Degree of Uncertainty of a Set and of a Mass

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Abstract.
In this paper we use extend Harley's measure of uncertainty of a set and of mass to the degree of uncertainty of a set and of a mass (bba).

Measure of Uncertainty of a Set.

In DST (Dempster-Shafer's Theory), Hartley defined the measure of uncertainty of a set A by:

$$I(A) = \log_2 |A|, \text{ for } A \in 2^{\theta} \setminus \{\Phi\},$$

where $|A|$ is the cardinal of the set A .

We can extend it to $DSmT$ in the same way:

$$I(A) = \log_2 |A|, \text{ for } A \in G^{\theta} \setminus \{\Phi\}$$

where G^{θ} is the super-power set, and $|A|$ means the DSm cardinal of the set A.

We even improve it to:

$$\bigcup_d^s : G^{\theta} \setminus \{\Phi\} \to [0,1]$$

If A is a singleton, i.e. $|A| = 1$, then $\bigcup_d^s(A) = 0$ (minimum degree of uncertainty of a set),

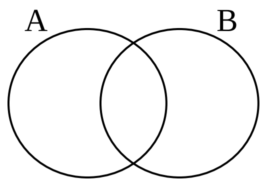
For the total ignorance I_t , since $|I_t|$ is the maximum cardinal, we get $\bigcup_d^s(I_t) = 1$ (maximum degree of uncertainty of a set).

For all other sets X from $G^{\theta} \setminus \{\Phi\}$, whose cardinal is in between 1 and $|I_t|$, we have $0 < \bigcup_d^s(X) < 1$.

We consider our degree of uncertainty of a set work better than Hartley Measure since it is referred to the frame of discernment.

Let's see an **Example 1**.

If $\theta = \{A, B\}$ and $A \cap B \neq \Phi$, we have the model

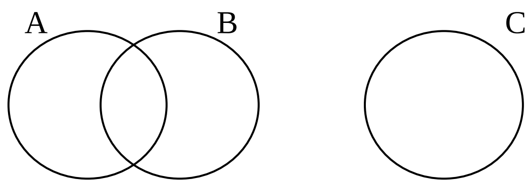


$$I(A) = \log_2 |A| = \log_2 2 = 1$$

While $\bigcup_d^s(A) = \frac{\log_2 |A|}{\log_2 |A \cup B|} = \frac{\log_2 2}{\log_2 3} = 0.63093$

Example 2.

If $\theta = \{A, B, C\}$, and $A \cap B \neq \Phi$, but $A \cap C = \Phi$, $B \cap C = \Phi$, we have the model



$I(A) = \log_2 |A| = 1$ as in Example 1.

While $\bigcup_d^s(A) = \frac{\log_2 |A|}{\log_2 |A \cup B \cup C|} = \frac{\log_2 2}{\log_2 4} = \frac{1}{2} = 0.5 < 0.63093$

It is normal to have a smaller degree of uncertainty of set A when the frame of discernment is larger, since herein the total ignorance has a bigger cardinal.

Generalized Hartley Measure of uncertainty for masses is defined as:

$$GH(m) = \sum_{A \in 2^\theta \setminus \{\Phi\}} m(A) \log_2 |A|$$

In DST we simply extend it in $DSmT$ as:

$$GH(m) = \sum_{A \in G^\theta \setminus \{\Phi\}} m(A) \log_2 |A|$$

Degree of Uncertainty of a mass.

We go further and define a degree of uncertainty of a mass m as

$$\bigcup_d^M(m) = \sum_{A \in G^\theta \setminus \{\Phi\}} m(A) \cdot \frac{\log_2 |A|}{\log_2 |I_t|}$$

where I_t is the total ignorance.

If $m(\cdot)$ is a mass whose focal elements are only singletons then $\bigcup_d^M(m) = 0$ (minimum uncertainty degree of a mass).

If $m(I_t) = 1$, then $\bigcup_d^M(m) = 1$ (maximum uncertainty degree of a mass).

For all other masses $m(\cdot)$ we have $0 < \bigcup_d^M(m) < 1$.

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