Exploration

From Acoustic Analog of Space to Acoustic Sachs-Wolfe Theorem: A Model of the Universe as a Guitar

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ABSTRACT

It has been known for long time that the cosmic sound wave was there since the early epoch of the Universe. Signatures of its existence are abundant. However, such an acoustic model of cosmology is rarely developed fully into a complete framework from the notion of space up to the sky. This paper may be the first attempt towards such a complete description of the Universe based on classical wave equation of sound. It is argued that one can arrive at a consistent description of space, elementary particles, Sachs-Wolfe acoustic theorem, starting from this simple classical wave equation of sound. We also discuss a plausible extension of Acoustic Sachs-Wolfe theorem based on its analogue with Klein-Gordon equation to a new equation. It is our hope that the new proposed equation can be verified with observation data. But we admit that our model is still in its infancy, more researches are needed to fill all the missing details.

Keywords: Acoustic metric, acoustic analogue of space, acoustic cosmology, Sachs-Wolfe theorem.

1. Introduction

In one of his papers, the late C.K. Thornhill wrote [1]:

> Relativists and cosmologists regularly refer to space-time without specifying precisely what they mean by this term. Here the two different forms of spacetime, real and imaginary, are introduced and contrasted. It is shown that, in real space-time (x, y, z, ct), Maxwell's equations have the same wave surfaces as those for sound waves in any uniform fluid at rest, and thus that Maxwell's equations are not general and invariant but, like the standard wave equation, only hold in one unique frame of reference. In other words, Maxwell's equations only apply to electromagnetic waves in a uniform ether at rest. But both Maxwell's equations and the standard wave equation, and their identical wave surfaces, transform quite properly, by Galilean transformation, into a

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> general invariant form which applies to waves in any uniform medium moving at any constant velocity relative to the reference-frame. It was the mistaken idea, that Maxwell's equations and the standard wave equation should be invariant, which led, by a mathematical freak, to the Lorentz transform (which demands the non-ether concept and a universally constant wave-speed) and to special relativity. The mistake was further compounded by misinterpreting the differential equation for the wave hypercone through any point as the quadratic differential form of a Riemannian metric in imaginary space-time (x, y, z, ict). Further complications ensued when this imaginary space-time was generalised to encompass gravitation in general relativity.

In this paper, we will start with a simple premise that the space itself has an acoustic origin and it relates to Maxwell equations. Maxwell equations can be expressed in terms of vortex sound equation. So it will indicate a new interpretation of aether in acoustic terminology.

It is argued that, starting from this simple classical wave equation of sound, one can arrive at a consistent description of space, elementary particles and Sachs-Wolfe acoustic theorem. We also discuss a plausible extension of Acoustic Sachs-Wolfe theorem to a new equation based on its analogue with Klein-Gordon equation.

It is our hope that the proposed new equation can be verified with observation data. It should be noted that this model is still in its infancy.

2. Acoustic Analogue of Space

In this section, we borrow some important ideas from C.K. Thornhill and also Tsutomu Kambe. According to Thornhill, real space-time is a four dimensional space consisting of three-dimensional space plus a fourth length dimension obtained by multiplying time by a constant speed. (This is usually taken as the constant wave-speed c of electromagnetic waves). If the four lengths, which define a four-dimensional metric (x, y, z, ict), are thought of as measured in directions mutually at right-angles, then the quadratic differential form of this metric is [1]:

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - \bar{c}^2 (dt)^2 \tag{1}$$

When the non-differential terms are removed from Maxwell's equations, i.e. when there is no charge distribution or current density, it can easily be shown that the components (E1 ,E2 ,E3) of the electrical field-strength and the components (H1 ,H2 ,H3) of the magnetic field-strength all satisfy the standard wave equation:[1]

$$\nabla \phi = \left(\frac{1}{\bar{c}^2} \right) \frac{\partial^2 \phi}{\partial t^2} \tag{2}$$

