A recurrent formula inspired by Rowland's formula and based on Smarandache function which might be a criterion for primality

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. Studying the two well known recurrent relations with the exceptional property that they generate only values which are equal to 1 or are primes, id est the formula which belongs to Eric Rowland and the one that belongs to Benoit Cloitre, I managed to discover a formula based on Smarandache function, from the same family of recurrent relations, which, instead to give a prime value for any input, seems to give the same value, 2, if and only if the value of the input is a prime. I name this relation the Coman-Smarandache criterion for primality and the exceptions from this rule, if they exist, Coman-Smarandache pseudoprimes.

Conjecture

Let f(1) = 1 and f(n) = S(f(n – 1)) + lcm[n, S(f(n – 1))], where S is the Smarandache function and lcm the least common multiple. Then the value of the function g(n) = f(n)/S(f(n – 1)) is equal to 2 if and only if n is an odd prime.

Verifying the conjecture
(up to n = 17)

: f(2) = 1 + lcm[2, 1] = 3; then g(2) = 3/1 = 3;
: **f(3)** = 3 + lcm[3, 3] = 6; then g(3) = 6/3 = **2**;
: f(4) = 3 + lcm[4, 3] = 15; then g(4) = 15/3 = 5;
: **f(5)** = 5 + lcm[5, 5] = 10; then g(5) = 10/5 = **2**;
: f(6) = 5 + lcm[6, 5] = 35; then g(6) = 35/5 = 7;
: **f(7)** = 7 + lcm[7, 7] = 14; then g(7) = 14/7 = **2**;
: f(8) = 7 + lcm[8, 7] = 63; then g(8) = 63/7 = 9;
: f(9) = 7 + lcm[9, 7] = 70; then g(9) = 70/7 = 10;
: f(10) = 7 + lcm[10, 7] = 77; then g(10) = 77/7 = 11;
: **f(11)** = 11 + lcm[11, 11] = 22; then g(11) = 22/11 = **2**;
: f(12) = 11 + lcm[12, 11] = 143; then g(12) = 143/11 = 13;
: **f(13)** = 13 + lcm[13, 13] = 26; then g(13) = 26/13 = **2**;
: f(14) = 13 + lcm[14, 13] = 195; then g(14) = 195/13 = 15;
: f(15) = 13 + lcm[15, 13] = 208; then g(15) = 208/13 = 16;
: f(16) = 13 + lcm[16, 13] = 221; then g(16) = 221/13 = 17;
: **f(17)** = 17 + lcm[17, 17] = 17; then g(17) = 34/17 = **2**.

Note

It can be seen that, in the verified cases, the value of g(n) is equal to 2 if and only if n is odd prime; the value of g(n) in any other case (for any other n) beside f(1) = 1 and f(p) = 2, where p is odd prime, is equal to n + 1.

Note

The function g(n) = f(n)/S(f(n – 1)) – 1, where f(n) = f(n – 1) + lcm[n, f(n – 1)] might also be interesting to study as a prime generating formula, as it gives prime values (i.e. 5, 17, 23, 191, 383) for the following consecutive values of n: 4, 5, 6, 7, 8; however, for n = 9 the value obtained is a semiprime and for n = 10 is not even obtained an integer value, because m is not always divisible by S(m) so f(n), which is always divisible by f(n – 1), is not always divisible by S(f(n – 1)).

References:

1. Rowland, Eric, *A simple prime-generating recurrence*;
2. Peterson, Ivars, *A new formula for generating primes*;
3. Shevelev, Vladimir, *Generalizations of the Rowland Theorem*.