To be and Not to be – An introduction to Neutrosophy: A Novel Decision Paradigm

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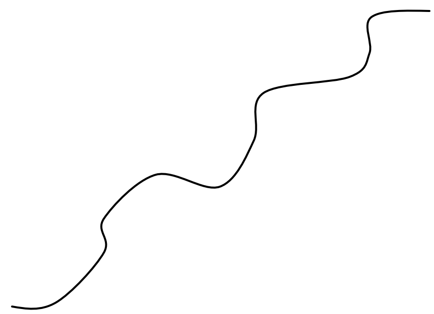
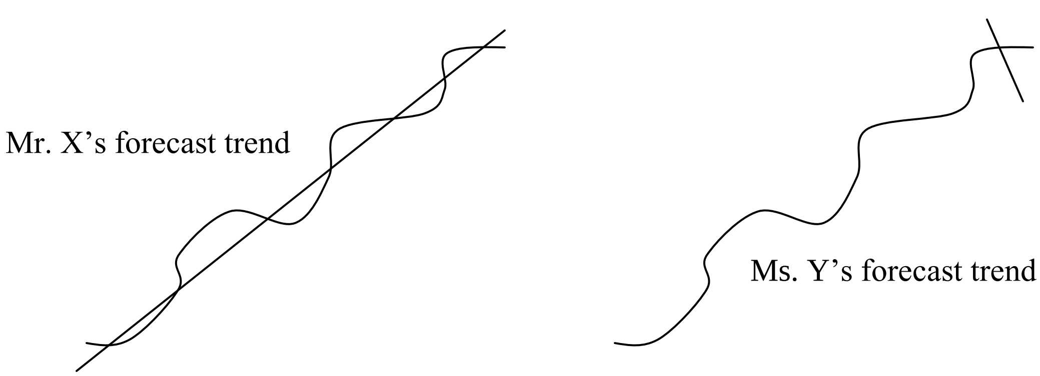
The Need for a Novel Decision Paradigm in Management

- The process of scientific decision-making necessarily follows and input-output system
- The primary input is in the form of raw data (quantitative, qualitative, or both)
- This raw data is subsequently "cleaned", "filtered", and "organized" to yield information
- The available information is then processed accordingly to either (a) very well-structures, "hard" rules, or (b) partially-structured "semi-soft" rules, or (c) almost completely unstructures "soft" rules
- The output is the final decision which may be a relatively simple and routine one such as deciding on an optimal inventory re-ordering level of a much more complex and involved such as discounting a product line or establishing a new SBU. It has been observed that most of these complex and involved decision problems are those that need to be worked out using the "soft" rules of information processing
- Besides being largely subjective, "soft" decision rules are often ambiguous, inconsistent and even contradictory
- The main reason is that the event spaces governing complex decision problems are not completely known. However, the human mind abhors incompleteness when it comes to complex cognitive processing. The mind invariably tries to "fill in the blanks" whenever it encounters incompleteness
- Therefore, when different people form their own opinions from a given set of incomplete information, it is only to be expected that there will be areas of inconsistency, because everybody will try to „complete the set" in their own individual ways, governed by their own subjective utility preferences

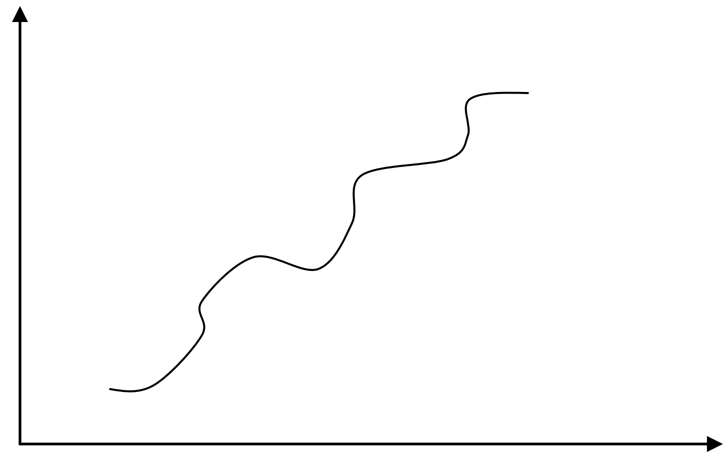
- Looking at the following temporal trajectory of the market price of a share in ABC Corp. over the past thirty days, would it be considered advisable to invest in this asset?
- The “hard” decision rule applicable in this case is that “one should buy and asset when its price is going up and one should sell an asset when its price is going down”



