Multi-Attribute Decision Making Based on Interval Neutrosophic Trapezoid Linguistic Aggregation Operators

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ABSTRACT

Multi-attribute decision making (MADM) play an important role in many applications, due to the efficiency to handle indeterminate and inconsistent information, interval neutrosophic sets is widely used to model indeterminate information. In this paper, a new MADM method based on interval neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation (INTrLWAA) operator and interval neutrosophic trapezoid linguistic weighted geometric aggregation (INTrLWGA) operatoris presented. A numerical example is presented to demonstrate the application and efficiency of the proposed method.

1. INTRODUCTION

Smarandache (1998) proposed the neutrosophic set (NS) by adding an independent indeterminacy-membership function. The concept of neutrosophic set is a generalization of classic set, fuzzy set (Zadeh, 1956), intuitionistic fuzzy set (Atanassov, 1989), interval intuitionistic fuzzy set(Atanassov et al., 1989; Atanassov, 1994) and so on. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. From scientific or engineering point of view, the neutrosophic set and set- theoretic view, operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang, et al.(2010) defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets. Furthermore, Wang, et al.(2005) proposed the set theoretic operations on an

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instance of neutrosophic set called interval valued neutrosophic set (IVNS) which is more flexible and practical than NS. The works on neutrosophic set (NS) and interval valued neutrosophic set (IVNS), in theories and application have been progressing rapidly (e.g, Kharal, 2013; Ansaria et al., 2013; Saha et al, 2013; Rabounski et al, 2005 ; Lupiáñez, 2008 ; Wang et al, 2010; Deli et al, 2014 ; Deli et al, 2014a, Ye, 2014 ; Ye, 2014a ; Ye, 2014b ; Ye, 2014c ; Ye, 2014d ; Ye, 2014e ; Ye, 2014f ; Ye, 2014g ; Zeng, 2006; Peide et al. 2014 ; Arora et al, 2011; Arora et al, 2010 ; Chi et al. 2013 ; Liu et al. 2014 ;Biswas et al. 2014 ; Şahin et al . 2014 ; Aggarwal et al, 2010 ; Broumi et al, 2013; Broumi et al, 2013a; Broumi et al, 2013b; Broumi et al, 2013c; Broumi et al, 2014d; Broumi et al, 2013e; Broumi et al, 2014f; Broumi et al, 2014g ; Broumi et al, 2014h, Broumi et al, 2015i ; Broumi et al, 2014j ; Broumi et al, 2015k ; Broumi et al, 2015*l ;* Broumi et al, 2015*m*)

Multiple attribute decision making (MADM) problem are of importance in most kind of fields such as engineering, economics, and management. In many situations decision makers have incomplete, indeterminate and inconsistent information about alternatives with respect to attributes. It is well known that the conventional and fuzzy or intuitionistic fuzzy decision making analysis (Zeng, 2006; Broumi et al., 2015; Broumi et al., 2014) using different techniques tools have been found to be inadequate to handle indeterminate an inconsistent data. So, Recently, neutrosophic multicriteria decision making problems have been proposed to deal with such situation.

In addition, because the aggregation operators are the important tools to process the neutrosophic decision making problems. Lately, Research on aggregation methods and multiple attribute decision making theories under neutrosophic environment is very active and lot of results have been obtained from neutrosophic information. Based on the aggregation operators, Ye (2013) developed some new weighted arithmetic averaging and weighted geometric averaging operators for simplified neutrosophic sets. Peide et al.(2014) present the generalized neutrosophic Hamacher aggregation operators such as Generalized neutrosophic number Hamacher weighted averaging (GNNHWA) operator, Generalized neutrosophic number Hamacher ordered weighted averaging (GNNHOWA) operator, and Generalized neutrosophic number Hamacher hybrid averaging (GNNHA) operator and studied some properties of these operators and analyzed some special cases and gave a decision-making method based on these operators for multiple attribute group decision making with neutrosophic numbers. Based on the idea of Bonferroni mean, Peide et al. (2014) proposed some Bonferroni mean operators such as the single-valued neutrosophic normalized weighted Bonferroni mean.

Based on the linguistic variable and the concept of interval neutrosophic sets, Ye (2014) defined interval neutrosophic linguistic variable, as well as its operation principles, and developed some new aggregation operators for the interval neutrosophic linguistic information, including interval neutrosophic linguistic arithmetic weighted average(INLAWA) operator, linguistic geometric weighted average(INLGWA) operator and discuss some properties. Furthermore, he proposed thedecision making method for multiple attribute decision making (MAGDM) problems with an illustrated example to show the process of decision making and the effectiveness of the proposed method.

In order to deal with the more complex neutrosophic information. Ye (2013), further proposed the interval neutrosophic uncertain linguistic variables by extending uncertain linguistic variables with an interval neutrosophic set, and proposed the operational rules, score function, accuracy function and certainty function of interval neutrosophic uncertain variables. Then, the interval neutrosophic uncertain weighted arithmetic averaging operator and interval neutrosophic neutrosophic uncertain linguistic weighted geometric averaging operator are developed, and a multiple attribute decision making method with interval neutrosophic linguistic information is proposed.

In order to process incomplete, indeterminate and inconsistent information more efficiency and precisely, it is necessary to make a further study on the extended form of the interval neutrosophic uncertain linguistic variables by combining trapezoid fuzzy linguistic variables and interval neutrosophic set. For example, we can evaluate the investment alternatives problem by the linguistic set: S= {s_0 (extremely low); s_1 (very low); s_2 (low); s_3 (medium); s_4 (high); s_5 (very high); s_6 (extermley high). Perhaps, we can use the trapezoid fuzzy linguistic $[s_\theta, s_\rho, s_\mu, s_v]$, $(0 \leq \theta \leq \rho \leq \mu \leq v \leq l - 1)$ to describe the evaluation result, but this is not accurate, because it merely provides a linguistic range. In this paper, we can use interval neutrosophic trapezoid linguistic (INTrL), $<[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{v(x)}], (T_A(x), I_A(x), F_A(x))>$to describe the investment problem giving the membership degree, indeterminacy degree, and non-membership degree interval to $[s_\theta, s_\rho, s_\mu, s_v]$. This is the motivation of our study. As a fact, INTrL avoids the information distortions and losing in decision making process, and overcomes the shortcomings of the interval neutrosophic linguistic variables by Ye (2014) and interval neutrosophic uncertain linguistic variables by Ye (2015).

To achieve the above purposes, The remainder of this paper is organized as follows: some basic definitions of trapezoid linguistic term set, neutrosophic set, and interval neutrosophic set are briefly reviewed in section 2. In section3, the concept, operational laws, score function, accuracy function and certainty function of including interval neutrosophic trapezoid linguistic elements are defined. In section 4, some interval neutrosophic trapezoid linguistic aggregation operators are proposed, such as interval neutrosophic trapezoid linguistic weighted average (INTrLWAA) operator, interval neutrosophic trapezoid linguistic weighted average (INTrLWGA) operators, then some desirable properties of the proposed operators are investigated. In section 5, we develop an approach for multiple attribute decision making problems with interval neutrosophic trapezoid linguistic information based on the proposed operators. In section 6, a numerical example is given to illustrate the application of the proposed method. The paper is concluded in section 7.

2. PRELIMINARIES

In the following section, we shall introduce some basic concepts related to trapezoidal fuzzy linguistic variables, neutrosophic set, single valued neutrosophic set, interval neutrosophic sets, Interval neutrosophic linguistic sets and interval neutrosophic uncertain linguistic sets.

2.1 Trapezoid Fuzzy Linguistic Variables

A linguistic set is defined as a finite and completely ordered discreet term set, $S = (s_0, s_1, \ldots, s_{l-1})$, where l is the odd value. For example, when l =7, the linguistic term set S can be defined as follows: S={s_0 (extremely low); s_1 (very low); s_2 (low); s_3 (medium); s_4 (high); s_5 (very high); s_6 (extremely high)}

Definition 2.1 (Chen et al.,(2011): Let $\overline{S} = \{s_\theta \mid s_0 \leq s_\theta \leq s_{l-1}\ \theta \in [0, l\text{-}1]\}$, which is the continuous form of linguistic set S. s_θ, s_ρ, s_μ, s_v are four linguistic terms in, and $s_0 \leq s_\theta \leq s_\rho \leq s_\mu \leq s_v \leq s_{l-1}$ if $0 \leq \theta \leq \rho \leq \mu \leq v \leq l{-}1$, then the trapezoid fuzzy linguistic variable is defined as $\hat{s} = [s_\theta, s_\rho, s_\mu, s_v]$, and $\hat{S}$ denotes a set of the trapezoid fuzzy linguistic variables.

