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Palindromic Numbers And Iterations of the Pseudo-Smarandache Function

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In his delightful little book[1] Kenichiro Kashara introduced the Pseudo-Smarandache function.

Definition: For any $n \geq 1$, the value of the Pseudo-Smarandache function Z(n) is the smallest integer m such that n evenly divides

$$\sum_{k=1}^{m} k.$$

And it is well-known that the sum is equivalent to $\frac{m(m+1)}{2}$.

Having been defined only recently, many of the properties of this function remain to be discovered. In this short paper, we will tentatively explore the connections between Z(n) and a subset of the integers known as the palindromic numbers.

Definition: A number is said to be a palindrome if it reads the same forwards and backwards. Examples of palindromes are

121, 34566543, 1111111111

There are some palindromic numbers n such that Z(n) is also palindromic. For example,

Z(909) = 404   Z(2222) = 1111

In this paper, we will not consider the trivial cases of the single digit numbers.

A simple computer program was used to search for values of n satisfying the above criteria. The range of the search was, $10 \leq n \leq 10000$. Of the 189 palindromic values of n within that range, 37, or slightly over 19%, satisfied the criteria.

Furthermore it is sometimes possible to repeat the function again and get another palindrome.

Z(909) = 404, Z(404) = 303

and once again, a computer program was run looking for values of n within the range

$1 \leq n \leq 10{,}000$. Of the 37 values found in the previous test, 9 or slightly over 24%, exhibited the above properties.

Repeating the program again, looking for values of n such that n, Z(n), Z(Z(n)) and Z(Z(Z(n))) are all palindromic, we find that of the 9 found in the previous test, 2 or roughly 22%, satisfy the new criteria.

Definition: Let $Z^k(n) = Z(Z(Z(\ldots(n))))$ where the Z function is executed k times. For notational purposes, let $Z^0(n) = n$.

Modifying the computer program to search for solutions for a value of n so that n and all iterations $Z^i(n)$ are palindromic for i = 1, 2, 3 and 4, we find that there are no solutions in the range $1 \leq n \leq 10{,}000$. Given the percentages already encountered, this should not be a surprise. In fact, by expanding the search up through 100,000 one solution was found.

Z(86868) = 17271, Z(17271) = 2222, Z(2222) = 1111, Z(1111) = 505

Since Z(505) = 100, this is the largest such sequence in this region.

Computer searches for larger such sequences can be more efficiently carried out by using only palindromic numbers for n.

Unsolved Question: What is the largest value of m so that for some n, $Z^k(n)$ is a palindrome for all $k = 0, 1, 2, \ldots, m$?

Unsolved Question: Do the percentages discussed previously accurately represent the general case?

Of course, an affirmative answer to the second question would mean that there is no largest value of m in the first.

Conjecture: There is no largest value of m such that for some n, $Z^k(n)$ is a palindrome for all $k = 0, 1, 2, 3, \ldots, m$.

There are solid arguments in support of the truth of this conjecture. Palindromes tend to be divisible by palindromic numbers, so if we take n palindromic, many of the numbers that it divides would also be palindromic. And that palindrome is often the product of two numbers, one of which is a different palindrome. Numbers like the repunits, 11. . . 111 and those with only a small number of different digits, like 1001 and 505 appeared quite regularly in the computer search.

Reference

1. K. Kashihara, **Comments and Topics on Smarandache Notions and Problems**, Erhus University Press, Vail, AZ., 1996.