

## I gpgt crk gf 'Kpwt xcnP gwt quqr j ke'Uqhw'Ugv'c'pf 'ku'F gekukqp'O cmlpi " Rt qdigo "

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*Cdwtcev'*– In this work, we introduce the concept of generalized interval neutrosophic soft set and study their operations. Finally, we present an application of generalized interval neutrosophic soft set in decision making problem.

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### 30 Kpvt qf wevkqp

Neutrosophic sets, founded by Smarandache [8] has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world. Neutrosophic set theory is a powerful tool which generalizes the concept of the classic set, fuzzy set [16], interval-valued fuzzy set [10], intuitionistic fuzzy set [13] interval-valued intuitionistic fuzzy set [14], and so on.

After the pioneering work of Smarandache, Wang [9] introduced the notion of interval neutrosophic set (INS) which is another extension of neutrosophic set. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval value (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set, and interval neutrosophic set provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information. The theory of neutrosophic sets and their hybrid structures has proven useful in many different fields such as control theory [25], databases [17,18], medical diagnosis problem [3,11], decision making problem [1,2,15,19,23,24,27,28,29,30,31,32,34], physics[7], and etc.

In 1999, a Russian researcher [5] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. Soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Recently, some authors have introduced new mathematical tools by generalizing and extending Molodtsov's classical soft set theory;

fuzzy soft set [22], vague soft set [35], intuitionistic fuzzy soft set [20], interval valued intuitionistic fuzzy set [36].

Similarity, combining neutrosophic set models with other mathematical models has attracted the attention of many researchers: neutrosophic soft set [21], intuitionistic neutrosophic soft set [26], generalized neutrosophic soft set [23], interval neutrosophic soft set [12].

Broumi et al. [33] presented the concept of rough neutrosophic set which is based on a combination of the neutrosophic set and rough set models. Recently, Şahin and Küçük [23] generalized the concept of neutrosophic soft set with a degree of which is attached with the parameterization of fuzzy sets while defining a neutrosophic soft set, and investigated some basic properties of the generalized neutrosophic soft sets.

In this paper our main objective is to extend the concept of generalized neutrosophic soft set introduced by Şahin and Küçük [23] to the case of interval neutrosophic soft set [12].

The paper is structured as follows. In Section 2, we first recall the necessary background on neutrosophic sets, soft set and generalized neutrosophic soft set. The concept of generalized interval neutrosophic soft sets and some of their properties are presented in Section 3. In Section 4, we present an application of generalized interval neutrosophic soft sets in decision making. Finally we conclude the paper.

## 40Rt gko kpct lgu

In this section, we will briefly recall the basic concepts of neutrosophic set, soft sets and generalized neutrosophic soft sets. Let  $U$  be an initial universe set of objects and  $E$  the set of parameters in relation to objects in  $U$ . Parameters are often attributes, characteristics or properties of objects. Let  $P(U)$  denote the power set of  $U$  and  $A \subseteq E$ .

## 40P gwt quqr j ke'Ugw'

**Fghplkqp'40'[8]** Let  $U$  be an universe of discourse. The neutrosophic set  $A$  is an object having the form  $A = \{ \langle x : u_A(x), w_A(x), v_A(x) \rangle : x \in U \}$ , where the functions  $u, w, v : U \rightarrow ]0^-, 1^+[$  define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element  $x \in U$  to the set  $A$  with the condition.

$$0^- \leq u_A(x) + w_A(x) + v_A(x) \leq 3^+$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of  $]0^-, 1^+[$ . So instead of  $]0^-, 1^+[$  we need to take the interval  $[0, 1]$  for technical applications, because  $]0^-, 1^+[$  will be difficult to apply in the real applications such as in scientific and engineering problems.

**Fghplkqp'40'[8]** A neutrosophic set  $A$  is contained in the other neutrosophic set  $B$ ,  $A \subseteq B$  iff  $\inf u_A(x) \leq \inf u_B(x)$ ,  $\sup u_A(x) \leq \sup u_B(x)$ ,  $\inf w_A(x) \geq \inf w_B(x)$ ,  $\sup w_A(x) \geq \sup w_B(x)$  and  $\inf v_A(x) \geq \inf v_B(x)$ ,  $\sup v_A(x) \geq \sup v_B(x)$  for all  $x \in U$ .

An INS is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the following, we introduce the definition of an INS.

**404 Kpwt xcrnP gwt quqr j le'Ugwu'**

**Fghpklqp'405'**[9] Let  $U$  be a space of points (objects) and  $\text{Int}[0,1]$  be the set of all closed subsets of  $[0,1]$ . An INS  $A$  in  $U$  is defined with the form

$$A = \{ \langle x, u_A(x), w_A(x), v_A(x) \rangle : x \in U \}$$

where  $u_A(x): U \rightarrow \text{int}[0,1]$ ,  $w_A(x): U \rightarrow \text{int}[0,1]$  and  $v_A(x): U \rightarrow \text{int}[0,1]$  with  $0 \leq \sup u_A(x) + \sup w_A(x) + \sup v_A(x) \leq 3$  for all  $x \in U$ . The intervals  $u_A(x), w_A(x)$  and  $v_A(x)$  denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of  $x$  to  $A$ , respectively.

For convenience,

if let  $u_A(x) = [u_A^-(x), u_A^+(x)]$ ,  $w_A(x) = [w_A^-(x), w_A^+(x)]$  and  $v_A(x) = [v_A^-(x), v_A^+(x)]$ , then

$$A = \{ \langle x, [u_A^-(x), u_A^+(x)], [w_A^-(x), w_A^+(x)], [v_A^-(x), v_A^+(x)] \rangle : x \in U \}$$

with the condition,  $0 \leq \sup u_A^+(x) + \sup w_A^+(x) + \sup v_A^+(x) \leq 3$  for all  $x \in U$ . Here, we only consider the sub-unitary interval of  $[0,1]$ . Therefore, an INS is clearly a neutrosophic set.

**Fghpklqp'406'**[9] Let  $A$  and  $B$  be two interval neutrosophic sets,

$$A = \{ \langle x, [u_A^-(x), u_A^+(x)], [w_A^-(x), w_A^+(x)], [v_A^-(x), v_A^+(x)] \rangle : x \in U \}$$

$$B = \{ \langle x, [u_B^-(x), u_B^+(x)], [w_B^-(x), w_B^+(x)], [v_B^-(x), v_B^+(x)] \rangle : x \in U \}$$

Then some operations can be defined as follows:

- (1)  $A \subseteq B$  iff  $u_A^-(x) \leq u_B^-(x), u_A^+(x) \leq u_B^+(x), w_A^-(x) \geq w_B^-(x), w_A^+(x) \geq w_B^+(x), v_A^-(x) \geq v_B^-(x), v_A^+(x) \geq v_B^+(x)$  for each  $x \in U$ .
- (2)  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$ .
- (3)  $A^c = \{ \langle x, [v_A^-(x), v_A^+(x)], [1 - w_A^+(x), 1 - w_A^-(x)], [u_A^-(x), u_A^+(x)] \rangle : x \in U \}$

**405'Uqhw'Ugwu'**

**Fghpklqp'407'**[5] A pair  $(F, A)$  is called a soft set over, where  $F$  is a mapping given by  $F : A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a mapping from parameters to the power set of  $U$ , and it is not a kind of set in ordinary sense, but a parameterized family of subsets of  $U$ . For any parameter  $e \in A$ ,  $F(e)$  may be considered as the set of  $e$ -approximate elements of the soft set  $(F, A)$ .

