

New Types of Soft Sets” HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set”: An Improved Version

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Abstract: This is an improved paper of [10], where we recall the definitions together with practical applications of the Soft Set and its extensions to HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set.

Keywords: Soft Set; HyperSoft Set; IndetermSoft Set; IndetermHyperSoft Set; MultiSoft Set; TreeSoft Set.

1. Introduction

The Soft Set was introduced by Molodtsov [1] in 1999.

Further on, the HyperSoft Set (2018), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), and TreeSoft Set (2022) were introduced by Smarandache [2-7]. The definitions of IndetermSoft Set and IndetermHyperSoft Set [9] are updated now.

The MultiSoft Set (2010) was introduced by Alkhazaleh et al. [8].

The soft set and its extensions have many applications in our real world.

Many hybrid version of the soft set have been proposed and used, combined with fuzzy or fuzzy extension sets, such as: fuzzy soft set, intuitionistic fuzzy soft set, neutrosophic soft set, picture fuzzy soft set, spherical fuzzy soft set, plithogenic soft set, and similarly for fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic hypersoftsoft set, picture fuzzy hypersoft set, spherical fuzzy hypersoft soft set, plithogenic sif set.

Future research may also investigate and apply the newly forms of soft sets, in combinations with fuzzy and fuzzy extension sets, that will result in fuzzy / intuitionistic fuzzy / neutrosophic / picture fuzzy / spherical fuzzy / Pythagorean fuzzy etc. / plithogenic IndetermSoft / IndetermHyperSoft / TreeSoft Set, respectively.

Let us recall their definitions together with real examples.

2. Definition of Soft Set

Let U be a universe of discourse, $P(U)$ the power set of U , and A a set of attributes. Then, the pair (F, U) , where $F: A \rightarrow P(U)$ is called a Soft Set over U .

3. Real Example of Soft Set

Let $U = \{\text{Helen, George, Mary, Richard}\}$ and a set $M = \{\text{Helen, Mary, Richard}\}$ included in U .

Let the attribute be: $a = \text{size}$, and its attribute' values respectively:

Size = $A_1 = \{\text{small, medium, tall}\}$.

Let the function be: $F: A_1 \rightarrow P(U)$.

Then, for example:

$$F(\text{tall}) = \{\text{Helen, Mary}\},$$

which means that both Helen and Mary are tall.

4. Definition of IndetermSoft Set

Let U be a universe of discourse, H a non-empty subset of U , and $P(H)$ be the powerset of H . Let a be an attribute, and A be a set of this attribute-values. Then $F: A \rightarrow P(H)$ is called an IndetermSoft Set if at least one of the bellow occurs:

- i) The set A has some indeterminacy;
- ii) At least one of the sets H or $P(H)$ has some indeterminacy;
- iii) The function F has some indeterminacy, i.e. there exist at least one relationship $F(a) = M$ where a or M have some indeterminacy (not unique, unclear, incomplete, unknown).

IndetermSoft Set, as an extension of the classical (determinate) Soft Set, deals with indeterminate data, because there are sources unable to provide exact or complete information on the sets A , H , or $P(H)$, nor on the function F . We did not add any indeterminacy, we found the indeterminacy in our real world. Because many sources give approximate/uncertain/incomplete/conflicting information, not exact information as in the Soft Set, as such we still need to deal with such situations.

Herein, 'Indeterm' stands for 'Indeterminate' (uncertain, conflicting, incomplete, not unique outcome).

Similarly, distinctions between determinate and indeterminate operators are taken into consideration. Afterwards, an IndetermSoft Algebra is built, using a determinate soft operator (joinAND), and three indeterminate soft operators (disjoinOR, exclusiveOR, NOT), whose properties are further on studied.

Smarandache has generalized the Soft Set to the HyperSoft set by transforming the function F into a multi-attribute function, and then he introduced the hybrids of Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, other fuzzy extensions, and Plithogenic HyperSoft Set.

The classical Soft Set is based on a determinate function (whose values are certain, and unique), but in our world there are many sources that, because of lack of information or ignorance, provide indeterminate (uncertain, and not unique – but hesitant or alternative) information. They can be modeled by operators having some degree of indeterminacy due to the imprecision of our world.

5. Real Example of IndetermSoft Set

The present study represents an application of modern material statistics, specifically leveraging the principles of neutrosophic statistics.

Assume a town has many houses.

- 1) Indeterminacy with respect to the set A of attributes.

You ask the source:

- What are all colors of the houses?

The source:

- I know for sure that there are houses of colors red, yellow, and blue, but I do not know if there are houses of other colors (?)

This is the IndetermSoft Set.

- 2) Indeterminacy with respect to the set H of houses.

You ask the source:

- How many houses are in the town?

The source:

- I never counted them, but I estimate their number to be between 100-120 houses.

- 3) Indeterminacy with respect to the function $F: A \rightarrow P(H)$.

3a) You ask a source:

- What houses have the red color in the town?

The source:

- I am not sure, I think the houses h_1 or h_2 .

