PROBLEM (1)

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Find a strictly increasing infinite series of integer numbers such that for any consecutive three of them the Smarandache Function is neither increasing nor decreasing.

*Find the largest strictly increasing series of integer numbers for which the Smarandache Function is strictly decreasing.

a) To solve the first part of this problem, we construct the following series:

p_3 , p_3+1, p_4 , p_4+1, ..., p_n , p_n+1, ...

where p_3 , p_4 , p_5 ... are the series of prime odd numbers 5, 7, 11 ...

Of course, $S(p_i) = p_i$ and $S(p_i+1) < p_i$, for any $i \geq 3$.

b) A way to look at this unsolved question is the following:
Because $S(p) = p$, for any prime number, we should get a large interval in between two prime numbers. A bigger chance is when p and q, the primes with that propriety, are very large (and $q \neq p + c$, where $c = 2, 4,$ or 6). In this case the series is finite. But this is not the optimum method!

The Smarandache Function is, generally speaking, increasing {we mean that for any positive integer k there is another integer $j > k$ such that $S(j) > S(k)$}. This property makes us to think that our series should be finite.

Calculating at random, for example, the series' width is at least seven, because:
for n = 43, 46, 57, 68, 70, 72, 120 then
S(n) = 43, 23, 19, 17, 10, 6, 5 respectively.
We are sure it's possible to find a larger series, but we worry if a maximum width does exist, and if this does: how much is it?
[Sorry, the author is not able to solve it!]

See: Mike Mudge, "The Smarandache Function" in the <Personal Computer World> journal, London, England, July 1992, page 420.