Applied Mechanics and Materials
ISSN: 1662-7482, Vol. 436, pp 511-517
doi:10.4028/www.scientific.net/AMM.436.511

Online: 2013-10-31

Correlation Coefficient of Interval Neutrosophic Set

Said Broumi[1,a], Florentin Smarandache[2,b]

[1] Faculty of Arts and Humanities, Hay El Baraka Ben M'sik Casablanca B.P. 7951, Hassan II University Mohammedia-Casablanca, Morocco

[2]Department of Mathematics, University of New Mexico,

705 Gurley Avenue, Gallup, NM 87301, USA

[a]broumisaid@gmail.com, [b]fsmarandache@gmail.com

Keywords: Neutrosophic Set, Correlation Coefficient of Interval Neutrosophic Set, Weighted Correlation Coefficient of Interval Neutrosophic Set.

Abstract. In this paper we introduce for the first time the concept of correlation coefficients of interval valued neutrosophic set (INS for short). Respective numerical examples are presented.

1. Introduction

Neutrosophy was pioneered by Smarandache [1]. It is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [2]. Neutrosophic set theory is a powerful formal framework which generalizes the concept of the classic set, fuzzy set [3], interval-valued fuzzy set [4], intuitionistic fuzzy set [5], interval-valued intuitionistic fuzzy set [6], and so on. Neutrosophy introduces a new concept called <NeutA> which represents indeterminacy with respect to <A>. It can solve certain problems that cannot be solved by fuzzy logic. For example, a paper is sent to two reviewers, one says it is 90% acceptable and another says it is 90% unacceptable. But the two reviewers may have different backgrounds. One is an expert, and another is a new comer in this field. The impacts on the final decision of the paper by the two reviewers should be different, even though they give the same grade level of the acceptance. There are many similar problems, such as weather forecasting, stock price prediction, and political elections containing indeterminate conditions that fuzzy set theory does not handle well. This theory deals with imprecise and vague situations where exact analysis is either difficult or impossible.
After the pioneering work of Smarandache. In 2005, Wang et al. [7] introduced the notion of interval neutrosophic set (INS) which is a particular case of the neutrosophic set (NS) that can be described by a membership interval, a non-membership interval, and an indeterminate interval, thus the NS is flexible and practical, and the NS provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information.
The theories of both neutrosophic set and interval neutrosophic set have achieved great success in various areas such as medical diagnosis [8], database [9,10], topology[11], image processing [12,13,14], and decision making problem [15].
Although several distance measures, similarity measures, and correlation measure of neutrosophic sets have been recently presented in [16, 17], there is a rare investigation on correlation of interval neutrosophic sets.
It is very common in statistical analysis of data to finding the correlation between variables or attributes, where the correlation coefficient is defined on ordinary crisp sets, fuzzy sets [18], intuitionistic fuzzy sets [19,20,21], and neutrosophic set [16,17] respectively. In this paper we first discuss and derive a formula for the correlation coefficient defined on the domain of interval neutrosophic sets. The paper unfolds as follows. The next section briefly introduces some definitions related to the method. Section III presents the correlation and weighted correlation coefficient of the interval neutrosophic set. Conclusions appear in the last section.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, and interval neutrosophic sets relevant to the present work. See especially [1, 7, 17] for further details and background.

2.1 Definition ([1]). Let U be an universe of discourse; then the neutrosophic set A is an object having the form A = {< x: $T_A(x)$,$I_A(x)$,$F_A(x)$>,x ∈ U}, where the functions T,I,F : U→$]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ U to the set A with the condition:

$$^-0 \leq T_A(x)+ I_A(x)+ F_A(x) \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$.So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

2.2 Definition ([7]). Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \subset [0,1]$.

Remark 1. An INS is clearly a NS.

2.3 Definition ([7]).

- An INS A is empty if $inf T_A(x) = sup T_A(x)$= 0, $inf I_A(x) = sup I_A(x)$= 1, $inf F_A(x) = sup F_A(x)$= 0, for all x in A.
- Let $\underline{0}$ = <0, 1 ,1> and $\underline{1}$ = <1, 0 ,0>

2.4 Correlation Coefficient of Neutrosophic Set ([17]).

Let A and B be two neutrosophic sets in the universe of discourse X = {x_1, x_2, …,x_n}.