**Gzco riq'408'** Suppose that  $U$  is the set of houses under consideration, say  $U = \{h_1, h_2, \dots, h_5\}$ . Let  $E$  be the set of some attributes of such houses, say  $E = \{e_1, e_2, e_3, e_4\}$ , where  $e_1, e_2, e_3, e_4$  stand for the attributes “beautiful”, “costly”, “in the green surroundings” and “moderate”, respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set  $(F, A)$  that describes the “attractiveness of the houses” in the opinion of a buyer, say Thomas, may be defined like this:

$$F(e_1) = \{h_2, h_3, h_5\}, F(e_2) = \{h_2, h_4\}, F(e_4) = \{h_3, h_5\} \text{ for } A = \{e_1, e_2, e_4\}.$$

**406'P gwt quqr j le'Uqhv'Ugvu'**

**Fghplkqp'40'[21]** Let  $U$  be an initial universe set and  $A \subset E$  be a set of parameters. Let  $NS(U)$  denotes the set of all neutrosophic subsets of  $U$ . The collection  $(F, A)$  is termed to be the neutrosophic soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow NS(U)$

**Gzco riq'40'[21]** Let  $U$  be the set of houses under consideration and  $E$  is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider  $E = \{\text{beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}\}$ . In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe  $U$  given by  $U = \{h_1, h_2, \dots, h_5\}$  and the set of parameters

$A = \{e_1, e_2, e_3, e_4\}$ , where  $e_1$  stands for the parameter 'beautiful',  $e_2$  stands for the parameter 'wooden',  $e_3$  stands for the parameter 'costly' and the parameter  $e_4$  stands for 'moderate'. Then the neutrosophic set  $(F, A)$  is defined as follows:

$$(F, A) = \left( \begin{aligned} & \left( e_1 \left\{ \frac{h_1}{(0.5, 0.6, 0.3)}, \frac{h_2}{(0.4, 0.7, 0.6)}, \frac{h_3}{(0.6, 0.2, 0.3)}, \frac{h_4}{(0.7, 0.3, 0.2)}, \frac{h_5}{(0.8, 0.2, 0.3)} \right\} \right) \\ & \left( e_2 \left\{ \frac{h_1}{(0.6, 0.3, 0.5)}, \frac{h_2}{(0.7, 0.4, 0.3)}, \frac{h_3}{(0.8, 0.1, 0.2)}, \frac{h_4}{(0.7, 0.1, 0.3)}, \frac{h_5}{(0.8, 0.3, 0.6)} \right\} \right) \\ & \left( e_3 \left\{ \frac{h_1}{(0.7, 0.4, 0.3)}, \frac{h_2}{(0.6, 0.7, 0.2)}, \frac{h_3}{(0.7, 0.2, 0.5)}, \frac{h_4}{(0.5, 0.2, 0.6)}, \frac{h_5}{(0.7, 0.3, 0.4)} \right\} \right) \\ & \left( e_4 \left\{ \frac{h_1}{(0.8, 0.6, 0.4)}, \frac{h_2}{(0.7, 0.9, 0.6)}, \frac{h_3}{(0.7, 0.6, 0.4)}, \frac{h_4}{(0.7, 0.8, 0.6)}, \frac{h_5}{(0.9, 0.5, 0.7)} \right\} \right) \end{aligned} \right)$$

**407'kpygtxcrlP gwt quqr j le'Uqhv'Ugvu'**

**Fghplkqp'40'[12]** Let  $U$  be an initial universe set and  $A \subset E$  be a set of parameters. Let  $INS(U)$  denotes the set of all interval neutrosophic subsets of  $U$ . The collection  $(F, A)$  is termed to be the interval neutrosophic soft set over  $U$ , where  $F$  is a mapping given by  $F: A \rightarrow INS(U)$

**Gzco riq'402'[12]** Let  $U = \{x_1, x_2\}$  be set of houses under consideration and  $E$  is a set of parameters which is a neutrosophic word. Let  $E$  be the set of some attributes of such houses, say  $E = \{e_1, e_2, e_3, e_4\}$ , where  $e_1, e_2, e_3, e_4$  stand for the attributes  $e_1 = \text{cheap}$ ,  $e_2 = \text{beautiful}$ ,  $e_3 = \text{in the green surroundings}$ ,  $e_4 = \text{costly}$  and  $e_5 = \text{large}$ , respectively. Then we define the interval neutrosophic soft set  $A$  as follows:

$$(F, A) = \left( \begin{aligned} & \left( e_1 \left\{ \frac{x_1}{[0.5, 0.8], [0.5, 0.9], [0.2, 0.5]}, \frac{x_2}{[0.4, 0.8], [0.2, 0.5], [0.5, 0.6]} \right\} \right) \\ & \left( e_2 \left\{ \frac{x_1}{[0.5, 0.8], [0.2, 0.8], [0.3, 0.7]}, \frac{x_2}{[0.1, 0.9], [0.6, 0.7], [0.2, 0.3]} \right\} \right) \\ & \left( e_3 \left\{ \frac{x_1}{[0.2, 0.7], [0.1, 0.5], [0.5, 0.8]}, \frac{x_2}{[0.5, 0.7], [0.1, 0.4], [0.6, 0.7]} \right\} \right) \\ & \left( e_4 \left\{ \frac{x_1}{[0.4, 0.5], [0.4, 0.9], [0.4, 0.9]}, \frac{x_2}{[0.3, 0.4], [0.6, 0.7], [0.1, 0.5]} \right\} \right) \\ & \left( e_5 \left\{ \frac{x_1}{[0.1, 0.7], [0.5, 0.6], [0.1, 0.5]}, \frac{x_2}{[0.6, 0.7], [0.2, 0.4], [0.3, 0.7]} \right\} \right) \end{aligned} \right)$$

**48'I gpgt crk gf 'P gwt quqr j le'Uqlh'Ugwu'**

The concept of generalized neutrosophic soft is defined by Şahin and Küçük [23] as follows:

**Fghplwqp'483'**[23] Let  $U$  be an initial universe and  $E$  be a set of parameters. Let  $NS(U)$  be the set of all neutrosophic sets of  $U$ . A generalized neutrosophic soft set  $F^\mu$  over  $U$  is defined by the set of ordered pairs"

$$F^\mu = \{(F(e), \mu(e)) : e \in E, F(e) \in NS(U), \mu(e) \in [0, 1]\},$$

where  $F$  is a mapping given by  $F: E \rightarrow NS(U) \times I$  and  $\mu$  is a fuzzy set such that  $\mu: E \rightarrow I = [0, 1]$ . Here,  $F^\mu$  is a mapping defined by  $F^\mu: E \rightarrow NS(U) \times I$ .

For any parameter  $e \in E$ ,  $F(e)$  is referred as the neutrosophic value set of parameter  $e$ , i.e.,

$$F(e) = \{(x, u_{F(e)}(x), w_{F(e)}(x), v_{F(e)}(x)) : x \in U\}$$

where  $u, w, v : U \rightarrow [0, 1]$  are the memberships functions of truth, indeterminacy and falsity respectively of the element  $x \in U$ . For any  $x \in U$  and  $e \in E$ ,

$$0 \leq u_{F(e)}(x) + w_{F(e)}(x) + v_{F(e)}(x) \leq 3.$$

In fact,  $F^\mu$  is a parameterized family of neutrosophic sets over  $U$ , which has the degree of possibility of the approximate value set which is represented by  $\mu(e)$  for each parameter  $e$ , so  $F^\mu$  can be expressed as follows:

$$F^\mu(e) = \left\{ \left( \frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \dots, \frac{x_n}{F(e)(x_n)} \right), \mu(e) \right\}.$$

**Fghplwqp'484'**[4] A binary operation  $\otimes: [0,1] \times [0,1] \rightarrow [0,1]$  is continuous  $t$ -norm if  $\otimes$  satisfies the following conditions:

- (1)  $\otimes$  is commutative and associative,
- (2)  $\otimes$  is continuous,
- (3)  $a \otimes 1 = a, \forall a \in [0,1]$ ,
- (4)  $a \otimes b \leq c \otimes d$  whenever  $a \leq c, b \leq d$  and  $a, b, c, d \in [0,1]$ .

**Fghplwqp'485'**[4] A binary operation  $\oplus: [0,1] \times [0,1] \rightarrow [0,1]$  is continuous  $t$ -conorm if  $\oplus$  satisfies the following conditions:

- (1)  $\oplus$  is commutative and associative,
- (2)  $\oplus$  is continuous,
- (3)  $a \oplus 0 = a, \forall a \in [0,1]$ ,
- (4)  $a \oplus b \leq c \oplus d$  whenever  $a \leq c, b \leq d$  and  $a, b, c, d \in [0,1]$ .

### 50I gpgt cñk gf 'Kpvt xcñP gwt quqr j le'Uqlh'Ugv

In this section, we define the generalized interval neutrosophic soft sets and investigate some basic properties.