It follows immediately, therefore, that the wave surfaces of Maxwell's equations are exactly the same as those for sound waves in any uniform fluid at rest, and that Maxwell's equations can only hold in one unique reference-frame and should not remain invariant when transformed into any other reference-frame. In particular, the equation for the envelope of all wave surfaces which pass through any point at any time is, for equation (2), and therefore also for Maxwell's equations,[1]

$$(dx)^2 + (dy)^2 + (dz)^2 = \bar{c}^2 (dt)^2 \tag{3}$$

or

$$\frac{(dx)^2}{(dt)^2} + \frac{(dy)^2}{(dt)^2} + \frac{(dz)^2}{(dt)^2} = \bar{c}^2 \tag{4}$$

It is by no means trivial, but it is, nevertheless, not very difficult to show, by elementary standard methods, that the general integral of the differential equation (4), which passes through (x1, y1, z1) at time t1, is the right spherical hypercone [1]:

$$(x - x_1)^2 + (y - y\ \ )^2 + (z - z_1)^2 = \bar{c}^2 (t - t_1)^2 \tag{5}$$

In other words, both Maxwell equations and space itself has the sound wave origin. We shall see later that this interpretation of Thornhill's work is consistent with the so-called acoustic Sachs-Wolfe theorem which is known in cosmology setting.

It is also interesting to remark here that Maxwell equations can be cast in the language of vortex sound theory, as follows.

T. Kambe from University of Tokyo has made a connection between the equation of vortex sound and fluid Maxwell equations. He wrote that it would be no exaggeration to say that any vortex motion excites *acoustic* waves. He considers the equation of vortex sound of the form: [2]:

$$\frac{1}{c^2}\partial_t^2 p - \nabla^2 p = \rho_0 \nabla . L = \rho_0 div(\omega \times v) \tag{6}$$

He also wrote that dipolar emission by the vortex-body interaction is:[3]

$$p_F(x,t) = -\frac{P_0}{4\pi c}\dot{\Pi}_i (t - \frac{x}{c})\frac{x_c}{x^2} \tag{7}$$

Then he obtained an expression of fluid Maxwell equations as follows [4]:

$$\begin{aligned}
&\nabla \cdot H = 0 \\
&\nabla \cdot E = q \\
&\nabla \times E + \partial_t H = 0 \\
&a_0^2 \nabla \times H - \partial_t E = J
\end{aligned} \tag{8}$$

where [4] a_0 denotes the sound speed, and

$$\begin{aligned}
&q = -\partial_t (\nabla \cdot \upsilon) - \nabla \hbar, \\
&J = \partial_t^2 v + \nabla \partial_t h + a_o^2 \nabla \times (\nabla \times \upsilon)
\end{aligned} \tag{9}$$

In our opinion, this new expression of fluid Maxwell equations suggests that there is a deep connection between vortex sound and electromagnetic fields. However, it should be noted that the above expressions based on fluid dynamics need to be verified with experiments. We should note also that in (8) and (9), the speed of sound a_0 is analogous of the speed of light in Maxwell equations, whereas in equation (6), the speed of sound is designated "c" (as analogous to the light speed in EM wave equation). For alternative hydrodynamics expression of electromagnetic fields, see [7].

The above interpretation of fluid Maxwell equations from vortex sound theory has been discussed in our recent paper, to appear in forthcoming issue of JCMNS [5].

3. Comparison between Schrödinger equation and Classical wave equation of sound

In the initial variant, the Schrodinger equation (SE) has the following form [8]:

$$\Delta\Psi + \frac{2m}{\hbar^2}\left(W + \frac{e^2}{4\pi\varepsilon_o r}\right)\Psi = 0 \tag{10}$$

The wave function satisfying the wave equation (10) is represented as:

$$\Psi = R(r)\Theta(\theta)\Phi(\varphi)T(t) = \psi(r,\theta,\varphi)T(t) \tag{11}$$

where $\psi(r,\theta,\varphi) = R(r)\Theta(\theta)\Phi(\varphi)$ is the complex amplitude of the wave function, because

$$\Phi_m(\varphi) = C_m e^{\pm im\varphi} \tag{12}$$

For standard method of separation of variables to solve spherical SE, see for example [11-13].

