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AN INEQUALITY CONCERNING THE SMARANDACHE FUNCTION

Maohua Le
Zhanjiang Normal College, Zhanjang, Guangdong, P.R.China

Abstract. For any positive integer n, let S(n) denote the Smarandache function of n. In this paper we prove that $S(mn) \le S(m)+S(n)$.

Let N be the set of all positive integers. For any positive integer n, let S(n) denote the Smarandache function of n. By [2], we have

(1) $$S(n)=\min\{k \mid k \in N,\ n \mid k!\}.$$

Recently, Jozsef[1] proved that

(2) $$S(mn) \le mS(n),\quad m,n \in N.$$

In this paper we give a considerable improvement for the upper bound (2). We prove the following result.

Theorem. For any positive integers m,n, we have
$$S(mn) \le S(m)+S(n).$$

Proof. Let a=S(m) and b=S(n). Then we have

(3) $$n \mid b!,$$

by (1). Let x be a positive integer with $x \ge a$, and let

(4) $$\binom{x}{a} = \frac{x(x-1)\dots(x-a+1)}{a!}$$

be a binomial coeficient. It is a well known fact that $\binom{x}{a}$ is a positive integer. So we have

(5) $$a! \mid x(x-1)\dots(x-a+1),$$

by (4). Further, since $m \mid a!$, we get from (5) that

(6) $$m \mid x(x-1)\dots(x-a+1),$$

for any positive integer x with $x \ge a$. Put x=a+b. We see from (3) and (6) that

(7) $$mn \mid b!(b+1)\dots(b+a)=(a+b)!.$$

Thus we get from (7) that $S(mn) \leq a+b=S(m)+S(n)$. The theorem is proved.

References

1. S.Jozsef, On certain inegualites involving the Smarandache function, Smarandache Notce J. 7(1996), No.1-3, 3-6.
2. F.Smarandache "A function in the number theory"," Smarandache Function J". 1(1990), No.1, 3-17.