Application of New Absolute and Relative Conditioning Rules in Threat Assessment

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***Abstract*—This paper presents new absolute and relative conditioning rules as possible solution of multi-level conditioning in threat assessment problem.**

An example of application of these rules with respect to target observation threat model has been provided.

The paper also presents useful directions in order to manage the implemented multiple rules of conditioning in the real system.

I. Introduction

Contemporary Command & Control systems operate with multiple sensors in order to elaborate consistent and complete information required for decision making [1]. These systems, however, must face another very important requirement, which is cooperation with other information systems. Dealing with information of different processing levels is inevitable consequence of the imposed demands, and requires specific tools for fusion in order to take this diversity into account effectively.

As Threat Assessment is one of the most important tasks imposed on C2 systems [2], [3], these systems must be able to deal with information obtained from uncertain and even unreliable sources, where the quality measures are often subjective. For this reason Theory of Evidence seems to be an appropriate approach.

Theory of Evidence known as Dempster-Shafer Theory (*DST*) [4] does not make any distinction to fusion operations regarding uncertainty of the gathered information. So called Dempster's rule of combination has been used in order to combine strong evidences from reliable sources, as well as poor evidences from unreliable sources, and hybrid (strong evidence with poor evidence). For many years researchers have been inventing diverse combination rules as alternative to Dempster's rule [5], [6], and [7]. These rules are different from each other mainly in the way the conflicting mass (referring to contradicting hypotheses) is distributed. However, according to knowledge of the authors, none of these rules takes into account possible different processing levels of the integrated information, which cannot be expressed with *basic belief assignments*.

Theory of Evidence by Dezert and Smarandache (*DSmT*) [8] distinguishes two operations: *combination* and *conditioning* for fusion of uncertain information and integration of uncertain pieces of information with confirmed i.e. certain evidence respectively. Aware of this fact, a certain idea of using *conditioning* operation (as an alternative of *combination* [9]) for the purpose of multiple level fusion has been published [10]. However, as it was presented in [11], [12], each of these solutions has its drawbacks, and in general neither is preferable over the other. The main disadvantage of *combination* as multiple level fusion operation is that it does not take into account the predominance of the conditioning information from the external system over the local sensor data, and in result it makes no distinction between the information processing levels. On the other hand, the main disadvantage of *conditioning* is that the condition is treated, by definition, as an absolute and literate fact, which is the assumption very hardly accepted in the real world.

For this reason another class of fusion rules, called *relative conditioning*, has been invented. In this type of rules the predominance of the condition over the uncertain evidence is stated explicitly, while the trust in the conditioning hypothesis is not absolute by definition.

In this paper two of these rules will be presented as possible solution of the multi-level conditioning [12] in threat assessment problem.

II. New Conditioning Rules

Let Θ be a frame of discernment formed by *n* singletons defined as:

$$\Theta = \{\theta_1, \theta_1, ..., \theta_n\}, n \geq 2 \tag{1}$$

and its Super-Power Set (or fusion space):

$$S^\Theta = (\Theta, \cup, \cap, C) \tag{2}$$

which means the set Θ is closed under union $\cup$, intersection $\cap$, and complement *C* respectively.

Let *m(.)* be a mass:

$$m(.) : S^\Theta \to [0, 1] \tag{3}$$

and a non-empty set A $\subseteq I_t$ where $I_t = \theta_1 \cup \theta_2 \cup ...\theta n$ is the total ignorance.

Conditioning of $m(.|.)$ becomes:

$$\forall X \in S^{\Theta}, m(X|A) = \sum_{\substack{Y \in S^{\Theta} \setminus \emptyset \\ Y \cap A = X}} m(Y) + \\ + \sum_{\substack{Y \in S^{\Theta} \setminus \emptyset \\ Y \cap A = \emptyset \\ X = A}} m(Y) \cdot \omega_A + \delta_X^A \cdot m(X) \cdot \omega_0 \quad (4)$$

where:

$$\delta_X^A = \begin{cases} 1, A = B \\ 0, A \neq B \end{cases} \quad (5)$$

and ω_0 and ω_A are the weights for all sets which are completely outside of A, and respectively for all sets which are inside or on the frontier of A.

