A BRIEF HISTORY OF THE "SMARANDACHE FUNCTION"

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This function is originated from the exiled Romanian professor Florentin Smarandache. It is defined as follows:

For any non-null integers n, S(n) is the smallest integer such that (S(n))! is divisible by n.

The importance of the notion is that it characterizes a prime number, i.e.:

Let $p > 4$, then: p is prime if and only if $S(p) = p$.

Another properties:

If $(a,b) = 1$, then $S(ab) = \max\{ S(a), S(b) \}$;

and

For any non-null integers, $S(ab) \le S(a) + S(b)$.

{All three found and proved by the author in 1979 (see [3], 15, 12-13, 65).}

If $n > 1$, then $S(n)$ and n have a proper common divisor.

{Found and proved by student Prodănescu in 1993: as a lemma needed to solve the conjecture formulated by the author in 1979 that:

the equation $S(n) = S(n + 1)$ has no solutions

(see [3], 37, and [30]).}

Etc.

Also, an infinity of open/unsolved problems, involving this function, provoked mathematicians around the world to study it and its applications (computational mathematics, simulation, quantum theory, etc.).

Thus, the unsolved question:

$$\text{Calculate } \lim_{n\to\infty} \left[1 + \sum_{k=2}^{n} \frac{1}{S(k)} - \log S(n) \right], \quad \text{(see [3], 29)}$$

made by the author in 1979, has been separately proved by J. Thompson from USA in 1992 (see [18], 1), by Nigel Backhouse from United Kingdom in 1993 (see [25]), and by Pål Grønås from Norway in 1993 (see [51]) that this limit is equal to ∞.

The author wonderred if it's posible to approach the function (see [3], 1979, 25-6), but Ian Parberry expressed that one can immediately find an algorithm that computes S(n) in $O(n\log n/\log\log n)$ time (see [38], 1993).

Some unsolved (by now!) other problems stated by the author in 1979 (see [3], 27-30):

a) To find a general form of the continued fraction expansion of $S(n)/n$, for all $n \ge 2$.

b) What is the smallest k such that for any integer n at least one of the numbers $S(n)$, $S(n+1)$, ..., $S(n+k-1)$ is a perfect square?

c) To build the largest arithmetical progression a_1, a_2, ..., a_r for which their images by the function are also an arithmetical progression.

Etc.

In 1975 Smarandache was a student at the University of Craiova, and he was attracted by the Number Theory. He created and published a lot of proposed problems of mathematics in various scientific journals. He liked to play with the numbers ...
Thus, in 1980 his research paper "A Function in the Number Theory", based on a special representation of integers, was published (for the first time) in <Analele Universităţii Timişoara>, Seria Ştiinţe Matematice, Vol. 18, pp. 79-88,
and was reviewed in <Zentralblatt fur Mathematik>, 471.10004, 1982, by P. Kiss, and in the <Mathematical Reviews>, 83c:10008, 1983, by R. Meyer.

In 1988 he escaped from the Ceauşescu's dictatorship, spent almost two years in a political refugee camp in Turkey (Istambul and Ankara), and finally emigrated to the United States.

Articles, notes, quickies, comments, proposals related to the Smarandache Function were presented to international conferences within the Mathematical Association of America or the American Mathematical Society at the New Mexico State University (Las Cruces), New Mexico Tech. (Socorro), University of Arizona (Tucson), University of San Antonio, University of Victoria (Canada) etc. or published in <Octogon> (Sacele), <Gazeta Matematică> (Bucharest), <The Mathematical Spectrum> (UK), <Elemente der Mathematik> (Switzerland), <The Fibonacci Quarterly> (USA) etc.

In 1992 Dr. J. R. Sutton from United Kingdom designed a BASIC PROCedure to calculate S(n) for all powers of a prime number up to a maximum. (see [26])

Jim Duncan from United Kingdom computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. (see [17])

Also, John McCarthy from United Kingdom estimated that his machine would take several years to just calculate and store S(n) to disk for the entire range of n it can handle (0<n<2^32), and using the compression detailed in ncld9207.c at least 12 Gigabytes of disk space would be needed. It took about 3 hours for his program to work out that 3,303,302 pages (!) would be needed to list the full range of n and S(n). (see [15])

In 1993 Henry Ibstedt from Sweden used a dtk-computer with 486/33MHz processor in Borland's Turbo Basic and calculated S(n) for n upto 10^6 which took 2 hours and 50 minutes! (see [52])

A group of professors (V. Seleacu, C. Dumitrescu, L. Tuţescu, I. Pătrascu, M. Mocanu) and scientific students from the University of Craiova, having a weekly meeting, are doing research on the function and its applicability.

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