A NOTE ON $S(p^r)$

by Pål Grønås
Enges Gate 12
7500 STaøRDAL
Norway

Problem (0). If $\prod_{i=1}^{k} p_i^{r_i}$ is the prime factorization of n, then it is easy to verify that

$$S(n) = S(\prod_{i=1}^{k} p_i^{r_i}) = \max\{\, S(p_i^{r_i}) \,\}_{i=1}^{k}.$$

From this formula we see that it is essensial to determine $S(p^r)$, where p is a prime and r is a natural number.

Legendres formula states that

$$n! = \prod_{i=1}^{k} p_i^{\sum_{m=1}^{\infty}[n/p_i^m]}.$$

This formula gives us a lower and an upper bound for $S(p^r)$, namely

(1) $$(p-1)r+1 \;\leq\; S(p^r) \;\leq\; pr.$$

It also implies that p divides $S(p^r)$, which means that

$$S(p^r) = p(r-i) \text{ for a particular } 0 \leq i \leq [\tfrac{r-1}{p}].$$