$$\omega_0, \omega_A \in [0,1], \omega_0 + \omega_A = 1 \quad (6)$$

For a more refined/ optimistic redistribution, all masses of the elements situated outside of A are redistributed, according to the formula (7).

$$\forall X \in S^{\Theta}, m(X|A) = \sum_{\substack{Y \in S^{\Theta} \setminus \emptyset \\ Y \cap A = X}} m(Y) + \\ + \frac{m(X)}{\sum_{\substack{Y \in S^{\Theta} \setminus \emptyset \\ Y \subseteq A \\ m(Y) \neq 0}} m(Y)} \sum_{\substack{Y \in S^{\Theta} \setminus \emptyset \\ Y \cap A = \emptyset \\ X = A}} m(Y) \cdot \omega_A + \delta_X^A \cdot m(X) \cdot \omega_0 \quad (7)$$

From the practitioner's point of view these formulas provide directions on how the mass of hypotheses not involved or partially involved in condition should be redistributed. In order to explain the idea of these rules it is suggested to consider a simple example of a model consisting of three hypotheses: A, B, and C, where A and B overlap each other, and C is disjoint. Assume the condition is A.

For this example, application of the rule (4) will cause the following action:

- former masses of A and A∩B remain unchanged, supplying A and A∩B hypotheses respectively,
- former mass of B is transferred to A∩B,
- former mass of C is transferred to A.

When applying the absolute version of the rule (4) all masses are transferred exactly as described above. Otherwise, i.e. relative conditioning, the mass of C is weighted according to the given ω_0 and ω_A.

Application of the rule (7) will cause the following action:

- former masses of A and A∩B remain unchanged, supplying A and A∩B hypotheses respectively,
- former mass of B is transferred to A∩B,
- former mass of C is transferred to A and A∩B proportionally to their masses

Similarly as for the rule (4) when applying the absolute version of the rule (7) all masses are transferred exactly as described above. Otherwise, i.e. relative conditioning, the mass C is weighted according to the given ω_0 and ω_A.

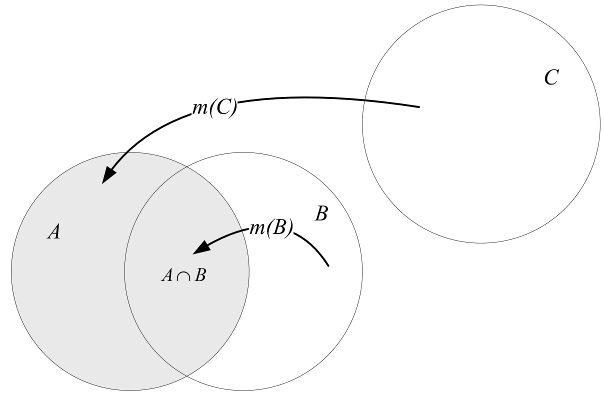

Figure 1. Mass transfer in case of application of the rule (4)

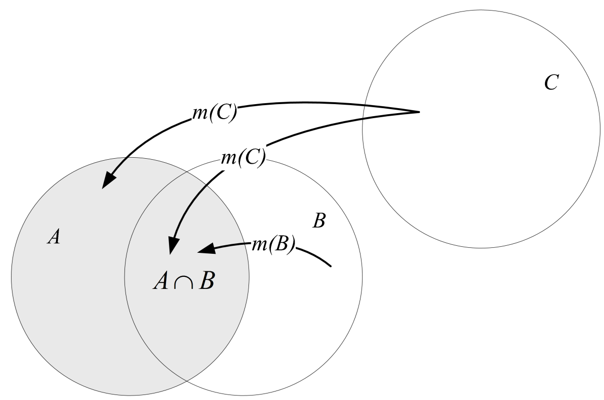

Figure 2. Mass transfer in case of application of the rule (7)

III. Threat Assessment Example

In order to illustrate application of the introduced rules it is suggested to consider the following conditioning example referring to the threat assessment problem. Assume the frame of discernment is defined as:

$$\Theta = \{F, H, U, N\} \quad (8)$$

where:

- F denotes FRIEND,
- H denotes HOSTILE,
- U denotes UNKNOWN,
- N denotes NEUTRAL.