In particular, if any two of s_θ, s_ρ, s_μ, s_ν are equal, then $\hat{s}$ is reduced to triangular fuzzy linguistic variable; if any three of s_θ, s_ρ, s_μ, s_ν are equal, then $\hat{s}$ is reduced to uncertain linguistic variable

2.2 The Expected Value of Trapezo Fuzzy Linguistic Variable

Let $\hat{s} = ([s_\theta, s_\rho, s_\mu, s_\nu])$ be a trapezoid fuzzy linguistic variable, then the expected value E($\hat{s}$) of $\hat{s}$ is defined as:

$$E(\hat{s}) = \frac{\theta + \rho + \mu + \nu}{4}$$

2.3 Neutrosophic Sets

Definition 2.2 (Smarandache, 1998): Let U be a universe of discourse then the neutrosophic set A is an object having the form

$A = \{< x: T_A(x), I_A(x),F_A(x) >, x \in X \}$,

where the functions T, I, F: U→]$^-$0,1+[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$^-0 \leq s$ up $T_A(x)$ +sup $I_A(x)$ +sup $F_A(x) \leq 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

2.4 Single Valued Neutrosophic Sets

Definition 2.3 (Wang, et al., 2010): Let X be an universe of discourse then the neutrosophic set A is an object having the form

$A = \{< x: T_A(x), I_A(x),F_A(x) >, x \in X \}$,

where the functions μ, ν, ω: U→[0,1]define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$0 \leq s$ up $T_A(x)$ +sup $I_A(x)$ +sup $F_A(x) \leq 3$

Definition 2.4 (Wang, et al., 2010): A single valued neutrosophic set A is contained in another single valued neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in U$, $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$.

2.5 Interval Valued Neutrosophic Sets

Definition 2.5 (Wang, et al., 2005): Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

For two IVNS,

$$A_{\mathrm{IVNS}} = \left\{\left\langle x, \left[T_A^L(x),\ T_A^U(x)\right],\ \left[I_A^L(x), I_A^U(x)\right],\ \left[F_A^L(x), F_A^U(x)\right]\right\rangle \mid x \in X\right\}$$

and

$$B_{\mathrm{IVNS}} = \left\{\left\langle x, \left[T_B^L(x),\ T_B^U(x)\right],\ \left[I_B^L(x), I_B^U(x)\right],\ \left[F_B^L(x), F_B^U(x)\right]\right\rangle \mid x \in X\right\}$$

the two relations are defined as follows:

1. $A_{\mathrm{IVNS}} \subseteq B_{\mathrm{IVNS}}$ if and only if $T_A^L(x) \le T_B^L(x)$, $T_A^U(x) \le T_B^U(x)$, $I_A^L(x) \ge I_B^L(x)$, $I_A^U(x) \ge I_B^U(x)$, $F_A^L(x) \ge F_B^L(x)$, $F_A^U(x) \ge F_B^U(x)$.
2. $A_{\mathrm{IVNS}} = B_{\mathrm{IVNS}}$ if and only if, $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$.

The complement of A_{IVNS} is denoted by A_{IVNS}^o and is defined by

$$A_{IVNS}^o = \left\{\left\langle x, \left[\mathrm{F}_\mathrm{A}^\mathrm{L}(\mathrm{x}), \mathrm{F}_\mathrm{A}^\mathrm{U}(\mathrm{x})\right],\ \left[1-\mathrm{I}_\mathrm{A}^\mathrm{U}(\mathrm{x}), 1-\mathrm{I}_\mathrm{A}^L(\mathrm{x})\right],\ \left[\mathrm{T}_\mathrm{A}^\mathrm{L}(\mathrm{x}),\ \mathrm{T}_\mathrm{A}^\mathrm{U}(\mathrm{x})\right] \mid x \in X\right\rangle\right\}$$

$$A \cap B = \left\{\left\langle\left[\min\left(T_A^L(x), T_B^L(x)\right), \min\left(T_A^U(x), T_B^U(x)\right)\right],\ \left[\max\left(I_A^L(x), I_B^L(x)\right),\right.\right.\right.$$
$$\left.\left.\max\left(I_A^U(x), I_B^U(x)\right)\right],\ \left[\max\left(F_A^L(x), F_B^L(x)\right), \max\left(F(x), F_B^U(x)\right)\right]\right\rangle : x \in X\}$$

$$A \cup B = \left\{\left\langle\left[\max\left(T_A^L(x), T_B^L(x)\right), \max\left(T_A^U(x), T_B^U(x)\right)\right],\ \left[\min\left(I_A^L(x), I_B^L(x)\right),\right.\right.\right.$$
$$\left.\left.\min\left(I_A^U(x), I_B^U(x)\right)\right],\ \left[\min\left(F_A^L(x), F_B^L(x)\right), \min\left(F_A^U(x), F_B^U(x)\right)\right]\right\rangle : x \in X\}$$

2.6 Interval Neutrosophic Linguistic Set

Based on interval neutrosophic set and linguistic variables, (Ye, 2014) presented the extension form of the linguistic set, i.e, interval neutrosophic linguistic set, which is shown as follows:

Definition2.6 (Ye, 2014): An interval neutrosophic linguistic set A in X can be defined as

$A = \{<x, s_{\theta(x)}, (T_A(x), I_A(x), F_A(x))>| \ x \in X\}$

where $s_{\theta(x)} \in \hat{s}$, $T_A(x) = \left[T_A^L(x), T_A^U(x)\right] \subseteq [0.1]$, $I_A(x) = \left[I_A^L(x), I_A^U(x)\right] \subseteq [0.1]$, and $F_A(x) = \left[F_A^L(x), F_A^U(x)\right] \subseteq [0.1]$ with the condition $0 \leq T_A^U(x) + I_A^U(x) + F_A^U(x) \leq 3$ for any $x \in X$. The function $T_A(x)$, $I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree, the indeterminacy – membership degree, and the falsity-membership degree with interval values of the element x in X to the linguistic variable $s_{\theta(x)}$.

2.7 Single Valued Neutrosophic Trapezoid Linguistic Sets

Definition2.7 (Broumi, et al., 2014). A single valued neutrosophic trapezoid linguistic set A in X can be defined as

$A = \{<x, [s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}], (T_A(x), I_A(x), F_A(x))>: x \in X\}$

where $s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)} \in I_A \ \hat{s}$, $T_A(x) \in [0.1]$, $(x) \in [0.1]$, and $F_A(x) \in [0.1]$ with the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for any $x \in X$. $[s_{\theta(x)}, T_A\ s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$ is a trapezoid fuzzy linguistic term, The function (x), $I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree, the indeterminacy – membership degree, and the falsity-membership degree of the element x in X belonging to the linguistic term $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$.

Definition2.8: Let $A = \{<x, [s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}], (T_A(x), I_A(x), F_A(x))>: x \in X\}$ be an SVNTrLN. Then the eight tuple $<[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}], (T_A(x), I_A(x), F_A(x))>$ is called a SVNTrLV and A can be viewed as a collection of SVNTrLVs. Thus, the SVNTrLVS can also be expressed as

$A = \{<x, [s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}], (T_A(x), I_A(x), F_A(x))>: x \in X\}$

For any two SVNTrLNs

$$\tilde{a}_1 = \left\langle \left[s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}, s_{\mu(\tilde{a}_1)}, s_{\nu(\tilde{a}_1)} \right], \left(T(\tilde{a}_1), I(\tilde{a}_1), F(\tilde{a}_1) \right) \right\rangle$$

and

$$\tilde{a}_2 = \left\langle \left[s_{\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_2)} \right], \left(T(\tilde{a}_2), I(\tilde{a}_2), F(\tilde{a}_2) \right) \right\rangle$$

and $\lambda \geq 0$, (Broumi et al. 2014) defined the following operational rules:

$$\tilde{a}_1 \oplus \tilde{a}_2 = \Big\langle \Big[s_{\theta(\tilde{a}_1)+\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)+\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_1)+\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_1)+\nu(\tilde{a}_2)} \Big]$$

$$\Big(\big(T(\tilde{a}_1) + T(\tilde{a}_2) - T(\tilde{a}_1), T(\tilde{a}_2) \big),\ I(\tilde{a}_1), I(\tilde{a}_2),\ F(\tilde{a}_1), F(\tilde{a}_2) \Big) \Big\rangle$$

$$\tilde{a}_1 \otimes \tilde{a}_2 = \Big\langle \Big[s_{\theta(\tilde{a}_1)\times\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)\times\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_1)\times\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_1)\times\nu(\tilde{a}_2)} \Big]$$

$$\big(T(\tilde{a}_1)T(\tilde{a}_2),\ \big(I(\tilde{a}_1) + I(\tilde{a}_2) - I(\tilde{a}_1)I(\tilde{a}_2) \big),\ \big(F(\tilde{a}_1) + F(\tilde{a}_2) - F(\tilde{a}_1)F(\tilde{a}_2) \big) \big) \Big\rangle$$

$$\lambda\tilde{a}_1 = \Big\langle \Big[s_{\lambda\theta(\tilde{a}_1)}, s_{\lambda\rho(\tilde{a}_1)}, s_{\lambda\mu(\tilde{a}_1)}, s_{\lambda\nu(\tilde{a}_1)} \Big],\ \Big(\big(1 - T(\tilde{a}_1)\big)^{\lambda},\ \big(I(\tilde{a}_1)\big)^{\lambda},\ \big(F(\tilde{a}_1)\big)^{\lambda} \Big) \Big\rangle$$

$$\tilde{a}_1^{\lambda} = \Big\langle \Big[s_{\theta^{\lambda}(\tilde{a}_1)}, s_{\rho^{\lambda}(\tilde{a}_1)}, s_{\mu^{\lambda}(\tilde{a}_1)}, s_{\nu^{\lambda}(\tilde{a}_1)} \Big],\ \Big(\big(T(\tilde{a}_1)\big)^{\lambda},\ \big(1 - I(\tilde{a}_1)\big)^{\lambda},\ \big(1 - F(\tilde{a}_1)\big)^{\lambda} \Big) \Big\rangle$$