**Definition 50I** Let  $U$  be an initial universe and  $E$  be a set of parameters. Suppose that  $INS(U)$  is the set of all interval neutrosophic sets over  $U$  and  $\text{int}[0,1]$  is the set of all closed subsets of  $[0,1]$ . A generalized interval neutrosophic soft set  $F^\mu$  over  $U$  is defined by the set of ordered pairs

$$F^\mu = \{(F(e), \mu(e)) : e \in E, F(e) \in INS(U), \mu(e) \in [0, 1]\},$$

where  $F$  is a mapping given by  $F: E \rightarrow INS(U) \times I$  and  $\mu$  is a fuzzy set such that  $\mu: E \rightarrow I = [0, 1]$ . Here,  $F^\mu$  is a mapping defined by  $F^\mu: E \rightarrow INS(U) \times I$ .

For any parameter  $e \in E, F(e)$  is referred as the interval neutrosophic value set of parameter  $e$ , i.e.,

$$F(e) = \{(x, u_{F(e)}(x), w_{F(e)}(x), v_{F(e)}(x)) : x \in U\}$$

where  $u_{F(e)}, w_{F(e)}, v_{F(e)}: U \rightarrow \text{int}[0, 1]$  with the condition

$$0 \leq \sup u_{F(e)}(x) + \sup w_{F(e)}(x) + \sup v_{F(e)}(x) \leq 3$$

for all  $x \in U$ .

The intervals  $u_{F(e)}(x), w_{F(e)}(x)$  and  $v_{F(e)}(x)$  are the interval memberships functions of truth, interval indeterminacy and interval falsity of the element  $x \in U$ , respectively.

For convenience, if let

$$\begin{aligned} u_{F(e)}(x) &= [u_{F(e)}^L(x), u_{F(e)}^U(x)] \\ w_{F(e)}(x) &= [w_{F(e)}^L(x), w_{F(e)}^U(x)] \\ v_{F(e)}(x) &= [v_{F(e)}^L(x), v_{F(e)}^U(x)] \end{aligned}$$

then

$$F(e) = \{(x, [u_{F(e)}^L(x), u_{F(e)}^U(x)], [w_{F(e)}^L(x), w_{F(e)}^U(x)], [v_{F(e)}^L(x), v_{F(e)}^U(x)]) : x \in U\}$$

In fact,  $F^\mu$  is a parameterized family of interval neutrosophic sets on  $U$ , which has the degree of possibility of the approximate value set which is represented by  $\mu(e)$  for each parameter  $e$ , so  $F^\mu$  can be expressed as follows:

$$F^\mu(e) = \left\{ \left( \frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \dots, \frac{x_n}{F(e)(x_n)} \right), \mu(e) \right\}$$

**Example 50I** Consider two generalized interval neutrosophic soft set  $F^\mu$  and  $G^\theta$ . Suppose that  $U = \{h_1, h_2, h_3\}$  is the set of house and  $E = \{e_1, e_2, e_3\}$  is the set of parameters where  $e_1 = \text{cheap}, e_2 = \text{moderate}, e_3 = \text{comfortable}$ . Suppose that  $F^\mu$  and  $G^\theta$  are given as follows, respectively:

$$F^\mu(e_1) = \left( \frac{h_1}{([0.2, 0.3], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])} \right), \quad (0.2)$$

$$F^\mu(e_2) = \left( \frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])} \right), \quad (0.5)$$

$$F^\mu(e_3) = \left( \frac{h_1}{([0.2, 0.6], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])} \right), \quad (0.6)$$

and

$$G^\theta(e_1) = \left( \frac{h_1}{([0.1, 0.2], [0.1, 0.2], [0.1, 0.2])}, \frac{h_2}{([0.4, 0.5], [0.2, 0.3], [0.3, 0.5])}, \frac{h_3}{([0.6, 0.7], [0.1, 0.3], [0.2, 0.3])} \right), \quad (0.4)$$

$$G^\theta(e_2) = \left( \frac{h_1}{([0.2, 0.5], [0.3, 0.4], [0.2, 0.3])}, \frac{h_2}{([0.7, 0.8], [0.3, 0.4], [0.4, 0.6])}, \frac{h_3}{([0.3, 0.6], [0.2, 0.5], [0.4, 0.6])} \right), \quad (0.7)$$

$$G^\theta(e_3) = \left( \frac{h_1}{([0.3, 0.5], [0.1, 0.3], [0.1, 0.3])}, \frac{h_2}{([0.4, 0.5], [0.1, 0.5], [0.2, 0.3])}, \frac{h_3}{([0.7, 0.9], [0.2, 0.3], [0.1, 0.2])} \right), \quad (0.8)$$

For the purpose of storing a generalized interval neutrosophic soft sets in a computer, we can present it in matrix form. For example, the matrix form of  $F^\mu$  can be expressed as follows;

$$\begin{pmatrix} ([0.2, 0.3], [0.3, 0.5], [0.2, 0.3]) & ([0.3, 0.4], [0.3, 0.4], [0.5, 0.6]) & ([0.5, 0.6], [0.2, 0.4], [0.5, 0.7]) \\ ([0.1, 0.4], [0.5, 0.6], [0.3, 0.4]) & ([0.6, 0.7], [0.4, 0.5], [0.5, 0.8]) & ([0.2, 0.4], [0.3, 0.6], [0.6, 0.9]) \\ ([0.2, 0.6], [0.2, 0.5], [0.1, 0.5]) & ([0.3, 0.5], [0.3, 0.6], [0.4, 0.5]) & ([0.6, 0.8], [0.3, 0.4], [0.2, 0.3]) \end{pmatrix}, \quad (0.2)$$

**Definition 5050** A generalized interval neutrosophic soft set  $F^\mu$  over  $U$  is said to be generalized null interval neutrosophic soft set, denoted by  $\emptyset^\mu$ , if  $\emptyset^\mu: E \rightarrow IN(U) \times I$  such that

$$\emptyset^\mu(e) = \{(F(e), \mu(e))\}, \text{ where } F(e) = \{ \langle x, ([0, 0], [1, 1], [1, 1]) \rangle \} \text{ and } \mu(e) = 0 \text{ for each } e \in E \text{ and } x \in U.$$

**Definition 5060** A generalized interval neutrosophic soft set  $F^\mu$  over  $U$  is said to be generalized absolute interval neutrosophic soft set, denoted by  $U^\mu$ , if  $U^\mu: E \rightarrow IN(U) \times I$  such that  $U^\mu(e) = \{(F(e), \mu(e))\}$ , where  $F(e) = \{ \langle x, ([1, 1], [0, 0], [0, 0]) \rangle \}$  and  $\mu(e) = 1$  for each  $e \in E$  and  $x \in U$ .

**Definition 5070** Let  $F^\mu$  be a generalized interval neutrosophic soft set over  $U$ , where

$$F^\mu(e) = \{(F(e), \mu(e))\}$$

and

$$F(e) = \{ \langle x, [u_{F(e)}^L(x), u_{F(e)}^U(x)], [w_{F(e)}^L(x), w_{F(e)}^U(x)], [v_{F(e)}^L(x), v_{F(e)}^U(x)] \rangle : x \in U \}$$

for all  $e \in E$ . Then, for  $e_m \in E$  and  $x_n \in U$ ;

- (1)  $F^* = [F_L^*, F_U^*]$  is said to be interval truth membership part of  $F^\mu$  where  $F^* = \{(F_{mn}^*(e_m), \mu(e_m))\}$  and  $F_{mn}^*(e_m) = \{ \langle x_n, [u_{F(e_m)}^L(x_n), u_{F(e_m)}^U(x_n)] \rangle \}$ ,
- (2)  $F^\dagger = [F_L^\dagger, F_U^\dagger]$  is said to be interval indeterminacy membership part of  $F^\mu$  where  $F^\dagger = \{(F_{mn}^\dagger(e_m), \mu(e_m))\}$  and  $F_{mn}^\dagger(e_m) = \{ \langle x_n, [w_{F(e_m)}^L(x_n), w_{F(e_m)}^U(x_n)] \rangle \}$ ,
- (3)  $F^\Delta = [F_L^\Delta, F_U^\Delta]$  is said to be interval falsity membership part of  $F^\mu$  where  $F^\Delta = \{(F_{mn}^\Delta(e_m), \mu(e_m))\}$  and  $F_{mn}^\Delta(e_m) = \{ \langle x_n, [v_{F(e_m)}^L(x_n), v_{F(e_m)}^U(x_n)] \rangle \}$ .

