Improvement In Estimating The Population Mean Using Exponential Estimator In Simple Random Sampling

Rajesh Singh, Pankaj Chauhan and Nirmala Sawan
School of Statistics, DAVV, Indore (M.P.), India
(rsinghstat@yahoo.co.in)
Florentin Smarandache
Department of Mathematics, University of New Mexico, Gallup, USA
fsmarandache@yahoo.com

Abstract

This study proposes some exponential ratio-type estimators for estimating the population mean of the variable under study, using known values of certain population parameter(s). Under simple random sampling without replacement (SRSWOR) scheme, mean square error (MSE) equations of all proposed estimators are obtained and compared with each other. The theoretical results are supported by a numerical illustration.

Keywords: Exponential estimator, auxiliary variable, simple random sampling, efficiency.

1. Introduction

Consider a finite population $U = U_1, U_2, ..., U_N$ of N unites. Let y and x stand for the variable under study and auxiliary variable respectively. Let (y_i, x_i), $i = 1,2,......,n$ denote the values of the units included in a sample s_n of size n drawn by simple random sampling without replacement (SRSWOR). The auxiliary information has been used in improving the precision of the estimate of a parameter (see Cochran (1977), Sukhatme and Sukhatme (1970) and the reference cited there in). Out of many methods, ratio and product methods of estimation are good illustrations in this context.

In order to have a survey estimate of the population mean $\overline{Y}$ of the study character y, assuming the knowledge of the population mean $\overline{X}$ of the auxiliary character x, the well-known ratio estimator is –

$$t_r = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.1}$$

Bahl and Tuteja (1991) suggested an exponential ratio type estimator as –

$$t_1 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{1.2}$$

Several authors have used prior value of certain population parameter(s) to find more precise estimates. Sisodiya and Dwivedi (1981), Sen (1978) and Upadhyaya and Singh (1984) used the known coefficient of variation (CV) of the auxiliary character for estimating population mean of a study character in ratio method of estimation. The use of prior value of coefficient of kurtosis in estimating the population variance of study character y was first made by Singh et.al.(1973). Later used by Singh and Kakaran (1993) in the estimation of population mean of study character. Singh and Tailor (2003) proposed a modified ratio estimator by using the known value of correlation coefficient. Kadilar and Cingi (2006(a)) and Khoshnevisan et.al.(2007) have suggested modified ratio estimators by using different pairs of known value of population parameter(s).

In this paper, under SRSWOR, we have suggested improved exponential ratio-type estimators for estimating population mean using some known value of population parameter(s).

2. The suggested estimator

Following Kadilar and Cingi (2006(a)) and Khoshnevisan (2007), we define modified exponential estimator for estimating $\bar{Y}$ as –

$$t = \bar{y}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right] \tag{2.1}$$

where $a(\neq 0)$, b are either real numbers or the functions of the known parameters of the auxiliary variable x such as coefficient of variation (C_x), coefficient of kurtosis $(\beta_2(x))$ and correlation coefficient (ρ).

To, obtain the bias and MSE of t, we write

$$\bar{y} = \bar{Y}(1+e_0),\ \bar{x} = \bar{X}(1+e_1)$$

such that

$$E(e_0) = E(e_1) = 0,$$

and $E(e_0^2) = f_1 C_y^2$, $E(e_1^2) = f_1 C_x^2$, $E(e_0 e_1) = f_1 \rho C_y C_x$,

where

$$f_1 = \frac{N-n}{nN},\ C_y^2 = \frac{S_y^2}{\overline{Y}^2},\ C_x^2 = \frac{S_x^2}{\overline{X}^2}.$$

Expressing t in terms of e's, we have

$$t = \overline{Y}(1+e_0)\exp\left[\frac{a\overline{X} - a\overline{X}(1+e_1)}{a\overline{X} + 2b + a\overline{X}(1+e_1)}\right]$$

$$t = \overline{Y}(1+e_0)\exp\left[-\theta e_1 (1+\theta e_1)^{-1}\right] \qquad (2.2)$$

where $\theta = \dfrac{a\overline{X}}{2(a\overline{X}+b)}$.