Definition 2.9: Let $\tilde{a}_i = \Big\langle \Big[s_{\theta(\tilde{a}_i)}, s_{\rho(\tilde{a}_i)}, s_{\mu(\tilde{a}_i)}, s_{\nu(\tilde{a}_i)} \Big],\ \big(T(\tilde{a}_i),\ I(\tilde{a}_i),\ F(\tilde{a}_i) \big) \Big\rangle$ be a SVNTrFLN, the expected function E($\tilde{a}_i$) and the accuracy H($\tilde{a}_i$) of are define as follows:

$$E(\tilde{a}) = \frac{1}{3}\Big(2 + T(\tilde{a}) - I(\tilde{a}) - F(\tilde{a}) \times S_{\left(\frac{\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})}{4}\right)}$$

$$= S_{\frac{1}{12}\left(2+T(\tilde{a})-I(\tilde{a})-F(\tilde{a})\right)\times\left(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})\right)} H(\tilde{a}) = \mathrm{T}(\tilde{a}) - F(\tilde{a}) \Big)$$

$$\times S_{\left(\frac{\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})}{4}\right)} = S_{\frac{1}{4}\left(T(\tilde{a})-F(\tilde{a})\times\left(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})\right)\right)}$$

Assume that $\tilde{a}_i$ and $\tilde{a}_j$ are two SVNTrLNs, they can be compared by the following rules:

1. If $E(\tilde{a}_i) > E(\tilde{a}_j)$, then $\tilde{a}_i > \tilde{a}_j$;
2. If $E(\tilde{a}_i) = E(\tilde{a}_j)$, then
   If $H(\tilde{a}_i) > H(\tilde{a}_j)$, then $\tilde{a}_i > \tilde{a}_j$,
   If $H(\tilde{a}_i) = H(\tilde{a}_j)$, then $\tilde{a}_i = \tilde{a}_j$,
   If $H(\tilde{a}_i) < H(\tilde{a}_j)$, then $\tilde{a}_i < \tilde{a}_j$.

3 INTERVAL NEUTROSOPHIC TRAPEZOID LINGUISTIC SETS

Based on the concept of INS and trapezoid fuzzy linguistic variable, we extend the NTrLS to define the INTrLS and INTrLNs. The operations and ranking method of INTrLN s are also given in this section

Definition3.1: Let X be a finite universal set and $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}] \in \hat{s}$ be trapezoid fuzzy linguistic variable. An INTrLS in X is defined as

A $=\{\langle x, [s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}], (T_A(x), I_A(x), F_A(x))\rangle | x \in X\}$

where $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}], \in \hat{s}$, T_A (x) = $\left[T_A^L(x), T_A^U(x)\right] \subseteq [0.1]$, I_A (x) = $\left[I_A^L(x), I_A^U(x)\right] \subseteq [0.1]$, and F_A (x) = $\left[F_A^L(x), F_A^U(x)\right] \subseteq [0.1]$ with the condition 0 ≤sup T_A (x)+sup I_A (x)+sup F_A (x) ≤3 for any x ∈ X. The function T_A (x), I_A (x) and F_A (x) express, respectively, the truth-membership degree interval, the indeterminacy –membership degree interval, and the falsity-membership degree interval of the element x in X belonging to the trapezoid linguistic variable $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}] \in \hat{s}$.

Definition 3.2: Let A =

$$\left\{\left\langle x,\ \left[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}\right],\ \left(\left[T_A^L(x), T_A^U(x)\right],\ \left[I_A^L(x), I_A^U(x)\right],\ \left[F_A^L(x), F_A^U(x)\right]\right)\right\rangle | x \in X\right\}$$

be an INTrL. Then the eight tuple

$$\left\langle\left[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}\right],\ \left(\left[T_A^L(x), T_A^U(x)\right],\ \left[I_A^L(x), I_A^U(x)\right],\ \left[F_A^L(x), F_A^U(x)\right]\right)\right\rangle$$

is called an INTrLV and A can be viewed as a collection of INTrLvs. Thus, the INTrLS can also be expressed as

$$A=\left\{\left\langle x,\ \left[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}\right],\ \left(\left[T_A^L(x), T_A^U(x)\right],\ \left[I_A^L(x), I_A^U(x)\right],\ \left[F_A^L(x), F_A^U(x)\right]\right)\right\rangle | x \in X\right\}$$

Definition 3.3: Let

$$\tilde{a}_1=\left\langle\left[s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}, s_{\mu(\tilde{a}_1)}, s_{\nu(\tilde{a}_1)}\right],\ \left(\left[T^L(\tilde{a}_1), T^U(\tilde{a}_1)\right],\ \left[I^L(\tilde{a}_1), I^U(\tilde{a}_1)\right],\ \left[F^L(\tilde{a}_1), F^U(\tilde{a}_1)\right]\right)\right\rangle$$

and

$$\tilde{a}_2=\left\{\left\langle x,\left[s_{\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_2)}\right],\ \left(\left[T^L(\tilde{a}_2), T^U(\tilde{a}_2)\right],\ \left[I^L(\tilde{a}_2), I^U(\tilde{a}_2)\right],\ \left[F^L(\tilde{a}_2), F^U(\tilde{a}_2)\right]\right)\right\rangle\right.$$

be two INTrLvs and λ≥ 0, then the operational laws of INTrLvs are defined as follows:

$$\tilde{a}_1 \oplus \tilde{a}_2=\left\langle\left[s_{\theta(\tilde{a}_1)+\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)+\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_1)+\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_1)+\nu(\tilde{a}_2)}\right],\ \left(\left[T^L(\tilde{a}_1)+T^L(\tilde{a}_2)-T^L(\tilde{a}_1)T^L(\tilde{a}_2),\ T^U(\tilde{a}_1)\right.\right.\right.$$
$$\left.\left.\left.+T^U(\tilde{a}_2)-T^U(\tilde{a}_1)T^U(\tilde{a}_2)\right],\left[I^L(\tilde{a}_1)I^L(\tilde{a}_2),\ I^U(\tilde{a}_1)I^U(\tilde{a}_2)\right],\left[F^L(\tilde{a}_1)F^L(\tilde{a}_2),\ F^U(\tilde{a}_1)F^U(\tilde{a}_2)\right]\right)\right\rangle$$

$$\tilde{a}_1 \otimes \tilde{a}_2 = \Big\langle \Big[s_{\theta(\tilde{a}_1)\times\theta(\tilde{a}_2)},\ s_{\rho(\tilde{a}_1)\times\rho(\tilde{a}_2)},\ s_{\mu(\tilde{a}_1)\times\mu(\tilde{a}_2)},\ s_{\nu(\tilde{a}_1)\times\nu(\tilde{a}_2)} \Big],\ \Big(\Big[T^L(\tilde{a}_1)T^L(\tilde{a}_2),\ T^U(\tilde{a}_1)T^U(\tilde{a}_2) \Big],\ \Big[I^L(\tilde{a}_1) + I^L(\tilde{a}_2) - I^L(\tilde{a}_1)I^L(\tilde{a}_2),\ I^U(\tilde{a}_1) + I^U(\tilde{a}_2) - I^U(\tilde{a}_1) + I^U(\tilde{a}_2) \Big],\ \Big[F^L(\tilde{a}_1) + F^L(\tilde{a}_2) - F^L(\tilde{a}_1)F^L(\tilde{a}_2),\ F^U(\tilde{a}_1) + F^U(\tilde{a}_2) - F^U(\tilde{a}_1)F^U(\tilde{a}_2) \Big] \Big) \Big\rangle$$

$$\lambda\tilde{a}_1 = \Big\langle \Big[s_{\lambda\theta(\tilde{a}_1)}, s_{\lambda\rho(\tilde{a}_1)}, s_{\lambda\mu(\tilde{a}_1)}, s_{\lambda\nu(\tilde{a}_1)} \Big], \Big(\Big[\big(1 - T^L(\tilde{a}_1)\big)^{\lambda}, \big(1 - T^U(\tilde{a}_1)\big)^{\lambda} \Big],\ \Big[\big(I^L(\tilde{a}_1)\big)^{\lambda}, \big(I^U(\tilde{a}_1)\big)^{\lambda} \Big], \Big[\big(F^L(\tilde{a}_1)\big)^{\lambda}, \big(F^U(\tilde{a}_1)\big)^{\lambda} \Big] \Big) \Big\rangle$$

$$\tilde{a}_1^{\lambda} = \Big\langle \Big[s_{\theta^{\lambda}(\tilde{a}_1)}, s_{\rho^{\lambda}(\tilde{a}_1)}, s_{\mu^{\lambda}(\tilde{a}_1)}, s_{\nu^{\lambda}(\tilde{a}_1)} \Big], \Big(\Big[\big(T^L(\tilde{a}_1)\big)^{\lambda}, \big(T^U(\tilde{a}_1)\big)^{\lambda} \Big],\ \Big[\big(1 - I^L(\tilde{a}_1)\big)^{\lambda}, \big(1 - I^U(\tilde{a}_1)\big)^{\lambda} \Big],\ \Big[\big(1 - F^L(\tilde{a}_1)\big)^{\lambda}, \big(1 - F^U(\tilde{a}_1)\big)^{\lambda} \Big] \Big) \Big\rangle$$

Obviously, the above operational results are still INTrLvs.

