How Many Points are there in a Line Segment? A New Answer from a Discrete Cellular Space Viewpoint

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ABSTRACT. While it is known that Euclids five axioms include a proposition that a line consists at least of two points, modern geometry consistently avoids any discussion on the precise definition of points, lines, etc. It is our aim to clarify one of notorious question in Euclidean geometry from discrete cellular space (DCS) viewpoint: How many points are there in a line segment? In retrospect, it may offer an alternative for quantum gravity, i.e., by exploring discrete gravitational theories. To elucidate our propositions, in the last section we will discuss some implications of the discrete cellular space model in several areas of interest: (a) cell biology, (b) cellular computing, (c) Maxwell equations, (d) low energy fusion, and (e) cosmology modelling.

I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In that case nothing remains of my entire castle in the air gravitation theory included, -and of- the rest of modern physics. - A. Einstein

INTRODUCTION

So many students of all ages have asked this question: how many points are there in a line segment? A good math teacher would answer politely: in the circumference of a circle there are an infinite number of points [1]. Similarly, one can also ask: how many lines are there in a rectangle? The answer again is known: there are infinite number of lines in given rectangle.

But a careful student would ask again: then what is the definition of a point and a line? The teacher would answer again: a point is a circle with a zero diameter, and a line is composed of infinite points.

If our beloved student persisted, he/she would continue to ask: but teacher, if a circle has a zero diameter, then an infinite number of zeroes will not make a finite line, right? At this time, there is fair chance that the teacher would feel upset and say: shut up and calculate! That is what usually happens in most primary and high school mathematics classrooms, and the situation is not getting better in the undergraduate classroom. Only in graduate math class are the students allowed to ask deeper questions concerning the foundations of mathematics, etc. A more serious debate among mathematicians over this notorious continuum problem has been recorded in reference [11].

Here, we will offer a simpler solution of the above posed question from a discrete cellular space (DCS) viewpoint, with wide implications, including more clarity in the distinction between quantization and discretization.

2 SOLUTION: SPACE CONSISTS OF CIRCLES WITH FINITE DIAMETERS (DISCRETE-CELLULAR MODEL)

The obvious paradox that we set in the introduction section can be simplified as follows:

$$0+0+0+.\text{ad infinitum} = 0$$

Therefore, the basic postulate that a line segment consists of circles with zero diameter is contradictory by itself. Our proposed solution is to assume that space consists of circles with small but finite diameter (z), therefore if a line segment consists of circles like that, we have:

$$z+z+z+ \text{ ad infinitum} = \text{finite line}$$

One implication of this proposition is that we should consider the geometry of space not as continuum, but as a discrete cellular space. We must also remember that the discretization of space is much more fundamental than quantization. Moreover, we can consider the following:

a. It can be shown that similar indeterminacy problems plague the very definition of differential calculus, as no one knows that actual size of dx. See H.J.M. Bos [2]:

2.15. I turn now to a difficulty which necessarily arises in any attempt to set up an infinitesimal calculus which takes the differential as fundamental concept, namely the indeterminancy of differentials.

The first differential dx of the variable x is infinitely small with respect to x, and it has the same dimension as x. These are the only conditions it has to satisfy, and they do not determine a unique dx, for if dx satisfies the conditions then clearly so do 2dx and $\frac{1}{2}dx$ and in general all adx for finite numbers a. That is, all quantities that have the same dimension and the same order of infinity as dx might serve as dx.

Moreover, there are elements not from this class which satisfy the conditions for dx; for instance dx^2/a and $\sqrt{adx}$, for finite positive a of the same dimension as x. dx^2/a is infinitely small with respect to dx and $\sqrt{adx}$ is infinitely large with respect to dx, so that there is even not a privileged class of infinite smallness from which dx has to be chosen; there is no first class of infinite smallness adjacent to ' finiteness.

Thus first-order differentials involve a fundamental indeterminancy.

b. Boyer has shown that Planck blackbody radiation can be derived from a discrete charge assumption (without partition as assumed by Planck). See [3].

c. Lee Smolin has described three approaches to quantum gravity in his book [4]. Considering our proposition above however, it seems that the notion of quantum gravity may be not necessary. Instead, we should consider discrete gravity theories.

d. Gary W. Gibbons and George F.R. Ellis have investigated a discrete Newtonian cosmology. That is a good start [5].

e. Gerard t Hooft has proposed a discrete deterministic interpretation of QM [6]. However, it seems the use of both discrete and quantum language are superfluous. We need to let go of quantum terminology that has its own excess baggage.

f. At the astronomical scale, Conrad Ranzan has proposed a cellular universe, which is essentially a Newtonian steady-state model with a discrete cellular space model [7]. In our view, such an approach needs to be explored and investigated further. See also our recent paper where we suggest an ultradiscrete KdV as a model of cosmology [8]. See also the Lindquist-Wheeler model [9][10]. We discuss this approach in the last example of the last section of this paper.

g. It may be possible for certain conditions to consider a partial continuum and partially discrete space. In other words, we may have a hybrid space. We have yet to investigate it, however.

