتقديم ورقات البحث في المناقشات والمؤتمرات. هذا الممارسة مفرغة و يجب أن تتوقف فالعلم لا يعترف بالانتماء. لا يجوز منع أي عالم من تقديم الأوراق البحثية في المؤتمرات أوالندوات أو الحلقات الدراسية أونشرها في أي من وسائل الإعلام، لا يجوز منع أي عالم من الوصول إلى المكتبات الأكاديمية أو المنشورات العلمية ولا يجوز منعه من حضور الاجتماعات العلمية أو من إعطاء المحاضرات لعدم وجود انتماء مع المؤسسة الأكاديمية أو المعاهد العلمية أوالحكومة أو المختبرات التجارية أو أي منظمة أخرى.

المادة ١١ : الحصول على المعلومات العلمية
كثير من الكتب المتخصصة في الأمور العلمية وكذلك المجلات العلمية تدر أرباح ضئيلة أو معدومة بحيث يكون الناشرين التجاريين غير راغبين في نشرها بدون مساهمة مالية من المؤسسات الأكاديمية أوالهيئات الحكومية أوالمؤسسات الخيرية وما شابه ذلك. في ظل هذه الظروف ينبغي على الناشرين التجاريين السماح بحرية الوصول إلى النسخ الالكترونية من المنشورات و السعى جاهدين للحفاظ على تكلفة المواد المطبوعة إلى الحد الأدنى.

على جميع العلماء الجهاد لضمان أن الأبحاث الخاصة بهم متاحة للمجتمع العلمي الدولي مجانا أو بدلا من ذلك - إذا كان لا يمكن تجنب عدم المجانية - أن تكون بأقل تكلفة. كما ينبغي على جميع العلماء اتخاذ تدابير فعالة لجعل كتبهم التقنية متاحة بأقل تكلفة ممكنة بحيث يمكن للمعلومات العلمية أن تكون متاحة على اوسع نطاق للمجتمع العلمي الدولي.

المادة ١٢ : المسؤولية الأخلاقية للعلماء
يشهد التاريخ ان الاكتشافات العلمية تستخدم لاغراض الخير و اغراض الشر على نحو سواء وأيضا لمنفعة البعض وضرر للآخرين. ولأن تقدم العلم والتكنولوجيا لا يمكن أن يتوقف فإن بعض وسائل احتواء التطبيقات المضرة ينبغي أن تنشأ. كما أن الحكومات المنتخبة ديمقراطيا فقط والخالية من التحيز الديني والعرقي وغيرها هى التى يمكنها الحفاظ على الحضارة فإن الإدارات المنتخبة ديمقراطيا ولجان التحكيم فقط يمكنها حماية حق حرية صناعة العلم. و اليوم دول مختلفة ربما غير ديمقراطية أو أنظمة شمولية تقوم بإجراء البحوث النشطة في الفيزياء النووية والكيمياء وعلم الفيروسات والهندسة الوراثية وما إلى ذلك من أجل إنتاج أسلحة نووية وكيماوية وبيولوجية. لا ينبغي لعالم أن يتعاون طوعا مع الدول غير الديمقراطية أو النظم الشمولية. وينبغي على أي عالم أُكره للعمل على تطوير الأسلحة إيجاد طرق أو وسائل ليبطئ التقدم المحرز في برامج البحوث والإنتاج العلمي للحد منه لكى تكون الحضارة والديمقراطية هى السائدة في نهاية المطاف.
يتحمل كل العلماء المسؤولية الأخلاقية لإبداعاتهم العلمية واكتشافاتهم. لا يجوز لأي عالم الانخراط طوعا في تصميم أو بناء أسلحة من أي نوع على الإطلاق وذلك لدول غير ديمقراطية أو الأنظمة الشمولية أو السماح بتطبيق المهارات العلمية له أو لها والمعرفة لتطوير أي شيء ضارة بالبشرية. للعالم أن يعيش تحت شعار أن كل حكومة غير ديمقراطية وانتهاك لحقوق الإنسان هو جريمة.

كتبه : ديمتري رابونسكى ،
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LETTERS TO PROGRESS IN PHYSICS

Scientists Deduced the Existence of Particles with Faster-than-Light Speeds Recently Discovered by CERN

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In this paper we present a short survey on Smarandache Hypothesis that there is no speed barrier in the universe and one can construct arbitrary speeds, hypothesis which has been partially confirmed by the recent CERN results of OPERA team led by Antonio Ereditato that experimentally found that neutrino particles travel faster than *c*.

Physicists at CERN have recently experimentally discovered particles traveling faster than light: the neutrinos! The OPERA experiment, which sent sprays of neutrinos from CERN, Geneva, Switzerland, to INFN Gran Sasso Laboratory, Italy, found that neutrinos traveled underground 730 km faster than light could do. Dr. Antonio Ereditato of University of Bern, leader of the OPERA scientist team, made the results public and invited scientists all over the world to discuss these astonishing results.

There are mediums where the light travels slower than some particles, for example in water and oil, but not in the vacuum. There also exist superluminal phenomena like wave phase velocity and wave group velocity, but in these cases no information or energy travels faster than the light. Similarly are the X-waves whose superluminal velocity of the peak is a transitory phenomenon, but their wavefronts move with the speed *c* [1].

In the breaking News on September 22, 2011, in the Live Science.com, it is said that proven true, the laws of physics have to be re-written [2].

Professor Florentin Smarandache from the University of New Mexico, United States, has deduced the existence of particles moving faster-than-light in a published paper called "There Is No Speed Barrier in the Universe" in 1998 [3], as an extension of a 1972 manuscript [4] that he also presented in 1993 at the conference *Paradoxism in Literature and Science* held in the Universidad de Blumenau, Brazil. His paper is based on the Einstein-Podolsky-Rosen Paradox [5], a Bohm's paper [6], and Bell's Inequalities [7]. For this goal known as *Smarandache Hypothesis*, and for his neutrosophic logic, set, and probability (which are the most general and powerful logic, and, respectively, set and probability theories today), Prof. Smarandache awarded the Telesio-Galilei Academy Gold Medal in 2010. Smarandache Hypothesis is also included in the Weinstein's *Encyclopedia of Physics* [8]. It is is enounced as follows:

- Suppose a certain physical process produces a pair of entangled particles A and B (having opposite or complementary characteristics), which fly off into space in the opposite direction and, when they are billions of miles apart, one measures particle A; because B is the opposite, the act of measuring A instantaneously tells B what to be; therefore those instructions would somehow have to traveled between A and B faster than the speed of light; hence, one can extend the Einstein-Podolsky-Rosen paradox and Bell's inequalities and assert that the light speed is not a speed barrier in the universe;
- Even more, one can construct any speed, even greater than the speed of light, *c*, by measuring particle A at various time intervals;
- Also, the information from particles A and B is transmitted instantaneously (thus, there is no speed barrier in the universe).

Although superluminal phenomena are in contradiction with Einstein's theory of Special Relativity (1905) that prevents energy, information and (real) mass from traveling faster than light, Smarandache (1972) considered that superluminal phenomena do not violate Causality Principle, neither produce time traveling, nor necessitating infinite energy for particles traveling at speeds greater than the speed of light.

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