- The share price as shown above is definitely trending in a particular direction. But will the observred trend over the past thirty days continue in the future? It is really very hard to say because most financial analysts will find this information rather inadequate to arrive at an informal judgement
- Although this illustration is purely anecdotal, it is nevertheless a matter of fact that the world of managerial decision-making is fraught with such inadequacies and „complete information” is often an unaffordable luxury
- The more statiscally minded decison-takers would try to forecast the future direction of the price trend of a share in ABC Corp. from the given (historical) information
- The implied logic is that the more accurate this forecast the more profitable will be the oucome resulting from the decision
- Let us take two financial analyst Mr. X and Ms Y trying to forecast the price of a share in ABC Corp. To fit their respective trendlines, Mr. X considers the entire thirty days of data while Ms Y (who knows about Markovian property of stock prices) considers only the price movement over a single day



- Who do you think is more likely to make the greater profit?



- Most people will have formed their opinions after having made spontaneous assumption about the orientation of the coordinate axes i.e. the temporal order of the price data! This is an example of how our minds sub-consciously complete an "incomplete set" of information prior to cognitive processing
- Obviously, without a definite knowledge about the orientatopm of the axws it is impossible to tell who is more likely to make a greater profit. This has nothing to do with which one of Mr. X or Ms. Y has the better forecasting model. In fact it is a somewhat paradoxical situation – we may know who among Mr. X and Ms. Y has a technically better forecasting model and yet not know who will make more profit! That will remain indeterminate as long as the exact orientation of the two coordinate axes is unknown!
- The neutrosophic probability approach makes a distinction between "relative sure event", event that is true only in certain world(s) and "absolute sure event", event that is true for all possible world(s)
- Similar relations can be drawn for "relative impossible event" / "absolute impossible event" and "relative indeterminate event" / "absolute indeterminate event"
- In case where the truth- and falsity- components are complimentary i.e. they sum up the unity and there is no indeterminacy, then one is reduced to classical probability. Therefore, neutrosophic probability may be viewed as a *three-way generalization* of classical and imprecise probabilities

- In our little anecdotal illustration, we may visualize a world where stock prices follow a Markovian path *and* Ms. Y knows the correct orientation of the coordinate axes. That Ms. Y will make a greater profit thereby becomes a *relative sure event* and that Mr. X will make a greater profit becomes a *relative impossible event*.
- Similarly we may visualize a different world where stock prices follow a linear path *and* Mr. X knows the correct orientation of the coordinate axes. That Mr. X will make a greater profit thereby becomes a *relative sure event* and that Ms. Y will make a greater profit becomes a *relative impossible event*
- Then there is our present world where we have no knowledge at all as to the correct orientation of the coordinate axes and hence both become relative indeterminate events!
- Because real-life managers have to mostly settle for "incomplete sets" of information, the arena of managerial decision-making is replete with such instances of paradoxes and inconsistencies. This is where neutrosophy can play a very significant role as a novel addition to the managerial decision paradigm!

Neutrosophy

A new branch of philosophy which studiues the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (1995);

Extension of dialectics;

The Fundamental Theory: Every idea <A> tends to be neutralized, diminished, balanced by <NonA> ideas (not only <AntiA> as Hegel asserted) – as a state of equilibrium

<NonA> = what is not <A>

<AntiA> = the opposite of <A>

<NeutA> = what is neither <A> nor <AntiA>;

Basement for Neutrosophical Logic, Neutrosophic Set, Neutrosophic Probability

Applications of Neutrosophy to Indian Philosophy

In India's VIIIth – IXth centuries one promulgated the Non-Duality (*Advaita*) through the non-differentiation between Individual Being (*Atman*) and Supreme Being (*Brahman*). The philosopher Sankaracharya (782-814 AC) was then considered the savior of Hinduism, just in the moment when the Buddhism and the Jainism were in a severe turmoil and India was in a spiritual crisis. Non-Duality means elimination of ego, in order to blend yourself with the Supreme Being (to reach the happiness).