Additionally assume:

$$S = H \cap U \quad (9)$$
$$A = F \cap U \quad (10)$$
$$K = F \cap H \quad (11)$$
$$J = F \cap H \cap U \quad (12)$$

where:

- S denotes SUSPECT,
- A denotes ASSUMED FRIEND,
- K denotes FAKER i.e. FRIEND acting as HOSTILE for training purposes, [13], [14], [15], and [16]

– J denotes JOKER i.e. FRIEND acting as SUSPECT for training purposes, [13], [14], [15], and [16].

Consider a scenario, where a local system, equipped with sensors and performing target threat observation and information fusion, gets informed by an external system about its decision, referring to the observed target. The decision transferred to the local system is that the target is FRIEND, which performs a conditioning information.

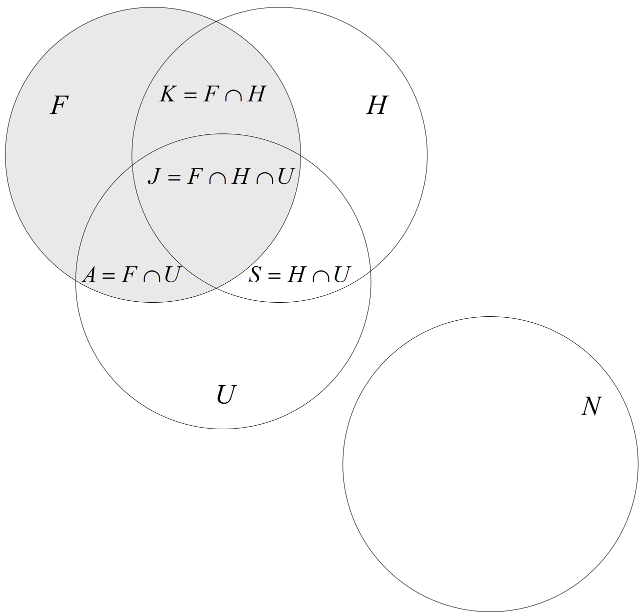

Figure 3. Venn's diagram of the observed target threat

Figure 3. shows a Venn's diagram describing the target threat observation model, where information obtained from the external system has been colored in gray. Notice that the model refers to observation of the target threat (not to the target threat in itself), which means it describes what the target looks like (not what the target really is). This is significant for justification why FAKER may be defined as the intersection of FRIEND and HOSTILE, not as a subset of FRIEND, which is by definition of FAKER in [13], [14], [15], and [16].

Consider that the local system has already performed sensor fusion and its results are summarized in basic belief assignment (*bba*) below:

$$\begin{array}{lll} m(F) = 0.2, & m(H) = 0.1, & m(U) = 0.1 \\ m(A) = 0.1, & m(S) = 0.1, & m(K) = 0.1 \\ m(J) = 0.2, & m(N) = 0.1 & \end{array}$$

Application of (4) leads to the following updated *bba* for absolute ($\omega_0 = 0$ and $\omega_A = 1$) conditioning:

$$\begin{array}{lll} m(F|F) = 0.3, & m(H|F) = 0, & m(U|F) = 0 \\ m(A|F) = 0.2, & m(S|F) = 0, & m(K|F) = 0.2 \\ m(J|F) = 0.3, & m(N|F) = 0 & \end{array}$$

For the relative conditioning with the following weights $\omega_0 = 0.3$ and $\omega_A = 0.7$ one should get:

$$\begin{array}{lll} m(F|F) = 0.27, & m(H|F) = 0, & m(U|F) = 0 \\ m(A|F) = 0.2, & m(S|F) = 0, & m(K|F) = 0.2 \\ m(J|F) = 0.3, & m(N|F) = 0.03 & \end{array}$$

For the absolute opposite ($\omega_0 = 1$ and $\omega_A = 0$) conditioning one should get:

$$\begin{array}{lll} m(F|F) = 0.2, & m(H|F) = 0, & m(U|F) = 0 \\ m(A|F) = 0.2, & m(S|F) = 0, & m(K|F) = 0.2 \\ m(J|F) = 0.3, & m(N|F) = 0.1 & \end{array}$$