The *correlation coefficient* of A and B is given by

$$R(A,B)=\frac{C(A,B)}{(E(A)\cdot E(B))^{\frac{1}{2}}} \quad (2)$$

where the *correlation* of two NSs A and B is given by

$$C(A,B) = \sum_{1=0}^{n}(T_A(x_i) T_B(x_i) + I_A(x_i) I_B(x_i) + F_A(x_i) F_B(x_i)) \quad (3)$$

And the *informational energy* of two NSs A and B are given by

$$E(A) = \sum_{1=0}^{n}(T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) \quad (4)$$

$$E(B) = \sum_{1=0}^{n}(T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)) \quad (5)$$

Respectively, the correlation coefficient of two neutrosophic sets A and B satisfies the following properties:

(1) $0 \leq R(A,B) \leq 1$ (6)
(2) R(A,B)= R(B,A) (7)
(3) R(A,B)= 1 if A=B (8)

3. Correlation of Two Interval Neutrosophic Sets

In this section, following the correlation between two neutrosophic sets defined by A. A. Salama in [17], we extend this definition to interval neutrosophic sets. If we have a random non-crisp set, with a triple membership form, for each of two interval neutrosophic sets, we get the interest in comparing the degree of their relationship. We check if there is any linear relationship between the two interval neutrosophic sets; thus we need a formula for the sample correlation coefficient of two interval neutrosophic sets in order to find the relationship between them.

3.1.Definition

Assume that two interval neutrosophic sets A and B in the universe of discourse $X = \{x_1, x_2, x_3, \ldots, x_n\}$ are denoted by

$$A=\frac{\sum_1^n <[\inf T_A(x_i)\ \sup T_A(x_i)],[\inf I_A(x_i)\ \sup I_A(x_i)],[\inf T_A(x_i)\ \sup F_A(x_i)],>}{x_i},\ x_i \in X, \text{ and} \tag{9}$$

$$B=\frac{\sum_1^n <[\inf T_B(x_i)\ \sup T_B(x_i)],[\inf I_B(x_i)\ \sup I_B(x_i)],[\inf T_B(x_i)\ \sup F_B(x_i)],>}{x_i},\ x_i \in X, \text{ where} \tag{10}$$

$\inf T_A(x_i) \le \sup T_A(x_i)$,$\inf F_A(x_i) \le \sup F_A(x_i)$, $\inf I_A(x_i) \le \sup I_A(x_i)$, $\inf T_B(x_i) \le \sup T_B(x_i)$, $\inf F_B(x_i) \le \sup F_B(x_i)$, $\inf I_B(x_i) \le \sup I_B(x_i)$, and they all belong to [0, 1];

then we define the correlation of the interval neutrosophic sets A and B in X by the formula

$$C_{INS}(A,B)= \sum_1^n\{\inf T_A(x_i)\cdot \inf T_B(x_i) + \sup T_A(x_i)\cdot \sup T_B(x_i) + \inf I_A(x_i)\cdot \inf I_B(x_i) + \sup I_A(x_i)\cdot \sup I_B(x_i) + \inf F_A(x_i)\cdot \inf F_B(x_i) + \sup F_A(x_i)\cdot \sup F_B(x_i)\} \tag{11}$$