Or, arriving to the Supreme was done by Prayer (*Bhakti*) or Cognition (*Jnana*). It is part of Sankaracharya's huge merit (*charya* means teacher) the originality of interpreting and synthesizing the Source of Cognition (*Vedas*, IVth century BC), the Epic (with many stories), and the *Upanishads* (principles of Hindu philosophy) concluding in Non-Duality.

Then Special Duality (Visishta Advaita) follows, which asserts that Individual Being and Supreme Being are different in the beginning, but end to blend themselves (Ramanujacharya, XIth century).

And later, to see that the neutrosophic scheme perfectly functions, Duality (Dvaita) ensues, through whom the Individual Being and Supreme Being were differentiated (Madvacharya, XIIIth – XIVth centuries).

Thus, Non-Duality converged to Duality, i.e. <NonA> converges through <NeutA> to <A>.

Introduction to Nonstandard Analysis

- Abraham Robinson developed the nonstandard analysis (1960s)
- x is called *infinitesimal* if $|x|<1/n$ for any positive n
- A *left monad* $({}^{-}a) = \{a-x\colon x \in R^*, x>0 \text{ infinitesimal}\} = a-\varepsilon$
  and a *right monad* $(b^{-}) = \{a+x\colon x \in R^*, x>0 \text{ infinitesimal}\} = b+\varepsilon$
  where $\varepsilon>0$ is infinitesimal;
  a, b called *standard parts*, ε called *nonstandard part*.
- Operations with nonstandard finite real numbers:

${}^{-}a*b = {}^{-}(a*b)$, $a*b^{+} = (a*b)^{+}$, ${}^{-}a*b^{+} = {}^{-}(a*b)^{+}$,
${}^{-}a*{}^{-}b = {}^{-}(a*b)$ [the left monads absorb themselves],
$a^{+}*b^{+} = (a*b)^{+}$ [the right monads absorb themselves],
where "*" can be addition, subtraction, multiplication, division, power.

Operations with Classical Sets

S1 and S2 two real standard or nonstandard sets.

→ Addition: $S_1 \oplus S_2 = \{x \mid x=s_1+s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$

→ Substraction: $S_2 = \{x \mid x=s_1+s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$

→ Multiplication: $S_1 \ominus S_2 = \{x \mid x=s_1-s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$

Let $k \in \mathbb{R}^*$, then $S_1 \oslash k = \{x \mid x=s_1/k, \text{ where } s_1 \in S_1\}$

Neutrosophic Logic

- Consider the *nonstandard unit interval* $]^-0, 1^+[$, with left and right borders vague, imprecise
- Let T, I, F be standard or nonstandard subsets of $]^-0, 1^+[$
- Neutrosophic Logic (NL) is a logic in which each proposition is T% true, I% indeterminate, and F% false
- $^-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+$
- T, I, F are not necessary intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.)
- Example: proposition P is between 30-40% or 45-50% true, 20% indeterminate, and 60% or between 66-70% false (according to various analyzers or parameters)
- NL is a generalization of Zadeh's *fuzzy logic* (FL), especially of Atanassov's *intuitionistic fuzzy logic* (IFL), and other logics

Differences between Neutrosophic Logic and Intuitionistic Fuzzy Logic

- In NL there is no restriction on T, I, F, while in IFL the sum of components (or their superior limits) = 1;
  thus NL can characterize the *incomplete information* (sum < 1), *paraconsistent information* (sum > 1).
- NL can distinguish, in philosophy, between *absolute truth* [NL(absolute truth)=1^+] and *relative truth* [NL(relative truth)=1], while IFL cannot;
  absolute truth is truth in all possible worlds (Leibniz),
  relative truth is truth in at least one world.
- In NL the components can be nonstandard, in IFL they don't.
- NL, like *dialetheism* [some contradictions are true], can deal with paradoxes, NL(paradox) = (1,I,1), while IFL cannot.