Theorem 3.4: Let

$$\tilde{a}_1 = \Big\langle \Big[s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}, s_{\mu(\tilde{a}_1)}, s_{\nu(\tilde{a}_1)} \Big],\ \Big(\Big[T^L(\tilde{a}_1), T^U(\tilde{a}_1) \Big],\ \Big[I^L(\tilde{a}_1), I^U(\tilde{a}_1) \Big],\ \Big[F^L(\tilde{a}_1), F^U(\tilde{a}_1) \Big] \Big) \Big\rangle$$

and

$$\tilde{a}_2 = \Big\langle \Big[s_{\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_2)} \Big],\ \Big(\Big[T^L(\tilde{a}_2), T^U(\tilde{a}_2) \Big],\ \Big[I^L(\tilde{a}_2), I^U(\tilde{a}_2) \Big],\ \Big[F^L(\tilde{a}_2), F^U(\tilde{a}_2) \Big] \Big) \Big\rangle$$

be any two interval neutrosophic trapezoid linguistic variables, and λ, λ_1, $\lambda_2 \geq 0$, then the characterstics of interval neutrosophic trapezoid linguistic variables are shown as follows:

1. $\tilde{a}_1 \oplus \tilde{a}_2 = \tilde{a}_2 \oplus \tilde{a}_1$;
2. $\tilde{a}_1 \otimes \tilde{a}_2 = \tilde{a}_2 \otimes \tilde{a}_1$;
3. $\lambda\left(\tilde{a}_1 \oplus \tilde{a}_2\right) = \lambda\tilde{a}_1 \oplus \lambda\tilde{a}_2$;
4. $\lambda\tilde{a}_1 \oplus \lambda\tilde{a}_2 = (\lambda_1 + \lambda_2)\tilde{a}_1$;
5. $\tilde{a}_1^{\lambda_1} \otimes \tilde{a}_1^{\lambda_2} = \tilde{a}_1^{\lambda_1+\lambda_2}$;
6. $\tilde{a}_1^{\lambda_1} \otimes \tilde{a}_2^{\lambda_1} = (\tilde{a}_1 \otimes \tilde{a}_2)^{\lambda_1}$.

Theorem 3.4 can be easily proven according to definition 3.3 (omitted).

To rank INTrLNs, we define the score function, accuracy function and certainty function of an INTrFLN based on (Ye, 2013 g), which are important indexes for ranking alternatives in decision making problems.

Definition 3.5: Let

$$\tilde{a} = \left\langle \left[s_{\theta(\tilde{a})}, s_{\rho(\tilde{a})}, s_{\mu(\tilde{a})}, s_{\nu(\tilde{a})} \right],\ \left(\left[T^L(\tilde{a}), T^U(\tilde{a}) \right],\ \left[I^L(\tilde{a}), I^U(\tilde{a}) \right],\ \left[F^L(\tilde{a}), F^U(\tilde{a}) \right] \right) \right\rangle$$

be an INTrFLV. Then, the score function, accuracy function and certainty function of an INTrFLN $\tilde{a}$ are defined, respectively, as follows:

$$E(\tilde{a}) = \frac{1}{6}\left(4 + T^L(\tilde{a}) - I^L(\tilde{a}) - F^L(\tilde{a}) + T^U(\tilde{a}) - I^U(\tilde{a}) - F^U(\tilde{a})\right) \times S_{\left(\frac{\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})}{4}\right)} = S_{\frac{1}{24}\left(4+T^L(\tilde{a})-I^L(\tilde{a})-F^L(\tilde{a})+T^U(\tilde{a})-I^U(\tilde{a})-F^U(\tilde{a})\right)\times\left(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})\right)} \quad (1)$$

$$H(\tilde{a}) = \frac{1}{2}\left(T^L(\tilde{a})F^U - F^L(\tilde{a}) + T^U(\tilde{a}) - (\tilde{a})\right) \times S_{\left(\frac{\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})}{4}\right)} = S_{\frac{1}{8}\left(T^L(\tilde{a})-F^L(\tilde{a})+T^U(\tilde{a})-F^U(\tilde{a})\right)\times\left(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})\right)} \quad (2)$$

$$C(\tilde{a}) = \frac{1}{2}\left(T^L(\tilde{a}) + T^U(\tilde{a})\right) \times S_{\left(\frac{\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})}{4}\right)} = S_{\frac{1}{8}\left(T^L(\tilde{a})+T^U(\tilde{a})\right)\times\left(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a})\right)} \quad (3)$$

Based on definition 3.5, a ranking method between INTrLvs can be given as follows.

Definition 3.6 Let $\tilde{a}_1$ and $\tilde{a}_2$ be two INTrLNs. Then, the ranking method can be defined as follows:

If $E(\tilde{a}_1) > E(\tilde{a}_2)$ then $\tilde{a}_1 > \tilde{a}_2$,

If $E(\tilde{a}_1) = E(\tilde{a}_2)$ and $H(\tilde{a}_1) > H(\tilde{a}_2)$ then $\tilde{a}_1 > \tilde{a}_2$,

If $E(\tilde{a}_1) = E(\tilde{a}_2)$ and $H(\tilde{a}_1) = H(\tilde{a}_2)$ and $C(\tilde{a}_1) > C(\tilde{a}_2)$ then $\tilde{a}_1 > \tilde{a}_2$,

If $E(\tilde{a}_1) = E(\tilde{a}_2)$ and $H(\tilde{a}_1) = H(\tilde{a}_2)$ and $C(\tilde{a}_1) = C(\tilde{a}_2)$ then $\tilde{a}_1 = \tilde{a}_2$.

4. INTERVAL NEUTROSOPHIC TRAPEZOID LINGUISTIC AGGREGATION OPERATORS

Based on the operational laws in definition 3.3, we can propose the following weighted arithmetic aggregation operator and weighted geometric aggregation operator for INTrLNs, which are usually utilized in decision making.

4.1 Interval Neutrosophic Trapezoid Linguistic Weighted Arithmetic Averaging Operator

Definition 4.1: Let $\tilde{a}_j = \left\langle \left[s_{\theta(\tilde{a})}, s_{\rho(\tilde{a})}, s_{\mu(\tilde{a})}, s_{\nu(\tilde{a})} \right], \left(\left[T^L_{a_j}, T^U_{a_j} \right], \left[I^L_{a_j}, I^U_{a_j} \right], \left[F^L_{a_j}, F^U_{a_j} \right] \right) \right\rangle$ (j=1,2,…,n) be a collection of INTrLNs. The interval neutrosophic trapezoid linguistic weighted arithmetic averaging average (INTrLWAA) operator can be defined as follows and INTrLWAA: $\Omega^n \rightarrow \Omega$

$$\text{INTrLWAA}\left(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n\right) = \sum_{j=1}^{n} \omega_j \tilde{a}_j \tag{4}$$

where, $\omega_j = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1, 2,…, n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Theorem 4.2: Let $\tilde{a}_j = \left\langle \left[s_{\theta(\tilde{a}_j)}, s_{\rho(\tilde{a}_j)}, s_{\mu(\tilde{a}_j)}, s_{\nu(\tilde{a}_j)} \right], \left(\left[T^L_{\tilde{a}_j}, T^U_{\tilde{a}_j} \right], \left[I^L_{\tilde{a}_j}, I^U_{\tilde{a}_j} \right], \left[F^L_{\tilde{a}_j}, F^U_{\tilde{a}_j} \right] \right) \right\rangle$ (j=1, 2,…,n) be a collection of INTrLNs, Then by Equation (4) and the operational laws in Definition 3.3, we have the following result

$$\begin{aligned}&\text{INTrLWAA}\\ &\left(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n\right) = \left\langle \left[s_{\sum_{j=1}^{n}\omega_j \theta(\tilde{a}_j)}, s_{\sum_{j=1}^{n}\omega_j \rho(\tilde{a}_j)}, s_{\sum_{j=1}^{n}\omega_j \mu(\tilde{a}_j)}, s_{\sum_{j=1}^{n}\omega_j \nu(\tilde{a}_j)} \right], \left[1 - \prod_{j=1}^{n}\left(1 - T^L\left(\tilde{a}_j\right)\right)^{\omega_j}, \right.\right.\\ &\left. 1 - \prod_{j=1}^{n}\left(1 - T^U\left(\tilde{a}_j\right)\right)^{\omega_j} \right], \left[\prod_{j=1}^{n}\left(I^L\left(\tilde{a}_j\right)\right)^{\omega_j}, \prod_{j=1}^{n}\left(I^U\left(\tilde{a}_j\right)\right)^{\omega_j} \right], \left.\left[\prod_{j=1}^{n}\left(F^L\left(\tilde{a}_j\right)\right)^{\omega_j}, \prod_{j=1}^{n}\left(F^U\left(\tilde{a}_j\right)\right)^{\omega_j} \right]\right\rangle\end{aligned} \tag{5}$$

where, $\omega_j = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1,2,…,n), $\omega_j \in [0,1]$and $\sum_{j=1}^{n} \omega_j = 1$.