Expanding the right hand side of (2.2) and retaining terms up to second power of e's, we have

$$t = \overline{Y}(1 + e_0 - \theta e_1 + \theta^2 e_1^2 + \theta e_0 e_1) \qquad (2.3)$$

Taking expectations of both sides of (2.3) and then subtracting $\overline{Y}$ from both sides, we get the bias of the estimator t, up to the first order of approximation, as

$$B(t) = f_1 \overline{Y}(\theta^2 C_y^2 + \theta \rho C_y C_x) \qquad (2.4)$$

From (2.3), we have

$$(t - \overline{Y}) \cong \overline{Y}(e_0 - \theta e_1) \qquad (2.5)$$

Squaring both sides of (2.5) and then taking expectation, we get MSE of the estimator t, up to the first order of approximation, as

$$MSE(t) = f_1 \overline{Y}^2 (C_y^2 + \theta^2 C_x^2 - 2\theta \rho C_y C_c) \qquad (2.6)$$

3. Some members of the suggested estimator 't'

The following scheme presents some estimators of the population mean which can be obtained by suitable choice of constants a and b.

Estimator	Values of	
	a	b
$t_0 = \overline{y}$ The mean per unit estimator	0	0

$t_1 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$ Bahl and Tuteja (1991) estimator	1	1
$t_2 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2\beta_2(x)}\right]$	1	$\beta_2(x)$
$t_3 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2C_c}\right]$	1	C_x
$t_4 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}+2\rho}\right]$	1	ρ
$t_5 = \bar{y}\exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2C_x}\right]$	$\beta_2(x)$	C_x
$t_6 = \bar{y}\exp\left[\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\beta_2(x)}\right]$	C_x	$\beta_2(x)$
$t_7 = \bar{y}\exp\left[\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\rho}\right]$	C_x	ρ
$t_8 = \bar{y}\exp\left[\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2C_x}\right]$	ρ	C_x
$t_9 = \bar{y}\exp\left[\frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2\rho}\right]$	$\beta_2(x)$	ρ
$t_{10} = \bar{y}\exp\left[\frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2\beta_2(x)}\right]$	ρ	$\beta_2(x)$

In addition to above estimators a large number of estimators can also be generated from the proposed estimator t at (2.1) just by putting different values of a and b.

It is observed that the expressions of the first order of approximation of bias and MSE of the given member of the family can be obtained by mere substituting the values of a and b in (2.4) and (2.6) respectively.

4. Modified estimators

Following Kadilar and Cingi (2006(b)), we propose modified estimator combining estimator t_1 and t_i $(i = 2,3,....,10)$ as follows

$$t_i^* = \alpha t_1 + (1-\alpha)t_i, \qquad (i = 2,3,....,10) \tag{4.1}$$

where α is a real constant to be determined such that the MSE of t_i^* is minimum and t_i $(i = 2,3,....,10)$ are estimators listed in section 3.

Following the procedure of section (2), we get the MSE of t_i^* to the first order of approximation as –

$$MSE(t_i^*) = f_1\overline{Y}^2\left[C_y^2 + C_x^2\left(\frac{\alpha}{2} + \theta_i - \alpha\theta_i\right)^2 - 2\rho C_y C_x\left(\frac{\alpha}{2} + \theta_i - \alpha\theta_i\right)\right] \tag{4.2}$$

where

$$\theta_2 = \frac{\overline{X}}{2(\overline{X} + \beta_2(x))}, \qquad \theta_3 = \frac{\overline{X}}{2(\overline{X} + C_x)},$$

$$\theta_4 = \frac{\overline{X}}{2(\overline{X} + \rho))}, \qquad \theta_5 = \frac{\beta_2(x)\overline{X}}{2(\beta_2(x)\overline{X} + C_x))},$$