Neutrosophic Logic generalizes many Logics - I

Let the components reduced to scalar numbers, t,i,f, with t+i+f=n; NL generalizes:

- the *Boolean logic* (for n = 1 and i = 0, with t, f either 0 or 1);
- the *multi-valued logic*, which supports the existence of many values between true and false [Lukasiewicz, 3 values; Post, m values] (for n = 1, i = 0, $0 \le t, f \le 1$);
- the *intuitionistic logic*, which supports incomplete theories, where $A \vee \neg A$ not always true, and $\exists x P(x)$ needs an algorithm constructing x [Brouwer, 1907]
  (for $0 < n < 1$ and i = 0, $0 \le t, f < 1$);
- the *fuzzy logic*, which supports degrees of truth [Zadeh, 1965] (for n = 1 and i = 0, $0 \le t, f \le 1$);
- the *intuitionistic fuzzy logic*, which supports degrees of truth and degrees of falsity while what's left is considered indeterminacy [Atanassov, 1982] (for n = 1);
- the *paraconsistent logic*, which supports conflicting information, and 'anything follows from contradictions' fails, i.e. $A \wedge \neg A \supset B$ fails; $A \wedge \neg A$ is not always false
  (for n > 1 and i = 0, with both $0 < t, f < 1$);
- the *dialetheism*, which says that some contradictions are true, $A \wedge \neg A$=true (for t = f = 1 and i = 0; some paradoxes can be denoted this way too);
- the *faillibilism*, which says that uncertainty belongs to every proposition (for i > 0);

Neutrosophic Logic Connectors

$A_1(T_1, I_1, F_1)$ and $A_2(T_2, I_2, F_2)$ are two propositions.

1. Negation:

$NL(\neg A_1) = (\{1\}\ominus T_1, \{1\}\ominus I_1, \{1\}\ominus F_1)$.

2. Conjunction:

$NL(A_1 \wedge A_2) = (T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2)$.
(And, in a similar way, generalized for n propositions.)

3. Weak or inclusive disjunction:

$NL(A_1 \vee A_2) = (T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2)$.
(And, in a similar way, generalized for n propositions.)

4. Strong or exclusive disjunction:

$$NL(A_1 \veebar A_2) = (T_1 \odot (\{1\}\ominus T_2) \oplus T_2 \odot (\{1\}\ominus T_1) \ominus T_1 \odot T_2 \odot (\{1\}\ominus T_1) \odot (\{1\}\ominus T_2),$$
$$I_1 \odot (\{1\}\ominus I_2) \oplus I_2 \odot (\{1\}\ominus I_1) \ominus I_1 \odot I_2 \odot (\{1\}\ominus I_1) \odot (\{1\}\ominus I_2),$$
$$F_1 \odot (\{1\}\ominus F_2) \oplus F_2 \odot (\{1\}\ominus F_1) \ominus F_1 \odot F_2 \odot (\{1\}\ominus F_1) \odot (\{1\}\ominus F_2)).$$
(And, in a similar way, generalized for n propositions.)

5. Material conditional (implication):

$NL(A_1 \mapsto A_2) = (\{1\}\ominus T_1 \oplus T_1 \odot T_2, \{1\}\ominus I_1 \oplus I_1 \odot I_2, \{1\}\ominus F_1 \oplus F_1 \odot F_2)$.

6. Material biconditional (equivalence):

$$NL(A_1 \leftrightarrow A_2) = ((\{1\}\ominus T_1 \oplus T_1 \odot T_2) \odot (\{1\}\ominus T_2 \oplus T_1 \odot T_2),$$
$$(\{1\}\ominus I_1 \oplus I_1 \odot I_2) \odot (\{1\}\ominus I_2 \oplus I_1 \odot I_2),$$
$$(\{1\}\ominus F_1 \oplus F_1 \odot F_2) \odot (\{1\}\ominus F_2 \oplus F_1 \odot F_2)).$$

7. Sheffer's connector:

$NL(A_1 \mid A_2) = NL(\neg A_1 \vee \neg A_2) = (\{1\}\ominus T_1 \odot T_2, \{1\}\ominus I_1 \odot I_2, \{1\}\ominus F_1 \odot F_2)$.

