Mike Mudge pays a return visit to the Florentin Smarandache Function.[1]

The originator of this function, Florentin Smarandache, an Eastern European mathematician, escaped from the country of his birth because the Communist authorities had prohibited the publication of his research papers and his participation in international congresses. After spending two years in a political refugee camp in Turkey, he emigrated to the United States.

Robert Muller of The Number Theory Publishing Company, PO Box 42561, Phoenix, Arizona 85080, USA, decided to publish a selection of his papers, mmencing with *The Smarandache Function Journal*, Vol 1, No 1, December 1990. ISSN 1053-4792.

PCW readers may have met this function before, in Numbers Count -112- July 1992, where a very encouraging response was generated. This article (February 1993) is complete in itself so don't worry if you have filed the July issue! It may be thought that those readers who attempted the previous problem-set will have an unfair advantage. However, it must be realised that no Numbers Count problems are completely original so previous work within a given subject area is always a possibility and the prize is awarded using 'suitable subjective criteria' anyway, *so please have a go and submit your results*, however trivial they may seem to yourself.

Definition For all non-null integers, n, the Smarandache Function, S(n), is defined to be the smallest integer such that (S(n))! (The Factorial Function with argument S(n).) is divisible by n. e.g. S(18) = 6 because 6! is divisible by 18 but 1!....5! are not.

Problem (0) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n upto a given n_{max}.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* n = $e p_1^{a_1} p_2^{a_2} p_3^{a_3} \ldots p_r^{a_r}$ where $e = \pm 1$, and $p_1, p_2, p_3 \ldots p_r$ denote the distinct prime factors of n and $a_1, a_2, a_3 \ldots a_r$ are their respective multiplicities.

NOTE S(n) is an even function, by which is meant S(-n) = S(n).

Problem (i) Using either graphical or finite difference technique (i.e. the construction of difference tables etc) or indeed anything else that comes to mind, address the following questions:

(a) Is there a closed expression (formula) for S(n)?

(b) Is there a good asymptotic expression for S(n)? (By which is meant a formula, which although never (in general) exact, becomes a better and better approximation to S(n) as n becomes larger and larger.)

Problem (ii) For a specified non-null integer m, under what conditions does S(n) divide the difference n - m?

Problem (iii) Investigate the possible integer solutions, (x,y,z) of $S(x^n) + S(y^n) = S(z^n)$ for any n greater than or equal to 1. e.g. examine the solution (5,7,2048) when n = 3.

(It can be proved that an infinity of solutions exist for any such n-value.) Compare with Fermat's Theorem re. $x^n + y^n = z^n$.

Problem (iv) Investigate the possibility of finding two integers n and k such that the LOGARITHM of $S(n^k)$ to the BASE $S(k^n)$ is an integer.

Problem (v) Recall that 'Gamma' defined as the limit as n tends to infinity of (1 + 1/2 + 1/3 + 1/4....+1/n - log(n)) exists, is known as Euler's Constant and is approximately 0.577.

Investigate the possible existence of 'Samma' defined as the limit as n tends to infinity of (1 + 1/S(2) + 1/S(3)....+1/S(n) - log(S(n)).

Problem (vi) Find the number of PARTITIONS of n as the sum of S(m) for $2 < m \leq n$. See *PCW* August 1989 and February 1990 for other problems involving PARTITIONS of n.

Review of 'Numbers Count -118- February 1993: a revisit to The Florentin Smarandache Function[2]

This produced a number of 'quite powerful' responses. As a note of related interest, the latest publication of Fl.Smarandache is 'A Numerical Function in Congruence Theory', *Libertas Mathematica* (American Romanian Academy of Arts and Science) vol 12, 1992, pp 181-185, Arlington, Texas.

Pal Gronas of Norway submitted theoretical results on both problems 0 & (v). However, the clear winner this month is a former regular respondent, now retired, Henry Ibstedt, Glimminge 2036, 280 60 Broby, Sweden. Henry used a dtk-computer with 486/33MHz processor in Borland's Turbo Basic. S(n) upto 10^6 took 2hr 50min. He completed a great deal of work on all problems except (vi): details of numerical results and conclusions available from Henry or myself to interested readers. What about problem (vi)?

---

1 Republished from <Personal Computer World>, No.118, 403, February 1993 (with the author permission), because some of the following research papers are referring to these open problems.

2 Republished from <Personal Computer World>, No.124, 495, August 1993 (with the author permission).