Sebastián Martín Ruiz

Applications of Smarandache Function, and Prime and Coprime Functions

$$C_k(n_1, n_2, \cdots, n_k) = \begin{cases} 0 & if \quad n_1, n_2, \cdots, n_k \quad are \quad coprime \quad numbers \\ 1 & \quad otherwise \end{cases}$$

American Research Press
Rehoboth
2002

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http://www.gallup.unm.edu/~smarandache/math.htm

ISBN: 1-931233-30-6

Standard Address Number 297-5092
Printed in the United States of America

Contents:

Chapter 1: Smarandache function applied to perfect numbers

The Smarandache function is defined as follows:

S(n)= the smallest positive integer such that S(n)! is divisible by n. [1]

In this article we are going to see that the value this function takes when n is a perfect number of the form $n = 2^{k-1} \cdot (2^k - 1)$, $p = 2^k - 1$ being a prime number.

Lemma 1: Let $n = 2^i \cdot p$ when p is an odd prime number and i an integer such that:

$$0 \le i \le E\left(\frac{p}{2}\right) + E\left(\frac{p}{2^2}\right) + E\left(\frac{p}{2^3}\right) + \cdots + E\left(\frac{p}{2^{E(\log_2 p)}}\right) = e_2(p!)$$

where $e_2(p!)$ is the exponent of 2 in the prime number decomposition of $p!$.
E(x) is the greatest integer less than or equal to x.

One has that $S(n) = p$.

Demonstration:
Given that $GCD(2^i, p) = 1$ (GCD= greatest common divisor) one has that $S(n) = \max\{S(2^i), S(p)\} \ge S(p) = p.$ Therefore $S(n) \ge p.$
If we prove that p! is divisible by n then one would have the equality.

$$p! = p_1^{e_{p1}(p!)} \cdot p_2^{e_{p2}(p!)} \cdots p_s^{e_{ps}(p!)}$$

where p_i is the $i-th$ prime of the prime number decomposition of $p!$. It is clear that
$p_1 = 2,\ \ p_s = p,\ \ e_{p_s}(p!) = 1$ for which:

$$p! = 2^{e_2(p!)} \cdot p_2^{e_{p2}(p!)} \cdots p_{s-1}^{e_{ps-1}(p!)} \cdot p$$

From where one can deduce that:

$$\frac{p!}{n} = 2^{e_2(p!)-i} \cdot p_2^{e_{p2}(p!)} \cdots p_{s-1}^{e_{p_{s-1}}(p!)}$$

is a positive integer since $e_2(p!) - i \geq 0$.

Therefore one has that $S(n) = p$

Proposition1: If n is a perfect number of the form $n = 2^{k-1} \cdot (2^k - 1)$ with k is a positive integer, $2^k - 1 = p$ prime, one has that $S(n) = p$.

Demonstration:

For the Lemma it is sufficient to prove that $k - 1 \leq e_2(p!)$.
If we can prove that:

$$k - 1 \leq 2^{k-1} - \frac{1}{2} \qquad (1)$$

we will have proof of the proposition since:

$$k - 1 \leq 2^{k-1} - \frac{1}{2} = \frac{2^k - 1}{2} = \frac{p}{2}$$

As $k - 1$ is an integer one has that $k - 1 \leq E\left(\frac{p}{2}\right) \leq e_2(p!)$

Proving (1) is the same as proving $k \leq 2^{k-1} + \frac{1}{2}$ at the same time, since k is integer, is equivalent to proving $k \leq 2^{k-1}$ (2).

In order to prove (2) we may consider the function: $f(x) = 2^{x-1} - x$ x real number.

This function may be derived and its derivate is $f'(x) = 2^{x-1} \ln 2 - 1$.

f will be increasing when $2^{x-1} \ln 2 - 1 > 0$ resolving x:

$$x > 1 - \frac{\ln(\ln 2)}{\ln 2} \cong 1'5287$$

In particular f will be increasing $\forall\ x \geq 2$.

Therefore $\forall\ x \geq 2$ $f(x) \geq f(2) = 0$ that is to say $2^{x-1} - x \geq 0$ $\forall x \geq 2$.