The Φ, Θ and T equations were known in the theory of wave fields. Hence these equations presented nothing new. Only the R was new. Its solution turned out to be *divergent*. However, Schrödinger together with H. Weyl (1885-1955), contrary to the logic of and all experience of theoretical physics, artificially cut off the divergent power series of the radial function R(r) at a κ-th term. This allowed them to obtain the radial solutions, which, as a result of the cut off operation, actually were the fictitious solutions [8].

Furthermore, it can be shown that the time-independent SE [9][10]:

$$\nabla\Psi + \frac{2m}{\hbar^2}(E-V)\Psi = 0, \tag{13}$$

can be written in the form of standard wave equation [8]:

$$\nabla\Psi + k^2\Psi = 0, \tag{14}$$

where

$$k = \pm\sqrt{\frac{2m}{\hbar^2}(E-V)}\,. \tag{15}$$

or if we compare (14) and (10), then we have [8]:

$$k = \pm\sqrt{\frac{2m}{\hbar^2}\left(W + \frac{e^2}{4\pi\varepsilon_o r}\right)}\,. \tag{16}$$

This means that the wave number k in Schrödinger's radial wave equation is a quantity that varies continuously in the radial direction. Is it possible to imagine a field where the wave number, and hence the frequency, change from one point to another in the space of the field? Of course, it is not possible. Such wave objects do not exist in Nature.

The unphysical nature of Schrödinger wave-function has created all confusing debates throughout 90 years. But it is rarely discussed in QM textbooks, on how he arrived at his equation. It is known that Schrodinger began with Einstein's mass-energy relation then he proceeded with Hamilton-Jacobian equation. At first he came to a similar fashion of Klein-Gordon equation, but then he arrived to a new equation which is non-relativistic. Logically speaking, he began with a relativistic assumption and he came to a nonrelativistic expression, and until now physicists remain debating on how to relativize Schrodinger equation. That is logically inconsistent and therefore unacceptable, and Schrodinger himself never knew where the problem lies. Until now people remain debating the problem of the meaning of his wavefunction, but it starts with unphysical nature of his equation. This is a common attitude of many young physicists who tend to neglect the process and logical implication of QM derivation, and they never asked about whether Schrodinger equation has deep logical inconsistency or not.

Moreover, there are some limitations in applying Schrödinger equation to experiments, although many textbooks on QM usually overlook existing problems on how to compare 3D spherical solution of Schrodinger equation with experimental data. The contradiction between QM and experiments are never discussed publicly, and this is why the most modern physicists hold the assertion that QM describes accurately "ALL" physical experiments; that is an unfounded assumption. George Shpenkov began with classical wave equation and he is able to derive a periodic table of elements which is very close to Mendeleev's table. And this is a remarkable achievement which cannot be done with standard wave mechanics.[1]

Nonetheless, equation (14) and (15) which suggests analogy between wave mechanics and sound wave equation has been discussed briefly by Hilbert & Batelaan [14]. And it seems worthy to explore further in experiments.

4. Derivation of Klein-Gordon equation from the Classical Wave equation

It is also possible to find theoretical correspondence between classical electromagnetic wave equation and Klein-Gordon equation. Such a correspondence has been discussed by David Ward & Sabine Volkmer [15]. They give a simple derivation of the KGE, which requires only knowledge of the electromagnetic wave equation and the basics of Einstein's special theory of relativity.

They begin with electromagnetic wave equation in one dimensional case:

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \,. \tag{17}$$

This equation is satisfied by plane wave solution:

$$E(x,t) = E_0 e^{i(kx-\omega t)} \,, \tag{18}$$

Where $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$ are the spatial and temporal frequencies, respectively. Substituting equation (18) into (17), then we obtain:

[1]For further discussion, it is advisable to check the website of Dr. George Shpenkov, at http://shpenkov.janmax.com. See especially Shpenkov, George P. 2013. *Dialectical View of the World: The Wave Model (Selected Lectures).* Volume I: Philosophical and Mathematical Background. URL: http://shpenkov.janmax.com/Vol.1.Dialectics.pdf

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_0 e^{i(kx-\omega t)}=0 \tag{19}$$

or

$$\left(k^2-\frac{\omega^2}{c^2}\right)E_0 e^{i(kx-\omega t)}=0 \tag{20}$$

Solving the wave vector, we arrive at dispersion relation for light in free space: $k=\frac{\omega}{c}$. Note that this is similar to wave number k in equation (14).