Proof: The proof of Equation (5) can be done by means of mathematical induction

When n=2, then

$$\begin{aligned}\omega_1 \tilde{a}_1 = &\left\langle \left[s_{\omega_1 \theta(\tilde{a}_1)}, s_{\omega_1 \rho(\tilde{a}_1)}, s_{\mu(\tilde{a}_1)}, s_{\nu(\tilde{a}_1)} \right], \left[1 - \left(1 - T^L\left(\tilde{a}_1\right)\right)^{\omega_1}, 1 - \left(1 - T^U\left(\tilde{a}_1\right)\right)^{\omega_1} \right], \right.\\ &\left. \left[\left(I^L\left(\tilde{a}_1\right)\right)^{\omega_1}, \left(I^U\left(\tilde{a}_1\right)\right)^{\omega_1} \right], \left[\left(F^L\left(\tilde{a}_1\right)\right)^{\omega_1}, \left(F^U\left(\tilde{a}_1\right)\right)^{\omega_1} \right] \right\rangle\\ \omega_2 \tilde{a}_2 = &\left\langle \left[s_{\omega_2 \theta(\tilde{a}_2)}, s_{\omega_2 \rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_2)} \right], \left[1 - \left(1 - T^L\left(\tilde{a}_2\right)\right)^{\omega_2}, 1 - \left(1 - T^U\left(\tilde{a}_2\right)\right)^{\omega_2} \right], \right.\\ &\left. \left[\left(I^L\left(\tilde{a}_2\right)\right)^{\omega_2}, \left(I^U\left(\tilde{a}_2\right)\right)^{\omega_2} \right], \left[\left(F^L\left(\tilde{a}_2\right)\right)^{\omega_2}, \left(F^U\left(\tilde{a}_2\right)\right)^{\omega_2} \right] \right\rangle\end{aligned}$$

Thus,

$$\begin{aligned}
&\text{INTrLWAA}\left(\tilde{a}_1,\tilde{a}_2\right)=\omega_1\tilde{a}_1\oplus\omega_2\tilde{a}_2=\left\langle\left[S_{\sum_{j=1}^{2}\omega_j\theta(\tilde{a}_j)},S_{\sum_{j=1}^{2}\omega_j\rho(\tilde{a}_j)},S_{\sum_{j=1}^{2}\omega_j\mu(\tilde{a}_j)},S_{\sum_{j=1}^{2}\omega_j\nu(\tilde{a}_j)}\right],\ \left[\left(1-T^L\left(\tilde{a}_1\right)\right)^{\omega_1}\right.\right.\\
&\quad+\left(1-T^L\left(\tilde{a}_2\right)\right)^{\omega_2}-\left(\left(1-T^L\left(\tilde{a}_1\right)\right)^{\omega_1}\left(1-T^L\left(\tilde{a}_2\right)\right)^{\omega_2}\right),\ \left(1-T^U\left(\tilde{a}_1\right)\right)^{\omega_1}+\left(1-T^U\left(\tilde{a}_2\right)\right)^{\omega_2}\\
&\quad\left.-\left(\left(1-T^u\left(\tilde{a}_1\right)\right)^{\omega_1},\left(1-T^U\left(\tilde{a}_2\right)\right)^{\omega_2}\right)\right],\ \left[\left(I^L\left(\tilde{a}_1\right)\right)^{\omega_1}\left(I^L\left(\tilde{a}_2\right)\right)^{\omega_2},\left(I^U\left(\tilde{a}_1\right)\right)^{\omega_1}\left(I^U\left(\tilde{a}_2\right)\right)^{\omega_2}\right],\\
&\quad\left.\left[\left(F^L\left(\tilde{a}_1\right)\right)^{\omega_1}\left(F^L\left(\tilde{a}_2\right)\right)^{\omega_2},\left(F^U\left(\tilde{a}_1\right)\right)^{\omega_1}\left(F^U\left(\tilde{a}_2\right)\right)^{\omega_2}\right]\right\rangle\\
&=\left\langle\left[S_{\sum_{j=1}^{2}\omega_j\theta(\tilde{a}_j)},S_{\sum_{j=1}^{2}\omega_j\rho(\tilde{a}_j)},S_{\sum_{j=1}^{2}\omega_j\mu(\tilde{a}_j)},S_{\sum_{j=1}^{2}\omega_j\nu(\tilde{a}_j)}\right],\left[\left(\left(1-T^L\left(\tilde{a}_1\right)\right)^{\omega_1}\left(1-T^L\left(\tilde{a}_2\right)\right)^{\omega_2}\right),\right.\right.\\
&\quad\left.\left(\left(1-T^u\left(\tilde{a}_1\right)\right)^{\omega_1}\left(1-T^U\left(\tilde{a}_2\right)\right)^{\omega_2}\right)\right],\ \left[\prod_{j=1}^{2}\left(I^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{2}\left(I^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\\
&\quad\left.\left[\prod_{j=1}^{2}\left(F^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{2}\left(F^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right]\right\rangle
\end{aligned}\tag{6}$$

2. When n=k, by applying Equation (5), we get

$$\begin{aligned}
&\text{INTrLWAA}\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_k\right)=\left\langle\left[S_{\sum_{j=1}^{k}\omega_j\theta(\tilde{a}_j)},S_{\sum_{j=1}^{k}\omega_j\rho(\tilde{a}_j)},S_{\sum_{j=1}^{k}\omega_j\mu(\tilde{a}_j)},S_{\sum_{j=1}^{k}\omega_j\nu(\tilde{a}_j)}\right],\ \left[1-\prod_{j=1}^{k}\left(1-T^L\left(\tilde{a}_j\right)\right)^{\omega_j},\right.\right.\\
&\qquad\left.1-\prod_{j=1}^{k}\left(1-T^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\ \left[\prod_{j=1}^{k}\left(I^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{k}\left(I^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\\
&\qquad\left.\left[\prod_{j=1}^{k}\left(F^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{k}\left(F^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right]\right\rangle
\end{aligned}\tag{7}$$

3. When n=k+1, by applying Equation (6) and Equation (7), we can get

INTrLWAA

$$\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_k,\tilde{a}_{k+1}\right)=\left\langle\left[S_{\sum_{j=1}^{k}\omega_j\theta\left(\tilde{a}_j\right)+\omega_{k+1}\theta\left(\tilde{a}_{k+1}\right)},S_{\sum_{j=1}^{k}\omega_j\rho\left(\tilde{a}_j\right)+\omega_{k+1}\rho\left(\tilde{a}_{k+1}\right)},S_{\sum_{j=1}^{k}\omega_j\mu\left(\tilde{a}_j\right)+\omega_{k+1}\mu\left(\tilde{a}_{k+1}\right)},S_{\sum_{j=1}^{k}\omega_j\nu\left(\tilde{a}_j\right)+\omega_{k+1}\nu\left(\tilde{a}_{k+1}\right)}\right],\right.$$

$$\left[1-\prod_{j=1}^{k}\left(1-T^L\left(\tilde{a}_j\right)\right)^{\omega_j}+1-\left(1-T^L\left(\tilde{a}_{k+1}\right)\right)^{\omega_{k+1}}\right],\ \left[1-\prod_{j=1}^{k}\left(1-T^L\left(\tilde{a}_j\right)\right)^{\omega_j}\left(1-T^L\left(\tilde{a}_{k+1}\right)\right)^{\omega_{k+1}}\right],$$

$$\left[1-\prod_{j=1}^{k}\left(1-T^U\left(\tilde{a}_j\right)\right)^{\omega_j}+1-\left(1-T^U\left(\tilde{a}_{k+1}\right)\right)^{\omega_{k+1}}\right],\ \left[1-\prod_{j=1}^{k}\left(1-T^U\left(\tilde{a}_j\right)\right)^{\omega_j}\left(1-T^U\left(\tilde{a}_{k+1}\right)\right)^{\omega_{k+1}}\right],$$

$$\left.\left[\prod_{j=1}^{k+1}\left(I^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{k+1}\left(I^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\left[\prod_{j=1}^{k+1}\left(F^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{k+1}\left(F^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right]\right\rangle$$

$$=\left\langle\left[S_{\sum_{j=1}^{k+1}\omega_j\theta\left(\tilde{a}_j\right)},\ S_{\sum_{j=1}^{k+1}\omega_j\rho\left(\tilde{a}_j\right)},\ S_{\sum_{j=1}^{k+1}\omega_j\mu\left(\tilde{a}_j\right)},\ S_{\sum_{j=1}^{k+1}\omega_j\nu\left(\tilde{a}_j\right)}\right],\ \left[1-\prod_{j=1}^{k+1}\left(1-T^L\left(\tilde{a}_j\right)\right)^{\omega_j},1-\prod_{j=1}^{k+1}\left(1-T^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\right.$$

$$\left.\left[\prod_{j=1}^{k+1}\left(I^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{k+1}\left(I^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\ \left[\prod_{j=1}^{k+1}\left(F^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{k+1}\left(F^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right]\right\rangle$$

Therefore, considering the above results, we have Equation (5) for any. This completes the proof.