3 A FEW PHILOSOPHICAL CONSIDERATIONS

According to Miguel Lorente, in order to better understand these models, it
would be useful to consider three levels of human knowledge in the
comprehension of the physical world [12]:

Level 1: Physical magnitudes such as distances, intervals, force, mass and
charge that are given by our sensation and perceptions.
Level 2: Mathematical structures that are the result of metrical properties
given by measurements and numerical relations among them.
Level 3: Fundamental concepts representing the ontological properties of the
physical world given by our intelligence in an attempt to know reality. This
level of knowledge is not accepted by some philosophical positions like logical
positivismus, conventionalismus and neokantismus.

There must be some connections among the three levels. In QM, the
theoretical models of microphysics in level 2 are related to observable
magnitudes in level 1 by correspondence laws. If we accept level 3, it should
be connected to level 2 and to level 1 (through level 2). In fact, the rules
governing the constructions of theoretical models in level 2 must be
grounded in some fundamental (ontological) properties of the physical world.
It is also worth noting that there are dierent interpretations of the concepts
of space and time. They are usually divided in three classes, as follows [12]:

(a) Dualistic theories: space is a container in which particles and waves are
moving. Time is also a separate entity with respect to the motion that takes
place. Therefore space and time are absolute and can be thought of in the
absence of particles (Newton).
(b) Monistic theories: spacetime is identied with some properties of matter
and cannot be conceivable without the existence of the latter. The eld of
forces and their sources are nothing more than geometrical deformations of
spacetime (Einstein, Kaluza-Klein, Wheeler).
(c) Relational theories: spacetime consists of the set of relations among some
fundamental objects, namely: monads (Leibniz), units (Penrose), processes
(Weisaecker, Finkelstein), preparticles (Bunge, Garca Sucre) and objects
(Hilbert).

In the present paper, following our argument in the previous section, we
assert that space consists of discrete cells with finite dimensions, which is the

most realistic model to the best of our knowledge. Next, we will discuss some implications of our assertion in different areas of interest.

4 PROOF OF CONCEPT: A FEW IMPLICATIONS OF DISCRETE CELLULAR SPACE

To elucidate our propositions, we will discuss some implications of the discrete cellular space model in several areas of interest: (a) cell biology, (b) cellular computing, (c) Maxwell equations, (d) low energy fusion, and (e) cosmology modelling.

(a) **Cell biology**. The mathematical modeling of cell populations can be, broadly speaking, split into two categories: continuum and discrete models. Discrete models treat cells as individual entities and hence provide a natural framework within which to make use of an increasing amount of experimental data available at the cellular and subcellular scales. There are now many different types of discrete cell-level models used to describe cell populations, e.g., cellular automata, cellular Potts models, cell-vertex, and off-lattice cell-based model [19].

(b) While continuum models have their own advantages, they also have certain limitations [20]:
Continuum models of the cell aim at capturing its passive dynamics. In addition to the limitations mentioned above, current models do not yet typically account for active biology: deformations and stresses experienced as a direct consequence of biochemical responses of the cell to mechanical load cannot be predicted by current continuum models. However, by contrasting the predicted purely mechanical cell response to experimental observations, one could isolate phenomena involving active biology, such as cell contraction or migration, from the passive mechanical response of the cell. Alternatively, continuum models might be envisioned that account for active processes through time-dependent properties or residual strains that are linked to biological processes.
Another limitation of continuum models stems from lack of description of cytoskeletal bers. As such, they are not applicable for micromanipulations of the cell with a probe of the same size or smaller than the cytoskeletal mesh (0.11.0 m). This includes most AFM experiments. In addition, the continuum models exclude small Brownian motions due to thermal uctuations of the cytoskeleton, which would correspond to uctuations of the network nodes in a continuum model and have been shown to play a key role

in cell motility (Mogilner and Oster, 1996). Finally, continuum models have
so far employed a limited number of time constants to characterize the cells
behavior. However, cells have recently been shown to exhibit behaviors with
power-law rheology, implying a continuous spectrum of time scales (Fabry et
al., 2001; Desprat et al., 2004). In the meantime, models involving a nite
number of time constants consistent with the time scale of the experimental
technique can be used, recognizing their limitations [20].