Therefore, $F(\text{red}) = h_1$ or h_2 (indeterminate / uncertain answer).

3b) You ask again:

- But, what houses are yellow?

The source:

- I do not know, the only thing I know is that the house h_5 is not yellow because I have visited it.

Therefore, $F(\text{yellow}) = \text{not } h_5$ (again indeterminate / uncertain answer).

3c) Another question you ask:

- Then what houses are blue?

The source:

- For sure, either h_8 or h_9 .

Therefore, $F(\text{blue}) = \text{either } h_8 \text{ or } h_9$ (again indeterminate / uncertain answer).

6. Definition of HyperSoft Set

The soft set was extended to the hypersoft set by transforming the function F into a multi-attribute function. Afterwards, the hybrids of HyperSoft Set with the Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, other fuzzy extensions, and Plithogenic Set were introduced.

Let U be a universe of discourse, $P(U)$ the power set of U . Let a_1, a_2, \dots, a_n , for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets A_1, A_2, \dots, A_n , with $A_i \cap A_j = \Phi$, for $i \neq j$, and i, j in $\{1, 2, \dots, n\}$. Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(U)$, is called a HyperSoft Set over U .

7. Real Example of HyperSoft Set

Let $U = \{\text{Helen, George, Mary, Richard}\}$ and a set $M = \{\text{Helen, Mary, Richard}\}$ included in U .

Let the attributes be: $a_1 = \text{size}$, $a_2 = \text{color}$, $a_3 = \text{gender}$, $a_4 = \text{nationality}$, and their attributes' values respectively:

Size = $A_1 = \{\text{small, medium, tall}\}$,

Color = $A_2 = \{\text{white, yellow, red, black}\}$,

Gender = $A_3 = \{\text{male, female}\}$,

Nationality = $A_4 = \{\text{American, French, Spanish, Italian, Chinese}\}$.

Let the function be: $F: A_1 \times A_2 \times A_3 \times A_4 \rightarrow P(U)$.

Then, for example:

$F(\{\text{tall, white, female, Italian}\}) = \{\text{Helen, Mary}\}$, which means that both Helen and Mary are tall, and white, and female, and Italian.

Notice that this is an extension of the previous Real Example of Soft Set.

8. Definition of IndetermHyperSoft Set

Let U be a universe of discourse, H a non-empty subset of U , and $P(H)$ the powerset of H . Let a_1, a_2, \dots, a_n , for $n \geq 1$, be n distinct attributes, whose corresponding attribute-values are respectively the sets A_1, A_2, \dots, A_n , with $A_i \cap A_j = \Phi$ for $i \neq j$, and i, j in $\{1, 2, \dots, n\}$. Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(H)$, is called an IndetermHyperSoft Set over U if at least one of the bellow occurs:

- i) at least one of the sets A_1, A_2, \dots, A_n has some indeterminacy;
- ii) at least one of the sets H or $P(H)$ has some indeterminacy;
- iii) the function F has some indeterminacy, i.e. there exist at least one relationship

$F(e_1, e_2, \dots, e_n) = M$, where some of e_1, e_2, \dots, e_n or M have indeterminacy (not unique, unclear, incomplete, unknown).

Similarly, IndetermHyperSoft Set is an extension of the HyperSoft Set, when there is indeterminate data, or indeterminate functions, or indeterminate sets.

9. Real Example of IndetermHyperSoft Set

Assume a town has many houses.

1) Indeterminacy with respect to the product set $A_1 \times A_2 \times \dots \times A_n$ of attributes.

You ask the source:

- What are all colors and sizes of the houses?

The source:

- I know for sure that there are houses of the following colors: red, yellow, and blue, but I do not know if there are houses of other colors (?)

About the size, I saw many houses that are small, but I do not remember to have seeing big houses.

2) Indeterminacy with respect to the set H of houses.

You ask the source:

- How many houses are in the town?

The source:

- I never counted them, but I estimate their number to be between 100-120 houses.

3) Indeterminacy with respect to the function $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(H)$.

3a) You ask a source:

- What houses are of red color and big size in the town?

The source:

- I am not sure, I think the houses h_1 or h_2 .

Therefore, $F(\text{red, big}) = h_1$ or h_2 (indeterminate / not unique / uncertain answer).

3b) You ask again:

- But, what houses are yellow and small?

The source:

- I do not know, the only thing I know is that the house h_5 is neither yellow nor small because I have visited it.

Therefore, $F(\text{yellow, small}) = \text{not } h_5$ (again indeterminate / uncertain answer).

3c) Another question you ask:

- Then what houses are blue and big?

The source:

- For sure, either h_8 or h_9 .

Therefore, $F(\text{blue, big}) = \text{either } h_8 \text{ or } h_9$ (again indeterminate / uncertain answer).

This is the IndetermHyperSoft Set.

10. Definition of TreeSoft Set

Let U be a universe of discourse, and H a non-empty subset of U , with $P(H)$ the powerset of H .

