FLORENTIN SMARANDACHE

On Solving Homogene Systems

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ON SOLVING HOMOGENE SYSTEMS

In the High School Algebra manual for grade IX (1981), pp. 103-104, is presented a method for solving systems of two homogenous equations of second degree, with two unknowns. In this article we'll present another method of solving them.

Let's have the homogenous system

$$\begin{cases} a_1x^2 + b_1xy + c_1y^2 = d_1 \\ a_2x^2 + b_2xy + c_2y^2 = d_2 \end{cases}$$

with real coefficients.

We will note $x = ty$, (or $y = tx$), and by substitution, the system becomes:

$$\begin{cases} y^2(a_1t^2 + b_1t + c_1) = d_1 & (1) \\ y^2(a_2t^2 + b_2t + c_2) = d_2 & (2) \end{cases}$$

Dividing (1) by (2) and grouping the terms, it results an equation of second degree of variable t:

$$(a_1d_2 - a_2d_1)t^2 + (b_1d_2 - b_2d_1)t + (c_1d_2 - c_2d_1) = 0$$

If $\Delta_t < 0$, the system doesn't have solutions.

If $\Delta_t \geq 0$, the initial system becomes equivalent with the following systems:

$$(S_1)\begin{cases} x = t_1y \\ a_1x^2 + b_1xy + c_1y^2 = d_1 \end{cases}$$

and

$$(S_2)\begin{cases} x = t_2y \\ a_1x^2 + b_1xy + c_1y^2 = d_1 \end{cases}$$

which can simply be resolved by substituting the value of x from the first equation into the second.

Further we will provide an extension of this method.

Let have the homogeneous system:

$$\sum_{i=0}^{n} a_{i,j}x^{n-i}y^i, \quad j = \overline{1,m}$$

To resolve this, we note $x = ty$, it results:

$$y^n \sum_{i=0}^{n} a_{i,j}t^{n-i} = b_j, \quad j = \overline{1,m}$$

By dividing in order the first equation to the rest of them, we obtain:

$$\left(\sum_{i=0}^{n} a_{i,1}t^{n-i}\right) \Big/ \left(\sum_{i=0}^{n} a_{i,j}t^{n-i}\right) = b_1 \ / \ b_j, \ \ j = \overline{2,m}$$

or:

$$\sum_{i=0}^{n} (a_{i,1}b_j - a_{i,j}b_1)t^{n-i}, \ \ j = \overline{2,m}$$

We will find the real values $t_1,...,t_p$ from this system.
The initial system is equivalent with the following systems

$$(S_h) \quad \begin{cases} x = t_h y \\ \sum_{i=0}^{n} a_{i,1} x^{n-1} y^i = b_1 \end{cases} \text{ where } h = \overline{1, p}.$$