(c) **Cellular computer**. Around 18 years ago, Sipper described a number of
interesting features of the cellular computer. He began his article by noting
that von Neumanns architecture - which is based upon the principle of one
complex processor that sequentially performs a single complex task at a
given moment - has dominated computing technology for the past 50 years.
Recently, however, researchers have begun exploring alternative
computational systems based on entirely different principles. Although
emerging from disparate domains, the work behind these systems shares a
common computational philosophy, which can be called cellular computing
[21]. Cellular computers are supposed to have three principles in common.
Combining these three principles results in the following definition: cellular
computing = simplicity + vast parallelism + locality. Because the three
principles are highly interrelated, attaining vast parallelism, for example, is
facilitated by the cells simplicity and local connectivity. Changing any single
term in the equation results in a different computational paradigm. So, for
example, foregoing the simplicity property results in the distributed
computing paradigm.

Cellular computing has been placed further along the parallelism axis to
emphasize the vastness aspect [21]. What specific application areas invite a
cellular computing approach?

Research has raised several possibilities: (1) Image processing. Applying
cellular computers to perform image-processing tasks arises as a natural
consequence of their architecture. For example, in a two-dimensional grid, a
cell (or group of cells) can correspond to an image pixel, with the machines
dynamics designed to perform a desired image-processing task. Research has
shown that cellular image processors can attain a high level of performance
and exhibit fast operation times for several problems. (2) Fast solutions to
NP-complete problems. Even if only a few such problems can be dealt with,
doing so may still prove highly worthwhile. NP-completeness implies that a
large number of hard problems can be efciently solved, given an efcient
solution to any one of them. The list of NP-complete problems includes

hundreds of cases from several domains, such as graph theory, network design, logic, program optimization, and scheduling, to mention but a few. (3) Generating long sequences of high-quality random numbers. This capability is of prime import in domains such as computational physics and computational chemistry. Cellular computers may prove a good solution to this problem. (4) Nanoscale calculating machines. Cellular computings ability to perform arithmetic operations raises the possibility of implementing rapid calculating machines on an incredibly small scale. These devices could exceed current models speed and memory capacity by many orders of magnitude. (5) Novel implementation platforms. Such platforms include recongurable digital and analog processors, molecular devices, and nanomachines. [21].

(d) Maxwells equations. While X.S. Wang [23] was able to derive the above mentioned Maxwells equations in vacuum based on a continuum mechanics model of vacuum and a singularity model of electric charges, Krasnoholovets managed to show in the meantime and quite remarkably that the very definition of charge can be modelled from the viewpoint of tessellated space [22]. He argued that Maxwells equations are the manifestation of hidden dynamics of surface fractals. He also concludes that James Clerk Maxwell was right when he utilized imaginary cogwheels constructing the equations of motion of an electromagnetic field [22, p. 128].

(e) Low energy fusion. Since the early years of condensed matter nuclear science (aka. LENR/cold fusion), Robert W. Bussard from the Energy/Matter Conversion Corporation has argued in favor of internal nuclear fusion in a metal lattice to explain the low energy reaction as reported by Pons and Fleischmann [24]. Subsequently, there are a number of researchers who have explored the implications of lattice vibration and lattice structure models from solid state physics in order to explain the CMNS process [25][26][27]. Such approaches seem to be quite promising and they are worthy to continue further. For a recent discussion on discrete and continuum modelling, see [28].

(f) Cosmology modelling. Many physicists and philosophers alike have debated a long-standing puzzle: whether space is continuous or discrete. It has been known for long time that most of the existing cosmology models rely on a pseudo-Riemannian metric as the cornerstone of the Einsteinian universe. However, the metric itself is based on continuum theory. It is

known that such models have led us to too many (monster) problems,
including dark matter and dark energy etc. Now, what if the universe is
discrete? Then perhaps we can solve these problems naturally.
Philosophically speaking, the notion of discrete space can be regarded as a
basic question in the definition of differential calculus and limit. If it is
supposed that space is continuous, then we can use standard differential
calculus, but if we assume it is finite and discrete, we should use a difference
equation or finite difference theories. This problem is particularly acute
when we want to compute our mathematical models in computers, because
all computers are based on discrete mathematics. Then we can ask: is it
possible that discrete mathematics can inspire cosmology theorizing too?
Despite the fact that the majority of cosmologists rely on a standard model
called Lambda-CDM theory, here we will explore redshift theory based on a
few lattice-cellular models, including Lindquist-Wheeler theory and beyond
it. We will also touch briefly on some peculiar models such as Voronoi
tessellation and Conrad Ranzans cellular model of the universe.