We say that every part of  $F^\mu$  is a component of itself and is denote by  $F^\mu = (F^*, F^\dagger, F^\Delta)$ . Then matrix forms of components of  $F^\mu$  in example 3.2 can be expressed as follows:

$$F^* = \begin{pmatrix} ([0.2, 0.3], [0.3, 0.6], [0.4, 0.5]) & (0.1) \\ ([0.2, 0.5], [0.3, 0.5], [0.4, 0.7]) & (0.4) \\ ([0.3, 0.4], [0.1, 0.3], [0.1, 0.4]) & (0.6) \end{pmatrix}$$

$$F^\iota = \begin{pmatrix} ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & (0.1) \\ ([0.2, 0.5], [0.4, 0.8], [0.3, 0.8]) & (0.4) \\ ([0.3, 0.4], [0.2, 0.5], [0.2, 0.3]) & (0.6) \end{pmatrix}$$

$$F^\Delta = \begin{pmatrix} ([0.2, 0.3], [0.2, 0.4], [0.2, 0.6]) & (0.1) \\ ([0.2, 0.5], [0.8, 0.9], [0.3, 0.4]) & (0.4) \\ ([0.7, 0.9], [0.3, 0.7], [0.5, 0.7]) & (0.6) \end{pmatrix}$$

where

$$\begin{aligned} F^*_{mn}(e_m) &= \{ \langle x_n, [u_{F(e_m)}^L(x_n), u_{F(e_m)}^U(x_n)] \rangle \} \\ F^\iota_{mn}(e_m) &= \{ \langle x_n, [w_{F(e_m)}^L(x_n), w_{F(e_m)}^U(x_n)] \rangle \} \\ F^\Delta_{mn}(e_m) &= \{ \langle x_n, [v_{F(e_m)}^L(x_n), v_{F(e_m)}^U(x_n)] \rangle \} \end{aligned}$$

are defined as the interval truth, interval indeterminacy and interval falsity values of  $n$  –th element the according to  $m$  –th parameter, respectively.

**Definition 5.80** Suppose that  $F^\mu$  is a generalized interval neutrosophic soft set over  $U$ . Then we say that each components of  $F^\mu$  can be seen as the generalized interval valued vague soft set [15]. Also if it is taken  $\mu(e) = 1$  for all  $e \in E$ , the our generalized interval neutrosophic soft set coincides with the interval neutrosophic soft set [12].

**Definition 5.90** Let  $U$  be an universe and  $E$  be a of parameters,  $F^\mu$  and  $G^\theta$  be two generalized interval neutrosophic soft sets, we say that  $F^\mu$  is a generalized interval neutrosophic soft subset  $G^\theta$  if

- (1)  $\mu$  is a fuzzy subset of  $\theta$ ,
- (2) For  $e \in E$ ,  $F(e)$  is an interval neutrosophic subset of  $G(e)$ , i.e, for all  $e_m \in E$  and  $m, n \in \Lambda$ ,

$$\begin{aligned} F^*_{mn}(e_m) \leq G^*_{mn}(e_m), F^\iota_{mn}(e_m) \geq G^\iota_{mn}(e_m) \text{ and } F^\Delta_{mn}(e_m) \geq G^\Delta_{mn}(e_m) \text{ where,} \\ u_{F(e_m)}^L(x_n) \leq u_{G(e_m)}^L(x_n), u_{F(e_m)}^U(x_n) \leq u_{G(e_m)}^U(x_n) \\ w_{F(e_m)}^L(x_n) \geq w_{G(e_m)}^L(x_n), w_{F(e_m)}^U(x_n) \geq w_{G(e_m)}^U(x_n) \\ v_{F(e_m)}^L(x_n) \geq v_{G(e_m)}^L(x_n), v_{F(e_m)}^U(x_n) \geq v_{G(e_m)}^U(x_n) \end{aligned}$$

For  $x_n \in U$ .

We denote this relationship by  $F^\mu \sqsubseteq G^\theta$ . Moreover if  $G^\theta$  is generalized interval neutrosophic soft subset of  $F^\mu$ , then  $F^\mu$  is called a generalized interval neutrosophic soft superset of  $G^\theta$  this relation is denoted by  $F^\mu \sqsupseteq G^\theta$ .

**Example 5.90** Consider two generalized interval neutrosophic soft set  $F^\mu$  and  $G^\theta$ . suppose that  $U = \{ h_1, h_2, h_3 \}$  is the set of houses and  $E = \{ e_1, e_2, e_3 \}$  is the set of parameters where  $e_1 = \text{cheap}, e_2 = \text{moderate}, e_3 = \text{comfortable}$ . Suppose that  $F^\mu$  and  $G^\theta$  are given as follows respectively:



$$F^\mu(e_1) = \left( \frac{h_1}{([0.1, 0.2], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])} \right), \quad (0.2)$$

$$F^\mu(e_2) = \left( \frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])} \right), \quad (0.5)$$

$$F^\mu(e_3) = \left( \frac{h_1}{([0.2, 0.6], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])} \right), \quad (0.6)$$

and

$$G^\theta(e_1) = \left( \frac{h_1}{([0.2, 0.3], [0.1, 0.2], [0.1, 0.2])}, \frac{h_2}{([0.4, 0.5], [0.2, 0.3], [0.3, 0.5])}, \frac{h_3}{([0.6, 0.7], [0.1, 0.3], [0.2, 0.3])} \right), \quad (0.4)$$

$$G^\theta(e_2) = \left( \frac{h_1}{([0.2, 0.5], [0.3, 0.4], [0.2, 0.3])}, \frac{h_2}{([0.7, 0.8], [0.3, 0.4], [0.4, 0.6])}, \frac{h_3}{([0.3, 0.6], [0.2, 0.5], [0.4, 0.6])} \right), \quad (0.7)$$

$$G^\theta(e_3) = \left( \frac{h_1}{([0.3, 0.7], [0.1, 0.3], [0.1, 0.3])}, \frac{h_2}{([0.4, 0.5], [0.1, 0.5], [0.2, 0.3])}, \frac{h_3}{([0.7, 0.9], [0.2, 0.3], [0.1, 0.2])} \right), \quad (0.8)$$

Then  $F^\mu$  is a generalized interval neutrosophic soft subset of  $G^\theta$ , that is  $F^\mu \sqsubseteq G^\theta$ .

**Definition 50** The union of two generalized interval neutrosophic soft sets  $F^\mu$  and  $G^\theta$  over  $U$ , denoted by  $H^\lambda = F^\mu \sqcup G^\theta$  is a generalized interval neutrosophic soft set  $H^\lambda$  defined by

$$H^\lambda = ([H_L^*, H_U^*], [H_L^\dagger, H_U^\dagger], [H_L^\Delta, H_U^\Delta])$$

where  $\lambda(e_m) = \mu(e_m) \oplus \theta(e_m)$ ,

$$H_{Lmn}^* = F_{Lmn}^*(e_m) \oplus G_{Lmn}^*(e_m)$$

$$H_{Lmn}^\dagger = F_{Lmn}^\dagger(e_m) \oplus G_{Lmn}^\dagger(e_m)$$

$$H_{Lmn}^\Delta = F_{Lmn}^\Delta(e_m) \oplus G_{Lmn}^\Delta(e_m)$$

and

$$H_{Umn}^* = F_{Umn}^*(e_m) \oplus G_{Umn}^*(e_m)$$

$$H_{Umn}^\dagger = F_{Umn}^\dagger(e_m) \oplus G_{Umn}^\dagger(e_m)$$

$$H_{Umn}^\Delta = F_{Umn}^\Delta(e_m) \oplus G_{Umn}^\Delta(e_m)$$

for all  $e_m \in E$  and  $m, n \in \Lambda$ .

