Generalizations in Extenics of the Location Value and Dependent Function from A Single Finite Interval to *2D*, *3D*, and *n-D* Spaces

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Abstract.
Qiao-Xing Li and Xing-Sen Li [1] have defined in *2011* the Location Value of a Point and the Dependent Function of a Point on a single finite or infinite interval. In this paper we extend their definitions from one dimension (*1D*) to *2D, 3D*, and in general *n-D* spaces.
Several examples are given in *2D* and *3D* spaces.

1. Short Introduction to Extenics.

In this paper we make a short description of Extenics, and then we present an extension of the Location Value of a Point and the Dependent Function of a Point from *1D* to *n-D*, with several examples in the particular cases of *2D* and *3D* spaces. Improvement of the Extenics website is given towards the end, followed by an Extenics what-to-do list.

Extenics is a science initiated by Professor Cai Wen in *1983*. It is at the intersection of mathematics, philosophy, and engineering. Extenics solves contradictory problems. It is based on modeling and remodeling, on transforming and retransforming until getting a reasonable solution to apparently an unreasonable problem.

Extenics solves unconventional and non-traditional problems and finding ingenious, perspicacious and novelty solutions.

Extenics helps in solving problems in hard conditions, incomplete conditions, conflicting conditions. Where mathematics doesn't work, i.e. for inconsistent problems where mathematics says that there is no solution, Extenics does work because it can obtain a solution.

Everything is dynamic; we have dynamic structure, dynamic classification, and dynamic change.

In Extenics a problem may have more solutions, some of them even contradictory with each other, but all of them can be valid solutions.

The five basic transformations are: substitution, increasing/decreasing, expansion/contraction, decomposition, and duplication.

Extenics studies:

- the antithetic properties of the matter: physical part (real) and non-physical part (imaginary), soft and hard parts of the matter, negative and positive parts of the matter;
- unfeasible problems are transformed to feasible problems;
- false propositions are transformed in true propositions;
- wrong inference is transformed into correct inference;
- transform non-conformity to conformity;
- in business non –customers are transformed to customers;
- there are qualitative and quantitative transformations;
- transformation of matter-element, transformation of affair-element, transformation of relation-element;
- transformation of the characteristics;
- one considers transformation of a single part too (not of the whole);
- Extenics deals with unconventional problems which are transformed into conventional;
- inconsistent problems are transformed into consistent;
- also one determines the composability and conductivity of transformations;
- Extenics finds rules and procedures of solving contradictory problems;
- get structures and patterns to deal with contradictions;
- get new methods of solving contradictions;
- reduces the degree of inconsistency of the problems;
- from divergent to less-divergent.

2. Location Value of a Point and the Dependent Function on a Single Finite Interval (on 1D-Space).

Suppose $S = <a, b>$ is a finite interval. By the notation $<a, b>$ one understands any type of interval: open (a, b), closed $[a, b]$, or semi-open/semi-closed $(a, b]$ and $[a, b)$.

a) For any real point $x_0 \in R$, Qiao-Xing Li and Xing-Sen Li have considered

$$D(x_0, S) = a\text{-}b \qquad (1)$$

as the <u>location value</u> of point $P(x_0)$ on the single finite interval $<a, b>$.
Of course $D(x_0, S) = D(P, S) < 0$, since $a < b$.
As we can see, a-b is the negative distance between the frontiers of the single finite interval S in the $1D$-space.

b) Afterwards, the above authors defined for any real point $P(x_0)$, with $x_0 \in S$, the elementary dependent function on the single interval S in the following way:

$$k(x_0) = \frac{\rho(x_0, S)}{D(x_0, S)} \qquad (2)$$

where $\rho(x_0, S)$ is the extension distance between point x_0 and the finite interval X in the *1D*-space. Or we can re-write the above formula as:

$$k(P) = \frac{\rho(P, S)}{D(P, S)}. \qquad (3)$$

3. We have introduced in [2] the **Attraction Point Principle**, which is the following:

Let S be a given set in the universe of discourse U, and the optimal point $O \in S$. Then each point $P(x_1, x_2, \ldots, x_n)$ from the universe of discourse tends towards, or is attracted by, the optimal point O, because the optimal point O is an ideal of each other point. There could be one or more linearly or non-linearly trajectories (curves) that the same point P may converge on towards O. Let's call all such points' trajectories as the **Network of Attraction Curves (NAC)**.