Application of (7) leads to the following updated bba for absolute conditioning:

$$\begin{array}{lll} m(F|F) = 0.233, & m(H|F) = 0, & m(U|F) = 0 \\ m(A|F) = 0.217, & m(S|F) = 0, & m(K|F) = 0.217 \\ m(J|F) = 0.333, & m(N|F) = 0 & \end{array}$$

For the relative conditioning with the following weights $\omega_0 = 0.3$ and $\omega_A = 0.7$ one should get

$$\begin{array}{lll} m(F|F) = 0.223, & m(H|F) = 0, & m(U|F) = 0 \\ m(A|F) = 0.212, & m(S|F) = 0, & m(K|F) = 0.212 \\ m(J|F) = 0.323, & m(N|F) = 0.03 & \end{array}$$

For the absolute opposite conditioning one should get:

$$\begin{array}{lll} m(F|F) = 0.2, & m(H|F) = 0, & m(U|F) = 0 \\ m(A|F) = 0.2, & m(S|F) = 0, & m(K|F) = 0.2 \\ m(J|F) = 0.3, & m(N|F) = 0.1 & \end{array}$$

Analysis of the obtained results shows that there are substantial differences in results between conditioning rules (4) and (7) for the considered case. Depending on the particular rate of belief (values of ω_0 and ω_A) in condition the mass of the condition (FRIEND), as well as subsequent masses of hypotheses contained in the hypothesis of the condition (FAKER, JOKER, ASSUMED FRIEND) have been supplied with masses of hypotheses not contained in the condition (HOSTILE, UNKNOWN, SUSPECT, and NEUTRAL).

For both of the rules, in the first place the absolute conditioning case has been considered as a specific circumstance of relative conditioning. As the second, the relative conditioning has been performed with given weights of ω_0 and ω_A. Then, the absolute opposite conditioning has been presented as another special circumstance of relative conditioning.

The reason for the absolute opposite conditioning in this case is purely illustrative. Theoretically, it could be useful if the condition hypothesis was complex (expressed as union or intersection of multiple hypotheses) and it was convenient to consider the complement of the condition. However, in most of the cases the condition, as output of the external system is simple. Thus, it is very unlikely that such kind of conditioning would be applied in threat assessment.

Regarding the distinction in the presented rules, in this case, the essential difference between conditioning rules (4) and (7) resides in the manner the mass of NEUTRAL hypothesis is redistributed. For the rule (4) the mass of NEUTRAL is transferred completely to the mass of FRIEND, while for the rule (7) the mass of NEUTRAL is transferred to FRIEND, JOKER, FAKER, and ASSUMED FRIEND proportionally to their masses. In other words, in case of the rule (7) the redistribution is performed with the higher degree of trust in the adequacy of the target threat observation model. Therefore it may be regarded as more optimistic in comparison to pessimistic rule (4).

IV. Choosing the proper conditioning rule

Choosing the proper rule is one of the most important questions related to application of any fusion techniques (conditioning and combination). Since there are many rules of combination and conditioning [8], [11], [9], [10], and even more possible fusion cases, the choice of any particular rule for the particular case could be a topic of papers for the next few decades. Moreover, since there are no existent standardized fusion cases for particular domains the choice of the optimal rule seems to be a philosophical problem.

Since in this paper there are two rules of conditioning proposed the problem of selection of the proper one still holds. Additionally, each of these rules introduces weights (ω_0 and ω_A) in order to establish the 'relativity' of the conditioning, and setting particular values to these weights requires a comment.

According to the knowledge of the authors [11], [9], [10], and [12], in most of the cases selection of the particular rule for conditioning (as well as combination) is done experimentally. For the particular fusion task e.g. threat assessment in Command and Control system one chooses the rule which returns the closest results to the expected values. However, even within the particular fusion task it is possible to find situations, where another rule returns results substantially better than the previously selected one. That means two things:

– there is no universal rule of conditioning, correct in every conditions,
– if that is so, the particular fusion task should be split for at least two subtasks.

In other words, the particular rule of conditioning should be selected dynamically according to specified circumstances of information integration process.