Then, recall from Einstein and Compton that the energy of a photon is $\varepsilon=h\nu=\hbar\omega$ and the momentum of a photon is $p=\frac{h}{\lambda}=\hbar k$. We can rewrite equation (18) using these relations:

$$E(x,t)=E_0 e^{\frac{i}{\hbar}(px-\varepsilon t)}, \tag{21}$$

Substituting this equation into (17) we find:

$$-\frac{1}{\hbar^2}\left(p^2-\frac{\varepsilon^2}{c^2}\right)E_0 e^{\frac{i}{\hbar}(px-\varepsilon t)}=0 \tag{22}$$

Then we get an expression of relativistic total energy for a particle with zero rest mass:

$$\varepsilon^2=p^2c^2. \tag{23}$$

We now assume with de Broglie that frequency and energy, and wavelength and momentum, are related in the same way for classical particles as for photons, and consider a wave equation for non-zero rest mass particles. So we want to end up with:

$$\varepsilon^2=p^2c^2+m^2c^4. \tag{24}$$

Inserting this equation (24) into equation (22), it is straightforward from (19), that we get:

$$\left(\nabla^2-\frac{m^2c^2}{\hbar^2}\right)\Psi=\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} \tag{25}$$

which is the Klein-Gordon equation for a free particle [15].

Having derived KGE from classical electromagnetic wave equation, now we are ready to discuss its implication in description of elementary particles. This will be discussed in the next section.

Interestingly, it can be shown that by using KGE one can describe hydrogen atom including electron spin without having to resort to the complicated Dirac equation [16]. It also appears worthnoting here that Meessen workout a description of elementary particles from excitation of spacetime, by starting from KGE and a novel assumption of quantized spacetime dx=n.a.[17] However, we will not discuss Ducharme's and Meessen's approach here, instead we will put more attention on how to extend Acoustic Sachs-Wolfe theorem by virtue of KGE.

5. Acoustic Sachs-Wolfe theorem and its plausible extension

According to Czaja, Golda, and Woszczyna [19], if one considers the acoustic field propagating in the radiation-dominated ($p=\varepsilon/3$) universe of arbitrary space curvature ($K=0,\pm1$), then the field equations are reduced to the d'Alembert equation in an auxiliary static Robertson-Walker spacetime. This is related to the so-called *Sachs-Wolfe acoustic* theorem, which can be found useful in the observation and analysis of Cosmic Microwave Background anisotropies.

In the meantime, there are papers suggesting that the integrated Sachs-Wolfe theorem may be useful to study dark energy, but we do not enter in such a discussion. See [22] for instance.
The Sachs–Wolfe acoustic theorem refers to the spatially flat ($K=0$), hot ($p=\varepsilon/3$) Friedmann–Robertson–Walker universe and the scalar perturbation propagating in it. The theorem states that with the appropriate choice of the perturbation variable, one can express the propagation equation in the form of d'Alembert's equation in Minkowski spacetime. Scalar perturbations in the flat, early universe propagate like electromagnetic or gravitational waves ([18], p. 79).
On the other hand, the wave equation for the scalar field of the dust ($p=0$) cosmological model can be transformed into the d'Alembert equation in the static Robertson–Walker spacetime, regardless of the universe's space curvature (see [18]). Therefore, we can suppose that the flatness assumption in the Sachs–Wolfe theorem is not needed and that the theorem is true in the general case. The proof of this fact, formulated as a symbolic computation, is presented in the first section of this paper.