4. Generalizations of the Location Value of a Point and the Dependent Function on a Single Finite Set on the n-D-Space.

In general, in a universe of discourse U, let's have an *n-D*-set S and a point $P \in U$.

a) The **Generalized Location Value of Point *P* on the Single Finite Set *S* in *n-D* Space,** $D_{nD}(x_0, S)$, is the classical geometric distance (yet taken with a negative sign in front of it) between the set frontiers, distance taken on the line (or in general taken on the curve or geodesic) passing through the optimal point O and the given point P.
In there are many distinct curves passing through both O and P in the Network of Attraction Curves, then one takes that curve for which one gets the maximum geometric distance (and one assigns a negative sign in front of this distance).
We can also denote it as $D_{nD}(P, S)$.

b) We geometrically studied the *1D*-Extension Distance $\rho(x_0, S)$ in our first Extenics paper [2] and we found out that the following principle was used by Prof. Cai Wen in *1983*:

$\rho(x_0, S)$ = the classical geometric distance between the point x_0 and the closest extremity point of the interval <a, b> to it (going in the direction that connects x_0 with the optimal point), distance taken as negative if $x_0 \in Int(<a, b>)$, as positive if $x_0 \in Ext(<a, b>)$, and as zero if $x_0 \in Fr(<a, b>)$,
where Int(<a, b>) = interior of <a, b>,
Ext(<a, b>) = exterior of <a, b>,
and Fr(<a, b>) = frontier of <a, b>. (4)

Thus we have defined the **Generalized Extension Linear/Non-Linear *n-D*-Distance** between point P and set S, as:

$$\rho_{nD}(P,S)=\begin{cases}-\max\limits_{c\in NAC}\ \underset{P'\in Fr(S)}{d}(P,P';c), & P\neq O, P\in c(OP');\\ \max\limits_{c\in NAC}\ \underset{P'\in Fr(S)}{d}(P,P';c), & P\neq O, P'\in c(OP);\\ -\max\limits_{c\in NAC, M\in Fr(S), M\in c(O)} d(P,M;c), & P=O.\end{cases} \tag{5}$$

where $\rho_{nD}(P,S)$ means the extension distance as measured along the curve *c* in the *n-D* space;
O is the optimal point (or non-linearly attraction point);
the points are attracting by the optimal point *O* on trajectories described by an injective curve *c*;
d(P,P';c) means the non-linearly *n-D*-distance between two points *P* and *P'* along the curve *c*, or the arclength of the curve *c* between the points *P* and *P'*;
Fr(S) means the frontier of set *S*;
and c(*OP')* means the curve segment between the points *O* and *P'* (the extremity points *O* and *P'* included), therefore *P*∈ *c(OP')* means that *P* lies on the curve *c* in between the points *O* and *P'*.
For *P* coinciding with *O*, one defined the distance between the optimal point *O* and the set *S* as the negatively maximum curvilinear distance (to be in concordance with the *1D*-definition).
In the same way, if there are many curves, *c* in the Network of Attraction Curves, passing through both *O* and *P,* then one chooses that curve which maximizes the geometric distance.

We do these maximizations in order to be consistent with the case when the point *P* coincides with the optimal point *O*.

We now proceed to defining the **Generalized Dependent Function on a Single Finite Set** ***S*** **in** ***n-D*-Space of Point *P***:

$$k_{nD}(P)=\max_{c\in NAC}\frac{\rho_{nD}(P,S;c)}{D_{nD}(P,S;c)} \tag{6}$$

or using words: the Generalized Dependent Function on a Single Finite Set *S* of point *P* is the geometric distance between point *P* and the closest frontier on the line (or in general on the curve/geodesic c that connects *P* with the optimal point *O)* in the same side of the optimal point, divided by the distance [taken along the line (or in general on the curve/geodesic c that connects P with the optimal point *O*)] between the set frontiers.
If there are more curves passing through *P* and *O*, then one takes that curve which maximizes the value of $k_{nD}(P)$.