Therefore: $2^{k-1} \geq k \ \forall k \geq 2$ integer.

And thus is proved the proposition.

EXAMPLES:

$6 = 2 \cdot 3$	S(6)=3
$28 = 2^2 \cdot 7$	S(28)=7
$496 = 2^4 \cdot 31$	S(496)=31
8128= $2^6 \cdot 127$	S(8128)=127

References:

[1] C. Dumitrescu and R. Müller: To Enjoy is a Permanent Component of Mathematics. SMARANDACHE NOTIONS JOURNAL Vol. 9 No 1-2, (1998) pp 21-26

Chapter 2: A result obtained using the Smarandache Function

Smarandache Function is defined as followed:
S(m)=The smallest positive integer so that S(m)! is divisible by m. [1]
Let's see the value which such function takes for $m = p^{p^n}$ with n integer, $n \geq 2$ and p prime number. To do so a Lemma required.

Lemma 1 $\forall \quad m, n \in \mathrm{N} \quad m, n \geq 2$

$$m^n = E\left[\frac{m^{n+1} - m^n + m}{m}\right] + E\left[\frac{m^{n+1} - m^n + m}{m^2}\right] + \cdots + E\left[\frac{m^{n+1} - m^n + m}{m^{E\left[\log_m\left(m^{n+1} - m^n + m\right)\right]}}\right]$$

where E(x) gives the greatest integer less than or equal to x.

Proof:

Let's see in the first place the value taken by $E\left[\log_m\left(m^{n+1} - m^n + m\right)\right]$.
If $n \geq 2$: $m^{n+1} - m^n + m < m^{n+1}$ and therefore
$\log_m\left(m^{n+1} - m^n + m\right) < \log_m m^{n+1} = n + 1$.
And if $m \geq 2$:
$mm^n \geq 2m^n \Rightarrow m^{n+1} \geq 2m^n \Rightarrow m^{n+1} + m \geq 2m^n \Rightarrow m^{n+1} - m^n + m \geq m^n$
$\Rightarrow \log_m\left(m^{n+1} - m^n + m\right) \geq \log_m m^n = n \Rightarrow E\left[\log_m\left(m^{n+1} - m^n + m\right)\right] \geq n$

As a result: $n \leq E\left[\log_m\left(m^{n+1} - m^n + m\right)\right] < n + 1$ therefore:

$$E\left[\log_m\left(m^{n+1} - m^n + m\right)\right] = n \quad if \;\; n, m \geq 2$$

Now let's see the value which it takes for $1 \leq k \leq n$: $E\left[\frac{m^{n+1} - m^n + m}{m^k}\right]$

$$E\left[\frac{m^{n+1} - m^n + m}{m^k}\right] = E\left[m^{n+1-k} - m^{n-k} + \frac{1}{m^{k-1}}\right]$$

If k=1: $E\left[\dfrac{m^{n+1}-m^n+m}{m^k}\right]=m^n-m^{n-1}+1$

If $1<k\le n$: $E\left[\dfrac{m^{n+1}-m^n+m}{m^k}\right]=m^{n+1-k}-m^{n-k}$

Let's see what is the value of the sum:

k=1 $\quad m^n \quad -m^{n-1} \quad \dots \quad \dots \quad \dots \quad \dots \quad +1$

k=2 $\quad m^{n-1} \quad -m^{n-2}$

k=3 $\quad m^{n-2} \quad -m^{n-3}$

.
.
.

k=n-1 $\quad m^2 \quad -m$

k=n $\quad m \quad -1$

Therefore:

$$\sum_{k=1}^{n} E\left[\frac{m^{n+1}-m^n+m}{m^k}\right]=m^n \qquad m,n\ge 2$$

<u>Proposition</u>: $\forall \quad p$ *prime number* $\forall n\ge 2$:

$$S(p^{p^n})=p^{n+1}-p^n+p$$

Proof:

Having $e_p(k)=$exponent of the prime number p in the prime decomposition of k.

