An Outline of Cellular Automaton Universe via Cosmological KdV equation

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V. CHRISTIANTO, F. SMARANDACHE, Y. UMNIYATI

A little caution

- This is a concept paper (outline)
- We will not present results, instead we offer a new *insight* in cosmology modelling

Our motivation

- It is known that there is quantization in astronomy, for example Tifft's redshift quantization is known since 1990s.
- Around 1960s V. Subramaniam suggested a quantized space, although his approach has been entirely forgotten.
- Now we consider that quantization in astrophysics phenomena can be explained by *discretization* in space.

What is cellular automaton?

- According to a dictionary:
- “one of a set of units in a mathematical model that have simple rules governing their replication and destruction. They are used to model complex systems composed of simple units such as living things or parallel processors.”

Another definition of CA

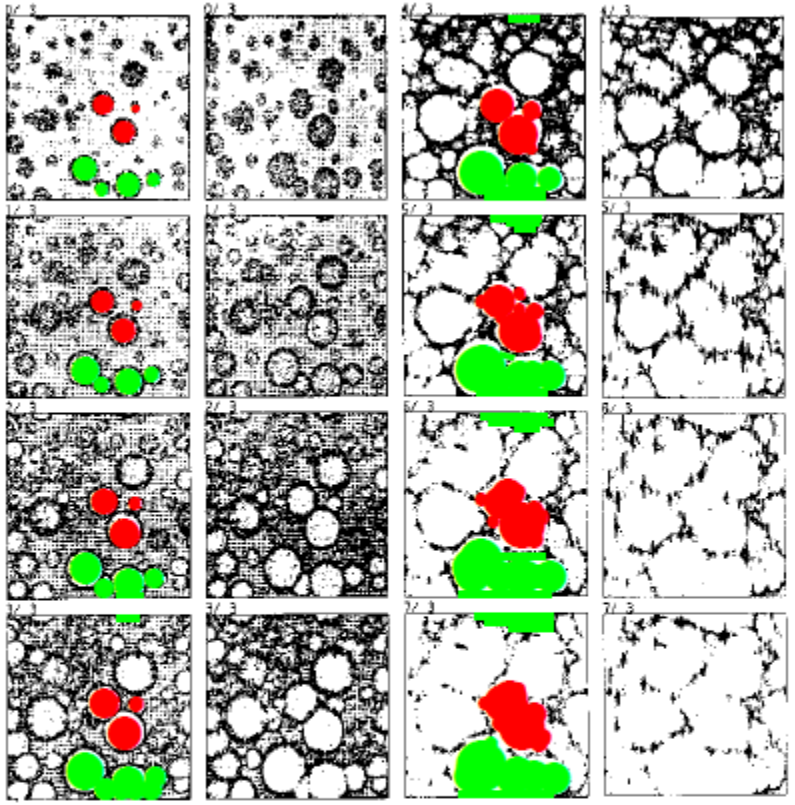
- A *cellular automaton* (pl. *cellular automata*, abbrev. CA) is a discrete model studied in computability theory, mathematics, physics, complexity science, theoretical biology and microstructure modeling. ... For each cell, a set of cells called its neighborhood is *defined* relative to the specified cell.
- Source: *https://en.wikipedia.org/wiki/Cellular_automaton*

A Computing universe?

