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A Generalization of the Inequality Cauchy-Bouniakovski-Schwarz

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Statement: Let us consider the real numbers $a_i^{(k)}$, $i \in \{1,2,...,n\}$, $k \in \{1,2,...,m\}$, with $m \geq 2$. Then:

$$\left(\sum_{i=1}^{n}\prod_{k=1}^{m} a_i^{(k)}\right)^2 \leq \prod_{k=1}^{m}\sum_{i=1}^{n}\left(a_i^{(k)}\right)^2 .$$

Proof:

One notes A the left member of the inequality and B the right member. One has:

$$A = \sum_{i=1}^{n}\left(a_i^{(1)}...a_i^{(m)}\right)^2 + 2\sum_{i=1}^{n-1}\sum_{k=i+1}^{n}\left(a_i^{(1)}...a_i^{(m)}\right)\left(a_k^{(1)}...a_k^{(m)}\right)$$

and

$$B = \sum_{(i_1,...,i_m)\in E}\left(a_{i_1}^{(1)}...a_{i_m}^{(m)}\right)^2,$$

where

$$E = \left\{(i_1,...,i_m)\,/\,i_k \in \{1,2,...,n\}, 1 \leq k \leq m\right\}.$$

From where:

$$B = \sum_{i=1}^{n}\left(a_i^{(1)}...a_i^{(m)}\right)^2 + \sum_{i=1}^{n-1}\sum_{k=i+1}^{n}\left[\left(a_i^{(1)}...a_i^{(m-1)}a_k^{(m)}\right)^2 + \left(a_k^{(1)}...a_k^{(m-1)}a_i^{(m)}\right)^2\right] +$$
$$+ \sum_{(i_1,...,i_m)\in E-(\Delta_E \cup L^m)}\left(a_{i_1}^{(1)}...a_{i_m}^{(m)}\right)^2$$

with

$$\Delta_E = \left\{\underbrace{(\gamma,...,\gamma}_{\text{m times}}\,/\,\gamma \in \{1,2,...,n\}\right\}$$

and

$$L = \left\{\underbrace{(\alpha,...,\alpha}_{\text{m-1 times}},\beta),\underbrace{(\beta,...,\beta}_{\text{m-1 times}},\alpha)\,/(\alpha,\beta) \in \{1,2,...,n\}^2 \text{ and } \alpha < \beta\right\}$$

Then

$$A - B = \sum_{i=1}^{n-1}\sum_{k=i+1}^{n}\left[-\left(a_i^{(1)}...a_i^{(m-1)}a_k^{(m)}\right)^2 - \left(a_k^{(1)}...a_k^{(m-1)}a_i^{(m)}\right)^2\right] -$$
$$- \sum_{(i_1,...,i_m)\in E-(\Delta_E \cup L)}\left(a_{i_1}^{(1)}...a_{i_m}^{(m)}\right)^2 \leq 0$$

Note: for $m = 2$ one obtains the inequality of Cauchy-Bouniakovski-Schwarz.