Especially when ω= $\left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$, then INTrLWAA operator reduces to an interval neutrosophic trapezoid linguistic arithmetic averaging operator for INTrLvs.

It is obvious that the INTrLWAA operator satisfies the following properties:

1. **Idempotency**: Let $\tilde{a}_j$ (j=1, 2,…, n) be a collection of INTrLvs. If $\tilde{a}_j$ (j=1, 2,…,n) is equal, i.e $\tilde{a}_j=\tilde{a}$ for j=1,2,…,n, then INTrLWAA $\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)=\tilde{a}$.
2. **Boundedness:** Let $\tilde{a}_j$ (j=1, 2,…, n) be a collection of INTrLvs and $\tilde{a}_{min}=\min\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)$ and $\tilde{a}_{max}=\max\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)$ for j=1,2,…,n, $\tilde{a}_{min}\leq$INTrLWAA $\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)\leq\tilde{a}_{max}$ then be a collection of INTrLvs.
3. **Monotonity:** Let $\tilde{a}_j$ (j=1, 2,…, n) be a collection of INTrLvs. If $\tilde{a}_j\leq\tilde{a}_j^{\,*}$ for j= 1,2,…,n. Then INTrLWAA$\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)\leq$INTrLWAA$\left(\tilde{a}_1^{\,*},\tilde{a}_2^{\,*},...,\tilde{a}_n^{\,*}\right)$.

Proof:

1. Since $\tilde{a}_j=\tilde{a}$ for j=1, 2,…,n. we have

$$\text{INTrLWAA}\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)=\left\langle\left[s_{\sum_{j=1}^{n}\omega_j\theta(\tilde{a}_j)},s_{\sum_{j=1}^{n}\omega_j\rho(\tilde{a}_j)},s_{\sum_{j=1}^{n}\omega_j\mu(\tilde{a}_j)},s_{\sum_{j=1}^{n}\omega_j\nu(\tilde{a}_j)}\right],\left[1-\prod_{j=1}^{n}\left(1-T^L\left(\tilde{a}_j\right)\right)^{\omega_j},\right.\right.$$
$$\left.1-\prod_{j=1}^{n}\left(1-T^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],\left[\prod_{j=1}^{n}\left(I^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{n}\left(I^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right],$$
$$\left.\left[\prod_{j=1}^{n}\left(F^L\left(\tilde{a}_j\right)\right)^{\omega_j},\prod_{j=1}^{n}\left(F^U\left(\tilde{a}_j\right)\right)^{\omega_j}\right]\right\rangle$$
$$=\left\langle\left[s_{\theta(\tilde{a})\sum_{j=1}^{n}\omega_j},s_{\rho(\tilde{a})\sum_{j=1}^{n}\omega_j},s_{\mu(\tilde{a})\sum_{j=1}^{n}\omega_j},s_{\nu(\tilde{a})\sum_{j=1}^{n}\omega_j}\right],\left[1-\left(1-T^L\left(\tilde{a}\right)\right)^{\sum_{j=1}^{n}\omega_j},1-\left(1-T^u\left(\tilde{a}\right)\right)^{\sum_{j=1}^{n}\omega_j}\right],\right.$$
$$\left.\left[\left(I^L\left(\tilde{a}\right)\right)^{\sum_{j=1}^{n}\omega_j},\left(I^U\left(\tilde{a}\right)\right)^{\sum_{j=1}^{n}\omega_j}\right],\left[\left(F^L\left(\tilde{a}\right)\right)^{\sum_{j=1}^{n}\omega_j},\left(F^U\left(\tilde{a}\right)\right)^{\sum_{j=1}^{n}\omega_j}\right]\right\rangle$$
$$=\left\langle\left[s_{\theta(\tilde{a})},\ s_{\rho(\tilde{a})},s_{\mu(\tilde{a})},s_{\nu(\tilde{a})}\right],\ \left(\left[T_{\tilde{a}}^L,T_{\tilde{a}}^U\right],\ \left[I_{\tilde{a}}^L,I_{\tilde{a}}^U\right],\ \left[F_{\tilde{a}}^L,F_{\tilde{a}}^U\right]\right)\right\rangle=\tilde{a}$$

2. Since $\tilde{a}_{min}=\min\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)$ and $\tilde{a}_{max}=\max\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)$ for j=1,2,…,n, there is $\tilde{a}_{min}\le\tilde{a}_j\le\tilde{a}_{max}$. Thus,thereexistis $\sum_{j=1}^{n}\omega_j\tilde{a}_{min}\le\sum_{j=1}^{n}\omega_j\tilde{a}_j\le\sum_{j=1}^{n}\omega_j\tilde{a}_{max}$. Thisis $\tilde{a}_{min}\le\sum_{j=1}^{n}\omega_j\tilde{a}_j\le\tilde{a}_{max}$. i.e., $\tilde{a}_{min}\le$INTrLWAA $\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)\le\tilde{a}_{max}$.
3. Since $\tilde{a}_j\le\tilde{a}_j^*$ for j= 1, 2,…, n. There is $\sum_{j=1}^{n}\omega_j\tilde{a}_j\le\sum_{j=1}^{n}\omega_j\tilde{a}_j^*$ Then INTRLWAA$\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n\right)$ $\le$INTrLWAA $\left(\tilde{a}_1^*,\tilde{a}_2^*,...,\tilde{a}_n^*\right)$.

Thus, we complete the proofs of these properties

4.2 Interval Neutrosophic Trapezoid Linguistic Weighted Geometric Averaging Operator

Definition 4.3: Let

$$\tilde{a}_j=\left\langle\left[s_{\theta(\tilde{a}_j)},s_{\rho(\tilde{a}_j)},s_{\mu(\tilde{a}_j)},s_{\nu(\tilde{a}_j)}\right],\ \left(\left[T_{a_j}^L,T_{a_j}^U\right],\ \left[I_{a_j}^L,I_{a_j}^U\right],\ \left[F_{a_j}^L,F_{a_j}^U\right]\right)\right\rangle\ \text{(j=1,2,…,n)}$$

be a collection of INTrLNs. The interval neutrosophic trapezoid linguistic *weighted geometric* averaging (INTrLWGA) operator can be defined as follows and INTrLWGA: $\Omega^n\rightarrow\Omega$

$$\text{INTrLWGA}\left(\tilde{a}_1,\tilde{a}_2,...\tilde{a}_n\right)=\prod_{j=1}^{n}\tilde{a}_j^{\omega_j}\tag{8}$$

where, $\omega_j = (\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1,2,…,n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$.

Theorem 4.4: Let

$$\tilde{a}_j = \left\langle \left[s_{\theta(\tilde{a}_j)}, s_{\rho(\tilde{a}_j)}, s_{\mu(\tilde{a}_j)}, s_{\nu(\tilde{a}_j)} \right],\ \left(\left[T^L_{\tilde{a}_j}, T^U_{\tilde{a}_j} \right],\ \left[I^L_{\tilde{a}_j}, I^U_{\tilde{a}_j} \right],\ \left[F^L_{\tilde{a}_j}, F^U_{\tilde{a}_j} \right] \right) \right\rangle \text{ (j=1,2,…,n)}$$

be a collection of INTrLs, Then by Equation (8) and the operational laws in Definition 3.3, we have the following result

INTrLWGA

$$\begin{aligned}\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right) = \Bigg\langle & \left[s_{\prod_{j=1}^{n}\theta^{\omega_j}(\tilde{a}_j)},\ s_{\prod_{j=1}^{n}\rho^{\omega_j}(\tilde{a}_j)},\ s_{\prod_{j=1}^{n}\mu^{\omega_j}(\tilde{a}_j)},\ s_{\prod_{j=1}^{n}\nu^{\omega_j}(\tilde{a}_j)} \right],\ \left[\prod_{j=1}^{n} T^L\left(\tilde{a}_j\right)^{\omega_j}, \prod_{j=1}^{n} T^U\left(\tilde{a}_j\right)^{\omega_j} \right], \\ & \left[1-\prod_{j=1}^{n}\left(1-I^L\left(\tilde{a}_j\right)\right)^{\omega_j}, 1-\prod_{j=1}^{n}\left(1-I^U\left(\tilde{a}_j\right)\right)^{\omega_j} \right], \\ & \left[1-\prod_{j=1}^{n}\left(1-F^L\left(\tilde{a}_j\right)\right)^{\omega_j}, 1-\prod_{j=1}^{n}\left(1-F^U\left(\tilde{a}_j\right)\right)^{\omega_j} \right] \Bigg\rangle \end{aligned} \tag{9}$$

where, $\omega = (\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1, 2,…,n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$.

