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A Generalization Regarding The Extremes of a Trigonometrique Function

In Florentin Smarandache: "Collected Papers", vol. I (second edition). Ann Arbor (USA): InfoLearnQuest, 2007.

After a passionate lecture of this book [1] (Mathematics plus literature!) I stopped at one of the problems explained here:

At page 121, the problem 2 asks to determine the maximum of expression:

$$E(x) = (9 + \cos^2 x)(6 + \sin^2 x).$$

Analogue, in G. M. 7/1981, page 280, problem 18820*.

Here, we'll present a generalization of these problems, and we'll give a simpler solving method, as follows:

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = (a_1 \sin^2 x + b_1)(a_2 \cos^2 x + b_2)$;
find the function's extreme values.

To solve it, we'll take into account that we have the following relation:

$$\cos^2 x = 1 - \sin^2 x,$$

and we'll note $\sin^2 x = y$. Thus $y \in [0,1]$.

The function becomes:

$$f(y) - (a_1 y + b_1)(-a_2 y + a_2 + b_2) = -a_1 a_2 y^2 + (a_1 a_2 + a_1 b_2 - a_2 b_1)y + b_1 a_2 + b_1 b_2,$$

where $y \in [0,1]$.

Therefore f is a parabola.

If $a_1 a_2 = 0$, the problem becomes banal.

If $a_1 a_2 > 0$, $f(y_{\max}) = \frac{-\Delta}{4a}$, $y_{\max} = \frac{-b}{2a}$ (*)

a) when $-\frac{b}{2a} \in [0,1]$, the values that we are looking for are those from (*), and

$$y_{\min} = \max\left\{-\frac{b}{2a} - 0, 1 + \frac{b}{2a}\right\}$$

b) when $-\frac{b}{2a} > 1$, we have $y_{\max} = 1$, $y_{\min} = 0$. (it is evident that $f_{\max} = f(y_{\max})$ and $f_{\min} = f(y_{\min})$)

c) when $-\frac{b}{2a} < 0$, we have $y_{\max} = 0$, $y_{\min} = 1$.

If $a_1 a_2 < 0$, the function admits a minimum for

$$y_{\min} = -\frac{b}{2a}, \; f_{\min} \frac{-\Delta}{4a} \text{ (on the real axes)} \qquad (**)$$

a) when $-\frac{b}{2a} \in [0,1]$, the looked after solutions are those from (**). And

$$y_{\max} = \max\left\{-\frac{b}{2a}, 1 + \frac{b}{2a}\right\}$$

b) when $-\frac{b}{2a} > 1$, we have $y_{max} = 0$, $y_{min} = 1$

c) when $-\frac{b}{2a} < 0$, we have $y_{max} = 1$, $y_{min} = 0$.

Maybe the cases presented look complicated and unjustifiable, but if you plot the parabola (or the line), then the reasoning is evident.

REFERENCE

[1] Viorel Gh. Vod - Surprize în matematica elementar - Editura Albatros, Bucure ti, 1981.