Let us notice that this formula coincides with that given by A. A Salama [17] when $\inf T_A(x_i) = \sup T_A(x_i)$, $\inf F_A(x_i) = \sup F_A(x_i)$, $\inf I_A(x_i) = \sup I_A(x_i)$ and $\inf T_B(x_i) = \sup T_B(x_i)$, $\inf F_B(x_i) = \sup F_B(x_i)$, $\inf I_B(x_i) = \sup I_B(x_i)$
and the correlation coefficient of the interval neutrosophic sets A and B given by

$$K_{INS}(A,B) = \frac{C_{INS}(A,B)}{(E(A)\cdot E(B))^{1/2}} \in [0, 1^+[\tag{12}$$

where

$$E(A)=\sum_1^n[T_{AL}{}^2(x_i) + T_{AU}{}^2(x_i) + I_{AL}{}^2(x_i) + I_{AU}{}^2(x_i) + F_{AL}{}^2(x_i) + F_{AU}{}^2(x_i)] \tag{13}$$

$$E(B) = \sum_1^n[T_{BL}{}^2(x_i) + T_{BU}{}^2(x_i) + I_{BL}{}^2(x_i) + I_{BU}{}^2(x_i) + F_{BL}{}^2(x_i) + F_{BU}{}^2(x_i)] \tag{14}$$

express the so-called informational energy of the interval neutrosophic sets A and B respectively.

Remark 2: For the sake of simplicity we shall use the symbols:

$$\inf T_A(x_i)= T_{AL}\ ,\ \sup T_A(x_i) = T_{AU}, \tag{15}$$
$$\inf T_B(x_i)=T_{BL},\ \sup T_B(x_i) = T_{BU}, \tag{16}$$
$$\inf I_A(x_i)=I_{AL}\ ,\sup I_A(x_i)=I_{AU}, \tag{17}$$
$$\inf I_B(x_i)= I_{BL}\ ,\ \sup I_B(x_i)=I_{BU}, \tag{18}$$
$$\inf F_A(x_i)=F_{AL}\ ,\ \sup F_A(x_i)=F_{AU}, \tag{19}$$
$$\inf F_B(x_i)= F_{BL},\ \sup F_B(x_i)= F_{BU}, \tag{20}$$

For the correlation of interval neutrosophic set, the following proposition is immediate from the definition.

3.2. Proposition

For A, B $\in$ $INSs$ in the universe of discourse X={$x_1,x_2,x_3,\ldots,x_n$} the correlation of interval neutrosophic set have the following properties:

$$(1)\, C_{INS}(A,A) = E(A) \tag{21}$$

$$(2)\, C_{INS}(A,B)=C_{INS}(B,A) \tag{22}$$

3.3. Theorem. For all INSs A, B the correlation coefficient satisfies the following properties:

$$(3)\ \text{If } A = B \text{, then } K_{INS}(A,B) = 1. \tag{23}$$

$$(4)\ K_{INS}(A,B) = K_{INS}(B,A). \tag{24}$$

$$(5)\ 0 \le K_{INS}(A,B) \le 1. \tag{25}$$

Proof. Conditions (1) and (2) are evident; we shall prove condition (3). $\mathrm{K_{INS}}(\mathrm{A},\mathrm{B}) \ge 0$ is evident. We will prove that $\mathrm{K_{INS}}(\mathrm{A},\mathrm{B}) \le 1$. From the Schwartz inequality, we obtain

$$K_{INS}(A,B) = (\textstyle\sum_1^n[T_{AL}(x_i)T_{BL}(x_i) + T_{AU}(x_i)T_{BU}(x_i) + I_{AL}(x_i)I_{BL}(x_i) + I_{AU}(x_i)I_{BU}(x_i) + F_{AL}(x_i)F_{BL}(x_i) + F_{AU}(x_i)F_{BU}(x_i)])$$