We get:

$$e_p(k)=E\left(\frac{k}{p}\right)+E\left(\frac{k}{p^2}\right)+E\left(\frac{k}{p^3}\right)+\dots+E\left(\frac{k}{p^{E(\log_p k)}}\right)$$

And using the lemma we have

$$e_p\left[\left(p^{n+1}-p^n+p\right)!\right]=E\left[\frac{p^{n+1}-p^n+p}{p}\right]+E\left[\frac{p^{n+1}-p^n+p}{p^2}\right]+\cdots+E\left[\frac{p^{n+1}-p^n+p}{p^{E\left[\log_p\left(p^{n+1}-p^n+p\right)\right]}}\right]=p^n$$

Therefore:

$$\frac{\left(p^{n+1}-p^n+p\right)!}{p^{p^n}}\in \mathrm{N} \quad \textit{and} \quad \frac{\left(p^{n+1}-p^n+p-1\right)!}{p^{p^n}}\notin \mathrm{N}$$

And :

$$S\left(p^{p^n}\right)=p^{n+1}-p^n+p$$

References:

[1] C. Dumitrescu and R. Müller: To Enjoy is a Permanent Component of Mathematics. SMARANDACHE NOTIONS JOURNAL VOL 9:, No. 1-2 (1998) pp 21-26.

Chapter 3: A Congruence with the Smarandache function

Smarandache's function is defined thus:

S(n)= is the smallest integer such that S(n)! is divisible by n. [1]

In this article we are going to look at the value that has $S(2^k - 1) \pmod{k}$

For all integer, $2 \le k \le 97$.

k	$S(2^k-1)$	$S(2^k-1) \pmod{k}$
2	3	1
3	7	1
4	5	1
5	31	1
6	7	1
7	127	1
8	17	1
9	73	1
10	31	1
11	89	1
12	13	1
13	8191	1
14	127	1
15	151	1
16	257	1
17	131071	1
18	73	1
19	524287	1
20	41	1
21	337	1
22	683	1
23	178481	1
24	241	1
25	1801	1
26	8191	1
27	262657	1
28	127	15
29	2089	1
30	331	1

k	$S(2^k-1)$	$S(2^k-1)$ (mod k)
31	2147483647	1
32	65537	1
33	599479	1
34	131071	1
35	122921	1
36	109	1
37	616318177	1
38	524287	1
39	121369	1
40	61681	1
41	164511353	1
42	5419	1
43	2099863	1
44	2113	1
45	23311	1
46	2796203	1
47	13264529	1
48	673	1
49	4432676798593	1
50	4051	1
51	131071	1
52	8191	27
53	20394401	1
54	262657	1
55	201961	1
56	15790321	1
57	1212847	1
58	3033169	1
59	3203431780337	1
60	1321	1
61	2305843009213693951	1
62	2147483647	1
63	649657	1
64	6700417	1
65	145295143558111	1
66	599479	1
67	761838257287	1
68	131071	35

k	$S(2^k-1)$	$S(2^k-1) \pmod k$
69	10052678938039	1
70	122921	1
71	212885833	1
72	38737	1
73	9361973132609	1
74	616318177	1
75	10567201	1
76	525313	1
77	581283643249112959	1
78	22366891	1
79	1113491139767	1
80	4278255361	1
81	97685839	1
82	8831418697	1
83	57912614113275649087721	1
84	14449	1
85	9520972806333758431	1
86	2932031007403	1
87	9857737155463	1
88	2931542417	1
89	618970019642690137449562111	1
90	18837001	1
91	23140471537	1
92	2796203	47
93	658812288653553079	1
94	165768537521	1
95	30327152671	1
96	22253377	1
97	13842607235828485645766393	1

One can see from the table that there are only 4 exceptions for $2 \le k \le 97$

We can see in detail the 4 exceptions in a table:

$k=28=2^2\circ 7$	$S(2^{28}-1)\equiv 15 \pmod{28}$
$k=52=2^2\circ 13$	$S(2^{52}-1)\equiv 27 \pmod{52}$
$k=68=2^2\circ 17$	$S(2^{68}-1)\equiv 35 \pmod{68}$
$k=92=2^2\circ 23$	$S(2^{92}-1)\equiv 47 \pmod{92}$

One can observe in these 4 cases that $k=2^2p$ with p is a prime and more over $S(2^k-1)\equiv \frac{k}{2}+1 \pmod{k}$

UNSOLVED QUESTION:

One can obtain a general formula that gives us, in function of k the value $S(2^k-1) \pmod{k}$ for all positive integer values of k?.

