Graph Distance, Optimal Communication and Group Stability: A Preliminary Conjecture

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Introduction

In recent years, there has been a rapid increase in the literature which discusses new phenomenon associated to social network. One of the well-known phenomenon in this regards is known as 'six degrees of separation' [1], which implies that one can always keep a communication with each other anywhere within a six-step. A number of experiments has verified this hypothesis, either in the context of offline communication (postal mail), or online communication (email, etc.).

In this article, we argue that by introducing this known 'six degrees of separation' into the context of group instability problem, one can find a new type of wisdom in organization. Therefore, we offer a new conjecture, which may be called 'Group stability conjectures based on Graph/Network distance."

To our knowledge this conjecture has not been discussed elsewhere, and therefore may be useful for further research, in particular in the area of organization development and group stability studies. The purpose of this article was of course not to draw a conclusive theory, but to suggest further study of this proposed conjecture.

Graph Distance

Let $G(V,E)$ be a graph, where V is a set of vertices, and E a set of edges:

$$V=\{v_1,v_2,...\}, E=\{e_1,e_2,...\}.$$

We say that there is a route between vertices v_i and v_j. We define the distance between vertices v_i and v_j, noted by $d\left(v_i,v_j\right)$ as the shortest chain of edges that connects v_i with v_j.

In the graph $G(V,E)$ let's consider

$$d\left(v_i,v_j\right)=n\geq 1$$

where n is the number of edges connecting v_i with v_j, and for each such edge an equiprobability $\frac{1}{n}$.

Using Shannon's entropy

$$H(x) = -\sum_{i=1}^{n} P_i \log_2 (P_i)$$

In order to find the entropy of the distance between two vertices we get

$$H\left(d\left(v_i, v_j\right)\right) = -\sum_{i=1}^{n} P_i \log_2 (P_i) = -\sum_{i=1}^{n} \frac{1}{n} \log_2 (P_i) = -\log_2 \left(\frac{1}{n}\right) = \log_2 n,$$

since all $P_i = \frac{1}{n}$.

The longer is the distance between two vertices, the bigger is the entropy, since $\log_2 n_1 > \log_2 n_2$ when $n_1 > n_2 \geq 1$, therefore the more degree of disorder, of loss of information, as both ambiguity and imprecision increase.

A Conjecture of Group Stability Based on Graph Distance

A hierarchical structure is a widespread organization form in many areas.

The hidden assumption behind Small-world hypothesis is that everyone is around six-steps away from any other person on Earth, which is known as 'six degrees of separation' principle. A number of experiments have been conducted in order to prove this hypothesis. [2]

In this regards, apparently we can draw analogy from this 'six degrees of separation' to the concept of graph distance. In this context, graph distance can be viewed as the number of 'nodes' that one should reach to come to a destination. This study of graph distance and group stability is quite new, and only a number of published articles have appeared in journals, see for example [3], [4].

Once this analogy is set, it becomes apparent that the 'six degrees of separation' may be interpreted as an optimum graph distance, where any given organization can function in its best, provided we can consider an organization as an actual social-network which functions better if and only if communication can be preserved in optimal way.

In other words, any given organization which expands rapidly beyond these six-degrees of node separation (let say, between the CEO and its factory workers) will be more prone to instability. This phenomenon may also be viewed as another example of '*self-organized criticality*' process in any given organization/structure.

At this point, now we will write down our new conjecture of Group stability based on Graph distance:

(a) For a given organization in any industry, there is an optimal graph distance which will keep communication in organization in its optimum. We can call this as 'optimal graph distance number'.
(b) This optimal graph distance number is inversely proportional to the innovation cycle time in any given organization. This optimal graph distance corresponds to both the hierarchy of organization and also to the degrees of separation.
(c) There is tendency that any organization will increase its size such that the graph distance number always grows such that it exceeds its own level of incompetence (similar to Lawrence's principle).

(d) In order to keep internal and healthy communication for its own survival preservation, a good organization will keep its graph distance number at optimal level.
(e) If an organization has graph distance number which exceeds its optimal number (let say 5 or 6 degrees of separation), then it will be prone to instability.
(f) Group instability can take the form of de-formation of organization in order to meet the communication works again, in other words an organization has tendency to keep the graph distance number at optimal, in accordance with self-organized criticality phenomena.
(g) This is what can be called as Conjecture of Group Stability based on Graph Distance.

While the above conjecture may appear quite simple and obvious at first sight, it covers the phenomena corresponding to group stability in uniquer way, i.e. from the viewpoint of preservation of optimum communication. Therefore any organization has its own tendency to keep the size of its hierarchy such that the graph distance is kept optimal.

Let us mention a simple example here: Toyota has a unique management way, which is well-known in management literature. What is not quite well-studied is perhaps the fact that it has less hierarchical structure compared to other large automobile companies in USA. As a result, the innovation cycle time tends to be faster. For instance, Toyota has released its first generation of hybrid car (Prius) in 2007, while GM only expects to release a first version of hybrid cars by 2010. The simple lesson here (see point b) is that keep graph distance at minimum in order to reach faster innovation cycle.

The same lesson we often hear when an organization performs excellently in the past when its hierarchy remains small, but during the course of its history it tends to increase in ‘graph distance’ and then gradually it loses its ‘agility’. In this regards one can observe that group stability has deep link with cooperation level in any given organization, because in large organization there is strong tendency that coordination becomes very difficult [4]. From the viewpoint of game theory, it becomes very difficult to maintain the condition such that all of its members have optimal return [3].

In turn, a good communication in organization can be viewed as part of ‘social capital’, which can play significant role to keep its stability. This viewpoint has been discussed in [5].

Similarly, a city which becomes too large and exceeds its capacity to maintain good communication tends to form smaller-cities (just like sub-urbans areas) in order to keep its optimal size.

In our opinion this conjecture has not been discussed elsewhere.

References:

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