$$\theta_6 = \frac{C_x\overline{X}}{2(C_x\overline{X} + \beta_2(x))}, \qquad \theta_7 = \frac{C_x\overline{X}}{2(C_x\overline{X} + \rho)},$$

$$\theta_8 = \frac{\rho\overline{X}}{2(\rho\overline{X} + C_x)}, \qquad \theta_9 = \frac{\beta_2(x)\overline{X}}{2(\beta_2(x) + \rho)},$$

$$\theta_{10} = \frac{\rho\overline{X}}{2(\rho + \beta_2(x))}.$$

Minimization of (4.2) with respect to α yields its optimum value as

$$\alpha = \frac{2(K-\theta)}{(1-2\theta)} = \alpha_{opt}(say) \tag{4.3}$$

where $K = \rho\dfrac{C_y}{C_x}$.

Substitution of (4.3) in (4.10) gives optimum estimator $t_o^*(say)$, with minimum MSE as

$$\min MSE(t_i^*) = f_1\overline{Y}^2 C_y^2(1-\rho^2) = MSE(t^*)_o \tag{4.4}$$

The $\min MSE(t_i^*)$ at (4.4) is same as that of the approximate variance of the usual linear regression estimator.

5. Efficiency comparison

It is well known that under SRSWOR the variance of the sample mean is

$$Var(\overline{y}) = f_1\overline{Y}^2 C_y^2 \tag{5.1}$$

we first compare the MSE of the proposed estimators, given in (2.6) with the variance of the sample mean, we have the following condition:

$$K \leq \frac{\theta_i}{2}, \qquad i = 2,3,.....,10 \tag{5.2}$$

When this condition is satisfied, proposed estimators are more efficient than the sample mean.

Next we compare the MSE of proposed estimators t_i^* $(i = 2,3,.....,10)$ in (4.4) with the MSE of estimators listed in section 3. We obtain the following condition

$$(\theta C_x - \rho C_y)^2 \geq 0, \qquad i = 2,3,.....,10. \tag{5.3}$$

We can infer that all proposed estimators t_i^*, $(i = 2,3,.....,10)$ are more efficient than estimators proposed in section 3 in all conditions, because the condition given in (5.1) is always satisfied.

6. Numerical illustration

To illustrate the performance of various estimators of $\overline{Y}$, we consider the data given in Murthy (1967 pg-226). The variates are:

y : Output, x: number of workers

$\overline{X}$ =283.875, $\overline{Y}$ =5182.638, $C_y = 0.3520$, $C_x = 0.9430$, $\rho = 0.9136$, $\beta_2(x) = 3.65$.

We have computed the percent relative efficiency (PRE) of different estimators of $\overline{Y}$ with respect to usual estimator $\overline{y}$ and complied in table 6.1:

Table 6.1: PRE of different estimators of $\overline{Y}$ with respect to $\overline{y}$

Estimator	PRE
$\overline{y}$	100
t_1	366.96
t_2	385.72
t_3	368.27
t_4	371.74
t_5	386.87
t_6	368.27
t_7	372.03

t_8	372.05
t_9	368.27
t_{10}	386.91
t_o^*	877.54

7. Conclusion

We have developed some exponential ratio type estimators using some known value of the population parameter(s), listed in section 3. We have also suggested modified estimators t_i^* $(i = 2,3,.....,10)$. From table 6.1 we conclude that the proposed estimators are better than Bahl and Tuteja (1991) estimator t_1. Also, the modified estimator t_i^* $(i = 2,3,.....,10)$ under optimum condition performs better than the estimators proposed and listed in section 3 and than the Bahl and Tuteja (1991) estimator t_1. The choice of the estimator mainly depends upon the availability of information about known values of the parameter(s) (C_x, ρ, $\beta_2(x)$, etc.).

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