$$\left(\textstyle\sum_1^n T_{AL}{}^2(x_i) + T_{AU}{}^2(x_i) + I_{AL}{}^2(x_i) + I_{AU}{}^2(x_i) + F_{AL}{}^2(x_i) + F_{AU}{}^2(x_i)\right)^{\frac{-1}{2}} \cdot \left(\textstyle\sum_1^n T_{BL}{}^2(x_i) + T_{BU}{}^2(x_i) + I_{BL}{}^2(x_i) + I_{BU}{}^2(x_i) + F_{BL}{}^2(x_i) + F_{BU}{}^2(x_i)\right)^{\frac{-1}{2}} \le \Big\{\left(\textstyle\sum_{i=1}^n T_{AL}{}^2(x_i)\sum_{i=1}^n T_{BL}{}^2(x_i)\right)^{\frac{1}{2}} + \left(\textstyle\sum_{i=1}^n T_{AU}{}^2(x_i)\sum_{i=1}^n T_{BU}{}^2(x_i)\right)^{\frac{1}{2}} + \left(\textstyle\sum_{i=1}^n I_{AL}{}^2(x_i)\sum_{i=1}^n I_{BL}{}^2(x_i)\right)^{\frac{1}{2}} + \left(\textstyle\sum_{i=1}^n I_{AU}{}^2(x_i)\sum_{i=1}^n I_{BU}{}^2(x_i)\right)^{\frac{1}{2}} + \left(\textstyle\sum_{i=1}^n F_{AL}{}^2(x_i)\sum_{i=1}^n F_{BL}{}^2(x_i)\right)^{\frac{1}{2}} + \left(\textstyle\sum_{i=1}^n F_{AU}{}^2(x_i)\sum_{i=1}^n F_{BU}{}^2(x_i)\right)^{\frac{1}{2}}\Big\} \cdot \Big\{\Big(\textstyle\sum_{i=1}^n \boldsymbol{T_{AL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{T_{AU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{AL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{AU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{AL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{AU}}{}^2(\boldsymbol{x_i})\Big) \cdot \Big(\sum_{i=1}^n \boldsymbol{T_{BL}}{}^2(\boldsymbol{x_i}) +$$

$$\times\ \textstyle\sum_{i=1}^n \boldsymbol{T_{BU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{BL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{BU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{BL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{BU}}{}^2(\boldsymbol{x_i})\Big)\Big\}^{\frac{-1}{2}} \tag{26}$$

Let us adopt the following notations:

$$\textstyle\sum_{i=1}^n T_{AL}{}^2(x_i) = a \quad \sum_{i=1}^n T_{BL}{}^2(x_i) = b \quad \sum_{i=1}^n T_{AU}{}^2(x_i) = c \quad \sum_{i=1}^n T_{BU}{}^2(x_i) = d \tag{27}$$

$$\textstyle\sum_{i=1}^n I_{AL}{}^2(x_i) = e \quad \sum_{i=1}^n I_{BL}{}^2(x_i) = f \quad \sum_{i=1}^n I_{AU}{}^2(x_i) = g \quad \sum_{i=1}^n I_{BU}{}^2(x_i) = h \tag{28}$$

$$\textstyle\sum_{i=1}^n F_{AL}{}^2(x_i) = i \quad \sum_{i=1}^n F_{BL}{}^2(x_i) = j \quad \sum_{i=1}^n F_{AU}{}^2(x_i) = k \quad \sum_{i=1}^n F_{BU}{}^2(x_i) = l \tag{29}$$

The above inequality is equivalent to

$$K_{INS}(A,B) \le \frac{\sqrt{ab}+\sqrt{cd}+\sqrt{ef}+\sqrt{gh}+\sqrt{ij}+\sqrt{kl}}{\sqrt{(a+c+e+g+i+k)}\sqrt{b+d+f+h+j+l}} \tag{30}$$

Then, since $K_{INS}(A,B) \ge 0$ we have

$$K_{INS}^2(A,B) \le \frac{\left(\sqrt{ab}+\sqrt{cd}+\sqrt{ef}+\sqrt{gh}+\sqrt{ij}+\sqrt{kl}\right)^2}{(a+c+e+g+i+k)(b+d+f+h+j+l)}$$

$$=1-\Big\{\left(\sqrt{ad}-\sqrt{bc}\right)^2+\left(\sqrt{af}-\sqrt{be}\right)^2+\left(\sqrt{ah}-\sqrt{bg}\right)^2+\left(\sqrt{aj}-\left(\sqrt{bi}\right)^2+\sqrt{al}-\sqrt{bk}\right)^2+$$
$$\left(\sqrt{cf}-\sqrt{de}\right)^2+\left(\sqrt{ch}-\sqrt{dg}\right)^2+\left(\sqrt{cj}-\sqrt{di}\right)^2+\left(\sqrt{cl}-\left(\sqrt{dk}\right)^2+\sqrt{eh}-\sqrt{fg}\right)^2+$$
$$\left(\sqrt{ej}-\sqrt{fi}\right)^2+\left(\sqrt{el}-\sqrt{fk}\right)^2+\left(\sqrt{gi}-\sqrt{fk}\right)^2+\left(\sqrt{gl}-\left(\sqrt{hk}\right)^2+\sqrt{il}-\sqrt{jk}\right)^2\Big\}\times$$
$$\{(a+c+e+g+i+k)(b+d+h+j+l)\}^{-1}\leq 1. \tag{31}$$