By a similar proof manner of theorem 4.2, we can also give the proof of theorem 4.4 (omitted).

Especially when $\omega = \left(\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right)^T$, then INTrLWGA operator reduces to an interval neutrosophic trapezoid linguistic geometric averaging operator for INTrLvs.

It is obvious that the INTrLWGA operator satisfies the following properties:

1. **Idempotency**: Let $\tilde{a}_j$ (j=1, 2,…,n) be a collection of INTrLvs. If $\tilde{a}_j$ (j=1, 2,…, n) is equal, i.e $\tilde{a}_j = \tilde{a}$ for j=1, 2,…,n, then INTrLWGA $\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right) = \tilde{a}$.
2. **Boundedness:** Let $\tilde{a}_j$ (j=1, 2,…,n) be a collection of INTrLvs and $\tilde{a}_{min} = \min\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right)$ and $\tilde{a}_{max} = \max\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right)$ for j=1, 2,…,n, $\tilde{a}_{min} \leq$ INTrLWGA $\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right) \leq \tilde{a}_{max}$ then be a collection of INTrLvs.
3. **Monotonity:** Let $\tilde{a}_j$ (j=1, 2,…,n) be a collection of INTrLvs. If $\tilde{a}_j \leq \tilde{a}_j^*$ for j= 1,2,…,n. Then INTrLWGA$\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right) \leq$ INTrLWGA$\left(\tilde{a}_1^*,\tilde{a}_2^*,\ldots,\tilde{a}_n^*\right)$.

Since the proof process of these properties is similar to the above proofs, we do not repeat it here.

5. DECISION MAKING METHOD BY INTRLWAA AND INTRLWGA OPERATORS.

This section presents a method for multi attribute decision making problems based on the INTrLWAA and INTrLWGA operators ant the score, accuracy, and certainty functions of INTrLvs under interval neutrosophic trapezoid linguistic variable environment.

In a multiple attribute decision-making problem, assume that A={A_1, A_2, A_3,…,A_m} is a setoff alternatives and C ={C_1, C_2,…,C_n} is a set of attributes. The weight vector of the attributes C_j (j=1, 2,…,n), entered by the decision maker, is ω= $(\omega_1,\omega_2,\ldots,\omega_n)^T$ where $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\text{É}_j = 1$. In the decision process, the evaluation information of the alternatives A_i (i=1, 2,…,m) with respect to the attribute C_j (j=1, 2,…,n) is represented by the form of an INTrLS:

$$A_i = \left\langle \left[s_{\theta_i(C_j)}, s_{\rho_i(C_j)}, s_{\mu_i(C_j)}, s_{\nu_i(C_j)} \right],\ \left(T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j) \right) \right\rangle | C_j \in C$$

where

$$\left[s_{\theta_i(C_j)}, s_{\rho_i(C_j)}, s_{\mu_i(C_j)}, s_{\nu_i(C_j)} \right] \in \hat{s}\ ,\ T_{A_i}(C_j) = \left[T^L_{A_i}(x), T^U_{A_i}(C_j) \right] \subseteq [0.1],$$

$$I_{A_i}(C_j) = \left[I^L_{A_i}(C_j), I^U_{A_i}(C_j) \right] \subseteq [0.1], \text{ and } F_{A_i}(C_j) = \left[F^L_{A_i}(C_j), F^U_{A_i}(C_j) \right] \subseteq [0.1]$$

with the condition

$$0 \le T^U_{A_i}(C_j) + I^U_{A_i}(C_j) + F^U_{A_i}(C_j) \le 3$$

for j=1,2,..,n and i=1,2,…,m. For convenience, an INTrLv is an INTrLS is denoted by

$$\tilde{d}_{ij} = \left\langle \left[S_{\theta_{ij}}, S_{\rho_{ij}}, S_{\mu_{ij}}, S_{\nu_{ij}} \right],\ \left(\left[T^L_{ij}, T^U_{ij} \right],\ \left[I^L_{ij}, I^U_{ij} \right],\ \left[F^L_{ij}, F^U_{ij} \right] \right) \right\rangle \text{(i=1=1,2,..m) j=1,2,…,n)}$$

thus, one can establish an interval neutrosophic trapezoid linguistic decision matrix D=$\left(\tilde{d}_{ij}\right)_{m\times n}$.

The decision steps are described as follows

Step1: Calculate the individual overall value of the INTrLv $\tilde{d}_j$ for A_i (i=1,2,…,m) by the following aggregation formula:

$$\tilde{d}_i = \left\langle \left[S_{\theta_i}, S_{\rho_i}, S_{\mu_i}, S_{\nu_i} \right], \left(\left[T_i^L, T_i^U \right], \left[I_i^L, I_i^U \right], \left[F_i^L, F_i^U \right] \right) \right\rangle$$
$$= \text{INTrLWAA}\left(\tilde{d}_{i1}, \tilde{d}_{i2}, ..., \tilde{d}_{in} \right)$$
$$= \left\langle \left[s_{\sum_{j=1}^{n} \omega_j \theta_{ij}}, s_{\sum_{j=1}^{n} \omega_j \rho_{ij}}, s_{\sum_{j=1}^{n} \omega_j \mu_{ij}}, s_{\sum_{j=1}^{n} \omega_j \nu_{ij}} \right], \left[1 - \prod_{j=1}^{n} \left(1 - T_{ij}^L \right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - T_{ij}^U \right)^{\omega_j} \right] \right. \tag{10}$$
$$\left. \left[\prod_{j=1}^{n} \left(I_{ij}^L \right)^{\omega_j}, \prod_{j=1}^{n} \left(I_{ij}^U \right)^{\omega_j} \right], \left[\prod_{j=1}^{n} \left(F_{ij}^L \right)^{\omega_j}, \prod_{j=1}^{n} \left(F_{ij}^U \right)^{\omega_j} \right] \right\rangle$$

or

$$\tilde{d}_i = \left\langle \left[S_{\theta_i}, S_{\rho_i}, S_{\mu_i}, S_{\nu_i} \right], \left(\left[T_i^L, T_i^U \right], \left[I_i^L, I_i^U \right], \left[F_i^L, F_i^U \right] \right) \right\rangle$$
$$= \text{INTrLWGA}\left(\tilde{d}_{i1}, \tilde{d}_{i2}, ..., \tilde{d}_{in} \right)$$
$$= \left\langle \left[s_{\prod_{j=1}^{n} \theta_{ij}^{\omega_j}}, s_{\prod_{j=1}^{n} \rho_{ij}^{\omega_j}}, s_{\prod_{j=1}^{n} \mu_{ij}^{\omega_j}}, s_{\prod_{j=1}^{n} \nu_{ij}^{\omega_j}} \right], \left(\left[\prod_{j=1}^{n} (T_{ij}^L)^{\omega_j}, \prod_{j=1}^{n} (T_{ij}^U)^{\omega_j} \right], \left[1 - \prod_{j=1}^{n} \left(1 - I_{ij}^L \right)^{\omega_j}, \right. \right. \right. \tag{11}$$
$$\left. \left. \left. 1 - \prod_{j=1}^{n} \left(1 - I_{ij}^U \right)^{\omega_j} \right], \left[1 - \prod_{j=1}^{n} \left(1 - F_{ij}^L \right)^{\omega_j}, 1 - \prod_{j=1}^{n} \left(1 - F_{ij}^U \right)^{\omega_j} \right] \right) \right\rangle$$

Step2: Calculate the score function E($\tilde{d}_i$) (i=1, 2,…, m) (accuracy function H($\tilde{d}_i$) and certainty function C($\tilde{d}_i$) by applying Equation (1) (Equations (2) and (3)).

Step 3: Rank the alternatives according to the values of E($\tilde{d}_i$) (H($\tilde{d}_i$) and C($\tilde{d}_i$)) ((i=1,2,…,m) by the ranking method in Defintion 3.3, and then select the best one(s).

Step4: End.

6. ILLUSTRATIVE EXAMPLE

An illustrative example about investment alternatives problem adapted from (Ye, 2015) is used to demonstrate the applications of the proposed decision making method under interval neutrosophic trapezoid linguistic environment. There is an investment company, which wants to invest a sum of money in the best option. To invest the money, there is a panel with four possible alternatives: (1) A_1 is car company; (2) A_2 is food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investement company must take a decision according to the three attributes: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is a the environmental impact. The weight vector of the attributes is ω= (0.35, 0.25, 0.4)^T.The expert evaluates the four possible alternatives of A_i (i=1, 2, 3, 4) with respect to the three attributes of C_j (i=1,2,3), where the evaluation information is expressed by the form of INTrLV values under the linguistic term set S={s_0=extremely poor, s_1=very poor, s_2= poor, s_3= medium, s_4= good, s_5= very good, s_6= extremely good}.