a). Lindquist-Wheelers theory. In this model, matter content is assumed
to be discrete, identical spherically symmetric islands uniformly distributed
in a regular lattice. This attempt was first introduced in 1957 by Lindquist
and Wheeler (LW) in a seminal paper. While LW suggested that their global
dynamics are similar to those of the Friedmann universe for closed dust
dominated universe, Shalaby has shown that the LW-model can be extended
to yield a redshift equation, as follows [9]:

$$1 + z = 1 + \langle y \rangle \ln \left(\frac{a_r}{a_e} \right) = 1 + \langle \langle y \rangle \rangle \ln(1 + z_{FRW}) \simeq (1 + z_{FRW})^{\langle y \rangle} \qquad (1)$$

It can be shown that the value of $\langle y \rangle$ approximates geometrically to be 2/3,
however, numerically its value was estimated to be 7/10 [9]. Liu also
analyzed the LW model and he concludes that LW redshifts can differ from
their FLRW counterparts by as much as 30%, even though they increase
linearly with FLRW redshifts and they exhibit a non-zero integrated
Sachs-Wolfe effect, something that would not be possible in
matter-dominated FLRW universes without a cosmological constant [10].

b. Voronoi Tessellation model. Rien van de Weygaert describes a novel
model based on Voronoi tessellation. The spatial cosmic matter distribution
on scales of a few up to more than a hundred Megaparsecs displays a salient
and pervasive foam-like pattern [14]. Voronoi tessellations are a versatile and

flexible mathematical model for such web-like spatial patterns. Cellular patterns may be the source of an intrinsic geometrically biased clustering. However, so far we have not found a redshift equation from this model [13].

c. Non-expanding cellular universe. Conrad Ranzan suggests a DSSU cellular cosmology (dynamics steady state universe), which he claims to be problem-free. The cosmic redshift is shown to be a velocity-differential effect caused by a flow differential of the space medium. He obtains the cosmic redshift equation in its basic form [7]:

$$z = (1 + z_{GC})^N - 1 \tag{2}$$

There are other cellular cosmology models of course and some of them have been reviewed by Marmet, but this paper is not intended to be an exhaustive list of redshift models. See for example: Marmet [18].

CONCLUSION

An old question and paradox in Euclidean geometry may be resolved consistently once we accept and assume discrete cellular space instead of a continuum model, which is full of indeterminacies. Many implications and further developments can be expected both in the particle physics realm and also in cosmology theorizing. More observations and experiments are recommended to verify whether space is discrete, continuous, or hybrid.

In retrospect, it may offer an alternative for quantum gravity, i.e. by exploring discrete gravitational theories. To elucidate our propositions, we discussed some implications of the discrete cellular space model in several areas of interest: (a) cell biology, (b) cellular computing, (c) Maxwell equations, (d) low energy fusion, and (e) cosmology modelling.

REFERENCES

[1] https://math.stackexchange.com/questions/1799125/how-many-points-in-a-line-segment

[2] H.J.M. Bos. Differentials, Higher Order Differentials, and the Derivative in the Leibnizian calculus. Arch. Hist. Exact. Sci. vol. 14, p. 24

[3] Timothy H. Boyer. Connecting blackbody radiation, relativity, and discrete charge in classical electrodynamics. arXiv: physics/0605003 (2006)

[4] Lee Smolin. Three roads to quantum gravity. Basic Books, 2001.

[5] Gary Gibbons and George Ellis . Discrete Newtonian Cosmology. Classical and Quantum Gravity vol. 31, no. 2, (2013), see also arXiv: 1308.1852 (2013). url: http://iopscience.iop.org/article/10.1088/0264-9381/31/2/025003/meta

[6] Gerard t Hooft. A Deterministic Interpretation of Quantum Mechanics. url: www.rug.nl/research/vsi/events/groenewold/thooft.pdf

[7] Conrad Ranzan. Large scale structure of the Dynamic Steady State Universe. American Journal of Astronomy and Astrophysics 2016, 4(6): 65-77. http://www.sciencepublishinggroup.com. See also www.cellularuniverse.org

[8] Victor Christianto & Florentin Smarandache & Yunita Umniyati. An outline of Cellular Automaton Universe via Cosmological KdV equation. Published at Prespacetime Journal, and this paper was also presented at CTPNP 3rd, 1st Aug 2017, held in Bandung, Indonesia.

