On the Meaning of Imaginary Part of Solution of Biquaternion Klein-Gordon Equation

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In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we discuss some possible interpretation of this imaginary part of the solution of biquaternionic KGE (BQKGE). Further observation is of course recommended in order to refute or verify this proposition.

Some interpretations of preceding result of biquaternionic KGE

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = -m^2\varphi(x,t)\,, \tag{1}$$

Or this equation can be rewritten as:

$$\left(\Diamond\bar{\Diamond} + m^2\right)\varphi(x,t) = 0\,, \tag{2}$$

Provided we use this definition:

$$\Diamond = \nabla^q + i\nabla^q = \left(-i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) + i\left(-i\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right) \tag{3}$$

Where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: e_1=i, e_2=j , e_3 =k):

$$i^2 = j^2 = k^2 = -1\,,\ ij = -ji = k\,,$$
$$jk = -kj = i\,, ki = -ik = j\,. \tag{4}$$

And quaternion *Nabla operator* is defined as [5]:

$$\nabla^q = -i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z} \quad (5)$$

Note that equation (3) and (5) included partial time-differentiation.

It is worth nothing here that equation (2) yields solution containing imaginary part, which differs appreciably from known solution of KGE:

$$y(x,t) = \left(\frac{1}{4} - \frac{i}{4}\right)m^2t^2 + cons\tan t \quad (6)$$

Some possible alternative interpretations of this *imaginary part* of the solution of biquaternionic KGE (BQKGE) are:

(a) The imaginary part implies that there is exponential term of the wave solution, which is quite similar to the Ginzburg-Landau extension of London phenomenology [3]:

$$\psi(r) = |\psi(r)|e^{i\varphi(r)}, \quad (7)$$

because equation (6) can be rewritten (approximately) as:

$$y(x,t) = \frac{e^i}{4}m^2t^2 \quad (8)$$

(b) The aforementioned exponential term of the solution (8) can be interpreted as signature of vortices solution. Interestingly Navier-Stokes equation which implies vorticity equation can also be rewritten in terms of Yukawa equation [8].

(c) The imaginary part implies that there is a spiral wave, which suggests spiralling motion of meson or other particles. Interestingly it has been argued that one can explain electron phenomena by assuming spiralling electrons [5]. Alternatively this spiralling wave may already be known in the form of Bierkeland flow. For meson observation, this could be interpreted as another form of meson, which may be called 'supersymmetric-meson' [1].

(d) The imaginary part of solution of BQKGE also implies that it consists of standard solution of KGE [1], and its alteration because of imaginary differential operator. That would mean the resulting wave is composed of two complementary waves.

(e) Considering some recent proposals suggesting that neutrino can have *imaginary mass* [6], the aforementioned imaginary part of solution of BQKGE can also imply that the (supersymmetric-) meson may be composed of neutrino(s). This new proposition may require new thinking both on the nature of neutrino and also supersymmetric-meson. [7]

While some of these propositions remain to be seen, in deriving the preceding BQKGE we follow Dirac's phrase that '*One can always generalize his physics by generalizing his mathematics.*' More specifically, we focus on using a 'theorem' from this principle, i.e.: '*One can generalize his mathematics by generalizing his (differential) operator.*'

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

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