**Definition 51** The intersection of two generalized interval neutrosophic soft sets  $F^\mu$  and  $G^\theta$  over  $U$ , denoted by  $K^\varepsilon = F^\mu \sqcap G^\theta$  is a generalized interval neutrosophic soft set  $K^\varepsilon$  defined by

$$K^\varepsilon = ([K_L^*, K_U^*], [K_L^\dagger, K_U^\dagger], [K_L^\Delta, K_U^\Delta])$$

where  $\varepsilon(e_m) = \mu(e_m) \otimes \theta(e_m)$ ,

$$K_{Lmn}^* = F_{Lmn}^*(e_m) \otimes G_{Lmn}^*(e_m)$$

$$K_{Lmn}^\dagger = F_{Lmn}^\dagger(e_m) \otimes G_{Lmn}^\dagger(e_m)$$

$$K_{Lmn}^\Delta = F_{Lmn}^\Delta(e_m) \otimes G_{Lmn}^\Delta(e_m)$$

and

$$K_{Umn}^* = F_{Umn}^*(e_m) \otimes G_{Umn}^*(e_m)$$

$$K_{Umn}^\dagger = F_{Umn}^\dagger(e_m) \otimes G_{Umn}^\dagger(e_m)$$

$$K_{Umn}^\Delta = F_{Umn}^\Delta(e_m) \otimes G_{Umn}^\Delta(e_m)$$

for all  $e_m \in E$  and  $m, n \in \Lambda$ .

**Gzco rlg'5030** Let us consider the generalized interval neutrosophic soft sets  $F^\mu$  and  $G^\theta$  defined in Example 3.2. Suppose that the  $t$ -conorm is defined by  $\oplus(a, b) = \max\{a, b\}$  and the  $t$ -norm by  $\otimes(a, b) = \min\{a, b\}$  for  $a, b \in [0, 1]$ . Then  $H^\lambda = F^\mu \sqcup G^\theta$  is defined as follows:

$$H(e_1) = \left( \frac{h_1}{([0.2, 0.3], [0.1, 0.2], [0.1, 0.2])}, \frac{h_2}{([0.4, 0.5], [0.2, 0.3], [0.3, 0.5])}, \frac{h_3}{([0.6, 0.7], [0.1, 0.3], [0.2, 0.3])} \right), \quad (0.4)$$

$$H(e_2) = \left( \frac{h_1}{([0.2, 0.5], [0.3, 0.4], [0.2, 0.3])}, \frac{h_2}{([0.7, 0.8], [0.3, 0.4], [0.4, 0.6])}, \frac{h_3}{([0.3, 0.6], [0.2, 0.5], [0.4, 0.6])} \right), \quad (0.7)$$

$$H(e_3) = \left( \frac{h_1}{([0.3, 0.6], [0.1, 0.3], [0.1, 0.3])}, \frac{h_2}{([0.4, 0.5], [0.1, 0.5], [0.2, 0.3])}, \frac{h_3}{([0.7, 0.9], [0.2, 0.3], [0.1, 0.2])} \right), \quad (0.8)$$

**Gzco rlg'5040** Let us consider the generalized interval neutrosophic soft sets  $F^\mu$  and  $G^\theta$  defined in Example 3.2. Suppose that the  $t$ -conorm is defined by  $\oplus(a, b) = \max\{a, b\}$  and the  $t$ -norm by  $\otimes(a, b) = \min\{a, b\}$  for  $a, b \in [0, 1]$ . Then  $K^\varepsilon = F^\mu \sqcap G^\theta$  is defined as follows:

$$K(e_1) = \left( \frac{h_1}{([0.1, 0.2], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])} \right), \quad (0.2)$$

$$K(e_2) = \left( \frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])} \right), \quad (0.5)$$

$$K(e_3) = \left( \frac{h_1}{([0.2, 0.5], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])} \right), \quad (0.6)$$

**Rt qr quklqp'5050** Let  $F^\mu, G^\theta$  and  $H^\lambda$  be three generalized interval neutrosophic soft sets over  $U$ . Then

- (1)  $F^\mu \sqcup G^\theta = G^\theta \sqcup F^\mu,$
- (2)  $F^\mu \sqcap G^\theta = G^\theta \sqcap F^\mu,$
- (3)  $(F^\mu \sqcup G^\theta) \sqcup H^\lambda = F^\mu \sqcup (G^\theta \sqcup H^\lambda),$
- (4)  $(F^\mu \sqcap G^\theta) \sqcap H^\lambda = F^\mu \sqcap (G^\theta \sqcap H^\lambda).$

**Rt qqlf0** The proofs are trivial.

**Rt qr quklqp'5060** Let  $F^\mu, G^\theta$  and  $H^\lambda$  be three generalized interval neutrosophic soft sets over  $U$ . If we consider the  $t$ -conorm defined by  $\oplus(a, b) = \max\{a, b\}$  and the  $t$ -norm defined by  $\otimes(a, b) = \min\{a, b\}$  for  $a, b \in [0, 1]$ , then the following relations holds:

- (1)  $H^\lambda \sqcap (F^\mu \sqcup G^\theta) = (H^\lambda \sqcap F^\mu) \sqcup (H^\lambda \sqcap G^\theta),$
- (2)  $H^\lambda \sqcup (F^\mu \sqcap G^\theta) = (H^\lambda \sqcup F^\mu) \sqcap (H^\lambda \sqcup G^\theta).$

**Tgo ct m5070** The relations in above proposition does not hold in general.

**F g h p k l q p'5080** The complement of a generalized interval neutrosophic soft sets  $F^\mu$  over  $U$ , denoted by  $F^{\mu(c)}$  is defined by  $F^{\mu(c)} = ([F_L^{*(c)}, F_U^{*(c)}], [F_L^{i(c)}, F_U^{i(c)}], [F_L^{\Delta(c)}, F_U^{\Delta(c)}])$  where

$$\mu^{(c)}(e_m) = 1 - \mu(e_m)$$

and

$$F_{Lmn}^{*(c)} = F_{Lmn}^\Delta; F_{Lmn}^{i(c)} = 1 - F_{Umn}^i; F_{Lmn}^{\Delta(c)} = F_{Lmn}^*$$

$$F_{Umn}^{*(c)} = F_{Umn}^{\Delta}; F_{Umn}^{l(c)} = 1 - F_{Lmn}^l; F_{Umn}^{\Delta(c)} = F_{Umn}^*$$

**Example 3.2.** Complement of the generalized interval neutrosophic soft set  $F^\mu$  denoted by  $F^{\mu(c)}$  is given as follows:

$$F^{\mu(c)}(e_1) = \left( \frac{h_1}{([0.2, 0.3], [0.5, 0.7], [0.2, 0.3])}, \frac{h_2}{([0.5, 0.6], [0.6, 0.7], [0.3, 0.4])}, \frac{h_3}{([0.5, 0.7], [0.6, 0.8], [0.5, 0.6])} \right), \quad (0.8)$$

$$F^{\mu(c)}(e_2) = \left( \frac{h_1}{([0.3, 0.4], [0.4, 0.5], [0.1, 0.4])}, \frac{h_2}{([0.5, 0.8], [0.5, 0.6], [0.6, 0.7])}, \frac{h_3}{([0.6, 0.9], [0.4, 0.7], [0.2, 0.4])} \right), \quad (0.5)$$

$$F^{\mu(c)}(e_3) = \left( \frac{h_1}{([0.1, 0.5], [0.5, 0.5], [0.2, 0.6])}, \frac{h_2}{([0.4, 0.5], [0.4, 0.7], [0.3, 0.5])}, \frac{h_3}{([0.2, 0.3], [0.6, 0.7], [0.6, 0.8])} \right), \quad (0.4)$$

**Proposition 5.8:** Let  $F^\mu$  and  $G^\theta$  be two generalized interval neutrosophic soft sets over  $U$ . Then,

- (1)  $F^\mu$  is a generalized interval neutrosophic soft subset of  $F^\mu \sqcup F^{\mu(c)}$
- (2)  $F^\mu \cap F^{\mu(c)}$  is a generalized interval neutrosophic soft subset of  $F^\mu$ .