5. Examples of 2D-Dependent Function on a Single Finite Set.

Let's retake a previous example with two rectangles, $A_0M_0B_0N_0$ and *AMBN,* whose sides are parallel to the axes of the Cartesian system of coordinates, such that they have no common end points, and $A_0M_0B_0N_0 \subset AMBN$. The optimal point is *O* located in their center of symmetry.

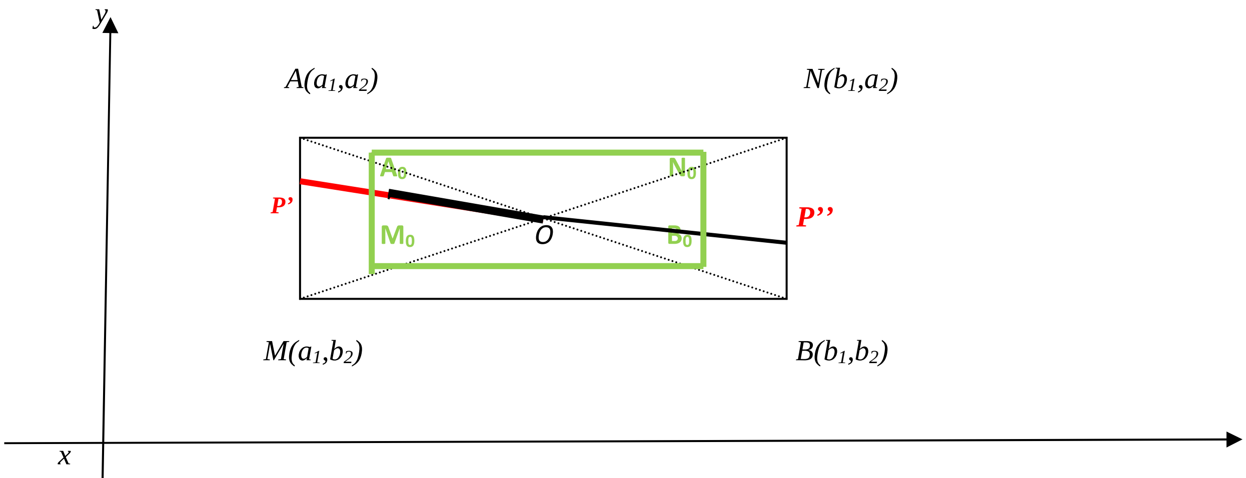

Fig. 1. The small rectangle shrinks until it vanishes.

If there is only a single finite set *AMBN*, this means that the other set $A_0M_0B_0N_0$ (which is included in *AMBN*) is shrinking little by little until it vanishing, thus the *(0, 1)* value of the dependent function of two nested sets increases until occupying the whole interior of the big set *AMBN*:

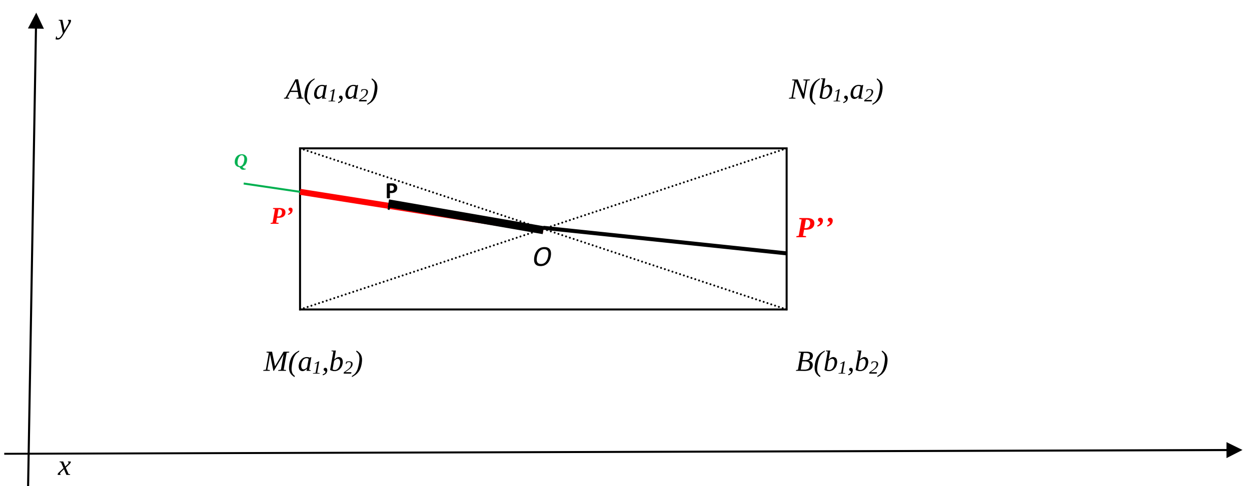

Fig. 2. The Dependent Function of a Point on a Single Rectangle.

The dependent function of interior point *P* with respect to the single rectangle *AMBN* is:

$$k(P) = +\frac{|PP'|}{|P''P'|} \qquad (7)$$

i.e. the distance between P and the closest frontier of the rectangle *{ =|PP'| }*, divided by the distance between the frontiers of the rectangle *{ =|P''P| }*.
The dependent function of exterior point *Q* with respect to the single rectangle *AMBN* is:

$$k(Q) = -\frac{|QP'|}{|P"P'|}. \tag{8}$$

And the dependent function of frontier point *P'* with respect to the single rectangle *AMBN* is:

$$k(P') = \frac{|P'P'|}{|P"P'|} = 0. \tag{9}$$

In this example we have considered only one curve of convergence for each point in the Network of Attraction Curves.