[9] Mohamad Shalaby. Dynamics and Light Propagation in a Universe with Discrete-Matter Content. arXiv: 1205.68773 [gr-qc]

[10] Rex G. Liu. The Lindquist-Wheeler formulation of lattice universes. arXiv: 1501.05169 [gr-qc]

[11] https://www.ias.edu/ideas/2011/kennedy-continuum-hypothesis

[12] Miguel Lorente. A realistic interpretation of lattice gauge theories. arXiv: hep-lat/0312044v1 (2003)

[13] Rien van de Weygaert. Voronoi Tessellations and the Cosmic Web: spatial patterns and clustering across the Universe. Proc. of the 4th International Symposium on Voronoi Diagrams in Science and Engineering, p.230-239, July 09-11, 2007 [doi¿10.1109/ISVD.2007.48]. url: http://www.astro.rug.nl/ weygaert/tim1publication/vorwey.isvd07.pdf

[14] A.A. Kirillov & D. Turaev. Foam-like structure of the Universe. Phys. Lett. B 656 (2007) 1-9.

[15] Conrad Ranzan. Cosmic redshift in the non-expanding cellular universe: velocity-differential theory of cosmic redshift. American Journal of Astronomy and Astrophysics 2014, 2(5): 47-60

[16] Large scale structure of the Dynamic Steady State Universe. American Journal of Astronomy and Astrophysics 2016, 4(6): 65-77. http://www.sciencepublishinggroup.com

[17] Chris Hand. Simple cellular automata on a spreadsheet. CHEER, vol. 17, 2005. url: http://economicsnetwork.ac.uk/cheer/ch17/hand.htm

[18] Louis Marmet. On the Interpretation of Red-Shifts: A Quantitative Comparison of Red-Shift Mechanisms II. http://www.marmet.org

[19] Philip J. Murray et al. From a discrete to a continuum model of cell dynamics in one dimension. Phys. Rev. E80 (2009).

[20] Mohammad R. K. Mofrad, Helene Karcher, and Roger D. Kamm. Continuum elastic or viscoelastic models of the cell. Chapter 4 in Cytoskeletal Mechanics: Models and Measurements in Cell Mechanics. https://books.google.co.id/books?isbn=1139458108

[21] M. Sipper, The Emergence of Cellular Computing, IEEE Computer, vol. 32, no. 7, pp. 18-26, July 1999. url: https://www.cs.bgu.ac.il/ sipper/publications/cellcomp.pdf

[22] Volodymyr Krasnoholovets. Structure of Space and the Submicroscopic Deterministic Concept of Physics. USA: Apple Academic Press & CRC, 2017. P. 118-145

[23] Xiao-Song Wang. Derivation of the Maxwells Equations Based on a Continuum Mechanical Model of Vacuum and a Singularity Model of Electric Charges. arXiv: physics/0609027 (2006)

[24] Robert W. Bussard. Virtual state internal nuclear fusion in metal lattice. Fusion Technology, Volume 16, September 1989.

[25] Steven E. Jones, E.P. Palmer, J.B. Czirr, D.L. Decker, G.L. Jensen, J.M. Thorne, S.F. Taylor, and J. Rafelski; Observations of Cold Nuclear Fusion in Condensed Matter, Nature 338: 737, 740, April 1989

[26] Martin Fleischmann and Stanley Pons; Electrochemically Induced Nuclear Fusion of Deuterium, Journal Electroanalytical Chemistry, 261, 301, 1989.

[26] V.B. Belyaev et al. Nuclear fusion induced by X-rays in a crystal. arXiv: 1603.01723 (2016)

[27] Yoshiaki Arata & Yue-Chang Zhang. Formation of condensed metallic deuterium lattice and nuclear fusion. Proc. Japan Acad., 78, Ser. B (2002), 57-60.

[28] V. A. Kuzkin, A. M. Krivtsov, E. A. Podolskaya & M. L. Kachanov (2016): Lattice with vacancies: elastic fields and effective properties in frameworks of discrete and continuum models, Philosophical Magazine, DOI:

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