8. Peirce's connector:

$$NL(A_1 \downarrow A_2) = NL(\neg A_1 \wedge \neg A_2) =$$
$$= ((\{1\}\ominus T_1) \odot (\{1\}\ominus T_2), (\{1\}\ominus I_1) \odot (\{1\}\ominus I_2), (\{1\}\ominus F_1) \odot (\{1\}\ominus F_2)).$$

Many properties of the classical logic operators do not apply in neutrosophic logic.

Neutrosophic logic operators (connectors) can be defined in many ways according to the needs of applications or of the problem solving.

Neutrosophic Set (NS)

- Let U be a universe of discourse, M a set included in U. An element x from U is noted with respect to the **neutrosophic set** M as x(T, I, F) and belongs to M in the following way:
it is t% true in the set (*degree of membership*),
i% indeterminate (unknown if it is in the set) (*degree of indeterminacy*),
and f% false (*degree of non-membership*),
where t varies in T, i varies in I, f varies in F.
- Definition analogue to NL
- Generalizes the fuzzy set (FS), especially the intuitionistic fuzzy set (IFS), intuitionistic set (IS), paraconsistent set (PS)
- Example: x(50,20,40)∈A means: with a believe of 50% x is in A, with a believe of 40% x is not in A, and the 20% is undecidable

Neutrosophic Set Operators

A and B two sets over the universe U.
An element $x(T_1, I_1, F_1) \in A$ and $x(T_2, I_2, F_2) \in B$ [*neutrosophic membership appurtenance* to A and respectively to B].
NS operators (similar to NL connectors) can also be defined in many ways.

1. Complement of A:

If $x(T_1, I_1, F_1) \in A$,
then $x(\{1^+\} \ominus T_1, \{1^+\} \ominus I_1, \{1^+\} \ominus F_1) \in C(A)$.

2. Intersection:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2) \in A \cap B$.

3. Union:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2) \in A \cup B$.

4. Difference:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \ominus T_1 \odot T_2, I_1 \ominus I_1 \odot I_2, F_1 \ominus F_1 \odot F_2) \in A \setminus B$,
because $A \setminus B = A \cap C(B)$.

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- NS can distinguish, in philosophy, between *absolute membership* [NS(absolute membership)=1^+] and *relative membership* [NS(relativemembership)=1], while IFS cannot;
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- In NS the components can be nonstandard, in IFS they don't.
- NS, like *dialetheism* [some contradictions are true], can deal with paradoxes, NS(paradox element) = (1,I,1), while IFS cannot.
- NS operators can be defined with respect to T,I,F while IFS operators are defined with respect to T and F only
- I can be split in NS in more subcomponents (for example in Belnap's four-valued logic (1977) indeterminacy is split into uncertainty and contradiction), but in IFS it cannot

Applications of Neutrosophic Logic - I

Voting (pro, contra, neuter):

- The candidate C, who runs for election in a metropolis M of p people with right to vote, will win.

  This proposition is, say, 20-25% true (percentage of people voting for him), 35-45% false (percentage of people voting against him), and 40% or 50% indeterminate (percentage of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Epistemic/subjective uncertainty (which has hidden/unknown parameters).

- Tomorrow it will rain.

  This proposition is, say, 50% true according to meteorologists who have investigated the past years' weather, between 20-30% false according to today's very sunny and droughty summer, and 40% undecided.

Paradoxes:

- This is a heap (Sorites Paradox).

  We may now say that this proposition is 80% true, 40% false, and 25-35% indeterminate (the neutrality comes for we don't know exactly where is the difference between a heap and a non-heap; and, if we approximate the border, our 'accuracy' is subjective). Vagueness plays here an important role.
- The Medieval paradox, called Buridan's Ass after Jean Buridan (near 1295-1356), is a perfect example of complete indeterminacy. An ass, equidistantly from two quantitatively and qualitatively heaps of grain, starves to death because there is no ground for preferring one heap to another.

