Back and Forth Summands

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Let n>k≥1 be two integers. Then a Back and Forth Summand is defined as:

$$S(n, {}_k) = \sum_{\substack{0<|n-k\cdot i|\le n \\ i=0,1,2,\dots}} (n-k\cdot i) \quad \text{[for signed numbers]}$$

$$S|n, {}_k| = \sum_{\substack{0<|n-k\cdot i|\le n \\ i=0,1,2,\dots}} |n-k\cdot i| \quad \text{[for absolute value numbers]}$$

which are duals and semi-duals respectively of Smarandacheials.

$S(n, {}_1)$ and $S(n, {}_2)$ with corresponding $S|n, {}_1|$ and $S|n, {}_2|$ are trivial.

a) In the case k=3:

$$S(n, {}_3) = \sum_{\substack{0<|n-3i|\le n \\ i=0,1,2,\dots}} (n-3i) = n+(n-3)+(n-6)+\dots \text{ ; [for signed numbers].}$$

$$S|n, {}_3| = \sum_{\substack{0<|n-3i|\le n \\ i=0,1,2,\dots}} |n-3i| = n+|n-3|+|n-6|+\dots \text{ ; [for absolute value numbers].}$$

Thus $S(7, {}_3) = 7+(7-3)+(7-6)+(7-9)+(7-12) = 7+(4)+(1)+(-2)+(-5) = 5$; [for signed numbers].
Thus $S|7, {}_3| = 7+|7-3|+|7-6|+|7-9|+|7-12| = 7+4+1+2+5 = 19$; [for absolute value numbers].

The sequence is $S(n, {}_3)$: 3, 2, 0, 5, 3, 0, 7, 4, 0, 9, 5, 0, … ; [for signed numbers].
The sequence is $S|n, {}_3|$: 7, 12, 18, 19, 27, 36, 37, 48, … ; [for absolute value numbers].

4) In the case k=4:

$$S(n, {}_4) = \sum_{\substack{0<|n-4i|\le n \\ i=0,1,2,\dots}} (n-4i) = n+(n-4)+(n-8)\dots \text{ ; [for signed numbers].}$$

$$S|n, {}_4| = \sum_{\substack{0<|n-4i|\le n \\ i=0,1,2,\dots}} |n-4i| = n+|n-4|+|n-8|\dots \text{ ; [for absolute value numbers].}$$

Thus $S(9, {}_4) = 9+(9-4)+(9-8)+(9-12)+(9-16) = 9+(5)+(1)+(-3)+(-7) = 5$; for signed numbers.
Thus $S|9, {}_4| = 9+|9-4|+|9-8|+|9-12|+|9-16| = 9+5+1+3+7 = 25$; [for absolute value numbers].

The sequence is $S(n, {}_4) =$ 3, 0, 4, 0, 5, 0, 6, 0, 7, 0, 8, 0, 9, 0, 10, 0, 11, … .
The sequence is $S|n, {}_4| =$ 9, 16, 16, 24, 25, 36, 36, 48, 49, 64, 64, 80, 81, 100, 100, … .

5) In the case k=5:

$$S(n, {}_5) = \sum_{\substack{0<|n-5i|\leq n \\ i=0,1,2,\ldots}} (n-5i) = n+(n-5)+(n-10)\ldots$$

$$S|n, {}_5| = \sum_{\substack{0<|n-5i|\leq n \\ i=0,1,2,\ldots}} |n-5i| = n+|n-5|+|n-10|\ldots$$

Thus $S(11, {}_5) = 11+(11-5)+(11-10)+(11-15)+(11-20) = 11+6+1+(-4)+(-9) = 5$.
Thus $S|11, {}_5| = 11+|11-5|+|11-10|+|11-15|+|11-20| = 11+6+1+4+9 = 31$.

The sequence is $S(n, {}_5)$: 3, 6, 2, 6, 0, 5, 10, 3, 9, 0, 7, 14, 4, 12, 0, … .
The sequence is $S|n, {}_5|$: 11, 12, 20, 20, 30, 31, 32, 33, 45, 60, 61, 62, 80, 80, 100, … .

More general:
Let $n>k\geq 1$ be two integers and $m\geq 0$ another integer.
Then the Generalized Back and Forth Summand is defined as:

$$S(n, m, {}_k) = \sum_{i=0,1,2,\ldots,\mathrm{floor}[(n+m)/k]} (n-k\cdot i) \quad \text{[for signed numbers].}$$

$$S|n, m, {}_k| = \sum_{i=0,1,2,\ldots,\mathrm{floor}[(n+m)/k]} |n-k\cdot i| \quad \text{[for absolute value numbers].}$$

For examples:
$S(7, 9, {}_2) = 7+(7-2)+(7-4)+(7-6)+(7-8)+(7-10)+(7-12)+(7-14)+(7-16)$
$= 7+(5)+(3)+(1)+(-1)+(-3)+(-5)+(-7)+(-9) = -2$.
$S|7, 3, {}_2| = 7+|7-2|+|7-4|+|7-6|+|7-8|+|7-10| = 7+5+3+1+1+3 = 20$.

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