In accordance with Czaja, Golda, and Woszczyna [19], we begin with Robertson–Walker metrics in spherical coordinates $x^\sigma=\{\eta,\chi,\vartheta,\phi\}$:

$$g_{(\mathcal{RW})} = a^2(\eta)\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\sin^2(\sqrt{K}\chi)}{K} & 0 \\ 0 & 0 & 0 & \frac{\sin^2(\sqrt{K}\chi)\sin^2(\vartheta)}{K} \end{bmatrix} \tag{26}$$

with the scale factor a(□) appropriate for the equation of state p=□/3,

$$a(\eta) = \frac{\sin(\sqrt{K}\chi)}{\sqrt{K}}. \tag{27}$$

Let us define a new perturbation variable □ with the help of the second-order differential transformation of the density contrast □,

$$\Psi(x^{\sigma}) = \frac{1}{\cos(\sqrt{K}\chi)} \frac{\partial}{\partial\eta} \left(\frac{K}{\tan^2(\sqrt{K}\chi)} \frac{\partial}{\partial\eta} \left(\frac{\tan^2(\sqrt{K}\chi)}{K} \cos(\sqrt{K}\chi) \delta(x^{\sigma}) \right) \right). \tag{28}$$

The function $\Psi(x^{\sigma})$ is the solution of the d'Alembert equation:

$$\frac{\partial^2}{\partial\eta^2} \Psi(x^{\sigma}) - \frac{1}{3} \Delta \Psi(x^{\sigma}) = 0, \tag{29}$$

with the Beltrami–Laplace operator Δ acting in this space,

$$^{(3)}g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sin^2(\sqrt{K}\chi)}{K} & 0 \\ 0 & 0 & \frac{\sin^2(\sqrt{K}\chi)\sin^2(\vartheta)}{K} \end{bmatrix}. \tag{30}$$

The Beltrami–Laplace operator Δ is defined as follow

$$\Delta = {}^{(3)}g_{mn} \nabla^m \nabla^n . \tag{31}$$

And it can be considered as an extension of Laplace operator for curved space.

Now let us discuss a basic question: what is Laplace-Beltrami operator? In differential geometry, the Laplace operator can be generalized to operate on functions defined on surfaces in Euclidean space and, more generally, on Riemannian and pseudo-Riemannian manifolds. This more general operator goes by the name Laplace-Beltrami operator, after Pierre-Simon Laplace and Eugenio Beltrami. Like the Laplacian, the Laplace-Beltrami operator is defined as the divergence of the gradient, and is a linear operator taking functions into functions. The operator can be extended to operate on tensors as the divergence of the covariant derivative. Alternatively, the operator can be generalized to operate on differential forms using the divergence and exterior derivative. The resulting operator is called the Laplace-de Rham operator (named after Georges de Rham).

Now, considering the formal equivalence between the form of (29) with KGE (25), minus the mass term, then it seems reasonable to include the mass term into (29). Then the extended version of equation (29) may be written as:

$$\frac{\partial^2}{\partial \eta^2}\Psi\left(x^\sigma\right)-\frac{1}{3}\Delta\Psi\left(x^\sigma\right)=-I\,\frac{m^2c^2}{\hbar^2}\Psi \quad , \tag{32}$$

where I is identity matrix as follows:

$$I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{33}$$

The above equations (32) and (33) can be considered as a plausible extension of Acoustic Sachs-Wolfe theorem based on its analogue with Klein-Gordon equation to become Acoustic Sachs-Wolfe-Christianto-Smarandache-Umniyati (ASWoCSU) equation. Its usefulness remains to be verified with observation data.

6. Discussion and Concluding Remarks

It has been known for long time that the cosmic sound wave was there since the early epoch of the Universe. Signatures of its existence are abound.[24] However, such an acoustic model of cosmology is rarely developed fully into a complete framework from the notion of space, cancer therapy up to the sky. This paper may be the first attempt towards such a complete description of the Universe based on classical wave equation of sound.

We have discussed how the very definition of Newtonian space can be related to sound wave and also Maxwell equations, and also how fluid Maxwell equations can be formulated based on vortex sound theory.

We have also discussed the inadequacies of Schrodinger equation as a description of elementary particles, instead we established connection from classical electromagnetic wave equation to Klein-Gordon equation.

Then we discuss Acoustic Sachs-Wolfe theorem which is worthy to investigate further in the context of cosmology. We also propose an extension of Acoustic Sachs-Wolfe to become a new equation. In other words, it appears very reasonable to model the Universe and Cosmos in general in terms of sound wave equation.

To summarize, in this paper we tried our best to offer a novel picture of the Universe as a guitar. Further observation and experiments are recommended to verify the above propositions.

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