# Replacing the Conjunctive Rule and Disjunctive Rule with T-norms and T-conorms respectively (Tchamova-Smarandache)

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A **T-norm** is a function $T_n$: $[0, 1]^2 \to [0, 1]$, defined in fuzzy/neutrosophic set theory and fuzzy/neutrosophic logic to represent the "intersection" of two fuzzy/neutrosophic sets and the fuzzy/neutrosophic logical operator "and" respectively. Extended to the fusion theory the T-norm will be a substitute for the conjunctive rule.
The T-norm satisfies the conditions:
a) Boundary Conditions: $T_n(0, 0) = 0$, $T_n(x, 1) = x$.
b) Commutativity: $T_n(x, y) = T_n(y, x)$.
c) Monotonicity: If $x \leq u$ and $y \leq v$, then $T_n(x, y) \leq T_n(u, v)$.
d) Associativity: $T_n(T_n(x, y), z) = T_n(x, T_n(y, z))$.
There are many functions which satisfy the T-norm conditions. We present below the most known ones:
The Algebraic Product T-norm:
$T_{n\text{-algebraic}}(x, y) = x \cdot y$
The Bounded T-norm:
$T_{n\text{-bounded}}(x, y) = \max\{0, x+y-1\}$
The Default (min) T-norm (introduced by Zadeh):
$T_{n\text{-min}}(x, y) = \min\{x, y\}$.

Min rule can be interpreted as an optimistic lower bound for combination of bba and the below Max rule as a prudent/pessimistic upper bound. (Jean Dezert)

A **T-conorm** is a function $T_c$: $[0, 1]^2 \to [0, 1]$, defined in fuzzy/neutrosophic set theory and fuzzy/neutrosophic logic to represent the "union" of two fuzzy/neutrosophic sets and the fuzzy/neutrosophic logical operator "or" respectively. Extended to the fusion theory the T-conorm will be a substitute for the disjunctive rule.
The T-conorm satisfies the conditions:
a) Boundary Conditions: $T_c(1, 1) = 1$, $T_c(x, 0) = x$.
b) Commutativity: $T_c(x, y) = T_c(y, x)$.
c) Monotonicity: if $x \leq u$ and $y \leq v$, then $T_c(x, y) \leq T_c(u, v)$.
d) Associativity: $T_c(T_c(x, y), z) = T_c(x, T_c(y, z))$.

There are many functions which satisfy the T-conorm conditions. We present below the most known ones:
The Algebraic Product T-conorm:

$T_{c\text{-algebraic}}(x, y) = x+y-x\cdot y$

The Bounded T-conorm:

$T_{c\text{-bounded}}(x, y) = \min\{1, x+y\}$

The Default (max) T-conorm (introduced by Zadeh):

$T_{c\text{-max}}(x, y) = \max\{x, y\}$.

Then, the T-norm Fusion rule is defined as follows:

$$m_{\cap 12}(A) = \sum_{\substack{X,Y\in\Theta \\ X\cap Y=A}} Tn(m1(X), m2(Y))$$

and the T-conorm Fusion rule is defined as follows:

$$m_{\cup 12}(A) = \sum_{\substack{X,Y\in\Theta \\ X\cup Y=A}} Tc(m1(X), m2(Y))$$

The T-norms/conorms are commutative, associative, isotone, and have a neutral element.