The evaluation information of an alternative A_i (i=1,2,3,4) with respect to an attribute C_j (j=1,2,3) can be given by the expert. For example, the INTrL value of an alternative A_1 with respect to an attribute C_1 is given as <[$s_{1.4}$, $s_{2.7}$, s_3, $s_{5.3}$], ([0.4, 0.5], [0.2, 0.3], [0.3, 0.4])> by the expert, which indicates that the mark of the alternative A_1 with respect to the attribute C_1 is about the trapezoid fuzzy linguistic value [$s_{1.4}$, $s_{2.7}$, s_3, $s_{5.3}$] with the satisfaction degree interval [0.4, 0.5], indeterminacy degree interval [0.2, 0.3], and dissatisfaction degree interval [0.3, 0.4]. similarly, the four possible alternatives with respect to the three attributes can be evaluated by the expert, thus we can obtain the following interval neutrosophic trapezoid linguistic decision matrix:

$$D\left(\tilde{d}_{ij}\right)_{\mathrm{m}\times\mathrm{n}} =$$

$$\begin{bmatrix}
\left\langle\left(\left[s_{1.8},s_{3.4},s_{4.5},s_{5.5}\right],\left(\left[0.4,0.5\right],\left[0.2,0.3\right],\left[0.3,0.4\right]\right)\right\rangle\left\langle\left(\left[s_{1.3},s_{2.3},s_{4.4},s_{5.4}\right],\left(\left[0.4,0.6\right],\left[0.1,0.2\right],\left[0.2,0.4\right]\right)\right\rangle \\
\left\langle\left(\left[s_{0.8},s_{2.2},s_{3.8},s_{5.1}\right],\left(\left[0.2,0.3\right],\left[0.1,0.2\right],\left[0.5,0.6\right]\right)\right\rangle\left\langle\left(\left[s_{1.4},s_{2.8},s_{3.8},s_{5.1}\right],\left(\left[0.5,0.7\right],\left[0.1,0.2\right],\left[0.2,0.3\right]\right)\right\rangle \\
\left\langle\left(\left[s_{1.5},s_{2.5},s_{4.5},s_{5.5}\right],\left(\left[0.6,0.7\right],\left[0.1,0.2\right],\left[0.2,0.3\right]\right)\right\rangle\left\langle\left(\left[s_{1.1},s_{2.1},s_{1.4},s_{5.4}\right],\left(\left[0.5,0.7\right],\left[0.2,0.2\right],\left[0.1,0.2\right]\right)\right\rangle \\
\left\langle\left(\left[s_{2.1},s_{3.2},s_{4.5},s_{5.7}\right],\left(\left[0.3,0.5\right],\left[0.1,0.2\right],\left[0.3,0.4\right]\right)\right\rangle\left\langle\left(\left[s_{1.8},s_{2.8},s_{4.4},s_{5.5}\right],\left(\left[0.5,0.6\right],\left[0.1,0.3\right],\left[0.3,0.4\right]\right)\right\rangle \\
\left\langle\left(\left[s_{1.4},s_{2.7},s_{3},s_{5.3}\right],\left(\left[0.5,0.6\right],\left[0.1,0.3\right],\left[0.1,0.3\right]\right)\right\rangle\left\langle\left(\left[s_{1.5},s_{3.1},s_{4.7},s_{5.9}\right],\left(\left[0.7,0.8\right],\left[0.0,0.1\right],\left[0.1,0.2\right]\right)\right\rangle \\
\left\langle\left(\left[s_{1.7},s_{2.9},s_{4},s_{5.4}\right],\left(\left[0.5,0.7\right],\left[0.1,0.2\right],\left[0.2,0.3\right]\right)\right\rangle\left\langle\left(\left[s_{1.2},s_{1.8},s_{4.3},s_{5.3}\right],\left(\left[0.3,0.4\right],\left[0.1,0.2\right],\left[0.1,0.2\right]\right)\right\rangle
\end{bmatrix}$$

The proposed decision making method can handle this decision making problem according to the following calculation steps:

Step1: By applying Equation (10), we can obtain the individual overall value of the INTrLV $\tilde{d}_i$ for A_i (i=1,2,.3,4)

$$\tilde{d}_1 = \left\langle\left(\left[s_{1.275},s_{2.645},s_{4.195},s_{5.315}\right],\left(\left[0.3268,0.4590\right],\left[0.1275,0.2305\right],\left[0.3325,0.4704\right]\right)\right\rangle$$

$$\tilde{d}_2 = \left\langle\left(\left[s_{1.305},s_{2.445},s_{3.015},s_{5.320}\right],\left(\left[0.5271,0.7000\right],\left[0.1320,0.2759\right],\left[0.1516,0.3519\right]\right)\right\rangle$$

$$\tilde{d}_3 = \left\langle\left(\left[s_{1.745},s_{2.900},s_{3.875},s_{5.490}\right],\left(\left[0.4375,0.5275\right],\left[0.1000,0.2603\right],\left[0.1933,0.3565\right]\right)\right\rangle$$

$$\tilde{d}_4 = \left\langle\left(\left[s_{1.430},s_{2.530},s_{4.365},s_{5.535}\right],\left(\left[0.5216,0.6565\right],\left[0.000,0.1569\right],\left[0.1189,0.2213\right]\right)\right\rangle$$

Step2: By applying Equation (1), we can obtain the score value of E($\tilde{d}_i$) (i=1,2,3,4)

$$E(\tilde{d}_1) = s_{2.028},\ E(\tilde{d}_2) = s_{2.173},\ E(\tilde{d}_3) = s_{2.390},\ E(\tilde{d}_4) = s_{2.703}$$

Step 3: Since $E(\tilde{d}_4) > E(\tilde{d}_3) > E(\tilde{d}_2) > E(\tilde{d}_1)$, the ranking order of four alternatives. Therefore, we can see that the alternative A_4 is the best choice among all the alternative.

On the other hand, we can also utilize the INTrLWGA operator as the following computational steps:

Step1: By applying Equation (11), we can obtain the individual overall value of the INTrLV $\tilde{d}_i$ for A_i (i=1,2,.3,4)

$$\tilde{d}_1 = \left\langle \left(\left[s_{1.200}, s_{2.591}, s_{4.182}, s_{5.312} \right], \left(\left[0.3031, 0.4266 \right], \left[0.1363, 0.2365 \right], \left[0.3674, 0.4898 \right] \right) \right\rangle$$
$$\tilde{d}_2 = \left\langle \left(\left[s_{1.293}, s_{2.426}, s_{2.659}, s_{5.317} \right], \left(\left[0.5233, 0.7000 \right], \left[0.1414, 0.1635 \right], \left[0.1614, 0.2279 \right] \right) \right\rangle$$
$$\tilde{d}_3 = \left\langle \left(\left[s_{1.718}, s_{2.892}, s_{3.805}, s_{5.487} \right], \left(\left[0.4181, 0.5629 \right], \left[0.1000, 0.2665 \right], \left[0.2260, 0.3618 \right] \right) \right\rangle$$
$$\tilde{d}_4 = \left\langle \left(\left[s_{1.416}, s_{2.453}, s_{4.356}, s_{5.528} \right], \left(\left[0.4585, 0.5864 \right], \left[0.0662, 0.1663 \right], \left[0.1261, 0.2263 \right] \right) \right\rangle$$

Step2: By applying Equation (1), we can obtain the score value of E($\tilde{d}_i$) (i=1,2,3,4)

$$E(\tilde{d}_1) = s_{1.937},\ E(\tilde{d}_2) = s_{2.207},\ E(\tilde{d}_3) = s_{2.332},\ E(\tilde{d}_4) = s_{2.556}$$

Step 3: Since $E(\tilde{d}_4) > E(\tilde{d}_3) > E(\tilde{d}_2) > E(\tilde{d}_1)$, the ranking order of four alternatives. Therefore, we can see that the alternative A_4 is the best choice among all the alternative.

Obviously, we can see that the above two kinds of ranking orders of the alternatives are the same and the most desirable choice is the alternative A_3.

The INTrLS is a further generalization of interval neutrosophic linguistic set and interval neutrosophic uncertain linguistic set proposed by (Ye, 2015; Ye, 2013 g). So the decision –making method proposed in this paper is more typical in applications. Furthermore, the decision making approach proposed in this paper can be used to solve not only the interval neutrosophic linguistic information and interval neutrosophic uncertain linguistic information but also decision making problems with interval neutrosophic triangular and trapezoidal linguistic information. Therefore, the decision making method proposed in the paper is a generalization of existing decision method with interval neutrosophic linguistic information and interval neutrosophic uncertain linguistic information.

7. CONCLUSION

In this paper, we have proposed some interval neutrosophic trapezoidal linguistic operators such as interval neutrosophic trapezoid linguistic weighted arithmetic averaging INTrLWAA and interval neutrosophic trapezoid fuzzy linguistic weighted geometric averaging INTrLWGA. We have studied some desirable properties of the proposed operators, such as commutativity, idempotency and monotonicity, and applied the INTrLWAA and INTrLWGA operator to decision making with interval neutrosophic trapezoidal linguistic information. Finally, an illustrative example has been given to show the developed operators.

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