Let A be a set of attributes (parameters, factors, etc.),

$A = \{A_1, A_2, \dots, A_n\}$, for integer $n \geq 1$, where A_1, A_2, \dots, A_n are considered attributes of first level (since they have one-digit indexes).

Each attribute $A_i, 1 \leq i \leq n$, is formed by sub-attributes:

$$A_1 = \{A_{1,1}, A_{1,2}, \dots\}$$

$$A_2 = \{A_{2,1}, A_{2,2}, \dots\}$$

.....

$$A_n = \{A_{n,1}, A_{n,2}, \dots\}$$

where the above $A_{i,j}$ are sub-attributes (or attributes of second level) (since they have two-digit indexes).

Again, each sub-attribute $A_{i,j}$ is formed by sub-sub-attributes (or attributes of third level):

$$A_{i,j,k}$$

And so on, as much refinement as needed into each application, up to sub-sub-...-sub-attributes (or attributes of m-level) (or having m digits into the indexes):

$$A_{i_1, i_2, \dots, i_m}$$

Therefore, a graph-tree is formed, that we denote as $Tree(A)$, whose root is A (considered of level zero), then nodes of level 1, level 2, up to level m.

We call *leaves* of the graph-tree, all terminal nodes (nodes that have no descendants).

Then the TreeSoft Set is:

$$F: P(Tree(A)) \rightarrow P(H)$$

$Tree(A)$ is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and $P(Tree(A))$ is the powerset of the $Tree(A)$.

All node sets of the *TreeSoft Set of level m* are:

$$Tree(A) = \{A_{i_1} \mid i_1 = 1, 2, \dots\}$$

The first set is formed by the nodes of level 1, second set by the nodes of level 2, third set by the nodes of level 3, and so on, the last set is formed by the nodes of level m . If the graph-tree has only two levels ($m = 2$), then the TreeSoft Set is reduced to a MultiSoft Set [8].

11. Practical Example of TreeSoft Set of Level 3

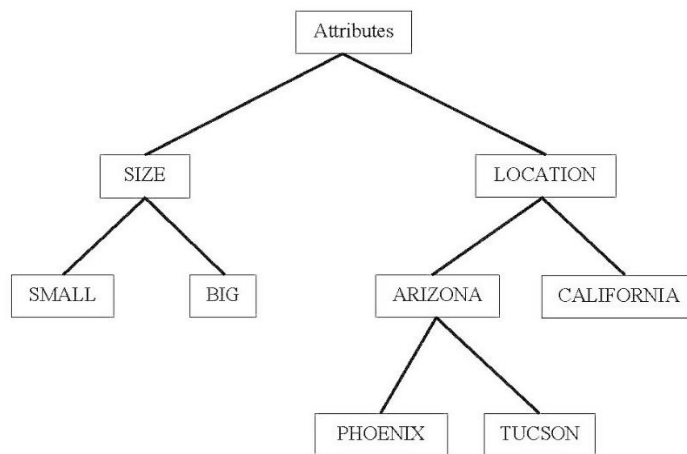


Figure 1. A TreeSoft Set of Level 3.

This is a classical tree, whose:

Level 0 (the root) is the node Attributes;

Level 1 is formed by the nodes: Size, Location;

Level 2 is formed by the nodes Small, Big, Arizona, California;

Level 3 is formed by the nodes Phoenix, Tucson.

Let's consider $H = \{h_1, h_2, \dots, h_{10}\}$ be a set of houses, and $P(H)$ the powerset of H .

And the set of Attributes: $A = \{A_1, A_2\}$, where $A_1 = \text{Size}$, $A_2 = \text{Location}$.

Then $A_1 = \{A_{11}, A_{12}\} = \{\text{Small}, \text{Big}\}$, $A_2 = \{A_{21}, A_{22}\} = \{\text{Arizona}, \text{California}\}$ as American states.

Further on, $A_{22} = \{A_{211}, A_{212}\} = \{\text{Phoenix}, \text{Tucson}\}$ as Arizonian cities.

Let's assume that the function F gets the following values:

$$F(\text{Big, Arizona, Phoenix}) = \{ h_9, h_{10} \}$$

$$F(\text{Big, Arizona, Tucson}) = \{ h_1, h_2, h_3, h_4 \}$$

$$F(\text{Big, Arizona}) = \text{all big houses from both cities, Phoenix and Tucson}$$

$$= F(\text{Big, Arizona, Phoenix}) \cup F(\text{Big, Arizona, Tucson}) = \{ h_1, h_2, h_3, h_4, h_9, h_{10} \}.$$

12. Conclusion

The present study represents an application of modern material statistics, specifically leveraging the principles of neutrosophic statistics.

The HyperSoft Set (2018) is a generalization of Soft Set (1999) and MultiSoft Set (2010), from a uni-variate function to a multi-variate function F ;

IndetermSoft Set (2022) is an extension of the Soft Set, from the determinate data to indeterminate data;

IndetermHyperSoft Set (2022) is an extension of the HyperSoft Set, from the determinate data to indeterminate data;

and TreeSoft Set (2022) that is a generalization of the MultiSoft Set.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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