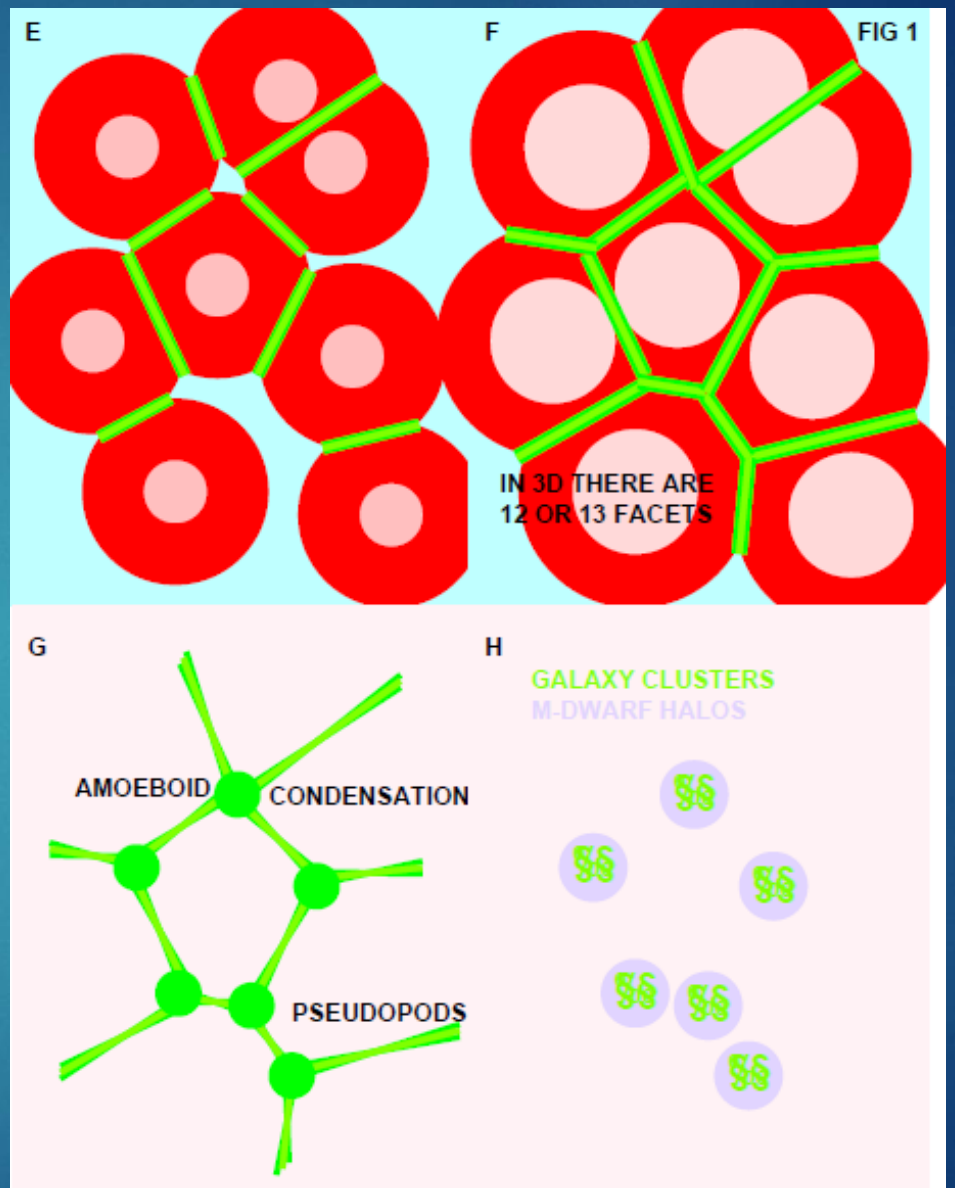
- Konrad Zuse is probably the first scholar who imagine a *Computing Universe*.
- In recent years, there are a few researchers who suggest similar vision in terms of cellular automata, for example Stephen Wolfram, Prof. Gerardus 't Hooft, and Robert L. Kurucz from Harvard Smithsonian of Astrophysics.

FIG 14

EVOLUTION OF VOIDS AND LARGE SCALE STRUCTURE



REGÕS AND GELLER (1991) MODELLED COSMOLOGICAL EVOLUTION OF VOIDS UP TO THE PRESENT TIME. THESE ARE 5% 2D SLICES THROUGH THE 3D PERIODIC CUBIC STRUCTURE. DENSE AREAS IN THE FINAL PANELS ARE CLUSTERS OF GALAXIES.



From R. Kurucz (2006), arXiv:astro-ph/0605469

Cosmic sound wave

- It has been known for long time that the cosmic sound wave was there since the early epoch of the Universe. Signatures of its existence are abound. However, such a sound wave model of cosmology is rarely developed fully into a complete framework.
- question: is it possible to model that cosmic sound wave using soliton as known solution of cosmological KdV equation?

What is Korteweg-deVries equation?