Reference:

[1] Smarandache Notions Journal, Vol. 9, No. 1-2, (1998), pp. 21-26.

Chapter 4: A functional recurrence to obtain the prime numbers using the Smarandache prime function.

Theorem: We are considering the function:

For n integer:

$$F(n) = n + 1 + \sum_{m=n+1}^{2n} \prod_{i=n+1}^{m} \left\lfloor - \left\lfloor - \frac{\sum_{j=1}^{i} \left(\left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor \right) - 2}{i} \right\rfloor \right\rfloor$$

one has: $p_{k+1} = F(p_k)$ for all $k \geq 1$ where $\{p_k\}_{k\geq 1}$ are the prime numbers and $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

Observe that the knowledge of p_{k+1} only depends on knowledge of p_k and the knowledge of the fore primes is unnecessary.

Proof:

Suppose that we have found a function $P(i)$ with the following property:

$$P(i) = \begin{cases} 1 \; if \;\; i \; is \; composite \\ 0 \; if \;\; i \; is \;\; prime \end{cases}$$

This function is called Smarandache prime function.(Ref.)

Consider the following product:

$$\prod_{i=p_k+1}^{m} P(i)$$

If $p_k < m < p_{k+1}$ $\prod_{i=p_k+1}^{m} P(i) = 1$ since $i : p_k + 1 \leq i \leq m$ are all composites.

If $m \geq p_{k+1}$ $\quad \prod_{i=p_k+1}^{m} P(i) = 0$ since $P(p_{k+1}) = 0$

Here is the sum:

$$\sum_{m=p_k+1}^{2p_k} \prod_{i=p_k+1}^{m} P(i) = \sum_{m=p_k+1}^{p_{k+1}-1} \prod_{i=p_k+1}^{m} P(i) + \sum_{m=p_{k+1}}^{2p_k} \prod_{i=p_k+1}^{m} P(i) = \sum_{m=p_k+1}^{p_{k+1}-1} 1 =$$

$$= p_{k+1} - 1 - (p_k + 1) + 1 = p_{k+1} - p_k - 1$$

The second sum is zero since all products have the factor $P(p_{k+1}) = 0$.

Therefore we have the following recurrence relation:

$$p_{k+1} = p_k + 1 + \sum_{m=p_k+1}^{2p_k} \prod_{i=p_k+1}^{m} P(i)$$

Let's now see we can find $P(i)$ with the asked property.

Consider:

$$\left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor = \begin{cases} 1 & si \quad j \mid i \\ 0 & si \quad j\, not \mid i \end{cases} \qquad j = 1,2,\cdots,i \quad i \geq 1$$

We deduce of this relation:

$$d(i) = \sum_{j=1}^{i} \left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor$$

where $d(i)$ is the number of divisors of i.

If i is prime $d(i)=2$ therefore:

$$-\left\lfloor -\frac{d(i)-2}{i} \right\rfloor = 0$$

If i is composite $d(i)>2$ therefore:

$$0<\frac{d(i)-2}{i}<1 \Rightarrow -\left\lfloor -\frac{d(i)-2}{i} \right\rfloor = 1$$

Therefore we have obtained the Smarandache Prime Function $P(i)$ which is:

$$P(i) = -\left\lfloor -\frac{\sum_{j=1}^{i}\left(\left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor\right) - 2}{i} \right\rfloor \qquad i \geq 2 \quad \text{integer}$$

With this, the theorem is already proved .

References:

[1] E. Burton, "Smarandache Prime and Coprime functions".
www.gallup.unm.edu/~Smarandache/primfnct.txt
[2]F. Smarandache, "Collected Papers", Vol II 200, p.p. 137,
Kishinev University Press, Kishinev, 1997.

Chapter 5: The general term of the prime number sequence and the Smarandache prime function.

Let is consider the function $d(i) =$ number of divisors of the positive integer number i . We have found the following expression for this function:

$$d(i) = \sum_{k=1}^{i} E\left(\frac{i}{k}\right) - E\left(\frac{i-1}{k}\right)$$

"E(x) =Floor[x]"

We proved this expression in the article "A functional recurrence to obtain the prime numbers using the Smarandache Prime Function".