And thus we have $0 \leq K_{INS}(A,B) \leq 1.$ (32)

Remark 3: From the following counter-example, we can easily check that

$K_{INS}(A,B)$ = 1 but A $\neq B.$ (33)

Remark 4:

Let A and B be two interval neutrosophic set defined on the universe X = {x_1}

A={ x_1: < [0.5, 0.5] [0.5, 0.5] [0.5, 0.5]>}

B={ x_1: < [0.25, 0.25] [0.25, 0.25] [0.25, 0.25]>}

$K_{INS}(A,B)$ = 1 but A$\neq B.$

3.4. Weighted Correlation Coefficient of Interval Neutrosophic Sets

In order to investigate the difference of importance considered in the elements in the universe of discourse, we need to take the weights of the elements x_i(i = 1, 2, 3, …, n). In the following we develop a weighted correlation coefficient between the interval neutrosophic sets as follows:

$$\boldsymbol{W_{INS}(A,B)} = \frac{\sum_1^n w_i\{T_{AL}(x_i)\cdot T_{BL}(x_i)+T_{AU}(x_i)\cdot T_{BU}(x_i)+I_{AL}(x_i)\cdot I_{BL}(x_i)+I_{AU}(x_i)\cdot I_{BU}(x_i)+F_{AL}(x_i)\cdot F_{BL}(x_i)+F_{AU}(x_i)\cdot F_{BU}(x_i)\}}{(\sum_1^n w_i\,(T_{AL}{}^2(x_i)+T_{AU}{}^2(x_i)+I_{AL}{}^2(x_i)+I_{AU}{}^2(x_i)+F_{AL}{}^2(x_i)+F_{AU}{}^2(x_i))\cdot\sum_1^n w_i\,(T_{BL}{}^2(x_i)+T_{BU}{}^2(x_i)+I_{BL}{}^2(x_i)+I_{BU}{}^2(x_i)+F_{BL}{}^2(x_i)+F_{BU}{}^2(x_i)))^{1/2}} \in [0, 1^+[\tag{34}$$

If w = $\{\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\}$, equation (34) is reduced to the correlation coefficient (12); it is easy to check that the weighted correlation coefficient $w_{INS}(A,B)$ between INSs A and B also satisfies the properties:

(1) $0 \leq w_{INS}(A,B) \leq 1$ (35)

(2) $w_{INS}(A,B) = w_{INS}(B,A)$ (36)

(3) $w_{INS}(A,B)$ = 1 if A = B (37)

3.5. Numerical Illustration.

In this section we present, an example to depict the method defined above, where the data is represented by an interval neutrosophic sets.

Example. For a finite universal set X = {x_1, x_2}, if two interval neutrosophic sets are written, respectively

A = {x_1:<[0.2, 0.3] [0.4, 0.5] [0.1, 0.2]>; x_2:<[0.3, 0.5] [0.1, 0.2] [0.4, 0.5]>}

B = {x_1:<[0.1, 0.2] [0.3, 0.4] [0.1, 0.3]>; x_2:<[0.4, 0.5] [0.2, 0.3] [0.1, 0.2]>}

Therefore, we have

$$K_{INS}(A,B) = \frac{1{,}06}{(1{,}39)^{\frac{1}{2}}(0{,}99)^{1/2}} \in \,]0, 1^{+}[$$

E(A) = 1,39

E(B) = 0,99

$$K_{INS}(A,B) = 0.90$$

It shows that the interval neutrosophic sets A and B have a good positively correlation.