- KdV is a non-linear wave equation plays a fundamental role in diverse branches of mathematical and theoretical physics. Its significance to cosmology has been discussed by a number of authors, such as Rosu and recently Lidsey.
- It is suggested that the KdV equation arises in a number of important scenarios, including inflationary cosmology etc.

A bit of history of KdV

- John Scott Russell, a Scottish naval engineer, reported in 1834 on his observation of a remarkable solitary water wave moving a considerable distance down a narrow channel.
- Korteweg and de Vries (1895) developed a theory to describe weakly nonlinear wave propagation in shallow water.

A bit of history

- Kruskal and Zabusky (1965) discovered that the KdV equation admits analytic solutions representing what they called "solitons"—propagating pulses or solitary waves that maintain their shape and can pass through one another. These are evidently waves that behave like particles!
- Several detailed analyses suggest that the coherence of solitons can be attributed to a compensation of nonlinear and dispersive effects.

KdV equation

- Korteweg-de Vries (KdV) equation is the completely integrable, third-order, non-linear partial differential equation (PDE):

$$\partial_t u + \partial_x^3 u + \frac{3}{u_0} u \partial_x u = 0$$

where u = u(x, t), $\partial_t = \partial/\partial t$, $\partial_x^3 = \partial^3/\partial x^3$, etc., u_0 is a constant and (x, t) represent space and time coordinates, respectively.

Soliton solution of KdV equation

- KdV equation was originally derived within the context of small-amplitude, non-linear water wave theory and it is well known that it admits a solitonic wave solution of the form

$$u = u_0 \lambda^2 \operatorname{sec} h^2 \left[\lambda \left(x - \lambda^2 t \right) / 2 \right],$$

- where the constant λ/2 represents the wavenumber of the soliton.

Cosmological KdV equation

- It can be shown that Friedmann equation after some steps yields to an equation which takes the form of KdV, as follows:

$$H^2(\phi) = H_0^2 \lambda^2 \sec h^2 \left[\lambda A / 2\right]$$

- Where:

$$A = \frac{\sqrt{8\pi}}{m_p}\phi.$$

CA model of KdV equation

- There are several methods to consider discretization of KdV equation into cellular automata models. Here we consider only 3 approaches.
- A. Steeb & Hardy's method:

$$u_j(t+1) = u_j(t)(u_{j+1}(t) - u_j(t)) + u_{j+2}(t) - u_{j+1}(t) - u_{j-1}(t).$$

CA model

- B. Hirota's model:
- The discrete analogue of the KdV equation is known thanks to the pioneering work of Hirota. It has the form:

$$\frac{1}{u_{l+1}^{t+1}} - \frac{1}{u_l^t} = \delta\left(u_{l+1}^t - u_l^{t+1}\right)$$

CA model

- C. Tokihiro et al.;
- They suggested that an integrable discretization (differential-difference equation) of the KdV equation is the Lotka-Volterra equation :

$$\frac{d}{dt} b_j(t) = b_j(t)\left[b_{j+t}(t) - b_{j-1}(t)\right]$$

CA Model

- In other words, it appears possible at least in theory to consider a Cellular Automaton-KdV Universe, based on discretization of the original KdV equation. Nonetheless, further analysis and numerical simulation is required to study its potential applications.

Concluding remarks

- It is our hope that the new proposed equations can be verified with observation data both at lab scale and also at large scale astronomy data.
- We also expect that the proposed theoretical models based on CA may offer a clue to answer the great mystery of our Universe: *the origins of life*. This problem remains missing in most existing physical cosmology models.
- Nonetheless, we admit that our model is still in its infancy, more researches are needed to fill all the missing details.

Future Works

- We plan to do computational simulation of the proposed CA equations of KdV, and then compare with existing data.
- We plan to investigate the role of acoustic soliton to study astrophysical phenomena.
- We will also investigate cellular automaton molecular model of elementary particles (the draft paper has been submitted to MDPI-Mathematics Journal).

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Thank you so much

- Bandung, 1 august 2017
- Presented by :
- Victor Christianto, DDiv.
- Founder of www.ketindo.com
- Email: victorchristianto@gmail.com
- http://researchgate.net/profile/Victor_Christianto