We deduce that the folowing function:

$$G(i) = -E\left[-\frac{d(i)-2}{i}\right]$$

This function is called the Smarandache Prime Function (Reference)
It takes the next values:

$$G(i) = \begin{cases} 0 & if \quad i\ is \quad prime \\ 1 & if \quad i\ is \quad composite \end{cases}$$

Let is consider now $\pi(n) =$ number of prime numbers smaller or equal than n.
It is simple to prove that:

$$\pi(n) = \sum_{i=2}^{n} (1 - G(i))$$

Let is have too:

$$If \quad 1 \le k \le p_n - 1 \quad \Rightarrow \quad E\left(\frac{\pi(k)}{n}\right) = 0$$

$$If \quad C_n \ge k \ge p_n \quad \Rightarrow \quad E\left(\frac{\pi(k)}{n}\right) = 1$$

We will see what conditions have to carry C_n.

Therefore we have the following expression for p_n n-th prime number:

$$p_n = 1 + \sum_{k=1}^{C_n} (1 - E\left(\frac{\pi(k)}{n}\right))$$

If we obtain C_n that only depends on n , this expression will be the general term of the prime numbers sequence, since π is in function with G and G does with d(i) that is expressed in function with i too. Therefore the expression only depends on n.

Let is consider $C_n = 2(E(n \log n) + 1)$
Since $p_n \approx n \log n$ from of a certain n_0 it will be true that

$$(1) \quad p_n \le 2(E(n \log n) + 1)$$

If n_0 it is not too big, we can prove that the inequality is true for smaller or equal values than n_0 .

It is necessary to that:

$$E\left[\frac{\pi(2(E(n \log n) + 1))}{n}\right] = 1$$

If we check the inequality:

$$(2) \qquad \pi(2(E(n \log n) + 1)) < 2n$$

We will obtain that:

$$\frac{\pi(C_n)}{n} < 2 \Rightarrow E\left[\frac{\pi(C_n)}{n}\right] \leq 1 \quad ; C_n \geq p_n \Rightarrow E\left[\frac{\pi(C_n)}{n}\right] = 1$$

We can experimentaly check this last inequality saying that it checks for a lot of values and the difference tends to increase, wich makes to think that it is true for all n .

Therefore if we prove that the (1) and (2) inequalities are true for all n which seems to be very probable; we will have that the general term of the prime numbers sequence is:

$$p_n = 1 + \sum_{k=1}^{2(E(n\log n)+1)} \left[1 - E\left[\frac{\sum_{j=2}^{k} \left[1 + E\left[- \frac{\sum_{s=1}^{j} (E(j/s) - E((j-1)/s)) - 2}{j} \right] \right]}{n} \right] \right]$$

Reference:

[1] E. Burton, "Smarandache Prime and Coprime Functions"
Http://www.gallup.unm.edu/~Smarandache/primfnct.txt

[2] F. Smarandache, "Collected Papers", Vol. II, 200 p.,p.137, Kishinev University Press.

Chapter 6: Expressions of the Smarandache Coprime Function

Smarandache Coprime function is defined this way:

$$C_k(n_1, n_2, \cdots, n_k) = \begin{cases} 0 & if \quad n_1, n_2, \cdots, n_k \quad are \quad coprime \quad numbers \\ 1 & \quad otherwise \end{cases}$$

We see two expressions of the Smarandache Coprime Function for k=2.

EXPRESSION 1:

$$C_2(n_1, n_2) = -\left\lfloor -\frac{n_1 n_2 - lcm(n_1, n_2)}{n_1 n_2} \right\rfloor$$

$\lfloor x \rfloor$ =the biggest integer number smaller or equal than x.