**Proof:** It is clear.

**Definition 5.8:** "AND" operation on two generalized interval neutrosophic soft sets  $F^\mu$  and  $G^\theta$  over  $U$ , denoted by  $H^\lambda = F^\mu \wedge G^\theta$  is the mapping  $H^\lambda: C \rightarrow \text{IN}(U) \times I$  defined by

$$H^\lambda = ([H_L^*, H_U^*], [H_L^l, H_U^l], [H_L^\Delta, H_U^\Delta])$$

where  $\lambda(e_m) = \min(\mu(e_k), \theta(e_h))$  and

$$\begin{aligned} H_L^*(e_m) &= \min\{F_L^*(e_{kn}), G_L^*(e_{hn})\} \\ H_U^*(e_m) &= \max\{F_U^*(e_{kn}), G_U^*(e_{hn})\} \\ H_L^l(e_m) &= \max\{F_L^l(e_{kn}), G_L^l(e_{hn})\} \\ H_U^l(e_m) &= \min\{F_U^l(e_{kn}), G_U^l(e_{hn})\} \\ H_L^\Delta(e_m) &= \max\{F_L^\Delta(e_{kn}), G_L^\Delta(e_{hn})\} \\ H_U^\Delta(e_m) &= \min\{F_U^\Delta(e_{kn}), G_U^\Delta(e_{hn})\} \end{aligned}$$

and

$$\begin{aligned} H_U^*(e_m) &= \min\{F_U^*(e_{kn}), G_U^*(e_{hn})\} \\ H_L^l(e_m) &= \max\{F_U^l(e_{kn}), G_U^l(e_{hn})\} \\ H_U^\Delta(e_m) &= \max\{F_U^\Delta(e_{kn}), G_U^\Delta(e_{hn})\} \end{aligned}$$

for all  $e_m = (e_k, e_h) \in C \subseteq E \times E$  and  $m, n, k, h \in A$ .

**Definition 5.20:** "OR" operation on two generalized interval neutrosophic soft sets  $F^\mu$  and  $G^\theta$  over  $U$ , denoted by  $K^\varepsilon = F^\mu \vee G^\theta$  is the mapping  $K^\varepsilon: C \rightarrow \text{IN}(U) \times I$  defined by

$$K^\varepsilon = ([K_L^*, K_U^*], [K_L^l, K_U^l], [K_L^\Delta, K_U^\Delta])$$

where  $\varepsilon(e_m) = \max(\mu(e_k), \theta(e_h))$  and

$$\begin{aligned} K_L^*(e_m) &= \max\{F_L^*(e_{kn}), G_L^*(e_{hn})\} \\ K_U^*(e_m) &= \min\{F_U^*(e_{kn}), G_U^*(e_{hn})\} \\ K_L^l(e_m) &= \min\{F_L^l(e_{kn}), G_L^l(e_{hn})\} \\ K_U^l(e_m) &= \max\{F_U^l(e_{kn}), G_U^l(e_{hn})\} \\ K_L^\Delta(e_m) &= \min\{F_L^\Delta(e_{kn}), G_L^\Delta(e_{hn})\} \\ K_U^\Delta(e_m) &= \max\{F_U^\Delta(e_{kn}), G_U^\Delta(e_{hn})\} \end{aligned}$$

and

$$\begin{aligned} K_U^*(e_m) &= \max\{F_U^*(e_{kn}), G_U^*(e_{hn})\} \\ K_L^l(e_m) &= \min\{F_U^l(e_{kn}), G_U^l(e_{hn})\} \\ K_U^\Delta(e_m) &= \min\{F_U^\Delta(e_{kn}), G_U^\Delta(e_{hn})\} \end{aligned}$$

for all  $e_m = (e_k, e_h) \in C \subseteq E \times E$  and  $m, n, k, h \in \Lambda$ .

**Definition 5043.** Let  $F^\mu$  and  $G^\theta$  be two generalized interval neutrosophic soft sets over  $U$  and  $C \subseteq E \times E$ , a function  $R: C \rightarrow \text{IN}(U) \times I$  defined by  $R = F^\mu \wedge G^\theta$  and  $R(e_m, e_h) = F^\mu(e_m) \wedge G^\theta(e_h)$  is said to be a interval neutrosophic relation from  $F^\mu$  to  $G^\theta$  for all  $(e_m, e_h) \in C$ .

### 60Cr r rlec v k p ' q h I g p g t c r k g f ' k p v g t x c r i P g w t q u r j k e ' U q h ' U g v

Now, we illustrate an application of generalized interval neutrosophic soft set in decision making problem.

**Example 6080.** Suppose that the universe consists of three machines, that is  $U = \{x_1, x_2, x_3\}$  and consider the set of parameters  $E = \{e_1, e_2, e_3\}$  which describe their performances according to certain specific task. Assume that a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations  $F^\mu$  and  $G^\theta$  by two experts A and B respectively, defined as follows:

$$F^\mu(e_1) = \left( \frac{h_1}{([0.2, 0.3], [0.2, 0.3], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.6], [0.3, 0.5], [0.2, 0.4])}, \frac{h_3}{([0.4, 0.5], [0.2, 0.5], [0.2, 0.6])} \right), \quad (0.2)$$

$$F^\mu(e_2) = \left( \frac{h_1}{([0.2, 0.5], [0.2, 0.5], [0.2, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.4, 0.8], [0.8, 0.9])}, \frac{h_3}{([0.4, 0.7], [0.3, 0.8], [0.3, 0.4])} \right), \quad (0.5)$$

$$F^\mu(e_3) = \left( \frac{h_1}{([0.3, 0.4], [0.3, 0.4], [0.7, 0.9])}, \frac{h_2}{([0.1, 0.3], [0.2, 0.5], [0.3, 0.7])}, \frac{h_3}{([0.1, 0.4], [0.2, 0.3], [0.5, 0.7])} \right), \quad (0.6)$$

$$G^\theta(e_1) = \left( \frac{h_1}{([0.2, 0.3], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])} \right), \quad (0.3)$$

$$G^\theta(e_2) = \left( \frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])} \right), \quad (0.6)$$

$$G^\theta(e_3) = \left( \frac{h_1}{([0.2, 0.6], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])} \right), \quad (0.4)$$

To find the “AND” between the two GINSSs, we have  $F^\mu$  and  $G^\theta, R = F^\mu \wedge G^\theta$  where

$$(F^\mu)^* = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.6], [0.4, 0.5]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.3, 0.5], [0.4, 0.7]) & (0.5) \\ e_3 & ([0.3, 0.4], [0.1, 0.3], [0.1, 0.4]) & (0.6) \end{pmatrix}$$

$$(F^\mu)' = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.4, 0.8], [0.3, 0.8]) & (0.5) \\ e_3 & ([0.3, 0.4], [0.2, 0.5], [0.2, 0.3]) & (0.6) \end{pmatrix}$$

$$(F^\mu)^\Delta = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.2, 0.4], [0.2, 0.6]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.8, 0.9], [0.3, 0.4]) & (0.5) \\ e_3 & ([0.7, 0.9], [0.3, 0.7], [0.5, 0.7]) & (0.6) \end{pmatrix}$$

$$(G^\theta)^* = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.4], [0.5, 0.6]) & (0.3) \\ e_2 & ([0.1, 0.4], [0.6, 0.7], [0.2, 0.4]) & (0.6) \\ e_3 & ([0.2, 0.6], [0.3, 0.5], [0.6, 0.8]) & (0.4) \end{pmatrix}$$

$$(G^\theta)' = \begin{pmatrix} e_1 & ([0.3, 0.5], [0.3, 0.4], [0.2, 0.4]) & (0.3) \\ e_2 & ([0.5, 0.6], [0.4, 0.5], [0.3, 0.6]) & (0.6) \\ e_2 & ([0.2, 0.5], [0.3, 0.6], [0.3, 0.4]) & (0.4) \end{pmatrix}$$

$$(G^\theta)^\Delta = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.5, 0.6], [0.5, 0.7]) & (0.3) \\ e_2 & ([0.3, 0.4], [0.5, 0.8], [0.6, 0.9]) & (0.6) \\ e_3 & ([0.1, 0.5], [0.4, 0.5], [0.2, 0.3]) & (0.4) \end{pmatrix}$$

We present the table of three basic component of  $R$ , which are interval truth –membership, Interval indeterminacy membership and interval falsity-membership part. To choose the best candidate, we firstly propose the induced interval neutrosophic membership functions by taking the arithmetic average of the end point of the range, and mark the highest numerical grade (underline) in each row of each table. But here, since the last column is the grade of such belongingness of a candidate for each pair of parameters, its not taken into account while making. Then we calculate the score of each component of  $R$  by taking the sum of products of these numerical grades with the corresponding values of . Next, we calculate the final score by subtracting the score of falsity-membership part of  $R$  from the sum of scores of truth-membership part and of indeterminacy membership part of  $R$ . The machine with the highest score is the desired machine by company.