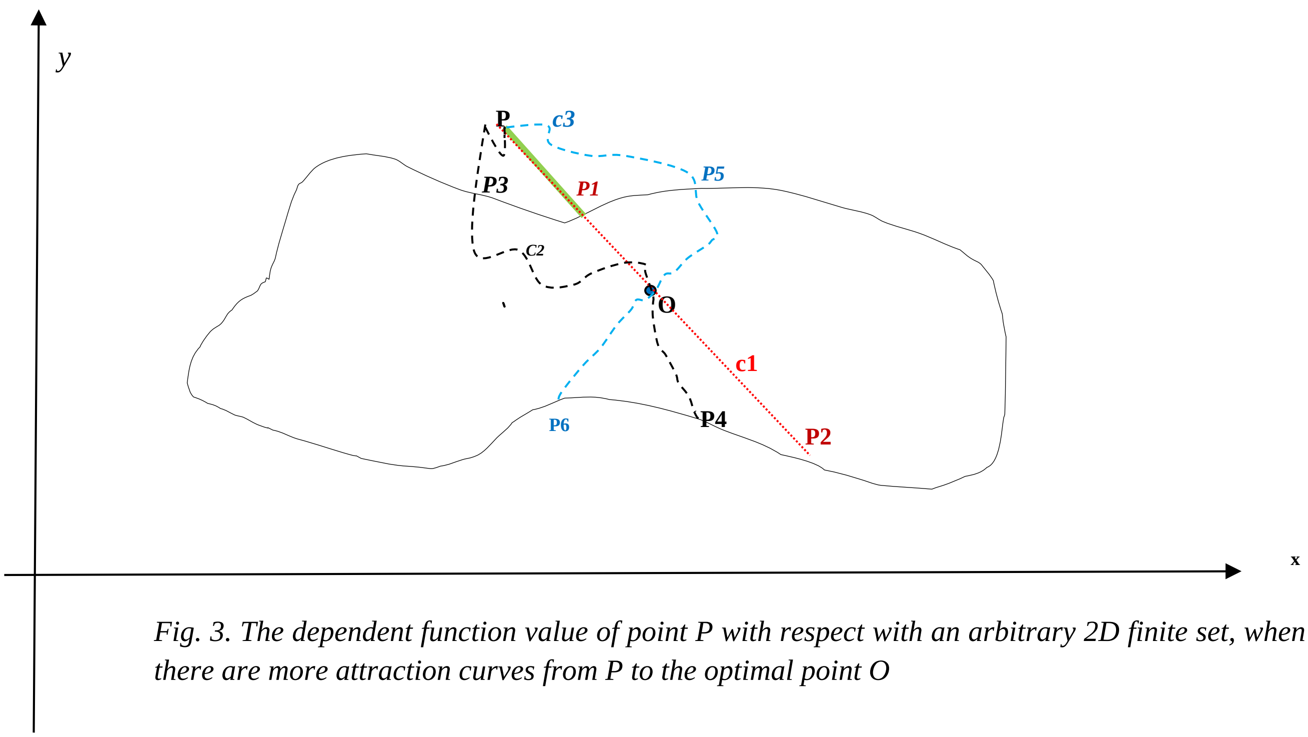

Fig. 3. The dependent function value of point P with respect with an arbitrary 2D finite set, when there are more attraction curves from P to the optimal point O

The dependent function value of point *P* is:

$$k(P) = -\max\{\frac{c1(PP1)}{c1(P1P2)} = \frac{|PP1|}{|P1P2|}, \frac{c2(PP3)}{|c2(P3P4)}, \frac{c3(PP5)}{c3(P5P6)}\} \tag{10}$$

where *c1(PP1)* means the arclength between the points *P* and *P1* on the curve *c1* (which happens in this case to be just a line segment), and similarly *c2(.,.)* and *c3(.,.)*.