  The neutrosophic value of ass's decision, NL = (0, 1, 0).

Games (win, defeated, tied).

Electrical charge, temperature, altitude, numbers, and other 3-valued systems (positive, negative, zero)

Business (M. Khoshnevisan, S. Bhattacharya):

- Investors who are: Conservative and security-oriented (*risk shy*), Chance-oriented and progressive (*risk happy*), or Growth-oriented and dynamic (*risk neutral*).

Applications of Neutrosophic Sets

Philosophical Applications:

- Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?
- In Eastern Philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.
- How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

Physics Applications:

- How to describe a particle ξ in the infinite micro-universe of Quantum Physics that belongs to two distinct places P_1 and P_2 in the same time? $\xi \in P_1$ and $\xi \notin P_1$ as a true contradiction, or $\xi \in P_1$ and $\xi \in \neg P_1$.
- Don't we better describe, using the attribute "neutrosophic" than "fuzzy" and others, a quantum particle that neither exists nor non-exists? [high degree of indeterminacy]
- In Schroedinger's Equation on the behavior of electromagnetic waves and "matter waves" in Quantum Theory, the wave function Psi which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).
- A cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set (e.g. there are a kind of separated water drops, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud).

More Applications of Neutrosophic Logic and Neutrosophic Set

- Mohammad Khoshnevisan and Sukanto Bhattacharya in finance, business
- Haibin Wang, F. Smarandache, Yanqing Zhang, Rajshekhar Sunderraman in engineering
- A. Tchamova, F. Smarandache, J. Dezert in information fusion, cybernetics, medicine, military
- Anne-Laure Jousselme, Patrick Maupin in situation analysis

A few specific applications of neutrosophics in business and economics

- **Application of Neutrosophics as a situation analysis tool**
- **Application of Neutrosophics in the reconciliation of financial market information**
- **Application of Neutrosophics in Production Facility Layout Planning and Design**

Neutrosophics as a situation analysis tool

- In situation analysis (SA), an agent observing a scene receives information from heterogeneous sources of information including for example remote sensing devices, human reports and databases. The aim of this agent is to reach a certain awareness about the situation in order to take decisions
- Considering the logical connection between belief and knowledge, the challenge for the designer is to transform the raw, imprecise, conflicting and often paradoxical information received from the different sources into statements understandable by both man and machines
- Hence, two levels of processing coexist in SA: measuring of the world and reasoning about the world. Another great challenge in SA is the reconciliation of both aspects. As a consequence, SA applications need frameworks general enough to take into account the different types of uncertainty present in the SA context, doubled with a semantics allowing reasoning on situations
- A particularity of SA is that most of the time it is impossible to list every possible situation that can occur. Corresponding frames of discernment cannot, thus, be considered as exhaustive
- Furthermore, in SA situations are not clear-cut elements of the frames of discernment. Considering these particular aspects of SA, a neutrosophic logic paradigm incorporating the *Dezert-Smarandache theory* (DSmT) appears as an appropriate modeling tool
- It has been recently shown that the neutrosophic logic paradigm does have the capacity to cope with the epistemic and uncertainty-related problems of SA
- In particular, it has been formally demonstrated that the neutrosophic logic paradigm incorporating DSmT has the ability to process symbolic and numerical statements on belief and knowledge using the *possible worlds* semantics (Jousselme and Maupin, 2004)

Applications of Neutrosophics in the reconciliation of financial market information

- In the classical, *Black-Scholes* world, if stock price volatility is known *a priori*, the prices of long-term option contracts are completely determinable and any deviations are deemed to be quickly arbitraged away. Therefore, when a long-term option priced by the collective action of the market agents is observed to be deviating from the theoretical price, the following three possibilities must be considered:

- (1) The theoretical price is obtained by an inadequate pricing model, which means that the observed market price may well *be* the true price, i.e. ***Observed Price = Theoretical Price*** $\pm \varepsilon$; (where ε is the ***systematic error*** in valuation that will result whenever any option is evaluated by the inadequate pricing model)

