Unification of Fusion Rules (UFR)

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In this short note we give a formula for the unification of a class of fusion rules based on the conjunctive and/or disjunctive rule at the first step, and afterwards the redistribution of the conflicting and/or non-conflicting mass to the non-empty sets at the second step.

Fusion of masses $m_1(.)$ and $m_2(.)$ is done directly proportional with some parameters and inversely proportional with other parameters (parameters that the hypotheses depend upon). The resulting mass is noted by $m_{UFR}(.)$.

a) If variable y is <u>directly proportional</u> with variable p, then $y=k_1 \cdot p$, where $k_1 \neq 0$ is a constant.
b) If variable y is <u>inversely proportional</u> with variable q, then $y=k_2 \cdot (1/q)$, where $k_2 \neq 0$ is a constant; we can also say herein that y is directly proportional with variable $1/q$.

In a general way, we say that if y is directly proportional with variables $p_1, p_2, \ldots, p_m$ and inversely proportionally with variables $q_1, q_2, \ldots, q_n$, then:

$y = k \cdot (p_1 \cdot p_2 \cdot \ldots \cdot p_m)/(q_1 \cdot q_2 \cdot \ldots \cdot q_n) = k \cdot P/Q$, where $P=\prod_{i=1}^{m} p_i$, $Q=\prod_{j=1}^{n} q_j$, and $k \neq 0$ is a constant.

With such notations we have a general formula for a **UFR rule**:

$$m_{UFR}(\phi) = 0, \text{ and } \forall A \in S^{\Theta} \setminus \phi \text{ one has:}$$

$$m_{UFR}(A) = \sum_{\substack{X_1, X_2 \in S^{\wedge}\Theta \\ X_1 * X_2 = A}} d(X_1 * X_2) T(X_1, X_2)$$

$$+ \frac{P(A)}{Q(A)} \sum_{\substack{X \in S^{\wedge}\Theta \setminus A \\ X * A \in Tr}} d(X * A) \frac{T(A, X)}{P(A)/Q(A) + P(X)/Q(X)}$$

where * can be an intersection or a union of sets,
$d(X*Y)$ is the degree of intersection or union,
$T(X,Y)$ is a *T-norm* fusion combination rule (extension of conjunctive or disjunctive rules),
Tr is the ensemble of sets (in majority cases they are empty sets) whose masses must be transferred,
$P(A)$ is the product of all parameters directly proportional with A,
while $Q(A)$ the product of all parameters inversely proportional with A,
$S^{\wedge}\Theta$ is the fusion space (i.e. the frame of discernment closed under union, intersection, and complement of the sets).
At the end we normalize the result.

Reference:

F. Smarandache, *Unification of Fusion Theories (UFT)*, International Journal of Applied Mathematics & Statistics, Roorkee, India, Vol. 2, 1-14, 2004.