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Non-Geometry

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NON-GEOMETRY

It's a lot easier to deny the Euclid's five postulates than Hilbert's twenty thorough axioms.

1. It is not always possible to draw a line from an arbitrary point to another arbitrary point.

   For example: this axiom can be denied only if the model's space has at least a discontinuity point; (in our bellow model MD, one takes an isolated point I in between f_1 and f_2, the only one which will not verify the axiom).

2. It is not always possible to extend by continuity a finite line to an infinite line.

   For example: consider the bellow Model, and the segment AB, the both A and B lie on f_1, A in between P and N, while B on the left side of N; one can not at all extend AB either beyond A or beyond B, because the resulted curve, noted say $A' - A - B - B'$, would not be a geodesic (i.e. line in our Model) anymore.

   If A and B lie in $\delta_1 - f_1$, both of them closer to f_1, A in the left side of P, while B in the right side of P, then the segment AB, which is in fact $A - P - B$, can be extended beyond A and also beyond B only up to f_1 (therefore one gets a finite line too, $A'_A - P - B - B'$), where A', B' are the intersections of PA, PB respectively with f_1).

   If A, B lie in $\delta_1 - f_1$, far enough from f_1 and P, such that AB is parallel to f_1, then AB verifies this postulate.

3. It is not always possible to draw a circle from an arbitrary point and of an arbitrary interval.

   For example: same as for the first axiom; the isolated point I, and a very small interval not reaching f_1 neither f_2, will deny this axiom.

4. Not all the right angles are congruent. (See example of the Anti-Geometry, explained bellow.)

5. If a line, cutting two other lines, forms the interior angles of the same side of it strictly less than two right angles, then not always the two lines extended towards infinite cut each other in the side where the angles are strictly less than two right angles.

   For example: let h_1, h_2, l be three lines in $\delta_1 - \delta_2$, where h_1 intersects f_1 in A, and h_2 intersects f_1 in B, with A, B, P different each other, such that h_1 and h_2 do not intersect, but l cuts h_1 and h_2 and forms the interior angles of one of its side (towards f_1) strictly less than two right angles;

the assumption of the fifth postulate is fulfilled, but the consequence does not hold, because h_1 and h_2 do not cut each other (they may not be extended beyond A and B respectively, because the lines would not be geodesics anymore).

Question 29

Find a more convincing midel for this non-geometry.