6. Example of 3D-Dependent Function on a Single Finite Set.

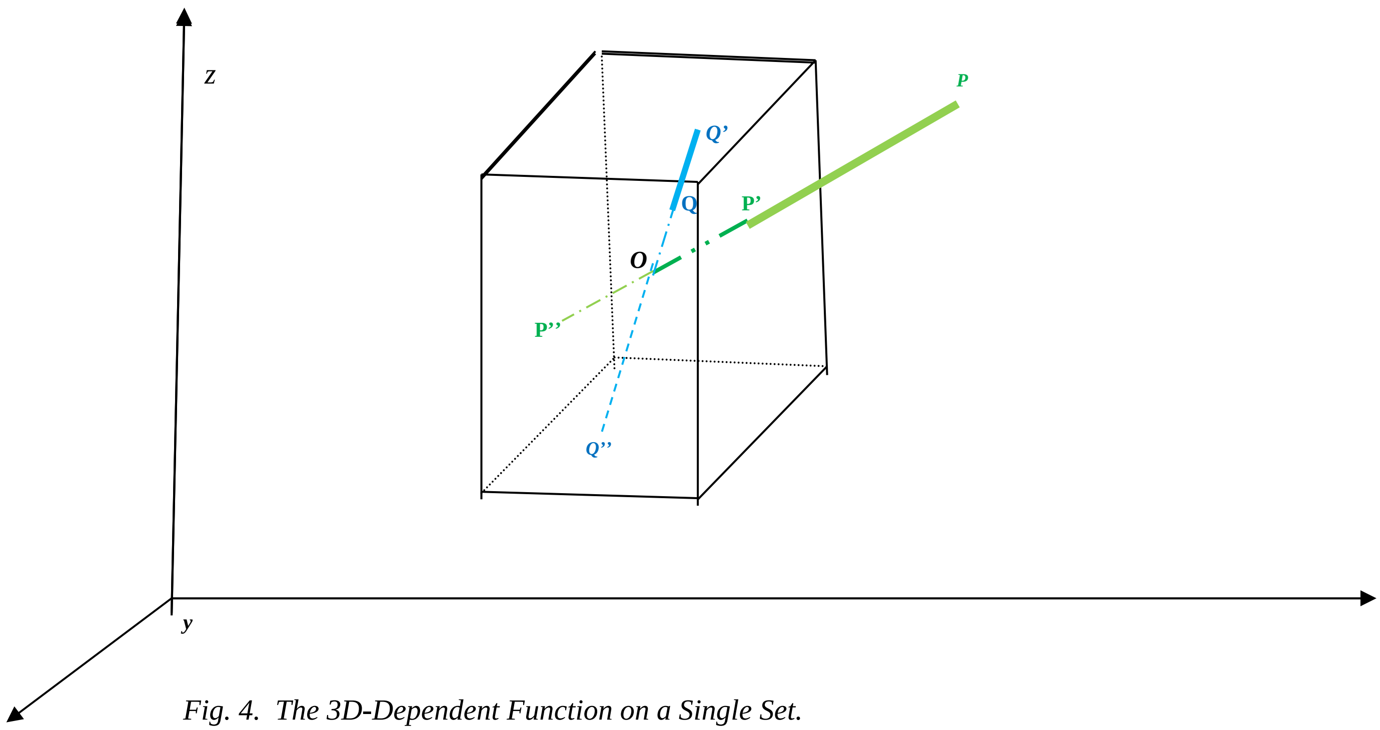

Fig. 4. The 3D-Dependent Function on a Single Set.

The dependent values on the single 3D-set is calculated for the following points:

$$k(P) = -\frac{|PP'|}{|P"P'|}, k(Q) = +\frac{|QQ'|}{|Q"Q'|}, k(P') = k(Q') = 0. \tag{11}$$

7. Extenics Web Development.

Web Developer Mihai Liviu Smarandache worked for the Research Institute of Extenics and Innovation Methods, the Guangdong University of Technology, from Guangzhou, P. R. China, in August 2012, together with Prof. Cai Wen, Prof. Xingsen Li, Prof. Weihua Li, Prof. Xiaomei Li, Prof. Yang Chunyan, Prof. Li Qiao-Xing, Prof. Florentin Smarandache, Research Assistant Jianming Li, and graduate student Zhiming Li.

a) He proposed the improvement of the Extenics website's layout to provide a better user experience. Currently, the Extenics website has many articles that the user can click on and that will show up on the page. If the user wants to save or print the articles they will have to copy and paste the article text into Microsoft Word and print.
The new layout gives the user the ability to click on the article and have it export directly to a *pdf* file for easy processing (saving/printing).
The new layout also provides a contact Extenics page where the user can send a direct email to the Extenics department.

b) He proposed the including of an interactive Tutorial on Extenics, so more people around the globe learn about it. The tutorial can have games that the user can play as part of the

Extenics learning. The tutorial can also include tests to test the user to see how much they have learned.

c) He proposed an Automatic Email Sending, such that when new Extenics publications, presentations, conferences, events occur an automatic email system will send the new information to all Extenics members.
   He wrote the code and incorporated it into the Extenics control panel, but the *php* mail function was not supported. He checked the version of *php* installed on the server and it was *5.2*, which is a very old version. He suspected that this could be the problem as to why the emails are not working.
d) He also proposed a Calendar of Extenics Events to be incorporated to the Extenics website. This calendar can let an Extenics administrator add important upcoming events from a control panel. The user can visit the Extenics website and click on the calendar and view the events. If a user chooses to do so, the website can send email reminders about these events.

8. Extenics What-to-do List.

- So far there have been done applications of Extenics in one-dimensional space. Now there are needed generalizations of the applications of Extenics in *2D, 3D,* and in general in *n-D* spaces in all previous fields done in *1D* space: i.e. in data mining, control theory, management, design, information theory, etc.
  One has to use the <u>*n-D* extension distance</u> between a point and a set, and the <u>*n-D* extension dependent function</u> of a point with respect to a nested set without common ending points and with common ending points.
- Single infinite interval dependent function to be generalized from one-dimensional space to *2D, 3D* and in general *n-D* spaces.
- Applications of Extenics if possible in new fields not yet approached in the one-dimensional space yet, such as: in physics, chemistry, biology, geology, etc.
- More software related to Extenics.
- Tutorials related to Extenics.
- Improving the Extenics website. Introducing the automatic email.
  Also, adding more papers and books (especially in English) to the Extenics website.

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