In this section, the authors would like to define the factors which may influence on the choice of the particular rule of conditioning.

Quality of gathered information could be regarded as a basic parameter that affects selection of conditioning rules. Further, this parameter may be decomposed for two components referring to attribute (observation) model and data. Thus, the quality aggregates both: model adequacy and data precision. The fundamental question is how these model adequacy and data precision may be assessed and transformed into the quality in order to make choice of conditioning rule?

Possible solution of this problem may reside in analysis of *bba* subjected to conditioning. *Bba*, by definition, performs a kind of distribution, where subsequent masses reflect the degree of belief in particular hypotheses. If sensors are not reliable relatively high mass will be transferred to hypothesis describing complete ignorance. For instance, for the considered case it could be I = F ∪ H ∪ U ∪ N. By implication if the sensors are reliable the mass referring to the complete ignorance is zero. That may be regarded as the first insight in data precision. Another inference on data precision may be done by overview of distribution of mass over the rest of the hypotheses. Conciseness of the distribution means higher precision. Adequacy of the attribute (observation) model, on the other hand, may be defined by compliance of hypothesis of the highest mass with the hypothesis of the condition. If there exists any relation between the highest mass hypothesis and the condition, e.g. including or intersecting they may be regarded as compliant. On the other hand if they are disjoint they are regarded as noncompliant.

Referring the deductions above to the features of the presented rules a simple logic (briefly described in Table I) may be applied in order to choose the proper conditioning rule.

Table I
Choice of the conditioning rule based on model adequacy and data reliability

Model	Data	Quality	Description	Conditioning
poor	poor	poor	$m_{max} \neq Cond$, $m(\Theta)\uparrow$	absolute, (4)
poor	good	poor	$m_{max} \neq Cond$, $m(\Theta)\downarrow$	absolute, (4)
good	poor	poor	$m_{max} \cong Cond$, $m(\Theta)\uparrow$	absolute, (7)
good	good	good	$m_{max} \cong Cond$, $m(\Theta)\downarrow$	relative, (7)

If the highest mass hypothesis is not compliant with the condition, which means the attribute (observation) model is not adequate, no matter if the data are precise or not, in such case absolute conditioning should be applied with no respect to the attribute (observation) model. This may be achieved by using the rule (4) with $\omega_0 = 0$ and $\omega_A = 1$.

If the mass referring to total ignorance is relatively high and the highest mass hypothesis is compliant with the condition that means that the sensor data are poor and the attribute (observation) model is adequate. In such case absolute conditioning should be applied with respect to the attribute (observation) model which may be achieved by using the rule (7) with $\omega_0 = 0$ and $\omega_A = 1$.

Finally, if the mass referring to total ignorance is relatively low and the highest mass hypothesis is compliant with the condition that means that the sensor data are reliable (good) and the attribute (observation) model is adequate. In such case relative conditioning should be applied with respect to the attribute (observation) model which may be achieved by using the rule (7) with $\omega_0, \omega_A \in (0, 1)$, where: $\omega_0 + \omega_A = 1$.

As a summary of this section it is worth of notice that particular values of the 'relativity' weights (ω_0 and ω_A) depend only on the specific configuration of the fusion system. In the authors' opinion it is pointelss to discuss any specific values without reference to the particular system since there are no general guidelines for presetting.

V. Selection of conditioning rules - examples

In order to illustrate the selection mechanism few more examples have been delivered. However, in the first place, it is suggested to reconsider the example from Section III. Table II presents the summarized *bba* (before and after conditioning). In this very case, before conditioning performed, the dominant masses had referred to FRIEND and FAKER hypotheses, which was compliant with the condition hypothesis (FRIEND). That means the model was adequate. Additionally, the total ignorance mass has not been defined (as nonzero), which means the data were reliable. According to Table I, in such case the relative version of the rule (7) should be selected, which was exactly what was decided.