- (2) The theoretical pricing model is valid but a largely irrational buying/selling behavior by a group of market agents has temporarily pushed the market price of a particular option j 'out of sync' with its true price, i.e. ***(Observed Price)$_j$ = (True Price)$_j$*** $\pm \varepsilon_j$; (where ε_j is the ***unsystematic error*** in valuation specific to option j) or

- (3) The nature of the deviation is indeterminate and could be due to either (a) or (b) or even a super-position of both (a) and (b) and/or due to some random white noise

- With T, I, F as the neutrosophic components, let us now define the following two apparently mutually inconsistent events:

H = {p: p is the true option price determined by the theoretical pricing model}; and

M = {p: p is the true option price determined by the prevailing market price}

- Then there is a t% chance that the event ($H \cap M^c$) is true, or corollarily, the corresponding complimentary event ($H^c \cap M$) is untrue i.e. the best detereminant of the true market price is the theoretical pricing model

- There is a f% chance that the event ($M^c \cap H$) is untrue, or corollarily, the complimentary event ($M \cap H^c$) is true i.e. the best detereminant of the true market price is the observed market price and;

- There is a i% chance that neither ($H \cap M^c$) nor ($M \cap H^c$) is true/untrue; i.e. the best determinant of the true market price is indeterminate!

- Illustratively, a set of AR1 models used to extract the mean reversion parameter driving the volatility process over time have *coefficients of determination* in the range say between 50%-70%, then we can say that t varies in the set T (50% - 70%).

- If the subjective probability assessments of well-informed market agents about the weight of the current excursions in implied volatility on short-term options lie in the range say between 40%-60%, then f varies in the set F (40% - 60%).

- The unexplained variation in the temporal volatility driving process together with the subjective assessment by the market agents will make the event indeterminate say by either 30% or 40%.

- Then the neutrosophic probability of the true price of the option being determined by the theoretical pricing model is formally representable as follows: **NP ($H \cap M^c$) = [(50 – 70), (40 – 60), {30, 40}]**. The *Dezert-Smarandache formula* can be used in cases like these to *fuse* the conflicting sources of information and arrive at a correct and computable probabilistic assessment of the true price of the long-term option

Applications of Neutrosophics in Production Facility Layout Planning and Design

- The original CRAFT (Computerized Relative Allocation of Facilities Technique) model for cost-optimal relative allocation of production facilities as well as many of its later extensions tend to be quite "heavy" in terms of CPU engagement time due to their heuristic nature

- A Modified Assignment (MASS) model (first proposed by Bhattacharya and Khoshnevisan in 2003) increases the computational efficiency by developing the facility layout problem as primarily a *Hungarian assignment* problem but becomes indistinguishable from the earlier CRAFT-type models beyond the initial configuration

- However, some amount of introspection will reveal that the production facilities layout problem is basically one of achieving *best interconnectivity by optimal fusion of spatial information*. In that sense, the problem may be better modeled in terms of mathematical information theory whereby *the best layout is obtainable as the one that maximizes relative entropy of the spatial configuration*

- Going a step further, one may hypothesize a neutrosophic dimension to the problem. Given a combination rule like the *Dezert-Smarandache formula*, the layout optimization problem may be formulated as a *normalized basic probability assignment* for optimally comparing between several alternative interconnectivities

- The neutrosophic argument can be justified by considering the very practical possibility of conflicting bodies of evidence for the structure of the load matrix possibly due to conflicting assessments of two or more design engineers

- If for example we consider two mutually conflicting bodies of evidence Ξ_1 and Ξ_2, characterized respectively by their basic probability assignments μ_1 and μ_2 and their cores k (μ_1) and k (μ_2) then one has to look for the optimal combination rule which maximizes the joint entropy of the two conflicting information sources

- Mathematically, it boils down to the general optimization problem of evaluating $-\min_{\mu}[-H(\mu)]$ subject to the constraints that (a) the marginal basic probability assignments μ_1 (.) and μ_2 (.) are obtainable by the summation over each column and summation over each row respectively of the relevant information matrix and (b) the sum of all cells of the information matrix is unity