If n_1, n_2 are coprime numbers:

$lcm(n_1, n_2) = n_1 n_2$ therefore: $C_2(n_1, n_2) = -\left\lfloor \frac{0}{n_1 n_2} \right\rfloor = 0$

If n_1, n_2 aren't coprime numbers:

$$lcm(n_1, n_2) < n_1 n_2 \Rightarrow 0 < \frac{n_1 n_2 - lcm(n_1, n_2)}{n_1 n_2} < 1 \Rightarrow C_2(n_1, n_2) = 1$$

EXPRESSION 2:

$$C_2(n_1, n_2) = 1 + \left\lfloor -\frac{\prod\limits_{\substack{d \mid n_1 \\ d > 1}} \prod\limits_{\substack{d' \mid n_2 \\ d' > 1}} |d - d'|}{\prod\limits_{d \mid n_1} \prod\limits_{d \mid n_2} (d + d')} \right\rfloor$$

If n_1, n_2 are coprime numbers then $d \neq d' \quad \forall d, d' \neq 1$

$$\Rightarrow 0 < \frac{\prod_{\substack{d|n_1 \\ d>1}} \prod_{\substack{d'|n_2 \\ d'>1}} |d - d'|}{\prod_{d|n_1} \prod_{d|n_2} (d + d')} < 1 \Rightarrow C_2(n_1, n_2) = 0$$

If n_1, n_2 aren't coprime numbers $\exists\, d = d' \quad d > 1, d' > 1 \Rightarrow C_2(n_1, n_2) = 1$

EXPRESSION 3:

Smarandache Coprime Function for $k \geq 2$:

$$C_k(n_1, n_2, \cdots, n_k) = -\left\lfloor \frac{1}{GCD(n_1, n_2, \cdots, n_k)} - 1 \right\rfloor$$

If $n_1, n_2, \cdots, n_k$ are coprime numbers:

$$GCD(n_1, n_2, \cdots, n_k) = 1 \Rightarrow C_k(n_1, n_2, \cdots, n_k) = 0$$

If $n_1, n_2, \cdots, n_k$ aren't coprime numbers: $GCD(n_1, n_2, \cdots, n_k) > 1$

$$0 < \frac{1}{GCD} < 1 \Rightarrow -\left\lfloor \frac{1}{GCD} - 1 \right\rfloor = 1 = C_k(n_1, n_2, \cdots, n_k)$$

References:

1. E. Burton, "Smarandache Prime and Coprime Function"
2. F. Smarandache, "Collected Papers", Vol II 22 p.p. 137,Kishinev University Press.

Chapter 7: New Prime Numbers

I have found some new prime numbers using the PROTH program of Yves Gallot.
This program in based on the following theorem:

Proth Theorem (1878):
Let $N = k \cdot 2^n + 1$ where $k < 2^n$. If there is an integer number a so that $a^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ therefore N is prime.

The Proth progam is a test for primality of greater numbers defined as $k \cdot b^n + 1$ or $k \cdot b^n - 1$. The program is made to look for numbers of less than 5.000000 digits and it is optimized for numbers of more than 1000 digits..

Using this Program, I have found the following prime numbers:

$3239 \cdot 2^{12345} + 1$	with 3720 digits	$a = 3, \quad a = 7$
$7551 \cdot 2^{12345} + 1$	with 3721 digits	$a = 5, \quad a = 7$
$7595 \cdot 2^{12345} + 1$	with 3721 digits	$a = 3, \quad a = 11$
$9363 \cdot 2^{12321} + 1$	with 3713 digits	$a = 5, \quad a = 7$

Since the exponents of the first three numbers are Smarandache number Sm(5)=12345 we can call this type of prime numbers, prime numbers of Smarandache .

Helped by the MATHEMATICA progam, I have also found new prime numbers which are a variant of prime numbers of Fermat. They are the following:

$$2^{2^n} \cdot 3^{2^n} - 2^{2^n} - 3^{2^n} \text{ for n=1, 4, 5, 7 .}$$

It is important to mention that for n=7 the number which is obtained has 100 digits.

Chris Nash has verified the values n=8 to n=20, this last one being a number of 815.951 digits, obtaining that they are all composite. All of them have a tiny factor except n=13.

References:

1. Micha Fleuren, "Smarandache Factors and Reverse Factors", Smarandache Notions Journal, Vol. 10, www.gallup.unm.edu/~smarandache/

2. Chris Caldwell, The Prime Pages, www.utm.edu/research/primes

A book for people who love numbers:
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