Table II
EXAMPLE 1: *BBA* BEFORE AND AFTER CONDITIONING OPERATION

Threat \ bba	m	$m_{(7)R}(\cdot\vert F)$
F	0.2	**0.223**
H	0.1	0
U	0.1	0
A = F ∩ U	0.1	0.212
S = H ∩ U	0.1	0
K = F ∩ H	0.1	0.212
J = F ∩ H ∩ U	0.2	0.323
N	0.1	0.03
I = F ∪ H ∪ U ∪ N	0	0

In the next example it is suggested to consider *bba* given in the second column of the Table III. In this case the biggest mass has been assigned to total ignorance. Furthermore, there is no predominance of any particular primary hypotheses [17] (FRIEND, HOSTILE, UNKNOWN, NEUTRAL) or secondary hypotheses [17] (ASSUMED FRIEND, SUSPECT, FAKER, JOKER). That means that the gathered data are not reliable and and the model adequacy has not been proven. Therefore the absolute version of the rule (4) should be chosen.

Table III
EXAMPLE 2: *BBA* BEFORE AND AFTER CONDITIONING OPERATION

Threat \ bba	m	$m_{(4)A}(\cdot\vert F)$
F	0.1	**0.56**
H	0.1	0
U	0.1	0
A = F ∩ U	0.06	0.16
S = H ∩ U	0.06	0
K = F ∩ H	0.06	0.16
J = F ∩ H ∩ U	0.06	0.12
N	0.06	0
I = F ∪ H ∪ U ∪ N	0.4	0

In the last example it is suggested to consider *bba* given in the second column of the Table IV. In this case the biggest mass has also been assigned to total ignorance, which proves relatively low sensor reliability. However, except *m(I)*, there is a predominance of FRIEND hypothesis over the other hypotheses. Thus the model be regarded as adequate. Therefore the absolute version of the rule (7) should be chosen.

In the presented procedure of selection of the conditioning rules *bba* provides qualitative information on data reliability as well as model adequacy. Analyzing the above examples, some harsh reader could regard reasoning about the adequacy of the model based on the *bba* as vague, due to the fact *bba*s are affected with measuring errors, and it is possible these errors influence on the decision whether a particular model is adequate or not. However, it is important to notice that in real systems these *bba*s are updated regularly, which enables to improve statistically the reference for decision making. That means that any predominance of a certain hypothesis may be confirmed by the subsequent version of updated *bba*.

Table IV
EXAMPLE 3: *BBA* BEFORE AND AFTER CONDITIONING OPERATION

Threat \ bba	m	$m_{(7)A}(\cdot\vert F)$
F	0.16	**0.422**
H	0.1	0
U	0.1	0
A = F ∩ U	0	0.1
S = H ∩ U	0.06	0
K = F ∩ H	0.06	0.259
J = F ∩ H ∩ U	0.06	0.219
N	0.06	0
I = F ∪ H ∪ U ∪ N	0.4	0

It is also a matter of convention how to deal with a particular case when $m(\Theta) = m(I)$ is the maximal mass in the *bba*. Assuming that the condition hypothesis does not refer to total ignorance: On one hand, since *bba* influences both data reliability and model adequacy it is justified to select the absolute version of the rule (4). On the other hand, it is reasonable to exclude the total ignorance hypothesis $m(\Theta) = m(I)$ while deciding about the adequacy of the model, in order to distinguish two aspects (qualitative features) of the gathered *bba*, which is preferable by the authors.

VI. CONCLUSION

The introduced new rules of conditioning have been invented as a response for problems emerging while applying the existing absolute conditioning techniques in the real world. Considering the condition as identical with the ground truth may be useful in theory, however in practice it often performs an assumption hard to accept [11]. Updating attribute fusion results with evidence from the external system is an excellent example for that. Each time the highly processed information is used, no matter how good the system is, there is a risk that the output information is corrupted or at least slightly changed [18], [19].

The presented conditioning rules enable to set weights in order to define the degree of belief in the external system output. These weights should be treated as tactical and technical parameters of the system performing combination and conditioning. Certainly, depending on the actual needs, they may be fixed or changeable dynamically. However the exact values should result from the particular system configuration thus no theoretical preference is made.

In case of choosing a particular rule of conditioning it is different, and some general guidelines may be established. The proposed method of selection of the conditioning rules may be applied in Command and Control systems, where multiple rules may be implemented. In such case the choice of the proper conditioning rule may perform an element of so called Conditioning Management.

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