# A BASIC PROCedure to calculate S(p^i)

by John Sutton
16A Overland Rd.
Mumbles, SWANSEA
SA3 4LP, U. K.

Integer function of a single variable S%(N%)

S is the least integer such that S! is divisible by N.

Obviously for a prime S(p)=p since this is the least factorial to include p.

It is easy to see that for two primes p1>p2 S(p1*p2)=p1 since this factorial is necessary to include p1 and already includes p2. This generalizes to the product of any number of primes.

In fact it generalizes to the product of relatively prime numbers n1 and· n2. S(n1*n2)=Max(s(n1),S(n2)).

Therefore we can simplify the general case to:

S(∏ni^pi)=Max(S(ni^pi))

All we need now is a way of calculating S for powers of primes.

Start with the inverse problem: for a given factorial and a given prime what is the maximum power of the prime included ?

Consider p=2. All even numbers contribute a factor, all multiples of 4 contribute another, all multiples of 8 contribute yet another ...etc. So the answer is got by summing succesive DIV 2 results (DIV p in general).

Returning to the calculation of S. To do this for a single N would require factorisation of N first. A program to calculate S for all integers up to N can avoid this by doing powers of 2, then powers of 3 and their products with powers of 2 then powers of 5 etc. Calculating S for all powers of a prime up to a maximum is straightforward. A BASIC PROCedure is attached. The main program requires some care and I have not been able to finish in time.

```
  10REM TEST PROC TOCALC S(P^I) FOR VALUES UPTO N
  20:
  30:
  40:
  50:
  60INPUT"UP TO",N%
  70DIM SPP%(100)
  80DIM NPP%(100)
  90INPUT"WHICH PRIME",P%
100PROCSpp(P%,N%)
110FOR I%=0 TO 100
120PRINT SPP%(I%),NPP%(I%)
130NEXT I%
140GOTO 60
150END
160DEF PROCSpp(P%,N%)
170I%=1
180NPP%(0)=1
190SPP%(0)=1
200J%=1
210PJ%=0
220REPEAT
230PJ%=PJ%+P%
240X%=FNinvSpp(P%,PJ%)
250REPEAT
260SPP%(I%)=PJ%
270NPP%(I%)=P%*NPP%(I%-1)
280I%=I%+1
290UNTIL I%>X%
300J%=J%+1
310UNTIL NPP%(I%-1)>N%
320ENDPROC
330DEF FNinvSpp(P%,N%)
340LOCAL S%,T%
350S%=0
360T%=N%
370REPEAT
380T%=T% DIV P%
390S%=S%+T%
400UNTIL T%<=1
410=S%
```