For the interval truth membership function components we have:

$$(F^\mu)^* = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.6], [0.4, 0.5]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.3, 0.5], [0.4, 0.7]) & (0.5) \\ e_3 & ([0.3, 0.4], [0.1, 0.3], [0.1, 0.4]) & (0.6) \end{pmatrix}$$

$$(G^\theta)^* = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.4], [0.5, 0.6]) & (0.3) \\ e_2 & ([0.1, 0.4], [0.6, 0.7], [0.2, 0.4]) & (0.6) \\ e_3 & ([0.2, 0.6], [0.3, 0.5], [0.6, 0.8]) & (0.4) \end{pmatrix}$$

$$(R)^* =$$

$$(R)^*(e_1, e_1) = \left\{ \left( \frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.3, 0.4]}, \frac{x_3}{[0.4, 0.5]} \right), 0.2 \right\}$$

$$(R)^*(e_1, e_2) = \left\{ \left( \frac{x_1}{[0.1, 0.3]}, \frac{x_2}{[0.3, 0.6]}, \frac{x_3}{[0.2, 0.5]} \right), 0.2 \right\}$$

$$(R)^*(e_1, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.4]} \right), 0.2 \right\}$$

$$(R)^*(e_2, e_1) = \left\{ \left( \frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.3, 0.4]}, \frac{x_3}{[0.4, 0.6]} \right), 0.3 \right\}$$

$$(R)^*(e_2, e_2) = \left\{ \left( \frac{x_1}{[0.1, 0.4]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.4]} \right), 0.5 \right\}$$

$$(R)^*(e_2, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.4, 0.7]} \right), 0.4 \right\}$$

$$(R)^*(e_3, e_1) = \left\{ \left( \frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.1, 0.3]}, \frac{x_3}{[0.1, 0.4]} \right), 0.3 \right\}$$

$$(R)^*(e_3, e_2) = \left\{ \left( \frac{x_1}{[0.1, 0.4]}, \frac{x_2}{[0.1, 0.3]}, \frac{x_3}{[0.1, 0.4]} \right), 0.6 \right\}$$

$$(R)^*(e_3, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.4]}, \frac{x_2}{[0.1, 0.3]}, \frac{x_3}{[0.1, 0.4]} \right), 0.4 \right\}$$

	$x_1$	$x_2$	$x_3$	$\mu$
$(e_1, e_1)$	[0.2, 0.3]	[0.3, 0.4]	[0.4, 0.5]	0.2
$(e_1, e_2)$	[0.1, 0.3]	[0.3, 0.6]	[0.2, 0.5]	0.2
$(e_1, e_3)$	[0.2, 0.3]	[0.3, 0.5]	[0.2, 0.4]	0.2
$(e_2, e_1)$	[0.2, 0.3]	[0.3, 0.4]	[0.4, 0.6]	0.3
$(e_2, e_2)$	[0.1, 0.4]	[0.3, 0.5]	[0.2, 0.4]	0.5
$(e_2, e_3)$	[0.2, 0.5]	[0.3, 0.5]	[0.4, 0.7]	0.4
$(e_3, e_1)$	[0.2, 0.3]	[0.1, 0.3]	[0.1, 0.4]	0.3
$(e_3, e_1)$	[0.1, 0.4]	[0.1, 0.3]	[0.1, 0.4]	0.6
$(e_3, e_2)$	[0.2, 0.4]	[0.1, 0.3]	[0.1, 0.4]	0.4

**Vcdig'3**<Interval truth membership function.

	$x_1$	$x_2$	$x_3$	$\mu$
$(e_1, e_1)$	0.25	0.35	<u>0.45</u>	0.2
$(e_1, e_2)$	0.2	<u>0.45</u>	0.35	0.2
$(e_1, e_3)$	0.25	<u>0.4</u>	0.3	0.2
$(e_2, e_1)$	0.25	0.35	<u>0.5</u>	0.3
$(e_2, e_2)$	0.25	<u>0.4</u>	0.3	0.5
$(e_2, e_3)$	0.35	0.4	<u>0.55</u>	0.4
$(e_3, e_1)$	<u>0.25</u>	0.2	<u>0.25</u>	0.3
$(e_3, e_1)$	<u>0.25</u>	0.2	<u>0.25</u>	0.6
$(e_3, e_2)$	<u>0.3</u>	0.2	0.25	0.4

**Vcdig'4**<Induced interval truth membership function.

The value of representation interval truth membership function  $[a, b]$  are obtained using mean value. Then, the scores of interval truth membership function of  $x_1, x_2$  and  $x_3$  are:

$$\begin{aligned}
 S_{(R)^*}(x_1) &= (0.25 \times 0.3) + (0.25 \times 0.6) + (0.3 \times 0.4) = \mathbf{0.325} \\
 S_{(R)^*}(x_2) &= (0.45 \times 0.2) + (0.4 \times 0.2) + (0.4 \times 0.5) = \mathbf{0.37} \\
 S_{(R)^*}(x_3) &= (0.45 \times 0.2) + (0.5 \times 0.3) + (0.55 \times 0.4) + (0.25 \times 0.3) + (0.25 \times 0.6) \\
 &= \mathbf{0.685}.
 \end{aligned}$$

For the interval indeterminacy membership function components we have:

$$(F^\mu)^\iota = \begin{pmatrix} ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & (0.2) \\ ([0.2, 0.5], [0.4, 0.8], [0.3, 0.8]) & (0.5) \\ ([0.3, 0.4], [0.2, 0.5], [0.2, 0.3]) & (0.6) \end{pmatrix}$$

$$(G^\theta)^\iota = \begin{pmatrix} ([0.3, 0.5], [0.3, 0.4], [0.2, 0.4]) & (0.3) \\ ([0.5, 0.6], [0.4, 0.5], [0.3, 0.6]) & (0.6) \\ ([0.2, 0.5], [0.3, 0.6], [0.3, 0.4]) & (0.4) \end{pmatrix}$$

$$\begin{aligned}
 (R)^\iota &= \\
 (R)^\iota(e_1, e_1) &= \left\{ \left( \frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.5]} \right), 0.3 \right\}
 \end{aligned}$$

$$(R)'(e_1, e_2) = \left\{ \left( \frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.3, 0.6]} \right), 0.6 \right\}$$

$$(R)'(e_1, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.3, 0.6]}, \frac{x_3}{[0.3, 0.5]} \right), 0.4 \right\}$$

$$(R)'(e_2, e_1) = \left\{ \left( \frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.4, 0.8]}, \frac{x_3}{[0.3, 0.8]} \right), 0.5 \right\}$$

$$(R)'(e_2, e_2) = \left\{ \left( \frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.8]}, \frac{x_3}{[0.3, 0.8]} \right), 0.6 \right\}$$

$$(R)'(e_2, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.4, 0.8]}, \frac{x_3}{[0.3, 0.8]} \right), 0.5 \right\}$$

$$(R)'(e_3, e_1) = \left\{ \left( \frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.4]} \right), 0.6 \right\}$$

$$(R)'(e_3, e_2) = \left\{ \left( \frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.3, 0.6]} \right), 0.6 \right\}$$

$$(R)'(e_3, e_3) = \left\{ \left( \frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.3, 0.6]}, \frac{x_3}{[0.3, 0.4]} \right), 0.6 \right\}$$

	$x_1$	$x_2$	$x_3$	$\mu$
$(e_1, e_1)$	[0.3, 0.5]	[0.3, 0.5]	[0.2, 0.5]	0.3
$(e_1, e_2)$	[0.5, 0.6]	[0.4, 0.5]	[0.3, 0.6]	0.6
$(e_1, e_3)$	[0.2, 0.5]	[0.3, 0.6]	[0.3, 0.5]	0.4
$(e_2, e_1)$	[0.3, 0.5]	[0.4, 0.8]	[0.3, 0.8]	0.5
$(e_2, e_2)$	[0.5, 0.6]	[0.4, 0.8]	[0.3, 0.8]	0.6
$(e_2, e_3)$	[0.2, 0.5]	[0.4, 0.8]	[0.3, 0.8]	0.5
$(e_3, e_1)$	[0.3, 0.5]	[0.3, 0.5]	[0.2, 0.4]	0.6
$(e_3, e_2)$	[0.5, 0.6]	[0.4, 0.5]	[0.3, 0.6]	0.6
$(e_3, e_3)$	[0.3, 0.5]	[0.3, 0.6]	[0.3, 0.4]	0.6