Conclusion:
In this paper we introduced a method to calculate the correlation coefficient of two interval neutrosophic sets.

References

[1] F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth, American Research Press, 1999.

[2] F. Smarandache, Linguistic Paradoxists and Tautologies, Libertas Mathematica, University of Texas at Arlington, Vol. XIX (1999), 143-154.

[3] L. A. Zadeh,, Fuzzy sets, Information and Control, 8 (1965), 338-353.

[4] Turksen, Interval valued fuzzy sets based on normal forms, Fuzzy Sets and Systems, 20 (1968), 191–210.

[5] Atanassov, K., Intuitionistic fuzzy sets".Fuzzy Sets and Systems, 20 (1986), 87–96.

[6] Atanassov, K., & Gargov, G., Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems.International Journal of General Systems, 393, 31 (1998), 343–349.

[7] Wang, H., Smarandache, F., Zhang, Y.-Q. and Sunderraman, R.,, Interval Neutrosophic Sets and Logic: Theory and Applications in Computing, Hexis, Phoenix, AZ, 2005.

[8] Ansari, Biswas, Aggarwal, Proposal for Applicability of Neutrosophic Set Theory in Medical AI, International Journal of Computer Applications (0975 – 8887),Vo 27, No.5 2011), 5-11.

[9] M. Arora, R. Biswas,U. S. Pandy, Neutrosophic Relational Database Decomposition, International Journal of Advanced Computer Science and Applications, Vol. 2, No. 8 (2011), 121-125.

[10] M. Arora and R. Biswas, Deployment of Neutrosophic technology to retrieve answers for queries posed in natural language, 3rd International Conference on Computer Science and Information Technology ICCSIT, IEEE catalog Number CFP1057E-art, Vol. 3 (2010), 435-439.

[11] F. G. Lupiáñez, On neutrosophic topology, Kybernetes, Vol. 37, Iss. 6 (2008), 797 - 800, Doi:10.1108/03684920810876990.

[12] H. D. Cheng & Y Guo, A new neutrosophic approach to image thresholding, New Mathematics and Natural Computation, 4(3), (2008), 291–308.

[13] Y. Guo & H. D. Cheng, New neutrosophic approach to image segmentation, Pattern Recognition, 42 (2009), 587–595.

[14] M.Zhang, L.Zhang, and H.D.Cheng, A neutrosophic approach to image segmentation based on watershed method, Signal Processing 5, 90 (2010), 1510-1517.

[15] A. Kharal, A Neutrosophic Multicriteria Decision Making Method, New Mathematics & Natural Computation, to appear in Nov 2013.

[16] I. M. Hanafy, A. A. Salama, K. M. Mahfouz, Correlation Coefficients of Neutrosophic Sets by Centroid Method, International Journal of Probability and Statistics, 2(1), (2013), 9-12, DOI: 10.5923/j.ijps.20130201.02.

[17] I. M. Hanafy, A. A. Salamaand, K. Mahfouz, Correlation of Neutrosophic Data, International Refereed Journal of Engineering and Science (IRJES), ISSN (Online) 2319-183X, (Print) 2319-1821,Volume 1, Issue 2, (2012), 39-43.

[18] Ding-An Chiang, Nancy P. Lin, Correlation of fuzzy sets, Fuzzy Sets and Systems,Volume 102, Issue 2, (1999), 221–226, http://dx.doi.org/10.1016/S0165-0114(97)00127-9.

[19] E. Szmidt, J. Kacprzyk, Correlation of Intuitionistic Fuzzy Sets, Computational Intelligence for Knowledge-Based Systems Design, Lecture Notes in Computer Science, Volume 6178, (2010), 169-177.

[20] Gerstenkorn, T. & Manko, J., Correlation of intuitionistic fuzzy sets, Fuzzy Sets and Systems, 44 (1991), 39–43.

[21] Hong, D. H., Hwang, S. Y., Correlation of intuitionistic fuzzy sets in probability spaces, Fuzzy Sets and Systems, 75 (1995), 77–81.