**Vcdig'5**<Interval indeterminacy membership function

	$x_1$	$x_2$	$x_3$	$\mu$
$(e_1, e_1)$	<u>0.4</u>	<u>0.4</u>	0.35	0.3
$(e_1, e_2)$	<u>0.55</u>	0.45	0.45	0.6
$(e_1, e_3)$	0.35	<u>0.45</u>	0.4	0.4
$(e_2, e_1)$	0.4	<u>0.6</u>	0.55	0.5
$(e_2, e_2)$	0.55	<u>0.6</u>	0.55	0.6
$(e_2, e_3)$	0.35	<u>0.6</u>	0.55	0.5
$(e_3, e_1)$	<u>0.4</u>	<u>0.4</u>	0.3	0.6
$(e_3, e_2)$	<u>0.55</u>	0.45	0.45	0.6
$(e_3, e_3)$	0.4	<u>0.45</u>	0.35	0.6

**Vcdig'6**<Induced interval indeterminacy membership function

The value of representation interval indeterminacy membership function  $[a, b]$  are obtained using mean value. Then, the scores of interval indeterminacy membership function of  $x_1, x_2$  and  $x_3$  are:

$$S_{(R)'}(x_1) = (0.4 \times 0.3) + (0.55 \times 0.6) + (0.4 \times 0.6) + (0.55 \times 0.6) = \mathbf{1.02}$$

$$S_{(R)'}(x_2) = (0.4 \times 0.3) + (0.45 \times 0.4) + (0.6 \times 0.5) + (0.6 \times 0.6) + (0.6 \times 0.5) + (0.4 \times 0.60) + (0.45 \times 0.6) + = \mathbf{1.77''}$$

$$S_{I(R)'}(x_2) = \mathbf{0.}$$

For the interval indeterminacy membership function components we have:

$$(F^\mu)^\Delta = \begin{pmatrix} ([0.2, 0.3], [0.2, 0.4], [0.2, 0.6]) & (0.2) \\ ([0.2, 0.5], [0.8, 0.9], [0.3, 0.4]) & (0.5) \\ ([0.7, 0.9], [0.3, 0.7], [0.5, 0.7]) & (0.6) \end{pmatrix}$$

$$(G^\theta)^\Delta = \begin{pmatrix} ([0.2, 0.3], [0.5, 0.6], [0.5, 0.7]) & (0.3) \\ ([0.3, 0.4], [0.5, 0.8], [0.6, 0.9]) & (0.6) \\ ([0.1, 0.5], [0.4, 0.5], [0.2, 0.3]) & (0.4) \end{pmatrix}$$

$$(R)^\Delta =$$

$$(R)^\Delta (e_1, e_1) = \left\{ \left( \frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.5, 0.6]}, \frac{x_3}{[0.5, 0.7]} \right), 0.3 \right\}$$

$$(R)^\Delta (e_1, e_2) = \left\{ \left( \frac{x_1}{[0.3, 0.4]}, \frac{x_2}{[0.5, 0.8]}, \frac{x_3}{[0.6, 0.9]} \right), 0.6 \right\}$$

$$(R)^\Delta (e_1, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.2, 0.6]} \right), 0.4 \right\}$$

$$(R)^\Delta (e_2, e_1) = \left\{ \left( \frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.5, 0.7]} \right), 0.5 \right\}$$

$$(R)^\Delta (e_2, e_2) = \left\{ \left( \frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.6, 0.9]} \right), 0.6 \right\}$$

$$(R)^\Delta (e_2, e_3) = \left\{ \left( \frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.3, 0.4]} \right), 0.5 \right\}$$

$$(R)^\Delta (e_3, e_1) = \left\{ \left( \frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.5, 0.7]}, \frac{x_3}{[0.5, 0.7]} \right), 0.6 \right\}$$

$$(R)^\Delta (e_3, e_2) = \left\{ \left( \frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.5, 0.8]}, \frac{x_3}{[0.6, 0.9]} \right), 0.6 \right\}$$

$$(R)^\Delta (e_3, e_3) = \left\{ \left( \frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.4, 0.7]}, \frac{x_3}{[0.5, 0.7]} \right), 0.6 \right\}$$



	$x_1$	$x_2$	$x_3$	$\mu$
$(e_1, e_1)$	[0.2, 0.3]	[0.5, 0.6]	[0.5, 0.7]	0.3
$(e_1, e_2)$	[0.3, 0.4]	[0.5, 0.8]	[0.6, 0.9]	0.6
$(e_1, e_3)$	[0.2, 0.5]	[0.4, 0.5]	[0.2, 0.6]	0.4
$(e_2, e_1)$	[0.2, 0.5]	[0.8, 0.9]	[0.5, 0.7]	0.5
$(e_2, e_2)$	[0.3, 0.5]	[0.8, 0.9]	[0.6, 0.9]	0.6
$(e_2, e_3)$	[0.2, 0.5]	[0.8, 0.9]	[0.3, 0.4]	0.5
$(e_3, e_1)$	[0.7, 0.9]	[0.5, 0.7]	[0.5, 0.7]	0.6
$(e_3, e_2)$	[0.7, 0.9]	[0.5, 0.8]	[0.6, 0.9]	0.6
$(e_3, e_3)$	[0.7, 0.9]	[0.4, 0.7]	[0.5, 0.7]	0.6

**Vcdig'7** <Interval falsity membership function.

	$x_1$	$x_2$	$x_3$	$\mu$
$(e_1, e_1)$	0.25	0.55	<u>0.6</u>	0.3
$(e_1, e_2)$	0.35	0.43	<u>0.75</u>	0.6
$(e_1, e_3)$	0.35	<u>0.45</u>	0.4	0.4
$(e_2, e_1)$	0.35	<u>0.85</u>	0.6	0.5
$(e_2, e_2)$	0.4	<u>0.85</u>	0.75	0.6
$(e_2, e_3)$	0.35	<u>0.85''</u>	0.35	0.5
$(e_3, e_1)$	<u>0.8</u>	0.6	0.6	0.6
$(e_3, e_2)$	<u>0.8</u>	0.43''	0.75	0.6
$(e_3, e_3)$	<u>0.8</u>	0.55	0.6	0.6

**Vcdig'8** <Induced interval falsity membership function.

The value of representation interval falsity membership function  $[a, b]$  are obtained using mean value. Then, the scores of interval falsity membership function of  $x_1, x_2$  and  $x_3$  are:

$$S_{(R)\Delta}(x_1) = (0.8 \times 0.6) + (0.8 \times 0.6) + (0.8 \times 0.6) = \mathbf{1.44}$$

$$S_{(R)\Delta}(x_2) = (0.45 \times 0.4) + (0.85 \times 0.5) + (0.85 \times 0.6) + (0.85 \times 0.5) = \mathbf{1.54}$$

$$S_{(R)\Delta}(x_3) = (0.6 \times 0.3) + (0.75 \times 0.6) = \mathbf{0.63}.$$

Thus, we conclude the problem by calculating final score, using the following formula:

$$S(x_i) = S_{(R)^*}(x_i) + S_{(R)'}(x_i) - S_{(R)\Delta}(x_i)$$

so,

$$S(x_1) = 0.325 + 1.02 - 1.44 = -0.095$$

$$S(x_2) = 0.37 + 1.77 - 1.54 = 0.6$$

$$S(x_3) = 0.685 + 0 - 0.63 = 0.055.$$

Then the optimal selection for Mr.X is the  $x_2$ .

Table 1, Table 3 and Table 5 present the truth-membership function, indeterminacy-membership function and falsity-membership function in generalized interval neutrosophic soft set respectively.

## 70E qpenwukpu

This paper can be viewed as a continuation of the study of Sahin and Küçük [23]. We extended the generalized neutrosophic soft set to the case of interval valued neutrosophic soft set and also gave the application of GINSS in dealing with some decision making problems. In future work, will study another type of generalized interval neutrosophic soft set where the degree of possibility are interval.

## Cempqy igf i go gpw'

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